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# Dark matter as a Cosmic Bose-Einstein Condensate and possible Superfluid [pre-print]

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# Dark Matter as a Cosmic Bose-Einstein Condensate and Possible Superfluid[1]

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## Abstract

Dark matter arising from spontaneous symmetry breaking of a neutral scalar field coupled to gravity comprises ultra low mass bosons with a Bose-Einstein condensation temperature far above the present background temperature. Assuming galactic halos to consist of a Bose-Einstein condensate of astronomical extent, we calculate the condensate coherence length, transition temperatures, mass distribution, and orbital velocity curves, and deduce the particle mass and number density from the observed rotation curves for the Andromeda and Triangulum galaxies. We also consider the possibility of superfluid behaviour in the halos of rotating galaxies, and estimate the critical angular frequency and line density for formation of quantised vortices.

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## I. Introduction

When, some seventy years after Einstein's prediction of the existence of a Bose-Einstein condensate (BEC) [1], the first BEC gas was prepared from Rb vapour in 1995 [2] , project co-director Eric Cornell remarked: "This state could never have existed naturally anywhere in the universe. So the sample in our lab is the only chunk of this stuff in the universe..." [3] Our investigations [4] of the nature of dark matter have led us to the diametrically opposite conclusion, *viz.* that a BEC may well be the most abundant form of matter in the cosmos—and a viable solution to the problem of "missing mass". A similar conclusion has been reached by Hu et. al. [5].

Observational evidence based on the power spectrum of temperature fluctuations in the cosmic microwave background radiation (CBR) [6] and the red shifts of high-$z$ Type Ia supernovae [7] provide compelling support for a flat universe with total density parameter $\Omega = \Omega_M + \Omega_\Lambda$ close to unity, in accord with the predictions of an inflationary Big Bang cosmology. Separate contributions from matter and cosmological constant amount to $\Omega_M = \frac{\rho}{\rho_c}$ ~ 0.3 and $\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$ ~ 0.7 respectively, where $\rho_c = 3H_0^2 / 8\pi G \sim 7\times10^{-27}$ kg/m$^3$ is the critical mass density for closure of an Einstein-de Sitter universe, $H_0$ ~ 60 km/s per Mpc ($\sim 2\times10^{-18}$ s$^{-1}$) is the present value of the Hubble constant, and $G$ is Newton's constant of gravity. Cosmological nucleosynthesis of light elements limits the baryonic contribution to much less than $\Omega_M$; most recently the extensive 2dF Galaxy Redshift Survey [8] has confirmed that $\Omega_B/\Omega_M$ ~ 0.15 ± .07. An even smaller fraction, $\Omega_{lum}$ ~ 0.01, inferred from galactic rotation curves and the velocity distribution of matter within galactic clusters, is contributed by luminous matter [9]. Thus, the overwhelming preponderance of matter and energy in the universe is believed to be dark, i.e. unobservable by telescopes across the full spectrum of accessible electromagnetic frequencies.

Cold dark matter (CDM) models comprising weakly interacting massive particles (WIMPs) are presently favoured by theorists over hot dark matter (HDM) models comprising relativistic light neutrinos. CDM simulations, however, have led to too sharp mass density profiles within galactic cores, as well as to an overabundance of dwarf satellites [10]. Moreover, recent experimental searches for WIMPs, while not conclusive, have cast doubt on their existence [11].

As an alternative to standard HDM and CDM models, we have proposed [12] that dark matter comprises, at least in part, very low mass scalar bosons for which the critical temperature $T_{cr}$ for transition to the BEC phase and the condensation temperature $T_c \leq T_{cr}$ at the present epoch both lie well above the temperatures at which galaxies formed. Under these circumstances the particles constitute a relativistic gas (HDM) above $T_c$ for a time following the Big Bang short in comparison to the recombination and decoupling times, but condense into, and remain, a nonrelativistic degenerate quantum fluid (CDM) at lower temperatures engendered by cosmic expansion. As a consequence of the quantum uncertainty principle, the particles of a BEC cannot be localised to regions smaller than the condensate coherence length $\xi_c$ which, for particles of sufficiently low mass, corresponds to a size of the scale of the luminous core of galaxies. In this way BEC dark matter within a galactic halo can provide the nonluminous mass needed to keep the galaxy together, yet not give rise to spike-like structures in the core or an excessive number of satellite structures.

In the following section we show how BEC dark matter arises from spontaneous symmetry breaking (SSB) of the reflection symmetry of a Ginzburg-Landau potential and calculate the critical and condensation temperatures for the BEC phase transition. In section 3 we deduce the condensate coherence length from the balance between quantum pressure and gravitational attraction, and show that this length is equivalent to the Jean's scale $\lambda_J$ for the onset of gravitationally unstable mass density perturbations. By application to M31 (Andromeda galaxy) and M33 (Triangulum galaxy) we estimate numerically the mass $m$ of the scalar boson, coherence length $\xi_c$, and temperatures $T_{cr}$ and $T_c$.

In section 4 we solve the nonlinear Schrödinger-Gross-Pitaevskii equation for self-gravitating BEC dark matter to obtain the (approximate) condensate wave function $\psi(r)$, dark mass distribution function $M(r)$, and rotation velocity curve $v(r)$ which we compare with the observed rotation curves for M31 and M33.

In the final section 5 we discuss the possibility of superfluid behaviour of BEC dark matter in rotating galaxies, and estimate the critical angular frequency and resulting line density for formation of quantised vortices.

