2001

Coherent Degenerate Dark Matter: A Galactic Superfluid? [pre-print]

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Published in Class. Quantum Grav. Vol 18 (2001) L103-L108

Coherent Degenerate Dark Matter: A Galactic Superfluid?[1]

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Abstract

Dark matter comprising a Bose-Einstein condensate (BEC) forms structures of the size of its coherence length, as determined by equilibrium between quantum pressure and gravitational attraction. This is also the core size of quantum vortices in a BEC superfluid. From the density and rotation curve of the Andromeda Galaxy (M31) we estimate the particle mass, particle density, coherence length, critical temperature, critical angular frequency, and vortex line density of the dark matter condensate composing the halo.

PACS: 04.20.Cv (General relativity and gravitation); 05.30.Jp (Boson systems) 93.95.+d (Dark matter)

Key Words: Gravity, General Relativity, Symmetry breaking, Bose-Einstein condensate, Dark matter

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[1]The essay upon which this Letter is based received honourable mention from the Gravity Research Foundation for 2001.

Observational evidence based on the power spectrum of temperature fluctuations in the cosmic microwave background (CMB)[1] and the red shifts of high-z Type Ia supernovae[2] provide compelling support for a flat universe with total density parameter $\Omega = \Omega_M + \Omega_\Lambda$ close to unity, in accord with predictions of inflationary cosmology, and separate contributions from matter and cosmological constant amounting to $\Omega_M = \frac{\rho}{\rho_c} \sim 0.3$ and $\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2} \sim 0.7$ respectively[3]. Here the critical mass density for closure of an Einstein-de Sitter universe is $\rho_c = 3H_0^2 / 8\pi G$ in which $H_0 \sim 60$ km/s-Mpc ($\sim 2\times10^{-18}$ s^{-1}) is the present value of the Hubble constant, and G is the universal constant of gravity.

Because of constraints posed by cosmological nucleosynthesis of light elements (principally isotopes of hydrogen, helium, and lithium), only a fraction, $\Omega_B \sim 0.1$, of the mass density presumed to affect such cosmological processes as the rate of universal expansion and the anisotropies in the CMB can be attributed to familiar baryonic matter. Furthermore, studies of galactic rotation curves and the velocity distribution of matter within galactic clusters indicate that a considerably smaller fraction, $\Omega_{lum} < 0.05$, of matter is luminous, i.e. directly observable to detectors over the full range of accessible electromagnetic frequencies[4]. Thus, the overwhelming preponderance of matter in the universe is believed to be dark.

The origin of nonluminous, nonbaryonic matter has been ascribed to a variety of hypothetical entities classified roughly as either cold dark matter (CDM), comprising nonrelativistic weakly interacting massive particles (WIMPs), or hot dark matter (HDM), including relativistic particles like massive neutrinos. HDM tends to free stream out of regions of mass overdensity and is not bound in galaxies; such models are presently not favoured by theorists. By contrast CDM tends to clump on small scales, giving rise to too steep density profiles within galactic cores and to excessive substructure, such as numbers of dwarf satellite galaxies. Recent experimental searches for WIMPs, although by no means conclusive, have cast doubt on their existence.[5]

As an alternative to standard HDM and CDM models, we have proposed[6,7] that dark matter comprises, at least in part, ultra-low mass scalar bosons for which the temperature T_c for a phase transition to a Bose-Einstein condensate (BEC) lies well above the temperatures at which galaxies formed. Under these circumstances the particles constitute a relativistic gas (HDM) above T_c

shortly following the initial singularity and form a nonrelativistic degenerate quantum fluid (CDM) for the ensuing lower temperatures engendered by cosmic expansion.

In this paper we deduce an expression for the coherence length ξ_c of a cosmic BEC as modeled by a Ginzburg-Landau (G-L) potential, and show that it is equivalent to the Jean's scale λ_J for gravitational stability. ξ_c sets the scale of size of the smallest structures formed by the condensate; it also characterises the size of the core of a quantised vortex in a rotating BEC superfluid[8]. By applying our results to the Andromeda Galaxy (M31) whose size and rotation curve are well known, we estimate the coherence length, particle mass, particle density, critical condensation temperature, critical angular frequency, vortex line density, and G-L potential parameters.

