Hindawi
Journal of Advanced Transportation
Volume 2018, Article ID 9364129, 12 pages
https://doi.org/10.1155/2018/9364129

Research Article

Distribution-Free Model for Ambulance Location Problem with Ambiguous Demand

Feng Chu,[1] Lu Wang,[2] Xin Liu ,[3] Chengbin Chu,[3,4] and Yang Sui[5]

[1] *Management Engineering Research Center, Xihua University, Chengdu 610039, China*
[2] *Department of Aviation Transportation, Shanghai Civil Aviation College, Shanghai 200232, China*
[3] *School of Economics & Management, Tongji University, Shanghai 200092, China*
[4] *Laboratoire Génie Industriel, Centrale Supélec, Université Paris-Saclay, Grande Voie des Vignes, Chatenay-Malabry, France*
[5] *Glorious Sun School of Business & Management, Donghua University, Shanghai 200051, China*

Correspondence should be addressed to Xin Liu; liuxin9735@126.com

Received 11 June 2018; Accepted 28 August 2018; Published 13 September 2018

Academic Editor: Oded Cats

Ambulance location problem is a key issue in Emergency Medical Service (EMS) system, which is to determine where to locate ambulances such that the emergency calls can be responded efficiently. Most related researches focus on deterministic problems or assume that the probability distribution of demand can be estimated. In practice, however, it is difficult to obtain perfect information on probability distribution. This paper investigates the ambulance location problem with partial demand information; i.e., only the mean and covariance matrix of the demands are known. The problem consists of determining base locations and the employment of ambulances, to minimize the total cost. A new distribution-free chance constrained model is proposed. Then two approximated mixed integer programming (MIP) formulations are developed to solve it. Finally, numerical experiments on benchmarks (Nickel et al., 2016) and 120 randomly generated instances are conducted, and computational results show that our proposed two formulations can ensure a high service level in a short time. Specifically, the second formulation takes less cost while guaranteeing an appropriate service level.

1. Introduction

The design of Emergency Medical Service (EMS) systems that affects people's health and life focuses on how to respond to emergencies rapidly. Ambulance location problem is one of the key problems in EMS system, which mainly consists of determining where to locate bases, also named as emergency service facilities, and the employment of ambulances in order to serve emergencies efficiently and to guarantee patient survivability. There have been various researches investigating ambulance location problem (e.g., [1, 2]) since it is introduced by Toregas et al. (1974).

Early studies addressing ambulance location problem mainly focus on deterministic environments, including set covering ambulance location problem that minimizes the number of ambulances to cover all demand points [3], maximal covering location problem to maximize the number of covered demand points with given number of ambulances [4], and double standard model (DSM) in which each demand point must be covered by one or more ambulances [5]. However, in practice, stochastic ambulance location problem is more realistic due to the inevitable uncertainties of emergency events (e.g., [6, 7], etc.). Usually, an emergency event has the following characteristics: (i) it is difficult to forecast precisely where an emergency will occur, and (ii) the required number of ambulances depends on the severity of the situation, which is also unforeseen [8].

Most existing works investigate stochastic ambulance location problem by assuming that the demand probability distribution at each possible emergency is known (e.g., [9, 10], Beraldi and Bruni, 2008; etc.). However, as stated by Wagner [11] and Delage and Ye [12], it is usually impossible to obtain the perfect information on probability distribution, due to (i) the lack of historical data and the fact that (ii) the given historical data may not be represented by probability distribution.

Moreover, how to cover as many emergencies as possible to guarantee a high patient service level (i.e., the portion of the satisfied emergency points among all emergency points) has always been a main goal of EMS operators. Motivated by the above observation, this work focuses on a stochastic ambulance location problem with only partial information of demand points, i.e., the mean and covariance matrix of the demands. Besides, we introduce a new individual chance constraint, which implies a minimum probability that each emergency demand has to be satisfied. For the problem, a new distribution-free model is presented, and then two approximated formulations are proposed. The contribution of this work mainly includes the following:

(1) For the studied stochastic ambulance location problem, we introduce a new individual chance constraint (i.e., safety level) guaranteeing each emergency demand satisfied with a least probability, while Nickel et al. [8] use a coverage constraint, where some service of individual emergencies may suffer insufficiency.

(2) A new distribution-free model based on individual chance constraints is proposed. To our best knowledge, it is the first distribution-free model for stochastic ambulance location problem.

(3) To solve the distribution-free model, two approximated mixed integer programming (MIP) formulations are developed. Experimental results show that our approximated formulations are efficient and effective for large size instances, compared to that proposed by Nickel et al. [8] in terms of both computational time and service level.

The remainder of this paper is organized as follows. Section 2 gives a brief literature review. In Section 3, we give the problem description and propose a new distribution-free model. In Section 4, two approximated MIP formulations are proposed. Computational results on benchmarks and 120 randomly generated instances are reported in Section 5. Section 6 summarizes this work and states future research directions.

2. Literature Review

The deterministic ambulance location problem has been well studied in literature (e.g., [2, 5, 13], etc.). Besides, there have been some works addressing ambulance location problem under uncertainty (e.g., [1, 6, 14], etc.). Since our study falls within the scope of stochastic ambulance location problem, in the following subsections, we first review existing studies on ambulance location problem with uncertain demand. Then we review the literature studying general and specific stochastic optimization problems with distribution-free approaches.

2.1. Ambulance Location Problem with Uncertain Demand. Ambulance location problem under uncertain demand has been investigated by many researchers. Most existing works address the uncertainty with given scenarios or known probability distribution.

Chapman and White [15] first investigate the ambulance location problem with uncertain demand, in which the complete information on probability distributions is assumed to be known. Beraldi et al. [9] study the emergency medical service location problem with uncertain demand to minimize the total cost with a marginal probability distribution. Beraldi and Bruni (2008) investigate the emergency medical service facilities location problem with stochastic demand based on given set of scenarios and known probability distribution. A stochastic programming formulation with probabilistic constraints is proposed. Noyan [10] studies the ambulance location problem with uncertain demand on given scenarios and probability distribution. Then, two stochastic optimization models and a heuristic are proposed.

Recently, Nickel et al. [8] first study the joint optimization of ambulance base location and ambulance employment. The objective is to minimize the total constructing base costs and ambulance employment costs. It is assumed that various scenarios are given beforehand. A coverage constraint is introduced in the paper. They propose a sampling approach that solves a finite number of scenario samples to obtain a feasible solution of the original problem. But with the coverage constraint, some emergency demands risk insufficient individual service or cannot be served. For the study, we propose a new distribution-free model for stochastic ambulance location problem.

2.2. Distribution-Free Approaches. In data-driven settings, the probability distributions of uncertain parameters may not always be perfectly estimated [16]. Therefore in the last decade, there have been many solution approaches developed to address stochastic problems under partial distributional knowledge. Most related researches focus on the distribution-free approach via considering chance constraints. Wagner [11] studies a stochastic 0-1 linear programming under partial distribution information, i.e., $\min_{X\in\{0,1\}^n}\{\mathbf{c}'\mathbf{x} : \mathbf{a}_j'\mathbf{x} \leq b_j, j = 1,\ldots,m\}$, where $\mathbf{a}_j$ are random vectors with unknown distributions. The only information on $\mathbf{a}_j$ are their moments, up to order k. A robust formulation, as a function of k, is given. Given the known second-order moment knowledge, i.e., $k = 2$, an approximated formulation is developed as $\min_{X\in\{0,1\}^n}\{\mathbf{c}'\mathbf{x} : \sqrt{\mathbf{x}'\mathbf{\Gamma}^j\mathbf{x}} \leq \sqrt{p_j/(1-p_j)}(b_j = \mathbb{E}[\mathbf{a}_j]'\mathbf{x}), j = 1,\ldots,m\}$, where $\mathbf{\Gamma}_{ik}^j = \mathbb{E}[(a_{ij} - \mathbb{E}[a_{ij}])(a_{kj} - \mathbb{E}[a_{kj}])], \forall i,k = 1,\ldots,n$. Delage and Ye [12] investigate the stochastic program with limited distribution information, and they propose a new moment-based ambiguity set, which is assumed to include the true probability distribution, to describe the uncertainties. There have been various works successfully applying the distribution-free approaches. Ng [17] investigates a stochastic vessel deployment problem for liner shipping, in which only the mean, standard deviation, and an upper bound of demand are known. A distribution-free optimization formulation is proposed. Based on that, Ng [18] studies a stochastic vessel deployment problem for the liner shipping industry, where only the mean and variance of the uncertain demands are known. New models are proposed, and the provided bounds are shown to be sharp under uncertain environment. The stochastic dependencies between the shipping demands are considered.