## 2. Formation of BEC Dark Matter by Spontaneous Symmetry Breaking

The simplest generally covariant Lagrangian density for a self-coupled neutral (and therefore real-valued) scalar field $\phi$ subject to gravity may be written as

$$L_\phi = \tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) \tag{1}$$

in which the gravitational coupling enters through the metric tensor elements of the kinetic energy term, $\partial_\mu\phi\,\partial^\mu\phi = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$, where $g_{\mu\nu}$ is the metric tensor of the general Riemannian manifold $\{x^\mu,\ \mu = 0,1,2,3\}$ with determinant $g$ (whose signature is negative). The complete action takes the form

$$I = \int\sqrt{-g}\left(L_R + L_\phi\right)d^4x \tag{2}$$

in which the gravitational Lagrangian density of general relativity is

$$L_R = \frac{1}{\kappa^2}R \qquad \left(\kappa^2 = \frac{16\pi G}{c^4}\right) \tag{3}$$

with curvature scalar $R = g^{\sigma\rho}R_{\sigma\rho}$ defined in terms of the affine connection $\Gamma^\alpha_{\beta\gamma}$ and its derivatives in the standard way (with Einstein summation convention)

$$R = g^{\sigma\rho}\left[\partial_\rho\Gamma^\alpha_{\sigma\alpha} - \partial_\alpha\Gamma^\alpha_{\sigma\rho} + \Gamma^\beta_{\sigma\alpha}\Gamma^\alpha_{\rho\beta} - \Gamma^\alpha_{\sigma\rho}\Gamma^\beta_{\alpha\beta}\right]. \tag{4}$$

We adopt for $V(\phi)$ the Ginzburg-Landau (G-L) free energy density

$$V(\phi) = \tfrac{1}{2}\left(a\phi^2 + b\phi^4\right) \tag{5}$$

widely employed in the phenomenological treatment of problems exhibiting a second-order phase transition (as, for example, superconductivity and superfluidity, which are manifestations of Bose-Einstein condensation). The quartic interaction parameter $b$ must be positive if $V(\phi)$ is to have a finite minimum (corresponding to the vacuum state in quantum theory), and the quadratic interaction parameter $a$ is positive above $T_{cr}$ and negative below $T_{cr}$. $V(\phi)$ is parabolic in the high-temperature phase with a minimum $V(\phi) = 0$ at $\phi = 0$. In the low-temperature phase (i.e. well below $T_{cr}$ or effectively at $T = 0$ K), $V(\phi)$ has two degenerate potential wells of minimum energy $V(\phi_{\pm}) = \frac{-a^2}{8b}$ at $\phi_{\pm} = \phi_0 e^{i\theta}$ where $\phi_0 = \sqrt{\frac{-a}{2b}}$ and $\theta = 0, \pi$. With the fall in temperature engendered by universal expansion, the global minimum at $\phi = 0$ becomes a local maximum, and gravitationally-induced SSB drives the system randomly into a true global minimum at $+\phi_0$ or $-\phi_0$. The geometry of the phase transition is illustrated in Figure 1. We will examine shortly the temperature dependence of the potential energy function.

Following established procedure [[13]], we express the Lagrangian in Eq. (1) in terms of the excitation

$$\overline{\phi} \equiv \phi - \phi_0 \tag{6}$$

about the asymmetric field and substitute it into the action integral (2) to obtain a total action of the form

$$I = \int \sqrt{-g}\left(L_{R+\Lambda} + L_{\overline{\phi}} + L_I\right) d^4x. \tag{7}$$

in which

$$L_{R+\Lambda} = \frac{1}{\kappa^2}(R + 2\Lambda) \tag{8}$$

is the Lagrangian density of general relativity with cosmological constant,

$$L_{\overline{\phi}} = \tfrac{1}{2}\partial_\mu \overline{\phi}\, \partial^\mu \overline{\phi} - \bar{\lambda}_c^{-2} \overline{\phi}^2 \tag{9}$$

is the Lagrangian density of a free massive scalar field, and

$$L_I = -\frac{\overline{\phi}^3}{\bar{\lambda}_c^2 \phi_0} - \frac{\overline{\phi}^4}{4\bar{\lambda}_c^2 \phi_0^2} \tag{10}$$

is the scalar field self-interaction Lagrangian density.

Thus, as a consequence of SSB two things occur: (a) the scalar bosons (corresponding to low-amplitude oscillations of the classical field about the asymmetric minimum) acquire real mass $m$ given by the reduced Compton wavelength ($\bar{\lambda}_c \equiv \lambda_c / 2\pi$)

$$\bar{\lambda}_c = \frac{\hbar}{mc} = \frac{1}{\sqrt{-2a}}, \tag{11}$$

and (b) spacetime acquires a cosmological constant [14]

$$\Lambda = \frac{\kappa^2 a^2}{16b} = \left(\frac{\kappa\phi_0}{4\lambda_c}\right)^2. \tag{12a}$$

For a universe with $\Omega = 1$ and $\Omega_M \sim 0.3$, $\Lambda$ is calculable from the critical mass density

$$\Lambda = (0.7)\frac{8\pi G\rho_c}{c^2} \sim 1.0\times 10^{-52}\ \mathrm{m}^{-2}. \tag{12b}$$

Variation of the action, $\delta I = 0$, leads to the Einstein gravitational field equations with cosmological constant,