The simplest generally covariant Lagrangian density for a self-coupled scalar field ϕ subject only to gravity can be written in the form

$$L_\phi = \tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) \tag{1}$$

in which the gravitational coupling enters through the metric tensor elements in the contraction of the partial derivatives of the kinetic energy term, i.e. through the relation $\partial_\mu\phi\,\partial^\mu\phi = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$. The potential energy function of the scalar field is modeled by the Ginzburg-Landau (G-L) potential

$$V(\phi) = a\phi^2 + b\phi^4 \tag{2}$$

employed in the phenomenological treatment of superconductivity and superfluidity, which are intimately related to the BEC. In Eq. (2) the quartic interaction parameter b must be positive if the potential is to have a finite minimum (corresponding to the vacuum state in a quantum field theory), and the quadratic interaction parameter a is positive above T_c and negative below T_c. In G-L theory $V(\phi)$ is the free energy per volume of the system and ϕ is a real-valued field.

In the high-temperature phase, the minimum of the potential (2) is $V(0) = 0$ at the field ϕ = 0. In the low-temperature phase, gravitationally-induced symmetry breaking causes the system to drop randomly into one of two degenerate potential wells of minimum energy $V(\phi_\pm) = \dfrac{-a^2}{4b}$ at $\phi_\pm = \pm\phi_0 = \phi_0 e^{i\theta}$ where $\phi_0 = \sqrt{\dfrac{-a}{2b}}$ and the phase assumes values $\theta = 0, \pi$. Oscillations about a minimum represent particles of mass m and Compton wavelength λ_c given by

$$\frac{\lambda_c}{2\pi} = \frac{\hbar}{mc} = \frac{1}{\sqrt{-2a}}. \qquad (3)$$

The essence of a BEC lies in the twin phenomena of broken symmetry and phase coherence[9]. Broken symmetry refers to the fact that when the condensate wave function is nonvanishing, the lowest-energy state (vacuum) must depend on its phase, even though the Lagrangian (or Hamiltonian) is invariant under a global phase change. Phase coherence refers to the fact that ϕ must be spatially correlated throughout the system.

As shown in Ref. [7], spontaneous symmetry breaking, besides endowing the scalar bosons with mass, results in a constant energy density term that becomes part of the gravitational Lagrangian

$$L_{R+\Lambda} = \frac{1}{16\pi G/c^4}(R + 2\Lambda) \qquad (4)$$

in which R is the Riemann curvature, and

$$\Lambda = \frac{2\pi G a^2}{bc^4} = \frac{16\pi G}{c^4}\left(\frac{\pi\phi_0}{\lambda_c}\right)^2 \qquad (5)$$

is the cosmological constant. For a universe with $\Omega = 1$ and $\Omega_M \sim 0.3$, Λ is determined by the critical mass density ($\rho_c \sim 7 \times 10^{-27}$ kg/m^3) by

$$\Lambda = (0.7)\frac{8\pi G \rho_c}{c^2} \sim 1.0 \times 10^{-52}\ \text{m}^{-2}. \qquad (6)$$

The preceding application of spontaneous symmetry breaking bears similarity to the Higgs mechanism leading to exponential expansion in inflationary cosmology. A seminal difference, however, is that the Higgs particles in inflationary cosmology are extremely massive (over 100 GeV), whereas the mass of the BEC scalar boson will turn out to be orders of magnitude below even the masses (few eV) postulated for neutrinos. T_c depends on particle mass, and is much lower than the temperature of the CMB for the Higgs bosons responsible for inflation. Higgs bosons, therefore, cannot condense into a BEC phase to form galactic halos.

The exact equations of motion for the gravitational interaction of the scalar bosons can be deduced from the Lagrangian introduced in Ref. [7].[10] Solution of these coupled nonlinear differential equations is beyond the scope of this Letter, however, and will be reported elsewhere.

Nevertheless, since the condensate is a nonrelativistic gas with the comparatively low mass density of a galactic halo, and since the pressure deriving from the contribution of the cosmological constant has negligible effect on the physics of systems small compared to the radius of curvature of the cosmological background[11], it is sufficient to employ the Newtonian gravitational potential to estimate the coherence length.