Jiang and Guan [19] develop approaches to solve stochastic programs with data-driven chance constraints. Two types of confidence sets for the possible probability distributions are proposed. For more distribution-free formulation applications, please see Lee and Hsu [20], Kwon and Cheong [21], etc., for the stochastic inventory problem, and Zhang et al. [23] for stochastic allocating surgeries problem in operating rooms, and Zheng et al. [24] for stochastic disassembly line balancing problem. To the best of our knowledge, there is no research for the stochastic ambulance location problem with only partial information on the uncertain demand.

3. Problem Description and Formulation

In this section, we first describe the considered problem and then propose a new distribution-free model.

3.1. Problem Description. There is a given set of candidate base locations $I = \{1, 2, \ldots, |I|\}$ and a set of potential emergency demand points $J = \{1, 2, \ldots, |J|\}$. The emergency points may refer to a road, a part of the urban area and a village. Base $i \in I$ is said to be covering an emergency point j if the driving time t_{ij} between i and j is no more than a predetermined value T. The set I_j of candidate bases covering emergency point j is denoted as $I_j = \{i \in I \mid t_{ij} \le T\}$.

The number of ambulances required by an emergency point $j \in J$ is denoted as d_j, which depends on the severity of the practical situation. We consider uncertain emergencies which are estimated or predicted by partial distributional information; thus the demand is regarded to be ambiguous. Besides, we focus on the case where the historical data cannot be represented by a precise probability distribution. It is assumed that the customer demands are independent of each other.

The objective of the problem is to select a subset of base locations and determine the number of ambulances at each constructed base in order to minimize the total base construction cost and ambulance cost. Throughout this paper we assume that only the mean and covariance matrix of demands are known. Moreover, a predefined *safety level* α_j is given for each potential emergency point $j \in J$. That is, the number of ambulances serving each emergency point $j \in J$ is larger than or equal to its demand with a least probability of α_j.

3.2. Chance Constraint Construction. In this section, we introduce a chance constraint to guarantee the safety level of each emergency demand point. In the following, y_{ij} is a decision variable denoting the number of ambulances serving point $j \in J$ from base $i \in I_j$ and d_j is the uncertain demand at point $j \in J$. The chance constrained inequality is presented as follows:

$$\mathrm{Prob}_{\mathbb{P}}\left(\sum_{i\in I_j} y_{ij} \ge d_j\right) \ge \alpha_j, \quad \forall j \in J \tag{1}$$

where $\mathrm{Prob}_{\mathbb{P}}(\cdot)$ denotes the probability of the event in parentheses under any potential probability distribution $\mathbb{P}$. Constraint (1) ensures that the number of ambulances serving emergency point $j \in J$ is no less than its demand with a least probability of α_j (i.e., safety level).

3.3. Distribution-Free Formulation. In the following, we give basic notations, define decision variables and propose the distribution-free formulation **DF** for the ambulance location problem with uncertain demand.

Parameters

i: index of base locations

j: index of emergency points

I: set of candidate base locations

J: set of possible emergency demand points

t_{ij}: driving time between base $i \in I$ and demand point $j \in J$

T: maximum driving time for serving any emergency call

I_j: set of candidate bases that can cover emergency point $j \in J$, i.e., $I_j = \{i \in I \mid t_{ij} \le T\}$

J_i: set of potential emergency points covered by base $i \in I$, i.e., $J_i = \{j \in J \mid t_{ij} \le T\}$

f_i: fixed construction cost for installing a base at location $i \in I$

g_i: fixed cost associated with an ambulance to be located at $i \in I$

d_j: number of (stochastic) ambulances requested by emergency point $j \in J$

$\mathscr{M}$: a sufficiently large positive number

Decision Variables

x_i: a binary variable equal to 1 if a base is constructed at $i \in I$; 0 otherwise

z_i: number of ambulances assigned to possible base location $i \in I$

y_{ij}: number of ambulances serving emergency point $j \in J$ from possible base location $i \in I_j$

Distribution-Free Model [DF]

[DF]:

$$\min \quad \left\{\sum_{i\in I} \left(f_i \cdot x_i + g_i \cdot z_i\right)\right\} \tag{2}$$

s.t. Constraint (1)

$$z_i \le \mathscr{M} \cdot x_i, \quad \forall i \in I \tag{3}$$

$$\sum_{j\in J_i} y_{ij} \le z_i, \quad \forall i \in I \tag{4}$$

$$x_i \in \{0, 1\}, \quad \forall i \in I \tag{5}$$

$$z_i \in \mathbb{Z}^+, \quad \forall i \in I \tag{6}$$

$$y_{ij} \in \mathbb{Z}^+, \quad \forall i \in I,\ j \in J_i \tag{7}$$

The objective function denotes the goal to minimize the total cost consisting of two parts: (i) the cost for constructing bases, i.e., $\sum_{i\in I} f_i \cdot x_i$, and (ii) the cost for deploying ambulances, i.e., $\sum_{i\in I} g_i \cdot z_i$.

Constraint (3) ensures that ambulances can only be located at the opened bases. Constraint (4) ensures that the number of ambulances sent to serve emergency points from base $i \in I$ does not exceed the total number of ambulances located at $i \in I$. Constraints (5)-(7) are the restrictions on decision variables.

4. Solution Approaches

The proposed distribution-free model is difficult to solve with the commercial software due to chance constraints. In this section, we propose two approximated MIP formulations based on those in Wagner [11] and in Delage and Ye [12], respectively. In the following subsections, we present the two approximated MIP formulations.

*4.1. Approximated MIP Formulation: **MIP-DF1**.* In this part, we first describe a widely used ambiguity set to describe the uncertainty. Then an approximated MIP formulation **MIP-DF1** is proposed.

Given a set of independent historical data samples $\{d^s\}_{s=1}^{|S|}$ of random vectors of demands, where S is the set of sample indexes, the mean vector μ and the covariance matrix Σ of demands can be estimated as follows:

$$\mu = \frac{1}{|S|}\sum_{s\in S} d^s, \qquad \Sigma = \frac{1}{|S|}\sum_{s\in S} (d^s - \mu)(d^s - \mu)^\top . \tag{8}$$

where $(\cdot)^\top$ implies the transposition of the vector in parentheses. Then, an ambiguity set of all probability distributions of demands $\mathscr{P}_1(\mu, \Sigma)$ can be described as the following:

$$\mathscr{P}_1(\mu,\Sigma) = \left\{ \mathbb{P} : \begin{array}{c} \mathbb{E}_{\mathbb{P}}[d] = \mu, \\ \mathbb{E}_{\mathbb{P}}\left[(d-\mu)(d-\mu)^\top\right] = \Sigma \end{array} \right\}, \tag{9}$$

where $\mathbb{P}$ denotes a possible probability distribution satisfying the given conditions, and $\mathbb{E}_{\mathbb{P}}[\cdot]$ denotes the expected value of the number in parentheses. Then the chance constraint (1) with the ambiguity set $\mathscr{P}_1$ can be presented as follows:

$$\text{Prob}_{\mathbb{P}}\left(\sum_{i\in I_j} y_{ij} \geq d_j\right) \geq \alpha_j, \quad \forall j \in J,\ \mathbb{P} \in \mathscr{P}_1. \tag{10}$$

According to Wagner (2010) and Ng [18], constraint (1) can be conservatively approximated by the following:

$$\sum_{i\in I_j} y_{ij} \geq \mu_j + \sigma_j \cdot \sqrt{\frac{\alpha_j}{(1-\alpha_j)}}, \quad \forall j \in J \tag{11}$$

where $\sigma_j = \sqrt{\Sigma_{jj}}$, and Σ_{jj} denotes the j-th element in the j-th column of matrix Σ. In terms of conservative approximation, all solutions satisfying constraint (11) must satisfy constraint (1). Then we describe the approximated MIP formulation **MIP-DF1** to **DF** combined with constraint (11):

[MIP-DF1]:

$$\begin{aligned} \min \quad & \left\{\sum_{i\in I} (f_i \cdot x_i + g_i \cdot z_i)\right\} \\ \text{s.t.} \quad & \text{Constraints (4) - (8), (12)}. \end{aligned} \tag{12}$$

MIP-DF1 can be exactly solved by the commercial software, such as CPLEX. As we can observe from the computational results in Section 5, it can obtain a relatively high service level. However, the system cost is also high. In order to save the system cost, another approximated MIP formulation of **DF** is then proposed in the next subsection.