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = -\tfrac{1}{2}\kappa^2 T_{\mu\nu} \tag{13}$$

and energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu\overline{\phi}\partial_\nu\overline{\phi} - \tfrac{1}{2}g_{\mu\nu}\partial_\sigma\overline{\phi}\partial^\sigma\overline{\phi} + \frac{1}{4\lambda_c{}^2}g_{\mu\nu}\overline{\phi}^2 + \frac{\kappa}{4\lambda_c{}^2}g_{\mu\nu}\overline{\phi}^3 + \frac{\kappa^2}{16\lambda_c{}^2}g_{\mu\nu}\overline{\phi}^4 \tag{14}$$

determined by the scalar field and its derivatives. The resulting static field equation of a gravitationally bound spherically symmetric scalar field [15]

$$\frac{d^2\overline{\phi}}{dr^2} + p(r)\frac{d\overline{\phi}}{dr} + \frac{1}{2\lambda_c{}^2}\overline{\phi} + \frac{\kappa}{2\lambda_c{}^2}\overline{\phi}^2 + \frac{\kappa^2}{8\lambda_c{}^2}\overline{\phi}^3 = 0 \tag{15}$$

depends on the (spherically symmetric) metric tensor

$$g_{\mu\nu} = \left(g_{tt}, g_{rr}, g_{\theta\theta}, g_{\varphi\varphi}\right) = \left(e^{\beta(r)}, -e^{-\delta(r)}, -r^2, r^2\sin^2\theta\right) \tag{16a}$$

with functions $p(r)$ and $f(r)$ defined by

$$\sqrt{-g} = f(r)\sin\theta \tag{16b}$$

$$p(r) = \frac{d\ln f(r)}{dr}g^{rr} + \frac{dg^{rr}}{dr}. \tag{16c}$$

To a good approximation, the explicit temperature dependence of the free energy density function (5) can be obtained by calculating the higher-order quantum corrections to the classical potential in the one-loop approximation [16]. This leads to the finite-temperature ($T > T_{cr}$) effective free energy density of the form

$$V_T(\phi) = \tfrac{1}{2}\left(a'\phi^2 + b\phi^4\right) - \frac{\pi^2 (k_B T)^4}{90(\hbar c)} \tag{17}$$

where $k_B$ is Boltzmann's constant and

$$a' = a + \frac{b(k_B T)^2}{2\hbar c} \tag{18a}$$

is the temperature-dependent quadratic parameter; the unprimed $a$ denotes specifically the quadratic parameter at $T = 0$ (which determines the particle mass $m$). The parameter $a'$ vanishes at the critical temperature $T_{cr}$, and the point $\phi = 0$ becomes an inflection, as shown in Figure 1. From Eqs. (11) and (17b) $T_{cr}$ is found to be

$$k_B T_c = \frac{mc^2}{\sqrt{b\hbar c}} = 2\phi_0\sqrt{\hbar c}. \tag{19}$$

Close to critical temperature $a'$ takes the form

$$a' \approx \frac{b k_B{}^2 T_{cr}}{\hbar c}\left(T - T_{cr}\right) \tag{18b}$$

exhibiting explicitly the temperature dependence of a Landau second-order phase transition.

As the ambient temperature continues to fall below $T_{cr}$, an increasing fraction of bosons, now confined within one of the two asymmetric ($\phi \neq 0$) potential minima, drops into the single-particle quantum ground state $\psi$. When the temperature reaches the condensation temperature $T_c$, the Gibbs free energy per particle has approached (from below) sufficiently closely to the energy of the first excited state that the ground-state population comprises virtually the entire macroscopic system of bosons, and a Bose-Einstein condensation is said to have occurred. $T_c$ is the temperature at which, for a given mean particle density $\bar{n}$, evaluation of the partition function as an integral over a density of states, as opposed to a summation over states, fails [17].

At the temperature $T_c$, the mean density of bosons $\bar{n} = \bar{\rho}/m$, where $\bar{\rho}$ is the mean mass density, is effectively one particle in a volume $\lambda_T{}^3$, in which $\lambda_T$ is the thermal de Broglie wavelength. Equating particle kinetic energy $K = \sqrt{p^2c^2 + m^2c^4} - mc^2$ and thermal energy $k_B T$, and expressing momentum $p$ in terms of the thermal de Broglie wavelength $\lambda_T = h / p$ leads to $\lambda_T = h\big/\sqrt{(k_B T / c)^2 + 2k_B T m}$. In the limit $mc^2 / k_B T << 1$, which will be shown to pertain in the present case for $T_c > T > T_{CBR} \sim 2.7$ K, the de Broglie wavelength reduces to $\lambda_T = hc/k_B T$. It then follows that

$$k_B T_c \sim hc\left(\frac{\overline{\rho}}{m}\right)^{1/3}. \tag{20}$$

Well below $T_c$ (but above $T_{CBR}$), the ratio of excited particles to the total number of particles declines as

$$\frac{N_e}{N} \sim \left(\lambda_{Tc}/\lambda_T\right)^3 = \left(T/T_c\right)^3 \tag{21}$$

where $\lambda_{Tc}$ is the de Broglie wavelength at $T_c$. In the following section we will show that the number of excited bosons is utterly negligible by the time galaxies formed after decoupling (red shift $z_d \sim 1100$). The scalar bosons presumed here to constitute dark matter in galactic halos were by then in virtually pure BEC ground states.

The defining nature of a BEC lies in the dual phenomena of broken symmetry and phase coherence [18]. Broken symmetry refers to the fact that when the condensate wave function is nonvanishing, the lowest-energy state (vacuum) must depend on its phase, even though the Lagrangian or Hamiltonian is invariant under a global phase change. Phase coherence refers to the fact that $\phi$ must be spatially correlated throughout the system. In the next section we calculate the coherence length $\xi_c$ which determines (a) the scale over which BEC matter is correlated, (b) the scale above which mass density perturbations become unstable, and (c) the size of the core of a quantised vortex in a rotating BEC superfluid [19].

## 3. Astrophysical Implications

The effects of gravity on the BEC phase follow rigorously from the coupled nonlinear differential equations of motion (13) and (15). We will report elsewhere a detailed account of these equations whose solution is beyond the scope of the present paper. Since the cosmic Bose-Einstein condensate is a nonrelativistic gas with a relatively low mean mass density (that of a galactic halo), it suffices for our present purposes to employ the Newtonian gravitational potential in the following discussion.