In a semiclassical approximation the energy of a quantum particle of mass m gravitationally bound at radial coordinate ξ within a coherent spherical system of isotropic density ρ (that decreases sufficiently fast with distance to be integrable over all space) and total mass $M = 4\pi\int_0^\infty \rho(\xi')\xi'^2 d\xi'$ takes the form

$$E(\xi) \approx \frac{h^2}{2m\xi^2} + m\Phi(\xi) \ , \tag{7a}$$

with gravitational potential

$$\Phi(\xi) = -4\pi G\left[\frac{1}{\xi}\int_0^\xi \rho(\xi')\xi'^2 \, d\xi' + \int_\xi^\infty \rho(\xi')\xi' \, d\xi'\right]. \tag{7b}$$

The kinetic energy term in (7a) reflects the quantum uncertainty principle in which a particle confined to a region of spatial extent ξ has a momentum uncertainty $p \sim h / \xi$. Because of the coherence of the condensate wave function, constituent particles cannot be localised to a region smaller than the coherence length, and therefore cannot all condense into a spike-like structure at the galactic centre (as occurs in CDM models). Because of gravitational attraction, the low-mass particles cannot free-stream away (as occurs in HDM models). The equilibrium ($dE / d\xi = 0$) between quantum pressure and gravitational attraction leads to a minimum size (the coherence length)

$$\xi_c = \frac{h^2}{GMm^2} = \left(\frac{3h^2}{4\pi G m^2 \overline{\rho}}\right)^{\frac{1}{4}} \tag{8}$$

determined by the boson mass and condensate mass irrespective of the radial variation in density. The second equality in (8) expresses ξ_c in terms of the mean density defined by $M = \frac{4\pi}{3}\xi_c{}^3\overline{\rho}$.

From the rotation curve of M31 and data from the Andromeda Atlas[12], one can estimate the mean mass density $\overline{\rho} \sim 2.0 \times 10^{-24}$ kg/m^3, which is about 280 times the critical background density ρ_c. If the dark matter in M31 is due primarily to scalar bosons with a coherence length of the order of the size of the M31 luminous core, $\xi_c \sim 30$ kpc, it then follows from Eq. (8) that the boson Compton wavelength is $\lambda_c \sim 7$ ly, corresponding to a particle mass $m \sim 2\times10^{-23}$ eV/c^2 and number density $n = \overline{\rho}/m \sim 6\times10^{34}$ m^{-3}. This is somewhat larger than the mass deduced in Ref. [7] on the basis of a BEC condensate with critical background density, but still orders of magnitude smaller than the masses attributed to neutrinos to account for neutrino oscillation phenomena. From Eqs. (5) and (6) the magnitude of the broken-symmetry field is estimated to be $\phi_0 \sim 1.4\times10^{21}\ (\mathrm{eV/m})^{1/2}$. The transition temperature, derived in Ref. [7], is given approximately by

$$T_c \sim \frac{hcn^{1/3}}{k_B}, \tag{9}$$

in which k_B is Boltzmann's constant. This is the temperature at which the boson density increases to one particle in a volume $\lambda_T{}^3$ where $\lambda_T = hc/k_BT$ is the thermal de Broglie wavelength of a relativistic free particle.[13] From the foregoing value of the number density, Eq. (9) leads to a temperature $T_c \sim 2\times10^9$ K which corresponds to the temperature of the universe at about 1 second after formation according to the big-bang cosmology with inflation.

According to the standard cosmological scenario, fluctuations in baryon density on sub-horizon scales could begin to grow only after recombination. Prior to recombination, formation of baryonic structure was inhibited by photon pressure (principally through Compton scattering from electrons) in the matter-radiation plasma. When the radiation remperature fell to approximately 0.26 eV, sustained H atom formation could occur, and matter decoupled from radiation. Spatial ripples in the gravitational potential at the surface of last scattering became imprinted on the relic radiation as temperature fluctuations whose angular distribution on the sky is related to the scale of mass density fluctuations. As a rough measure, this scale is provided by the Jeans wavelength $\lambda_J \sim \frac{v_s}{\sqrt{G\overline{\rho}}}$, in which v_s is the adiabatic sound velocity in a medium of mean density $\overline{\rho}$ [14]. Density perturbations of wavelength $\lambda < \lambda_J$ oscillate as acoustic modes, but perturbations of wavelength $\lambda > \lambda_J$ are gravitationally unstable and lead to exponential growth or decay.