*4.2. Approximated MIP Formulation: **MIP-DF2**.* Ambiguity set $\mathscr{P}_1$ focuses on exactly matching the mean and covariance matrix of uncertain parameters. However, in practice, there may be considerable estimation errors in the mean and covariance matrix. To take the inevitable estimation errors into consideration, Delage and Ye [12] introduce a new moment-based ambiguity set. Therefore, we in the following employ the moment-based ambiguity set:

$$\mathscr{P}_2(\mu,\Sigma,\gamma_1,\gamma_2) = \left\{ \mathbb{P} : \begin{array}{c} (\mathbb{E}_{\mathbb{P}}[d]-\mu)^\top \Sigma^{-1} (\mathbb{E}_{\mathbb{P}}[d]-\mu) \leq \gamma_1, \\ \mathbb{E}_{\mathbb{P}}\left[(d-\mu)(d-\mu)^\top\right] \preceq \gamma_2 \Sigma \end{array} \right\}, \tag{13}$$

where $\gamma_1 \geq 0$ and $\gamma_2 \geq 0$ are two parameters of ambiguity set $\mathscr{P}_2(\mu, \Sigma, \gamma_1, \gamma_2)$ with the following assumptions: (i) the true mean vector of demands is within an ellipsoid of size proportion to γ_1 centered at μ, and (ii) the true covariance matrix of demands is in a positive semidefinite cone bounded by a matrix inequality of $\gamma_2\Sigma$. The ambiguity set describes how likely the uncertain parameters are to be close to the mean in terms of the correlation [12]. Besides, under the moment-based ambiguity set, various stochastic programs with partial distributional information have been successfully modeled and solved [22, 23, 25].

According to the approximation method in Zhang et al. [23], chance constraint (1) can be approximated by

$$\sqrt{\frac{1}{1-a-b}} \cdot \left(1 + \sqrt{\frac{1-\alpha_j}{\alpha_j}} \cdot b\right) \cdot \sigma_j \leq \sqrt{\frac{1-\alpha_j}{\alpha_j}} \cdot \left(\sum_{i\in I_j} y_{ij} - \mu_j\right), \quad j \in J \tag{14}$$

where

$$a = 1 - \frac{\gamma_1 + 1}{\gamma_2 - \gamma_1}, \qquad b = \frac{\gamma_1}{\gamma_2 - \gamma_1}. \tag{15}$$

Table 1: $\alpha = 0.95$.

	$\|I\| = \|J\|$	Sampling approach			MIP-DF1			MIP-DF2		
		CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)
1	7	4.9	23	96.26	0.1	36	100.00	0.1	23	100.00
2	7	5.1	22	98.63	0.1	35	100.00	0.1	24	99.82
3	6	4.1	14	96.92	0.1	30	100.00	0.1	14	98.50
4	6	4.3	13	97.13	0.2	29	100.00	0.1	12	97.42
5	6	5.7	12	79.96	0.1	30	100.00	0.2	12	100.00
6	8	5.8	21	98.50	0.1	24	100.00	0.2	27	100.00
Average		5.0	17.5	94.57	0.1	30.7	100.00	0.1	18.7	99.29

The second approximated MIP formulation **MIP-DF2** to **DF** is presented as follows:

[MIP-DF2]:

$$\begin{aligned} \min \quad & \left\{ \sum_{i \in I} \left(f_i \cdot x_i + g_i \cdot z_i \right) \right\} \\ \text{s.t.} \quad & \text{Constraints } (4)\text{ - }(8), (15). \end{aligned} \tag{16}$$

Notice that when $\gamma_1 = 0$ and $\gamma_2 = 1$, inequality (14) reduces to (11) in **MIP-DF1**, which implies that **MIP-DF1** is a special case of **MIP-DF2**.

5. Computational Experiments

In this section, the performance of our proposed formulations first evaluated and compared to the sampling approach proposed in Nickel et al. [8]. Specifically, given a required service level α, we apply our MIP formulations by restricting the chance constraint for each emergency point to satisfy a safety level $\alpha_j = \alpha$, while we adopt the sampling approach by restricting the coverage constraint with α. Then computational results on 120 randomly generated instances based on the information on emergencies in Shanghai are reported. Our approximated MIP formulations and the sampling approach are solved by coding in MATLAB_2014b calling CPLEX 12.6 solver. All numerical experiments are conducted on a personal computer with Core I5 and 3.30GHz processor and 8GB RAM under windows 7 operating system. The computational time for sampling approach is limited to 3600 seconds.

5.1. Out-of-Sample Test. Following Zhang et al.'s [22] methodology, we examine the out-of-sample performance of solutions obtained by our proposed approximated MIP formulations and the sampling approach [8]. The out-of-sample test includes three steps:

(1) The base locations and the employment of ambulances at each base are determined with known mean vector and covariance matrix of demands, which is named as in-sample test.

(2) 1000 scenarios are sampled from the underlying true distribution, which represent realizing the uncertain demand.

(3) Based on the solution obtained in step (1), ambulances are assigned to serve emergency points in the 1000 scenarios to maximize the (out-of-sample) service level.

The (out-of-sample) service level is defined as the ratio of the number of emergency points with demand satisfied to the total number of emergency points in 1000 scenarios. In the following subsections, the benchmarks are tested and then three common distributions are employed, i.e., uniform distribution, Poisson distribution, and normal distribution, respectively.

5.2. Benchmark Instances. We first test the benchmarks in Nickel et al. [8] to compare our proposed two approximated MIP formulations with the sampling approach. In Nickel et al. [8], they assume that demand nodes are also potential base locations, and the number of ambulances required by each emergency point is chosen from set $\{0, 1, 2\}$ with given probability distribution. Computational results on benchmarks are reported in Tables 1 and 2.

Table 1 reports the computational results with service level requirement $\alpha = 0.95$. In Table 1, CT, Obj, and Sel denote the computational time, objective value (i.e., the system cost), and the service level, respectively. The first column includes the indexes of instances, and the second indicates the number of nodes. It shows that the average service level obtained by the sampling approach is 94.57%, which is smaller than the required service level, i.e., 85%. Specifically, the service level of instance 5 obtained by the sampling approach is only 79.96%. However, MIP-DF1 and MIP-DF2 can guarantee the service levels for all instances higher than the requirement. Concluding, from Table 1, we can observe that (i) the average computational time of sampling approach is about 50 times as those of the two approximated MIP formulations and (ii) MIP-DF1 and MIP-DF2 improve the average service level by about 5.74% and 4.99% compared with sampling approach, however, (iii) system costs obtained by MIP-DF1 and MIP-DF2 are, respectively, 75.43% and 6.86% higher than that of sampling approach.

Table 2 reports the computational results with service level requirement $\alpha = 0.99$. For instances 2-6, the service levels obtained by sampling approach cannot meet the requirement, i.e., 99%. Besides, it shows that the average computational time of sampling approach is 4.8, which is about 48 times greater than those of MIP-DF1 and MIP-DF2.

Table 2: $\alpha = 0.99$.

$\|I\| = \|J\|$		Sampling approach			MIP-DF1			MIP-DF2		
		CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)
1	7	4.9	24	99.79	0.1	64	100.00	0.1	25	100.00
2	7	4.8	23	98.56	0.1	63	100.00	0.1	24	100.00
3	6	5.1	15	97.17	0.1	55	100.00	0.1	16	100.00
4	6	4.7	15	92.75	0.1	52	100.00	0.1	13	100.00
5	6	4.0	14	90.00	0.1	55	100.00	0.1	14	100.00
6	8	5.2	23	97.85	0.1	45	100.00	0.1	27	100.00
Average		4.8	19.0	93.43	0.1	55.7	100.00	0.1	19.8	100.00

Table 3: Solutions to instance 4 in Nickel et al. [8] for the given scenario.

	Sampling approach							MIP-DF1							MIP-DF2						
	Emergency points	1	2	3	4	5	6	Emergency points	1	2	3	4	5	6	Emergency points	1	2	3	4	5	6
	Demand	1	0	2	0	2	0	Demand	1	0	2	0	2	0	Demand	1	0	2	0	2	0
Base locations	Ambulance employment							Ambulance employment							Ambulance employment						
1	0							0							0						
2	1			1				0							3	1		2			
3	1	1						26	1		2		2		0						
4	0							0							0						
5	0							6							0						
6	2					2		0							4					2	

The average service level is 93.43%. Although the average service levels obtained by MIP-DF1 and MIP-DF2 are both 100%, the average system cost obtained by MIP-DF1 is about 3 times larger than that of sampling approach. Besides, the average system cost obtained by MIP-DF2 is 4.21% higher than that of sampling approach.

Concluding, for the benchmarks described in Nickel et al. [8], we can observe that (1) MIP-DF1 and MIP-DF2 can be solved in far less computational time compared with the sampling approach; (2) the sampling approach cannot meet the requirement of service level for all benchmarks, while the two MIP formulations can; (3) the average system cost of MIP-DF2 is very close to that of sampling approach; (4) with the increase of α, the average system costs obtained by MIP-DF2 is getting closer to that obtained by sampling approach.