In a semiclassical approximation the energy of a quantum particle of mass $m$ gravitationally bound at radial coordinate $\xi$ within a coherent spherical system of isotropic density $\rho$ (that decreases sufficiently fast with distance to be integrable over all space) and total mass $M = 4\pi\int_0^\infty \rho(\xi')\xi'^2 d\xi'$ takes the form

$$E(\xi) \approx \frac{h^2}{2m\xi^2} + m\Phi(\xi) \ , \tag{22}$$

with gravitational potential

$$\Phi(\xi) = -4\pi G\left[\frac{1}{\xi}\int_0^\xi \rho(\xi')\xi'^2\, d\xi' + \int_\xi^\infty \rho(\xi')\xi'\, d\xi'\right]. \tag{23}$$

The kinetic energy term in (22) reflects the quantum uncertainty principle in which a particle confined to a region of spatial extent $\xi$ has a momentum uncertainty $p \sim h/\xi$. Bosons close to the centre have a high kinetic energy. Conversely, bosons sufficiently far from the centre to be outside the bulk of the central mass have a high potential energy. The equilibrium ($dE/d\xi = 0$) between quantum pressure and gravitational attraction leads to a minimum size (the coherence length)

$$\xi_c = \frac{h^2}{GMm^2} = \left(\frac{3h^2}{4\pi Gm^2\overline{\rho}}\right)^{\frac{1}{4}} = \left(\frac{6\lambda_c{}^2}{\Lambda_c}\right)^{\frac{1}{4}} \tag{24}$$

determined by the boson mass and condensate mass irrespective of the radial variation in density. The second equality in (24) expresses $\xi_c$ in terms of the mean density defined by $M = \frac{4\pi}{3}\xi_c{}^3\overline{\rho}$; the third equality expresses $\xi_c$ in terms of the boson Compton wavelength $\lambda_c$ and the condensate density parameter $\Lambda_c \equiv \frac{8\pi G\overline{\rho}}{c^2}$ (which differs from the cosmological constant $\Lambda$ since $\overline{\rho} >> \rho_c$).

According to the standard cosmological scenario, fluctuations in baryon density on sub-horizon-sized scales could begin to grow only after recombination. Prior to recombination, formation of baryonic structure was inhibited by photon pressure (principally through Compton scattering from electrons) in the matter-radiation plasma, which persisted for about 300,000 years following the initial Bang. When the radiation temperature fell to approximately 0.26 eV, sustained hydrogen atom formation could occur and matter decoupled from radiation. Spatial ripples in the gravitational potential at the surface of last scattering—actually, a layer of width $\Delta z_d / z_d \sim 0.1$—became imprinted on the relic radiation as temperature fluctuations whose angular distribution on the sky is related to the scale of mass density fluctuations. As a rough measure, this scale is provided by the Jeans wavelength $\lambda_J \sim \frac{v_s}{\sqrt{G\bar{\rho}}}$, in which $v_s$ is the adiabatic sound velocity in a medium of mean density $\bar{\rho}$ [20]. Density perturbations of wavelength $\lambda < \lambda_J$ oscillate as acoustic modes, but perturbations of wavelength $\lambda > \lambda_J$ are gravitationally unstable and lead to exponential growth or decay.

Since the scalar bosons of our model have no electroweak interactions, they do not participate in Compton scattering and, like WIMPs, would therefore have decoupled from radiation much earlier than baryonic matter. Perturbations in a nearly collisionless component are subject to Landau damping, also known as free streaming. Until the onset of the Jeans instability, collisionless particles can stream out of overdense regions and into underdense regions, thereby smoothing out inhomogeneities. Once a relativistic species decouples from the plasma, it travels in free fall in the expanding universe. However, in marked contrast to neutrinos and photons which always remain relativistic, low mass bosons undergo the BEC phase transition below $T_c$ to form a nonrelativistic self-gravitating degenerate gas. This transition occurs at a temperature well above that of recombination, in which case the matter-radiation plasma would have been suffused with a largely, but not perfectly, homogeneous condensate.

Since the pressure of an ideal BEC gas depends only on temperature [21], the adiabatic compressibility is zero and therefore the sound velocity vanishes. From the preceding discussion, it might seem that $\lambda_J = 0$, and hence density perturbations at *all* wavelength scales in a cosmic BEC would be gravitationally unstable. This is not the case, however, for two reasons. First, BEC dark matter does not constitute a truly ideal condensate, since there is a weak, but nonvanishing, gravitational interaction between individual bosons which must be taken into account in order to determine precisely the spectrum of density perturbations. Second, and of greater relevance to the astrophysical implications of our model, the derivation of the Jeans wavelength, based on the hydrodynamic equations for a classical fluid of noncoherent matter, do not strictly apply to a BEC fluid.

A simple heuristic argument can be given for the existence of a scale $\lambda_J$ separating gravitationally stable and gravitationally unstable modes. The dynamical time scale for gravitational collapse is given by $\tau_{grav} \sim 1/\sqrt{G\bar{\rho}}$. Conversely, the time scale for gas pressure to respond is $\tau_{pres} \sim \lambda/v$, where $\lambda$ is the size of the density fluctuation and—in the case of a classical fluid—$v$ is the sound velocity $v_s$. Setting $\tau_{grav} = \tau_{pres}$ and solving for the wavelength leads to the Jeans scale $\lambda_J \sim \frac{v_s}{\sqrt{G\bar{\rho}}}$. In the case of BEC dark matter, however, the relevant velocity is given by the nonrelativistic de Broglie wavelength of the constituent particles $v = h/m\lambda$ [22]. Substituting this velocity into the expression for $\tau_{pres}$, leads to the quantum Jeans scale $\lambda_Q \sim \left(\frac{h^2}{G\bar{\rho}m^2}\right)^{\frac{1}{4}}$, which, by comparison with Eq. (22), is seen to correspond to within a numerical factor of order unity to the coherence length $\xi_c$. Thus, density perturbations of a size less than $\lambda_Q \sim \xi_c$ are gravitationally *stable* , in accord with our earlier argument based on the quantum uncertainty principle. For fluctuations of a size greater than $\lambda_Q$, BEC dark matter behaves like CDM.