The derivation of the Jeans wavelength, based on the hydrodynamical equations for a classical fluid of noncoherent matter, does not strictly apply to a BEC fluid (for which the adiabatic sound velocity, in the case of an ideal BEC, vanishes). In this case, the relevant velocity is determined by the nonrelativistic de Broglie wavelength $\lambda = h/mv$ of the constituent particles. Upon identifying the Jean's scale with the de Broglie wavelength and substituting $v = h/m\lambda$ into the expression above for λ_J leads to the quantum Jeans scale $\lambda_J \sim \left(\frac{h^2}{G\bar{\rho}m^2} \right)^{\frac{1}{4}}$, which corresponds to within a numerical factor of order unity to the coherence length ξ_c of Eq. (8). Thus, density perturbations of a size less than ξ_c are gravitationally *stable* , in accord with our earlier argument based on the quantum uncertainty principle.

For fluctuations of a size much greater than ξ_c BEC dark matter behaves like CDM. Our model, therefore, is compatible with current observations of the angular distribution of CMB temperature fluctuations produced by density perturbations with wavelengths close to, or larger than, horizon size at the time of decoupling. The differences between the predictions of the proposed model (degenerate quantum gas of low-mass bosons) and traditional CDM models (incoherent gas of very massive WIMPs) lies in small-scale features of CMB fluctuations corresponding to density perturbations at decoupling $\leq \xi_c$. Present CMB detectors, which have only recently revealed the second and third peaks in the angular CMB power fluctuation spectrum[15], are not sensitive to perturbations of such small scale. To what extent, if any, such small-scale density perturbations affect CMB fluctuations at the smallest angular scales is not possible to say without detailed calculations. In general, however, Silk drag strongly damps oscillations whose wavelengths are small compared to the scale of the particle horizon at decoupling, and one might expect that any imprint on the CMB due to the quantum coherence of cold dark matter would be weak.

If dark matter should consist of low mass scalar bosons, then the preceding arguments lend support to the belief that these particles would form a degenerate Bose-Einstein gas of astronomical extent. Because the scalar bosons in our model interact only through gravity, their direct experimental detection would be difficult and require detection schemes quite different from those summarised in recent surveys of dark matter models with weak nuclear interactions. One intriguing possibility, however, by which the gravitational presence of degenerate dark matter might be discerned is by its superfluid vorticity.

The present understanding of superfluidity is much less complete than the understanding of Bose-Einstein condensation. A BEC need not automatically give rise to superfluidity, but recent studies of condensates of Rb and other metal vapours have shown that a rotating rarefied gas of weakly interacting Bose particles does give rise to superfluid vortices[16].

We examine here the interesting possibility of vortex formation in BEC halos of rotating galaxies. As is well known, the bulk of a stationary superfluid, in contrast to a normal fluid, will remain stationary when its container is rotated. However, if a sample of superfluid of size R is rotated at angular frequency ω, then localised vortices can form with circulation quantised in units of h/m provided that the frequency exceeds the critical frequency [17]

$$\omega_{cr} = \frac{h}{2\pi m R^2} \ln\left(\frac{R}{\xi_c}\right). \tag{10}$$

The studies of vortex creation in rotating BEC gases show that, rather than forming one vortex with a circulation of multiple units of h/m, the condensates give rise to multiple vortices, symmetrically disposed throughout the sample, with a circulation of one unit of h/m. The theoretical vortex line density for a circulation of one h/m is

$$n_v = \frac{2m\omega}{h}\ , \tag{11}$$

and the total number of vortices in a sample is

$$N = \pi R^2 n_v. \tag{12}$$

The implications of the preceding considerations for M31, for which the radius of the galactic halo is taken to be approximately $R \sim 150$ kpc, are striking. From Eq. (10) the critical frequency is approximately 2×10^{-19} rad/s. The rotational velocity of matter at 150 kpc is approximately 250 km/s, corresponding to a rotation rate of $\omega \sim 5 \times 10^{-17}$ rad/s which we take to be the vortex angular frequency over the long flat portion of the rotation curve. Since $\omega >> \omega_{cr}$, it would seem that it would actually be difficult to keep such vortices from forming in a dark matter superfluid. From Eq. (11) the estimated vortex line density for M31 is about 1 vortex per 208 kpc^2, or, from (12), approximately 340 vortices might be expected within the M31 halo.

The rotational motion of superfluid dark matter vortices would not show up as red- and blue-shifted subgalactic regions, since this form of matter, having no electroweak interactions, does not emit or scatter light. Evidence of dark matter vortices, however, could conceivably be sought in rotationally-induced frame-dragging effects manifested through gravitational lensing or variation in polarisation[18] of transmitted light from distant background sources.

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