From Tables 1 and 2, we can observe that, in spite of the lower system cost obtained by the sampling approach, there may exist extreme situations that demands of some emergency points are not satisfied. Take one benchmark in Nickel et al. [8], i.e., instance 4, where there are 6 nodes. A scenario in which demand at each emergency point is generated from the given probability distribution is shown in Figure 1. We can observe from Figure 1 that under the given scenario, the demand of each possible emergency point is $\{1, 0, 2, 0, 2, 0\}$. The solutions obtained by sampling approach, MIP-DF1 and MIP-DF2, are shown in Table 3, which includes base locations and ambulance employment obtained by in-sample test and ambulance assignment under given scenario.

In Table 3, the first column includes the indexes of potential base locations. The second column represents the base locations and the ambulance employment at each location, in which a location with 0 ambulance implies that it is not selected for base construction. From Table 3, we can observe that the in-sample solutions obtained by sampling approach is to construct bases at locations 2, 3 and 6, where the number of ambulances is 1, 1, and 2, respectively. Besides, under given scenario, there is one ambulance serving emergency point 1 from base location 3, and one ambulance serving emergency point from base location 2, and 2 ambulances serving emergency point 5 from base location 6. It shows that the requirement of ambulances at emergency point 3 is not fully satisfied. The insufficient service may cause serious loss of life. However, we can find that the solutions obtained by MIP-DF1 and MIP-DF2 can provide sufficient ambulances for all emergency points.

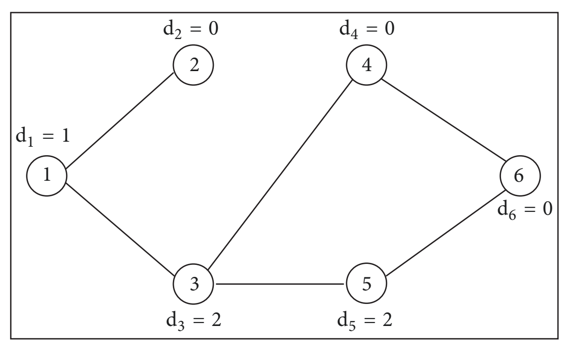


Figure 1: Instance 4 in Nickel et al. [8] under given scenario.

In sum, to guarantee the required service level with less cost, MIP-DF2 is recommended for the solutions with

Nickel et al. [8]'s data set. Moreover, in the benchmarks, it is assumed that all nodes are possible emergency points as well as potential base locations and the number of required ambulances is small, which is not always consistent with the situation in Shanghai. Therefore, we further test the two approximated MIP formulations and the sampling approach on randomly generated instances by analysing the information on emergencies in Shanghai [26].

5.3. Computational Tests on Randomly Generated Instances. In the following subsection, numerical experiments on 120 randomly generated instances are described.

5.3.1. Test Instances. By analysing the information on emergencies in Shanghai [26], the mean value μ_j of demand d_j in each emergency point are randomly generated from a discrete Uniform distribution on interval $[5, 25]$. The standard deviation σ_j is set to be 5. Since the sampling approach proposed in Nickel et al. [8] requires a sample of scenarios, in numerical experiments scenarios are produced with demands randomly generated from Uniform distribution, Poisson distribution and Normal distribution consistent with given mean and standard deviation.

According to the instance settings in Erkut and Ingolfsson (2008), response time from a base to an emergency point is randomly generated from a discrete Uniform distribution on interval $[3, 30]$ in units of minutes. A base covers an emergency point if the driving time between them is less than 10 minutes to ensure the survival probability (Erkut and Ingolfsson, 2006). For an emergency point in each instance, if there is no base with 10 minutes driving time to it, then the base covering this point is set to be the nearest one. The cost of constructing a base and that of locating one ambulance are both set to be 1, as in Nickel et al. [8]. Besides, the number of emergency points is 3 times that of potential base locations.

5.3.2. Results and Discussion. Different values of required service level α are tested, i.e., 0.85, 0.9, 0.95. Computational results on 120 randomly generated instances are reported in Tables 4-6.

Table 4 reports the numerical results of instances with demand generated from Uniform distribution. We observe that when $\alpha = 0.85, 0.9, 0.95$, the average computational times of the sampling approach are 448.2, 437.9 and 429.5 seconds, respectively. However, both approximated MIP formulations can produce solutions in much faster speeds, i.e., within 1.3 seconds on average. With the increase of α, (i) the out-of-sample average service level is getting higher, and (ii) the system costs of solutions obtained by all method are getting larger, and (iii) the system costs of solutions yielded by MIP-DF2 is getting closer to those of solutions obtained by the sampling approach. In terms of the average system cost, the sampling approach performs the best, while MIP-DF1 is the poorest. When $\alpha = 0.85$, for example, the average system cost of MIP-DF2 and the sampling approach are about 30.32% and 44.2% less than that of MIP-DF1, respectively. When $\alpha = 0.95$, the above gaps are even larger. Concluding, for the Uniform distribution, (1) the proposed two approximated MIP formulations are far more time saving than the sampling approach, (2) MIP-DF1 and MIP-DF2 can improve the service level by about 19.07% compared with the sampling approach, however, (3) despite of the highest service levels obtained by MIP-DF1, the system cost obtained by MIP-DF1 is also high, (4) MIP-DF2 can ensure an appropriate service level with much less cost than MIP-DF1, and (5) for the situations with small-scale instances and high service level requirement, MIP-DF1 is recommended.

Computational results of instances with demand at each emergency point generated from Poisson distribution are presented in Table 5. For MIP-DF1 and MIP-DF2, only mean and variance are involved. Therefore, with given mean and standard deviation of the demand at each point, computational results are the same for different probability distributions. We can obtain from Table 5 that the objective value obtained by the sampling approach increases with the increase of α. Besides, when the value of α equals 0.85, the average service level obtained by the sampling approach is about 81.43%, which is smaller than that of the two approximated MIP formulations. The highest service level 99.34% is obtained by MIP-DF1, and that obtained by MIP-DF2 is 97.08%. When $\alpha = 0.9$, the average service level of the sampling approach is 84.37%, and those obtained by MIP-DF1 and MIP-DF2 are 99.85% and 97.97%, respectively. When $\alpha = 0.95$, the average service level obtained by the sampling approach is 93.56%, and those yielded by MIP-DF1 and MIP-DF2 are 99.97% and 97.81%, respectively. For Poisson distribution, concluding, MIP-DF1 and MIP-DF2 can improve the service level compared with the sampling approach, especially, MIP-DF1 can obtain the highest service level.

Similarly, Table 6 reports the computational results for instances in which the demand at each emergency point is generated following Normal distribution. When $\alpha = 0.85$, the average service levels obtained by the sampling approach, MIP-DF1 and MIP-DF2 are 81.74%, 99.25%, and 97.42%, respectively. When $\alpha = 0.9$, the three values are 84.57%, 99.81%, and 97.43% respectively. Moreover, for $\alpha = 0.95$, the values are equal to 87.32%, 99.99%, and 97.94%, respectively. It can be observed that the performance of the sampling approach is the poorest in terms of the service level.

By observing the above computational results on benchmarks and randomly generated instances, we conclude that (1) The proposed approximated MIP formulations, i.e., MIP-DF1 and MIP-DF2 are very time saving compared with the sampling approach; (2) MIP-DF1 an MIP-DF2 can achieve the requirement of service level and improve the service level by about 15.47% and 12.29% on average, compared with that of the sampling approach; (3) MIP-DF1 is more conservative in terms of overhigh emergency service level close to 100%, and the system cost is also high, which is about 56.8% larger than that of MIP-DF2; (4) Compared with MIP-DF1, MIP-DF2 is less conservative and much more cost saving while ensuring an appropriately service level on average, which is about 97%.

In sum, for medium and large problem instances (e.g., Shanghai), which are common in practice, to guarantee the required service level with less cost, we recommend MIP-DF2 as solution method. However, for very small instances with

TABLE 4: Computational results on instances with data generated from uniform distribution.