From the rotation curve of M31 and data from the Andromeda Atlas [23], we estimate the mean mass density of the Andromeda halo to be $\bar{\rho} \sim 2.0\times10^{-24}$ kg/m$^3$, which is about 280 times the critical background density $\rho_c$. If the preponderance of this matter is assumed to be dark matter due to scalar bosons with a coherence length of the order of the size of the M31 luminous core, $\xi_c \sim 30$ kpc, it then follows from Eq. (24) that the boson Compton wavelength is $\lambda_c \sim 7$ ly, corresponding to a particle mass $m \sim 2\times10^{-23}$ eV/c$^2$ and mean number density $\bar{n} = \bar{\rho}/m \sim 6\times10^{34}$ m$^{-3}$. From Eqs. (12a) and (12b) the magnitude of the broken-symmetry field is estimated to be $\phi_0 \sim 1.4\times10^{21}$ (eV / m)$^{1/2}$. The critical temperature calculated from Eq. (19) is then $T_{cr} \sim 10^{22}$ K, and from Eq. (20) the condensation temperature in the present epoch is $T_c \sim 2\times10^9$ K, which corresponds to the temperature of primordial nucleosynthesis at about 1 second after the Bang. Thus, the present background temperature of 2.7 K is so far below $T_c$ that, according to Eq. (21), the fraction of excited bosons in the condensate is $\sim10^{-27}$.

In a matter-dominated universe described by a Robertson-Walker metric the temperature $T$ and density $\rho$ vary with cosmic scale factor $R$ and red shift $z$ according to

$$\rho \sim R^{-3} \sim (1+z)^3 \tag{25a}$$

$$T \sim R^{-1} \sim (1+z) \ . \tag{25b}$$

Thus, the equilibrium BEC temperature and condensation temperature remain in the same proportion, and the fraction of bosons in the coherent BEC ground state is virtually 100% at the time of decoupling, $z_d$ ~ 1100, when baryonic structure can begin to form. If the present background density of the universe is taken to be the critical density $\rho_c$ for $\Omega = 1$, then, from (25a), the quantum Jeans scale at the time of decoupling is calculated to be $\lambda_Q(z_d) \sim 4 \times 10^{19}$ m, and the corresponding Jeans mass, $M_J \sim \rho(z_d)\lambda_Q(z_d)^3$, is $\sim 2 \times 10^{11}$ solar masses. Upon decoupling, therefore, gravitationally unstable perturbations in BEC dark matter can give rise to galaxy-sized structures.

## 4. BEC Mass Distribution and Rotation Curve

Since a Bose-Einstein condensate is a uniquely quantum mechanical state, its attributes cannot be calculated from a classical field theory. The quantum mechanical wave function of a spherically symmetric, nonrelativistic self-gravitating condensate, which we write as $\psi(r)$ to distinguish it from the classical field $\phi(r)$, is governed by a nonlinear Schrödinger equation [24]. If the mass density, $\rho = Nm|\psi|^2$ where $M = Nm$ is the total condensate mass, varies slowly with $r$ as indicated by galactic rotation curves, the equation can be reduced approximately to the form of a Gross-Pitaevskii equation [25]

$$\frac{d^2}{dr^2}(r\psi(r)) + \left(\alpha + \beta|r\psi(r)|^2\right)(r\psi(r)) = 0 \tag{26}$$

with exact analytical solution

$$\psi(r) = \frac{\sqrt{\frac{-\alpha}{\beta}}\tanh(\sqrt{\alpha/2}r)}{r} . \tag{27}$$

with $\alpha = \frac{2mE}{\hbar^2}$, $\beta = \frac{8\pi GNm^3}{\hbar^2}$; $N$ is the total number of particles in the condensate, and $E$ is the ground-state energy. The ensuing mass distribution is then

$$M(u) = 4\pi Nm\int_0^r |\psi(r')|^2 dr' = \left(\frac{E}{Gm}\sqrt{\frac{2}{a}}\right)(u - \tanh u) \tag{28}$$

where $u = \sqrt{\alpha/2}\, r$ is a dimensionless measure of radial distance. Eq. (28) reveals a constant density ($M(u) \propto u^3$) within the core ($u << 1$) and a linear variation in mass ($M(u) \propto u$) well outside the core ($u >> 1$), in accordance with astronomical inferences [26]. Substitution of Eq. (28) into the Newtonian expression $v(r) = \sqrt{\frac{GM(r)}{r}}$ for the velocity of matter orbiting a central mass leads to the theoretical BEC dark matter rotation curve

$$v(u) = v_\infty \sqrt{1 - \frac{\tanh u}{u}} \tag{29}$$

in which $v_\infty = \sqrt{4\pi Gm|\alpha/\beta|} = \sqrt{E/m}$ is the velocity at $r = \infty$.

Figures 3a and 3b compare the observed M31 and M33 rotation curves with fits to (29). The resulting expressions, $v_{M31} \sim 249.2\sqrt{1 - \frac{\tanh(0.11r)}{.011r}}$ and $v_{M33} \sim 125.0\sqrt{1 - \frac{\tanh(1.2r)}{1.2r}}$, where $v$ is in km/s and $r$ is in kpc, lead consistently to a boson mass $m \sim 10^{-24} - 10^{-23} eV/c^2$, which is very close to that deduced previously and independently by assuming a coherence length of the size of the luminous core. The better match of theory with M31 than with the dwarf galaxy M33 may be understood as follows. M31 is a large ($M > 10^{11}$ Solar masses) isolated "island universe" like the Milky Way with a presumably spherical halo of dark matter consistent with the assumptions of our model. M33, by contrast, is a smaller galaxy ($M \sim 10^{10}$ Solar masses) for which the outlying distribution of dark matter is probably perturbed by its proximity to M31 and other galactic neighbours. This could account for the gradual rise in the M33 velocity curve.

## 5. Quantised Vortices

If dark matter should consist of low mass scalar bosons, then the preceding arguments lend strong support to the belief that these particles would form a degenerate Bose-Einstein gas of astronomical extent. Because the scalar bosons in our model interact only through gravity, their direct experimental detection would be difficult and require detection schemes quite different from those summarised in recent surveys of dark matter models. One intriguing possibility, however, by which the gravitational presence of degenerate dark matter might be discerned is by its superfluid vorticity. Present understanding of superfluidity is much less complete than the understanding of Bose-Einstein condensation, in part because the former depends on interparticle interactions, whereas the latter takes place ideally among particles whose correlations are governed only by quantum statistics. A BEC need not automatically give rise to superfluidity, but recent studies of the condensates of alkali atom vapours have shown that a rotating rarefied gas of weakly interacting Bose particles does indeed give rise to superfluid vortices [27].

We examine here the interesting possibility of vortex formation in BEC halos of rotating galaxies. As is well known, the bulk of a stationary superfluid, in contrast to a normal fluid, will remain stationary when its container is rotated. However, if a sample of superfluid of size $R$ is rotated at angular frequency $\omega$, then localised vortices can form with circulation quantised in units of $h/m$ provided that the frequency exceeds a critical frequency [28]

$$\omega_{cr} = \frac{h}{2\pi m R^2} \ln\left(\frac{R}{\xi_c}\right). \tag{30}$$

The studies of vortex creation in rotating BEC gases show that, rather than forming one vortex with a circulation of multiple units of $h/m$, the condensates give rise to multiple vortices, symmetrically disposed throughout the sample, each with a circulation of one unit of $h/m$. The theoretical vortex line density for a circulation of $h/m$ is

$$n_v = \frac{2m\omega}{h}, \tag{31}$$

and the total number of vortices in a sample is

$$N = \pi R^2 n_v. \tag{32}$$

The implications of the preceding considerations for M31, for which the radius of the galactic halo is taken to be approximately $R \sim 150$ kpc, are striking. From Eq. (30) the critical frequency is approximately $2\times10^{-19}$ rad/s. The rotational velocity of matter at 150 kpc is approximately 250 km/s, corresponding to a rotation rate of $\omega \sim 5\times10^{-17}$ rad/s which we take to be the vortex angular frequency over the long flat portion of the rotation curve. Since $\omega >> \omega_{cr}$, it would seem that it would actually be difficult to keep such vortices from forming in a dark matter superfluid comprising the Andromeda halo. From Eq. (31) the estimated vortex line density for M31 is about 1 vortex per 208 $kpc^2$, or, from (32), approximately 340 vortices might be expected within the M31 halo.

The rotational motion of superfluid dark matter vortices would not show up as red- and blue-shifted subgalactic regions, since this form of matter, having no electroweak interactions, does not emit or scatter light. Evidence of dark matter vortices, however, could conceivably be sought in rotationally-induced frame-dragging effects manifested through gravitational lensing or variation in polarisation [29] of transmitted light from distant background sources.

## 6. Concluding Remarks

Among nonbaryonic constituents of dark matter that have been suggested in the past, light neutrinos were regarded at first as an attractive candidate, in part because neutrinos are known to exist (although the question of neutrino mass remains open), and in part because a neutrino-based cosmology, such as the Zel'dovich pancake model [30], successfully accounted for the large-scale distribution of clusters and superclusters in sheet-like structures with large voids. The difficulty with neutrinos, however, is that particles of such high velocity would form structures on scales larger than those observed, and that the time for fragmentation into galaxy-sized structures would take an appreciable fraction of the age of the universe. Thus, in contrast to prevailing evidence, galaxies would have formed only recently. In view of these deficiencies, cosmologists turned instead to CDM models with slow, massive, weakly interacting particles.

The implications for galaxy formation of dark matter made up of ultra low mass scalar bosons are quite interesting, for the evolution of structure could conceivably entail features of both HDM and CDM models. In the period preceding condensation light bosons behave as hot dark matter. Like free-streaming photons, the de Broglie wavelength of the particles red shifts with universal expansion, and the particles cool. Unlike neutrinos, however, which are fermions and always remain relativistic, light bosons eventually undergo a phase transition to a Bose-Einstein condensate, forming structures down to the scale of the quantum coherence length. In the condensed phase, the particles may cool further by gravitational interactions amongst themselves and with ordinary baryonic matter. Although the existence of an astronomical BEC has not yet been detected, there is no evidence at present to our knowledge that would rule out its existence.

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radius of curvature of the cosmological background. See Cooperstock, F. L. et al, *Ast. J.* **503** (1998) 61

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## Figures

Figure 1: Finite-temperature effective scalar potential $V_T(\phi)$ as a function of $\phi$ for temperatures above, at, and below critical temperature.

Figure 2: Theoretical radial variation of condensate density and corresponding rotation curve of luminous matter.

Figure 3: Calculated (smooth line) and observed galactic rotation curves for (a) M31 (theoretical parameters $v_\infty$ = 249.2 km/s, $\sqrt{\alpha / 2}$ = 0.11 kpc$^{-1}$) and (b) M33 (theoretical parameters $v_\infty$ = 125.0 km/s, $\sqrt{\alpha / 2}$ = 1.20 kpc$^{-1}$). The dashed line in (b) shows the rotation curve expected for luminous matter.