	Set (|I|, |J|)	α = 0.85									α = 0.9									α = 0.95								
		Sampling approach			MIP-DF1			MIP-DF2			Sampling approach			MIP-DF1			MIP-DF2			Sampling approach			MIP-DF1			MIP-DF2		
		CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)	CT	Obj	Sel (%)
1	(2,6)	2.7	89	73.17	0.1	149	99.06	0.1	107	92.22	2.8	96	73.73	0.1	155	99.33	0.1	107	91.67	2.7	106	75.85	0.1	185	99.87	0.1	107	91.28
2	(4,12)	4.7	169	74.23	0.0	291	99.19	0.0	207	91.22	5.8	181	74.38	0.1	303	99.97	0.1	207	93.63	5.0	199	77.02	0.0	363	99.82	0.0	207	93.38
3	(6,18)	7.2	258	75.06	0.1	438	99.76	0.1	312	91.06	7.5	279	80.82	0.1	456	99.96	0.1	312	93.69	7.8	304	83.37	0.1	546	99.97	0.1	312	93.57
4	(8,24)	9.9	314	79.56	0.1	553	99.93	0.1	385	92.06	11.1	338	82.02	0.1	577	99.99	0.1	385	96.02	11.2	366	84.92	0.1	697	99.94	0.1	385	96.20
5	(10,30)	13.2	415	84.06	0.1	722	98.96	0.1	512	94.06	14.3	441	83.22	0.2	752	99.83	0.1	512	96.92	14.2	476	86.09	0.1	902	99.93	0.1	512	96.92
6	(12,36)	17.0	475	82.49	0.2	843	98.70	0.1	591	93.67	18.1	504	82.51	0.1	879	99.82	0.1	591	96.08	18.9	535	87.26	0.1	1059	99.97	0.1	591	96.09
7	(14,42)	21.5	573	81.11	0.2	1006	98.49	0.2	712	96.06	24.3	609	82.17	0.2	1048	99.53	0.2	712	95.89	23.3	644	86.08	0.2	1258	99.92	0.2	712	95.82
8	(16,48)	24.8	686	75.32	0.2	1185	98.96	0.2	849	97.25	28.4	727	83.07	0.2	1233	99.49	0.2	849	96.13	28.1	768	90.63	0.2	1473	99.99	0.3	849	96.22
9	(18,54)	31.3	691	83.53	0.3	1236	99.74	0.4	858	97.09	35.1	731	83.97	0.3	1290	99.88	0.3	858	97.65	35.2	760	89.06	0.3	1560	99.94	0.3	858	97.58
10	(20,60)	37.2	794	81.74	0.3	1409	98.87	0.3	989	96.57	42.8	843	84.89	0.3	1469	99.47	0.3	989	97.73	43.2	892	90.04	0.3	1769	99.97	0.3	989	97.38
11	(22,66)	45.1	830	80.72	0.4	1493	99.49	0.3	1031	97.58	51.8	880	83.28	0.4	1559	99.82	0.3	1031	97.23	51.6	930	89.67	0.5	1889	99.95	0.3	1031	97.27
12	(24,72)	54.3	1025	79.70	0.4	1771	99.27	0.4	1267	98.35	60.1	1083	84.82	0.5	1843	99.49	0.4	1267	97.76	63.7	1146	90.14	0.4	2203	99.93	0.4	1267	97.77
13	(26,78)	63.1	1026	81.42	0.5	1819	98.98	0.4	1274	97.82	74.3	1087	86.78	0.5	1897	99.36	0.4	1274	97.78	76.5	1146	90.23	0.4	2287	99.97	0.5	1273	98.37
14	(28,84)	78.2	1101	79.03	0.5	1949	98.69	0.8	1361	98.04	87.8	1165	87.90	0.5	2033	99.35	0.7	1361	98.89	87.0	1227	90.56	0.5	2453	99.98	0.6	1361	98.08
15	(30,90)	90.7	1091	80.98	0.6	1997	99.15	0.6	1367	96.29	101.8	1155	83.29	0.6	2087	99.81	0.6	1367	98.07	102.0	1225	89.86	0.6	2537	99.99	0.8	1367	98.17
16	(32,96)	106.2	1160	87.98	0.7	2123	99.44	0.7	1449	97.87	118.3	1229	83.83	0.7	2219	99.88	0.6	1449	98.49	119.3	1300	91.05	0.6	2698	99.98	0.6	1449	98.15
17	(34,102)	120.2	1298	75.72	0.7	2328	99.69	0.7	1614	98.21	137.7	1374	88.60	0.8	2430	99.97	0.7	1614	98.24	126.0	1452	90.85	0.7	2940	99.99	0.8	1614	98.18
18	(36,108)	139.4	1408	81.21	0.8	2509	99.47	0.8	1752	98.09	154.4	1492	89.28	0.8	2617	99.83	0.8	1752	97.99	154.8	1580	91.73	0.8	3156	99.99	0.8	1752	98.09
19	(38,114)	161.8	1508	81.70	0.9	2676	98.99	1.1	1877	97.42	176.0	1598	91.30	0.9	2790	99.34	0.8	1877	99.08	175.5	1688	91.86	0.9	3360	99.99	0.9	1877	98.72
20	(40,120)	220.6	1628	77.19	1.0	2869	99.89	1.0	2029	98.73	200.7	1732	87.53	1.1	2990	99.97	0.9	2029	98.79	190.4	1827	91.38	1.0	3590	99.98	0.9	2029	98.75
21	(42,126)	278.0	1622	83.54	1.0	2900	99.78	1.0	2017	97.32	260.9	1718	88.54	1.0	3026	98.76	1.0	2017	98.18	267.0	1812	90.89	1.2	3655	99.99	1.0	2017	98.48
22	(44,132)	318.9	1735	84.94	1.1	3088	99.85	1.5	2165	98.57	297.0	1840	89.22	1.2	3220	99.25	1.2	2165	98.86	287.3	1952	92.06	1.1	3880	99.99	1.1	2165	98.50
23	(46,138)	363.0	1937	81.41	1.2	3372	98.93	1.3	2406	97.96	322.2	2048	89.37	1.2	3510	99.34	1.4	2406	99.06	346.1	2170	89.89	1.2	4199	100.00	1.2	2406	98.09
24	(48,144)	422.6	1968	82.26	1.3	3461	99.32	1.3	2453	98.14	367.4	2085	88.46	1.3	3605	99.56	1.5	2453	99.12	355.6	2211	93.00	1.3	4326	99.99	1.3	2453	98.19
25	(50,150)	472.7	1919	81.11	1.4	3456	99.99	1.5	2407	96.29	401.9	2037	86.99	1.4	3606	99.99	1.4	2407	98.57	425.7	2161	92.59	1.5	4356	99.95	1.4	2407	98.19
26	(52,156)	530.0	1906	80.96	1.5	3484	98.78	1.7	2392	96.58	449.5	2021	89.52	1.4	3640	99.99	1.5	2392	95.68	448.6	2142	91.82	1.5	4420	99.99	1.5	2392	98.54
27	(54,162)	591.3	2096	83.87	1.6	3751	98.46	1.7	2617	97.61	511.7	2221	90.45	1.6	3913	99.98	1.7	2617	95.44	483.0	2355	93.46	1.8	4723	99.99	1.6	2617	98.93
28	(56,168)	631.0	2110	77.58	1.7	3819	99.33	1.9	2643	98.29	577.7	2234	91.57	1.8	3987	99.98	1.8	2643	95.26	555.0	2365	90.45	1.8	4827	99.99	1.7	2643	98.31
29	(58,174)	678.0	2175	79.98	1.8	3937	98.98	1.8	2720	97.14	630.3	2307	90.22	2.0	4111	99.69	1.9	2720	96.23	618.9	2444	91.28	1.9	4981	100.00	1.9	2720	99.14
30	(60,180)	759.3	2384	83.21	1.9	4240	99.67	2.0	2979	98.41	679.1	2527	89.78	1.9	4419	99.79	2.3	2979	97.21	687.8	2679	91.43	2.1	5316	99.99	2.0	2979	98.69
31	(62,186)	822.0	2281	76.04	2.0	4157	99.84	2.1	2856	97.96	773.4	2416	89.34	2.1	4343	99.97	2.1	2856	96.79	746.6	2557	91.58	2.2	5273	99.99	2.3	2856	99.02
32	(64,192)	876.6	2517	83.78	2.1	4489	99.32	2.5	3145	97.29	827.6	2671	82.96	2.3	4680	99.77	2.4	3145	98.25	789.7	2822	90.89	2.4	5640	99.99	2.2	3145	98.91
33	(66,198)	1012.3	2532	78.06	2.3	4555	99.67	2.3	3169	96.15	901.0	2682	89.47	2.4	4752	99.89	2.4	3169	97.03	905.1	2835	91.57	2.4	5742	99.98	2.4	3169	98.67
34	(68,204)	1101.0	2535	79.34	2.4	4602	99.99	2.9	3174	96.31	989.8	2685	88.58	3.0	4806	99.98	2.8	3174	97.62	986.3	2844	92.36	2.6	5824	99.99	2.5	3174	98.89
35	(70,210)	1166.6	2714	80.62	2.5	4862	99.86	2.9	3391	97.87	1066.6	2875	91.37	2.6	5072	99.99	2.9	3391	97.16	1082.2	3043	93.15	2.6	6122	99.99	2.9	3391	99.23
36	(72,216)	1201.1	2827	81.04	2.6	5047	99.91	2.8	3535	97.98	1246.5	2984	90.16	2.8	5263	99.99	2.8	3535	98.78	1166.7	3169	92.53	3.0	6343	99.99	2.9	3535	98.96
37	(74,222)	1143.7	2826	82.28	2.8	5096	99.33	2.9	3543	98.84	1358.2	3098	88.95	2.9	5318	99.99	3.2	3543	98.88	1265.4	3178	91.39	2.9	6427	100.00	3.1	3543	98.93
38	(76,228)	1243.5	2839	81.52	2.9	5161	98.97	2.9	3565	97.88	1397.6	3103	89.26	2.9	5388	99.99	3.1	3565	97.24	1383.7	3185	92.55	3.3	6528	100.00	3.1	3565	98.95
39	(78,234)	1464.3	2945	84.12	3.1	5331	99.98	3.2	3695	98.76	1479.4	3210	90.05	3.1	5565	99.99	3.4	3695	97.13	1456.1	3313	90.81	3.4	6735	100.00	3.3	3695	98.78
40	(80,240)	1604.2	2975	85.10	3.2	5414	99.57	3.6	3734	97.96	1626.3	3346	89.99	3.5	5654	99.99	3.5	3734	98.06	1586.9	3343	92.48	3.5	6854	100.00	3.5	3734	98.73
Average		**448.2**	**1584.6**	**80.67**	**1.2**	**2839.7**	**99.36**	**1.3**	**1978.7**	**96.82**	**437.9**	**1691.3**	**86.54**	**1.3**	**2962.6**	**99.75**	**1.3**	**1978.7**	**97.21**	**429.5**	**1778.8**	**89.85**	**1.3**	**3577.4**	**99.97**	**1.3**	**1978.7**	**97.75**