**(Source of experimental rotation curves and galactic images: E. Corbelli, P. Salucci (2000) Mon. Not. R. Astron. Soc. vol 311, 441.)**

**Figure 1**

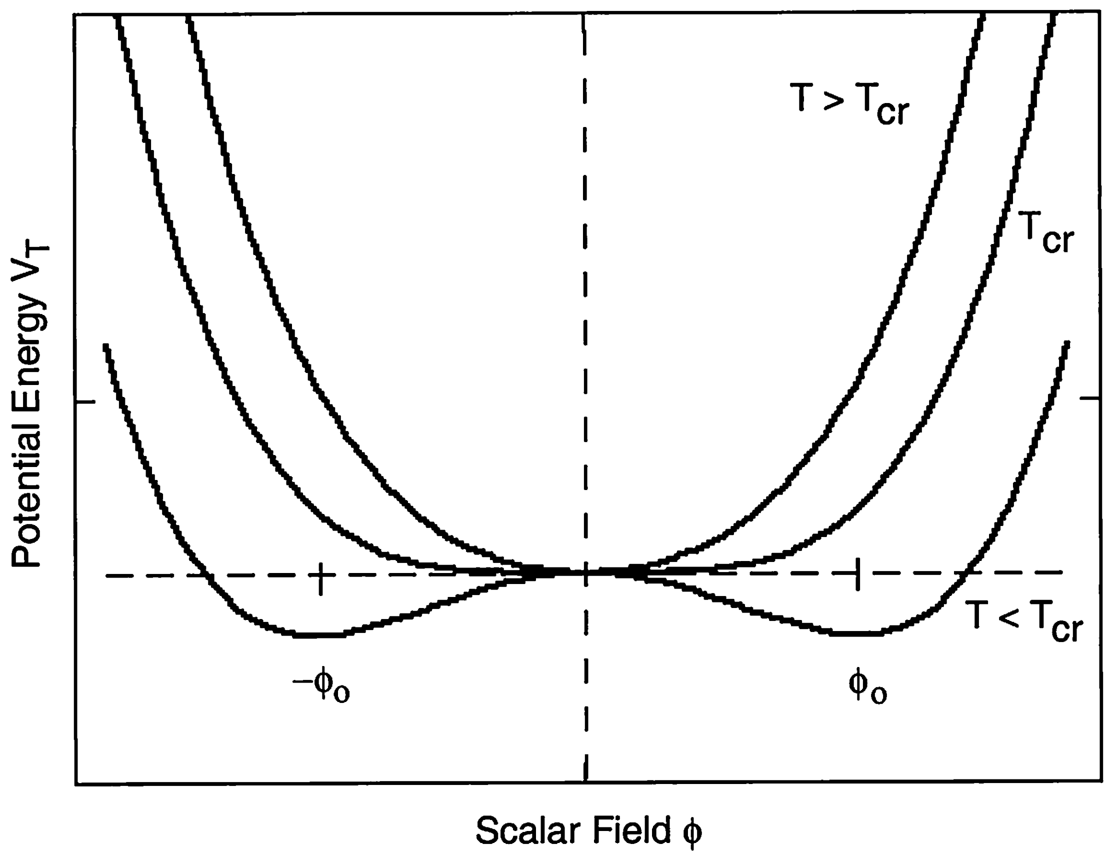


**Figure 2**

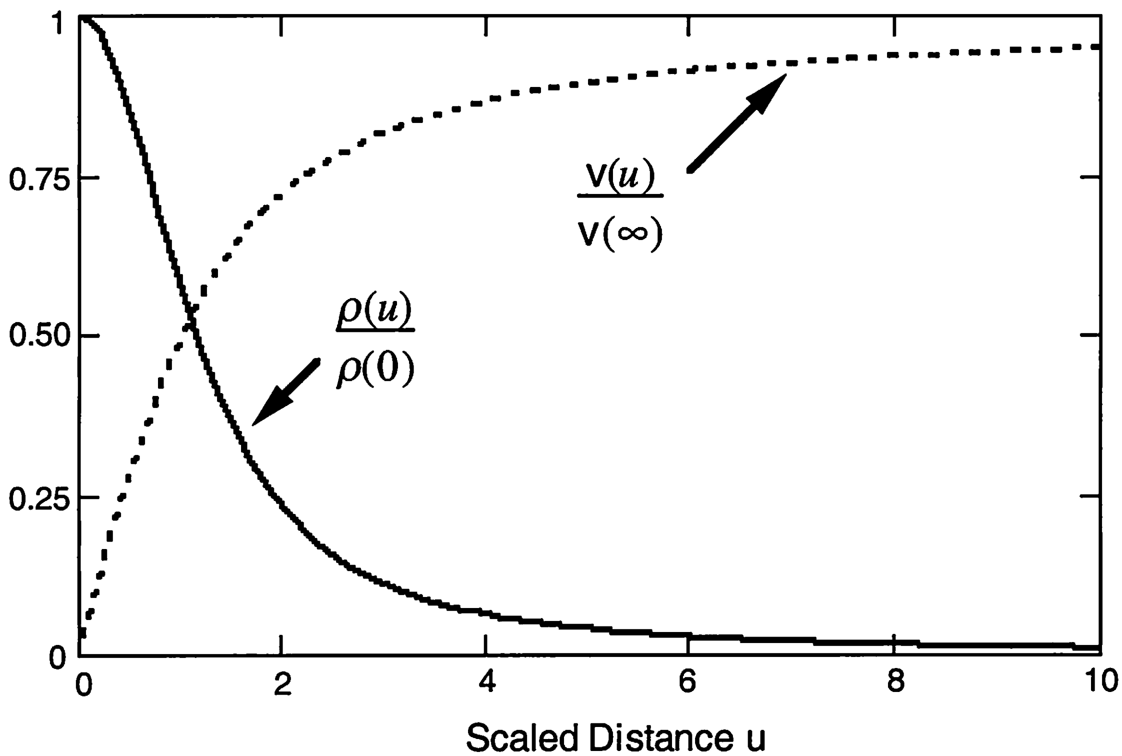