Table 5: Computational results on instances with data generated from Poisson distribution.

	set ($\|I\|, \|J\|$)	$\alpha = 0.85$					$\alpha = 0.9$					$\alpha = 0.95$				
		Sampling approach			MIP-DF1	MIP-DF2	Sampling approach			MIP-DF1	MIP-DF2	Sampling approach			MIP-DF1	MIP-DF2
		CT	Obj	Sel (%)	Sel (%)	Sel (%)	CT	Obj	Sel (%)	Sel (%)	Sel (%)	CT	Obj	Sel (%)	Sel (%)	Sel (%)
1	(2,6)	2.7	90	72.94	99.67	90.01	2.7	98	77.70	99.79	90.13	2.4	103	92.73	99.99	94.68
2	(4,12)	4.6	170	79.61	99.21	91.17	5.1	183	83.05	99.46	91.28	4.9	201	93.92	99.98	95.46
3	(6,18)	6.8	258	75.78	99.38	89.11	7.4	280	80.28	99.68	90.04	6.9	307	91.82	99.99	96.58
4	(8,24)	9.3	315	80.01	99.28	91.96	10.2	338	84.38	99.67	91.72	9.2	368	94.68	99.99	96.13
5	(10,30)	12.0	413	80.41	99.39	94.49	13.3	443	84.47	99.76	92.72	12.1	479	95.44	99.99	97.52
6	(12,36)	15.5	476	80.84	98.77	95.54	16.7	508	88.63	99.91	95.69	15.9	536	94.56	99.99	97.34
7	(14,42)	19.5	573	78.76	98.92	96.63	21.3	609	91.24	99.98	97.29	20.5	643	93.14	100.00	97.68
8	(16,48)	23.2	687	79.64	99.97	97.88	26.7	724	85.23	99.88	96.87	23.8	772	94.22	99.98	96.89
9	(18,54)	29.3	692	82.67	98.93	98.09	32.7	730	83.86	99.99	97.50	29.8	757	92.28	99.99	97.55
10	(20,60)	35.6	796	79.46	98.99	98.01	39.9	840	82.99	99.76	96.86	36.0	885	96.68	99.99	97.05
11	(22,66)	42.3	829	82.01	98.95	97.56	48.9	877	83.78	99.87	97.88	44.8	927	94.76	99.99	98.12
12	(24,72)	51.2	1023	83.38	99.13	98.53	55.4	1085	85.01	99.99	98.81	51.4	1148	93.98	99.99	98.88
13	(26,78)	59.2	1045	84.16	99.07	98.73	68.7	1088	79.98	99.96	98.02	60.7	1152	95.04	99.99	98.21
14	(28,84)	72.3	1099	79.46	99.15	97.88	83.8	1166	80.56	99.92	98.29	77.6	1230	95.46	99.99	98.39
15	(30,90)	85.5	1089	79.15	99.64	97.03	94.2	1156	84.32	99.84	97.78	90.3	1225	92.23	99.98	97.85
16	(32,96)	100.9	1158	80.24	98.97	97.69	116.3	1231	85.00	99.96	98.24	104.5	1222	91.99	99.99	98.35
17	(34,102)	116.5	1299	80.34	99.19	97.79	128.5	1375	84.95	99.37	98.08	119.0	1457	92.58	100.00	98.46
18	(36,108)	125.4	1411	81.76	99.43	98.85	147.9	1492	85.61	99.68	98.68	134.5	1579	92.86	99.99	98.75
19	(38,114)	150.4	1506	79.70	99.23	98.42	165.1	1594	81.67	99.72	98.46	156.4	1694	93.76	99.99	97.86
20	(40,120)	170.6	1635	78.19	99.16	97.83	185.4	1734	86.07	99.68	97.76	177.8	1825	92.85	99.04	97.18
21	(42,126)	205.4	1622	83.54	99.18	98.34	235.7	1720	82.97	99.78	98.33	222.6	1856	95.01	99.99	98.34
22	(44,132)	240.0	1736	84.94	99.22	98.46	285.2	1844	82.69	99.99	98.16	268.0	1946	94.86	99.99	98.20
23	(46,138)	265.1	1934	81.41	99.53	97.93	315.9	2049	84.92	99.69	97.99	307.2	2172	93.58	100.00	98.13
24	(48,144)	300.4	1961	82.26	99.47	96.81	352.2	2086	85.13	99.98	97.06	310.2	2209	94.88	99.99	97.24
25	(50,150)	330.8	1924	81.11	99.65	97.46	389.8	2038	82.64	99.99	97.88	363.9	2161	95.46	99.99	98.13
26	(52,156)	366.9	1911	80.96	99.39	98.55	430.5	2027	87.22	99.67	98.74	445.3	2142	91.85	99.99	98.76
27	(54,162)	441.8	2095	83.87	99.27	97.54	477.1	2219	83.41	99.89	97.49	480.6	2353	92.89	99.99	97.55
28	(56,168)	536.7	2113	84.58	99.68	98.48	535.6	2237	84.52	99.78	98.55	522.1	2369	92.28	100.00	98.68
29	(58,174)	621.7	2186	79.98	99.89	96.94	594.2	2315	83.16	99.96	96.99	585.3	2371	91.67	99.99	97.05
30	(60,180)	726.3	2382	83.21	99.91	98.37	652.7	2543	83.59	99.98	98.86	621.4	2678	93.69	99.97	98.86
31	(62,186)	821.4	2346	84.04	99.36	98.88	781.3	2430	83.21	99.99	98.46	687.6	2560	93.58	99.99	98.47
32	(64,192)	886.3	2518	83.78	99.38	98.07	810.6	2677	83.56	99.99	98.33	787.7	2822	94.08	100.00	98.35
33	(66,198)	998.5	2532	84.06	99.17	97.09	968.4	2689	84.97	99.96	97.46	808.8	2838	95.07	99.99	97.49
34	(68,204)	1126.4	2534	85.43	99.08	98.97	1098.6	2685	88.09	99.96	97.82	846.9	2843	88.44	99.98	97.80
35	(70,210)	1203.5	2715	82.58	99.37	97.99	1209.8	2886	85.69	99.91	98.09	994.6	3041	94.59	99.99	98.13
36	(72,216)	1280.3	2825	81.04	99.29	98.27	1307.6	3010	86.51	99.94	98.34	1101.3	3170	94.68	99.99	98.65
37	(74,222)	1302.1	2830	82.28	99.49	98.26	1326.1	3009	83.76	99.99	98.26	1201.4	3172	93.46	99.99	98.88
38	(76,228)	1386.2	2843	81.52	99.33	98.07	1399.5	3028	87.60	99.98	98.41	1213.3	3187	97.06	99.99	98.76
39	(78,234)	1538.7	2953	83.12	99.64	97.99	1482.7	3133	90.01	99.95	97.83	1338.1	3304	89.13	99.99	97.99
40	(80,240)	1697.4	2975	85.10	99.85	97.67	1525.6	3249	82.17	99.93	97.81	1586.7	3347	91.11	99.99	97.91
Average		**435.2**	**1587.5**	**81.43**	**99.34**	**97.08**	**436.2**	**1685.6**	**84.37**	**99.85**	**97.07**	**396.8**	**1776.3**	**93.56**	**99.97**	**97.81**

Table 6: Computational results on instances with data generated from normal distribution.

	set $(\|I\|, \|J\|)$	$\alpha = 0.85$ Sampling approach CT	Obj	Sel (%)	MIP-DF1 Sel (%)	MIP-DF2 Sel (%)	$\alpha = 0.9$ Sampling approach CT	Obj	Sel (%)	MIP-DF1 Sel (%)	MIP-DF2 Sel (%)	$\alpha = 0.95$ Sampling approach CT	Obj	Sel (%)	MIP-DF1 Sel (%)	MIP-DF2 Sel (%)
1	(2,6)	2.7	90	72.94	99.64	91.17	2.7	94	78.44	99.70	92.02	2.558	106	84.22	100.00	92.08
2	(4,12)	4.6	170	79.61	99.21	92.83	4.8	183	83.28	99.82	92.83	4.508	183	88.23	100.00	93.70
3	(6,18)	6.8	258	75.78	99.16	92.28	6.8	280	77.98	99.71	94.17	6.803	307	76.58	99.98	94.47
4	(8,24)	9.3	315	80.00	98.99	93.67	9.6	337	83.79	99.79	94.67	9.373	368	84.38	99.99	95.79
5	(10,30)	12.0	413	78.41	99.27	94.37	12.4	443	87.63	99.63	94.76	12.62	499	88.50	99.98	97.08
6	(12,36)	15.5	476	79.21	99.19	95.19	16.2	504	81.24	99.82	95.82	16.021	536	89.54	100.00	97.56
7	(14,42)	19.5	573	79.76	99.33	95.98	19.6	609	82.87	99.75	96.19	20.11	646	84.67	99.99	97.67
8	(16,48)	23.2	687	80.31	99.18	97.05	23.5	727	81.15	99.65	96.50	24.04	772	82.78	99.99	97.65
9	(18,54)	29.3	692	80.86	98.98	97.59	29.1	734	84.64	99.74	97.55	29.815	796	86.17	99.98	98.35
10	(20,60)	35.6	796	79.84	99.37	97.70	35.8	842	82.23	99.88	97.65	37.973	921	88.46	100.00	98.38
11	(22,66)	42.1	829	81.01	99.40	97.55	43.6	877	82.11	99.86	97.86	45.148	959	84.97	99.99	97.89
12	(24,72)	51.4	1023	80.33	99.51	97.85	50.1	1085	81.62	99.83	97.88	52.073	1147	83.21	99.99	98.24
13	(26,78)	59.2	1023	81.38	99.16	98.85	59.9	1087	84.11	99.70	98.45	62.937	1194	86.09	99.98	97.95
14	(28,84)	72.8	1099	82.35	99.13	99.15	72.2	1168	84.35	99.87	98.06	75.053	1270	85.67	99.97	98.34
15	(30,90)	85.5	1089	81.87	99.22	98.61	88.0	1157	83.76	99.76	97.96	89.222	1300	86.11	99.96	97.98
16	(32,96)	100.9	1158	87.70	98.97	97.88	96.6	1231	88.45	99.84	98.23	106.288	1359	89.97	99.99	98.48
17	(34,102)	115.4	1299	84.34	99.25	98.49	113.7	1377	86.01	99.80	97.98	123.869	1493	87.98	99.96	98.77
18	(36,108)	140.7	1411	83.21	99.27	98.66	133.9	1495	83.01	99.65	98.22	133.807	1601	85.67	99.99	98.34
19	(38,114)	170.2	1506	82.06	99.15	98.05	167.2	1598	89.77	99.78	98.45	152.999	1694	89.98	99.99	98.47
20	(40,120)	209.1	1635	85.28	99.18	98.89	191.6	1737	88.05	99.76	98.37	189.002	1833	90.12	99.99	97.91
21	(42,126)	284.3	1622	85.50	99.34	99.13	243.2	1718	88.86	99.74	98.13	239.913	1815	90.08	100.00	98.15
22	(44,132)	330.0	1736	84.49	99.40	98.59	279.5	1843	87.86	99.77	97.56	253.106	1948	89.97	99.99	98.26
23	(46,138)	374.1	1934	81.33	99.11	98.76	314.3	2050	88.76	99.81	97.85	274.583	2173	90.14	99.99	98.58
24	(48,144)	453.1	1961	82.45	99.09	98.46	351.5	2090	83.44	99.93	98.19	308.089	2211	86.61	99.99	98.83
25	(50,150)	471.5	1924	81.11	99.29	98.88	395.9	2041	82.79	99.96	98.37	351.093	2158	85.27	99.99	98.27
26	(52,156)	522.3	1911	80.96	98.98	98.57	430.1	2025	82.51	99.97	97.57	421.747	2146	84.97	99.99	99.14
27	(54,162)	599.4	2095	83.87	99.08	97.75	483.5	2226	84.97	99.74	97.80	461.443	2359	86.95	99.99	98.37
28	(56,168)	640.6	2111	84.11	99.06	97.98	533.6	2240	85.58	99.91	98.20	494.113	2371	92.49	99.99	98.16
29	(58,174)	689.7	2176	80.12	99.16	97.91	594.8	2311	82.11	99.84	97.65	624.835	2445	85.31	100.00	98.51
30	(60,180)	743.6	2391	81.11	99.13	97.66	655.8	2533	83.05	99.68	97.95	696.116	2682	85.75	99.99	97.99
31	(62,186)	811.8	2282	81.19	99.28	98.06	706.9	2422	84.17	99.73	98.73	797.768	2564	86.67	99.99	98.46
32	(64,192)	889.5	2519	82.23	99.19	97.57	785.7	2675	86.31	99.82	97.69	864.813	2827	89.76	100.00	99.05
33	(66,198)	1123.5	2533	85.63	99.28	98.11	916.2	2688	85.47	99.79	98.28	973.089	2843	86.57	99.99	98.46
34	(68,204)	1158.7	2534	82.49	99.26	98.47	954.9	2686	87.98	99.38	98.56	1052.3	3041	89.99	100.00	98.77
35	(70,210)	1164.9	2715	80.98	98.95	98.72	1034.5	2880	86.02	99.98	98.69	1127.2	3052	90.21	99.99	98.97
36	(72,216)	1222.4	2825	84.03	99.58	97.70	1116.5	3002	86.57	99.99	98.39	1245.6	3175	91.13	99.99	98.64
37	(74,222)	1297.8	2830	83.57	99.37	97.95	1185.0	3005	84.96	99.98	98.94	1345.8	3176	88.27	99.99	99.05
38	(76,228)	1342.6	2844	82.67	99.66	97.59	1279.5	3019	86.04	99.99	97.97	1515.5	3121	89.78	99.99	98.53
39	(78,234)	1500.3	2955	83.03	99.56	98.67	1381.6	3132	85.88	99.96	98.85	1504.1	3310	90.16	99.99	99.12
40	(80,240)	1649.5	2978	82.57	99.54	98.26	1489.6	3160	84.95	99.97	98.37	1656.3	3349	89.57	99.99	99.33
Average		**461.9**	**1585.5**	**81.74**	**99.25**	**97.42**	**407.7**	**1682.8**	**84.57**	**99.81**	**97.43**	**435.0**	**1793.8**	**87.32**	**99.99**	**97.94**

high service level requirement, we recommend MIP-DF1 for solutions.

6. Conclusion

This paper investigates an ambulance location problem with ambiguity set of demand, in which the probability distribution of the uncertain demand is unknown. In such a data-driven environment, only the mean and covariance demands at emergency points are known. The problem is to determine the base locations and the number of ambulances at each base, aiming at minimizing the total cost associated with locating bases and assigning ambulances. We propose a distribution-free model with chance constraints. Then two approximated MIP formulations with different approximation methods of chance constraints are proposed, which are based on different ambiguity sets of the unknown probability distribution. Numerical experiments on benchmarks and 120 randomly generated instances are conducted, and the computational results show that the first approximated MIP formulation is much conservative with overhigh system cost, while the second approximated MIP formulation is more cost saving with service level appropriately ensured.

In future research, we may consider the following relevant issues. First, the uncertainty of driving time from a base to an emergency point shall be analysed. Besides, unexpected or sudden disasters may be taken into consideration. Moreover, other uncertainties should be considered, such as travel time, the number of available vehicles and so on. The stochastic dependence should also further considered.

Data Availability

The data of the benchmark instances in our manuscript can be obtained in Nickel et al. [8]. For the test instances, the data is randomly generated and the way for generating data is stated in our manuscript. The datasets generated during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (NSFC) under Grants 71428002, 71531011, 71571134, and 71771048. This work was also supported by the Fundamental Research Funds for the Central Universities.

References

[1] L. Brotcorne, G. Laporte, and F. Semet, "Ambulance location and relocation models," *European Journal of Operational Research*, vol. 147, no. 3, pp. 451–463, 2003.

[2] X. Li, Z. Zhao, X. Zhu, and T. Wyatt, "Covering models and optimization techniques for emergency response facility location and planning: a review," *Mathematical Methods of Operations Research*, vol. 74, no. 3, pp. 281–310, 2011.