**Figure 3a**

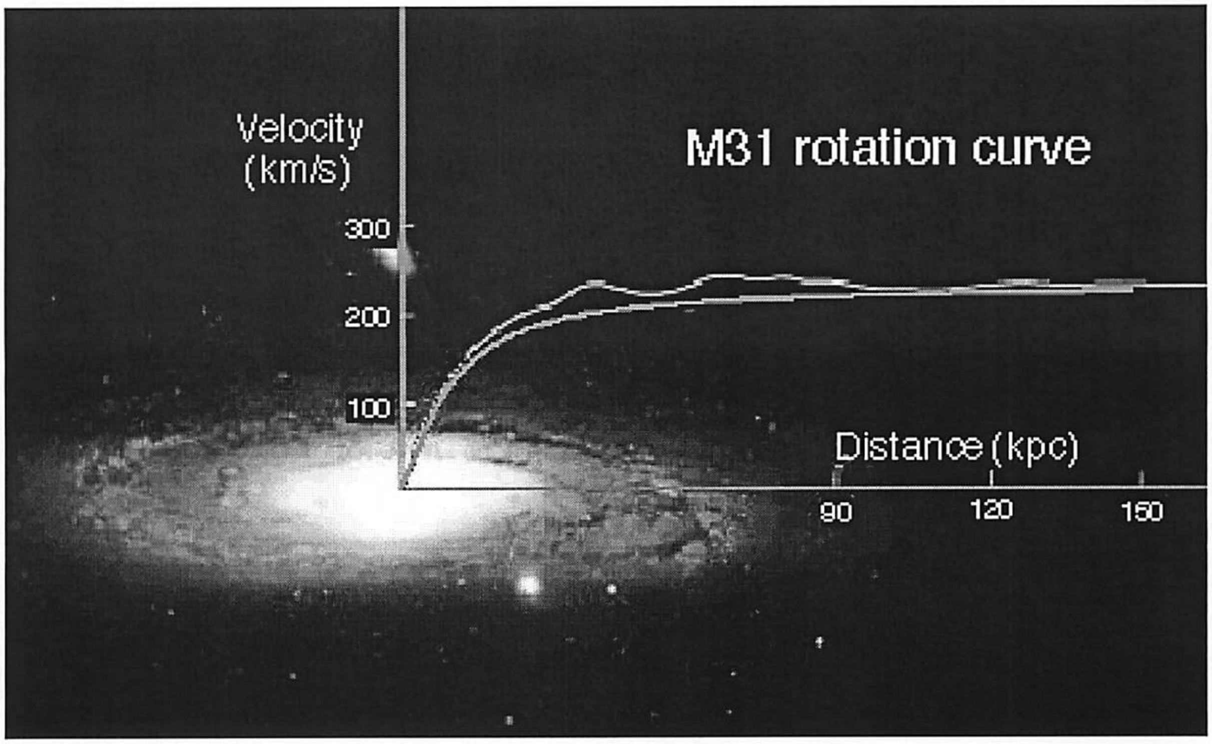
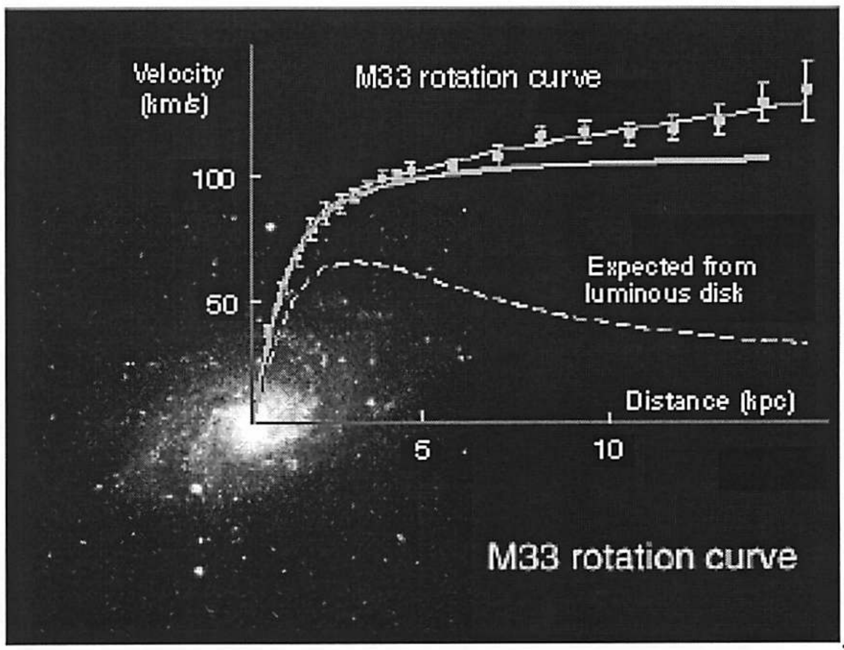

Velocity
(km/s)
M31 rotation curve
300
200
100
Distance (kpc)
90
120
150

**Figure 3b**


Velocity
(km/s)
M33 rotation curve
100
50
Expected from
luminous disk
Distance (kpc)
5
10
M33 rotation curve