[3] C. Toregas, R. Swain, C. ReVelle, and L. Bergman, "The location of emergency service facilities," *Operations Research*, vol. 19, no. 6, pp. 1363–1373, 1971.

[4] R. Church and C. ReVelle, "The maximal covering location problem," *Papers in Regional Science*, vol. 32, no. 1, pp. 101–118, 1974.

[5] M. Gendreau, G. Laporte, and F. Semet, "Solving an ambulance location model by tabu search," *Location Science*, vol. 5, no. 2, pp. 75–88, 1997.

[6] A. Ingolfsson, S. Budge, and E. Erkut, "Optimal ambulance location with random delays and travel times," *Health Care Management Science*, vol. 11, no. 3, pp. 262–274, 2008.

[7] M. Gendreau, G. Laporte, and F. Semet, "The maximal expected coverage relocation problem for emergency vehicles," *Journal of the Operational Research Society*, vol. 57, no. 1, pp. 22–28, 2006.

[8] S. Nickel, M. Reuter-Oppermann, and F. Saldanha-da-Gama, "Ambulance location under stochastic demand: A sampling approach," *Operations Research for Health Care*, vol. 8, pp. 24–32, 2016.

[9] P. Beraldi, M. E. Bruni, and D. Conforti, "Designing robust emergency medical service via stochastic programming," *European Journal of Operational Research*, vol. 158, no. 1, pp. 183–193, 2004.

[10] N. Noyan, "Alternate risk measures for emergency medical service system design," *Annals of Operations Research*, vol. 181, pp. 559–589, 2010.

[11] M. R. Wagner, "Stochastic 0-1 linear programming under limited distributional information," *Operations Research Letters*, vol. 36, no. 2, pp. 150–156, 2008.

[12] E. Delage and Y. Ye, "Distributionally robust optimization under moment uncertainty with application to data-driven problems," *Operations Research*, vol. 58, no. 3, pp. 595–612, 2010.

[13] J. T. Essen, J. L. Hurink, S. Nickel, and M. Reuter, "Models for ambulance planning on the strategic and the tactical level," *Beta Research School for Operations Management and Logistics*, p. 27, 2013.

[14] E. Erkut, A. Ingolfsson, and G. Erdogan, "Ambulance location for maximum survival," *Naval Research Logistics (NRL)*, vol. 55, no. 1, pp. 42–58, 2008.

[15] S. C. Chapman and J. A. White, "Probabilistic formulations of emergency service facilities location problems," in *Proceedings of ORSA/TIMS Conference*, San Juan, Puerto Rico, 1974.

[16] A. Ben-Tal, D. Den Hertog, A. De Waegenaere, B. Melenberg, and G. Rennen, "Robust solutions of optimization problems affected by uncertain probabilities," *Management Science*, vol. 59, no. 2, pp. 341–357, 2013.

[17] M. Ng, "Distribution-free vessel deployment for liner shipping," *European Journal of Operational Research*, vol. 238, no. 3, pp. 858–862, 2014.

[18] M. Ng, "Container vessel fleet deployment for liner shipping with stochastic dependencies in shipping demand," *Transportation Research Part B: Methodological*, vol. 74, pp. 79–87, 2015.

[19] R. Jiang and Y. Guan, "Data-driven chance constrained stochastic program," *Mathematical Programming*, vol. 158, no. 1-2, Ser. A, pp. 291–327, 2016.

[20] C.-M. Lee and S.-L. Hsu, "The effect of advertising on the distribution-free newsboy problem," *International Journal of Production Economics*, vol. 129, no. 1, pp. 217–224, 2011.

[21] K. Kwon and T. Cheong, "A minimax distribution-free procedure for a newsvendor problem with free shipping," *European Journal of Operational Research*, vol. 232, no. 1, pp. 234–240, 2014.

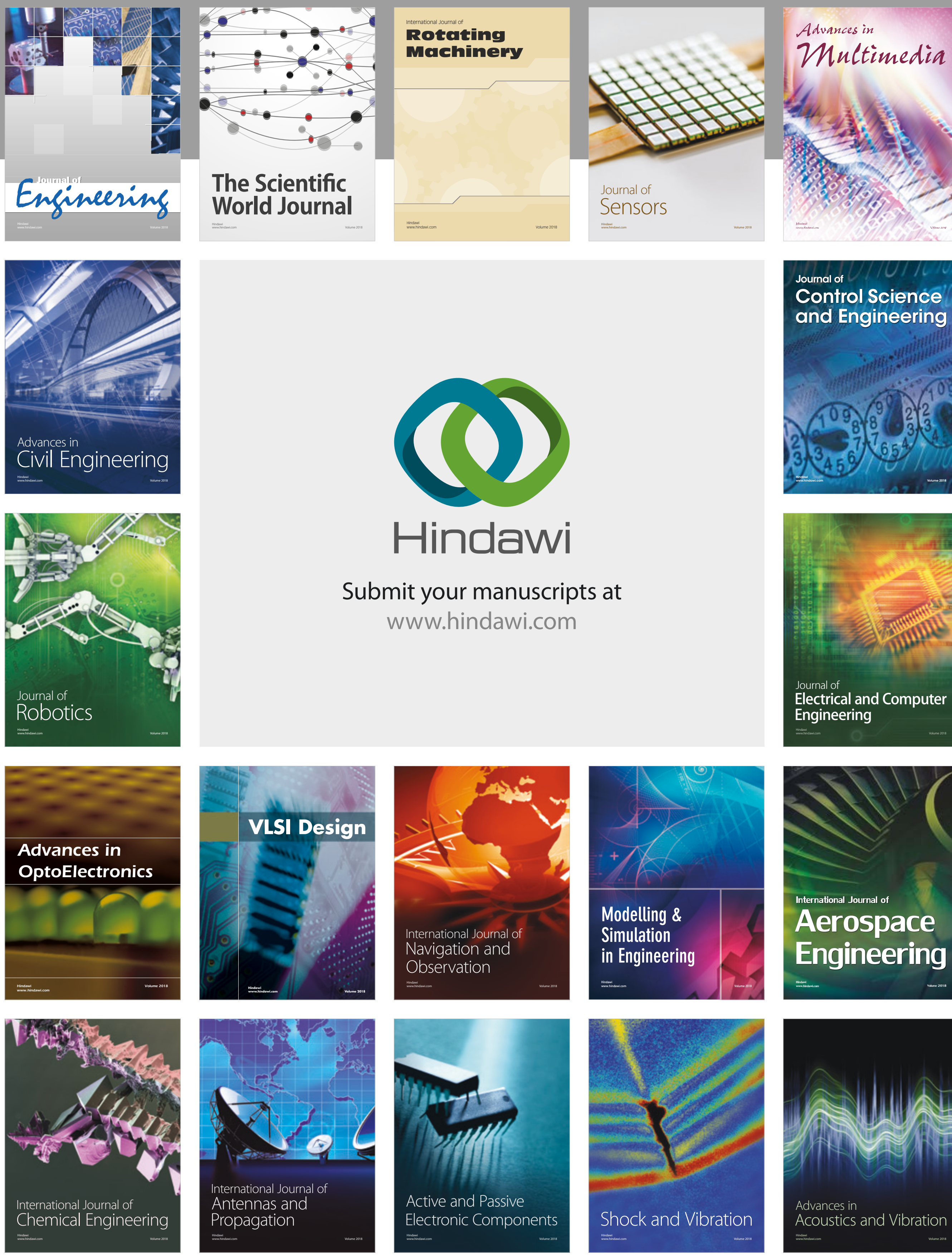
[22] Y. Zhang, S. Shen, and S. A. Erdogan, "Distributionally robust appointment scheduling with moment-based ambiguity set," *Operations Research Letters*, vol. 45, no. 2, pp. 139–144, 2017.

[23] Y. Zhang, S. Shen, and S. A. Erdogan, "Solving 0-1 semidefinite programs for distributionally robust allocation of surgery blocks," *Social Science Electronic Publishing*, 2017, https://ssrn.com/abstract=2997524.

[24] F. Zheng, J. He, F. Chu, and M. Liu, "A new distribution-free model for disassembly line balancing problem with stochastic task processing times," *International Journal of Production Research*, pp. 1–13, 2018.

[25] J. Cheng, E. Delage, and A. Lisser, "Distributionally robust stochastic knapsack problem," *SIAM Journal on Optimization*, vol. 24, no. 3, pp. 1485–1506, 2014.

[26] M. Liu, D. Yang, and F. Hao, "Optimization for the locations of ambulances under two-stage life rescue in the emergency medical service: a case study in Shanghai, China," *Mathematical Problems in Engineering*, vol. 2017, Article ID 1830480, 14 pages, 2017.


Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
Advances in Multimedia
Advances in Civil Engineering
Journal of Control Science and Engineering
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration