# Differential Equations in Stock Prediction Analysis

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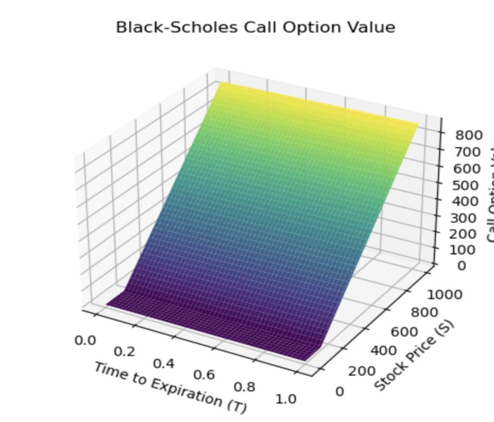



## Abstract

Stock price prediction plays a vital role in financial decision-making and has been an area of extensive research. In this research, we explore the effectiveness of differential equation of Brownian motion as a method for stock price prediction and compare its performance with two established techniques, ARIMA and XGBoost. Using historical data from the Yahoo Finance, we assess the predictive capabilities of these models and analyze their strengths and weaknesses. The findings of this study will shed light on the potential of Brownian motion as a viable approach in financial forecasting and provide valuable insights for investors and researchers in applying mathematics in social sciences. We also researched the application of this technique in option pricing and combine this with more complicated mathematical models.

## Introduction

Why stock price prediction?

**Inform Investors and Traders**
Investment strategies, risk management, and portfolio optimization

**Profit Maximizing**
identifying undervalued stocks and predicting potential price movements

**Risk Mitigation**
Provide insights into potential price declines and market downturns

**Brownian Motion Model**
It is a stochastic process that describes the continuous-time evolution of a variable, such as a stock price, over time.

**Model Configuration**

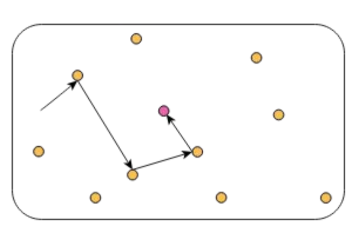
- The differential stochastic equation for the stock price in Brownian Motion contains the following parameters
  - μ: the mean of continuous compound percentage change in closed price during the time interval (drift term)
  - σ: the degree of variation or dispersion of the stock's returns from its average (volatility)
  - S(t): the stock price at time t.
- We calculate the drift term by the following formula:

$$\mu = \frac{\sum_{i=1}^{T} ln(\frac{S_t}{S_{t-1}})_i}{T}$$

- The differential stochastic equation for the stock price in Brownian motion is described below:

$$dS = \mu \cdot S \cdot dt + \sigma \cdot S \cdot dW$$

- The solution for the Brownian Motion Model is:



$$S(t) = S(t-1) \cdot e^{\left((\mu - \frac{\sigma^2}{2}) \cdot dt + \sigma \cdot \epsilon \cdot \sqrt{dt}\right)}$$

**Model Advantage**

➢ One of the key advantages of using GBM for stock prediction is its ability to generate a wide range of potential future paths for the stock price. By simulating multiple scenarios.

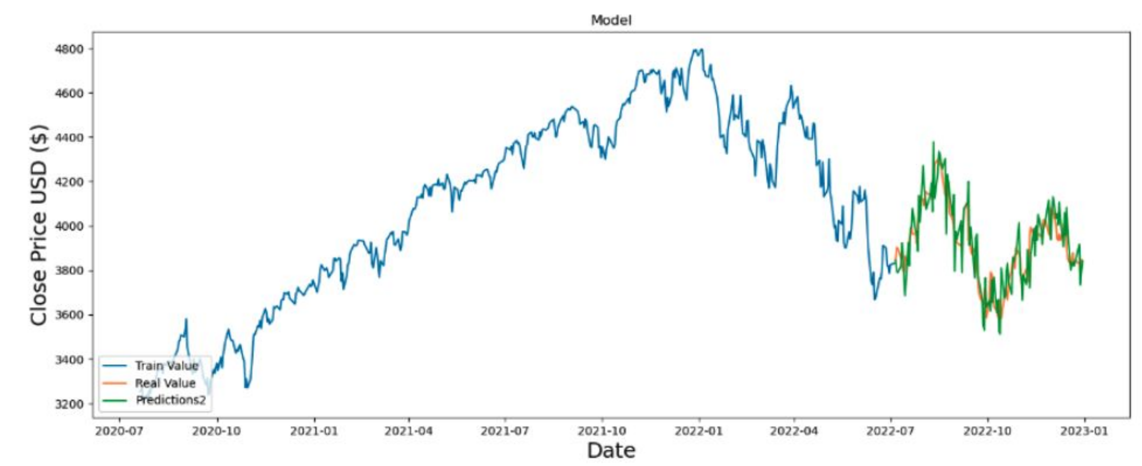
➢ Analysts can assess the probability distribution of future stock price movements and estimate the associated risks.

**Model Disadvantage**

➢ GBM is a simplified model and has limitations.

➢ Real financial markets can be influenced by various factors beyond GBM's assumptions, such as market sentiment, news events, and external shocks. Therefore, while GBM provides a useful framework for stock prediction, it should be used in conjunction with other models and analyses to make well-informed investment decisions.

## Methods of Stock Prediction




**Model Implementation**

- The Wiener process is simulated using the inverse transform method, with random samples drawn from the standard normal distribution.
- The diffusion factor, indicative of stochastic behavior, is computed as the product of volatility and random normal samples.

**Metrics Formulas**

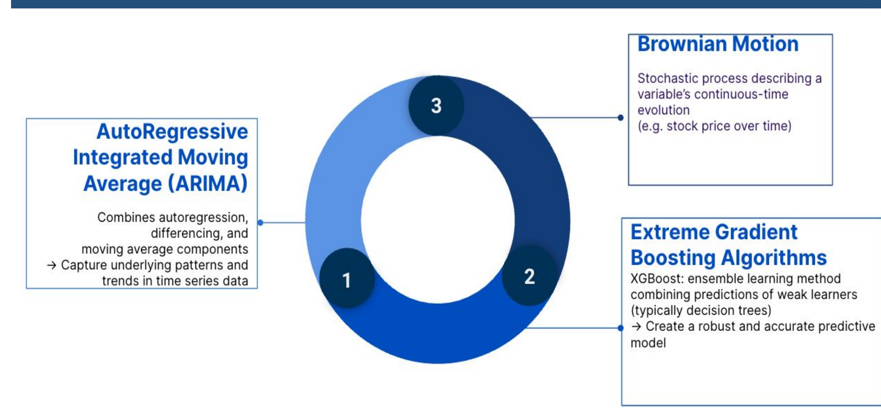
$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}$$

**Evaluation Metrics**

- We use **Root Mean Square Error (RMSE)** and **Mean Absolute Percentage Error (MAPE)** to evaluate model performance.
- RMSE measures the **average difference** between the predicted values and the actual values
- MAPE measures the **percentage difference** between the predicted values and the actual (observed) values

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100$$

## Comparison

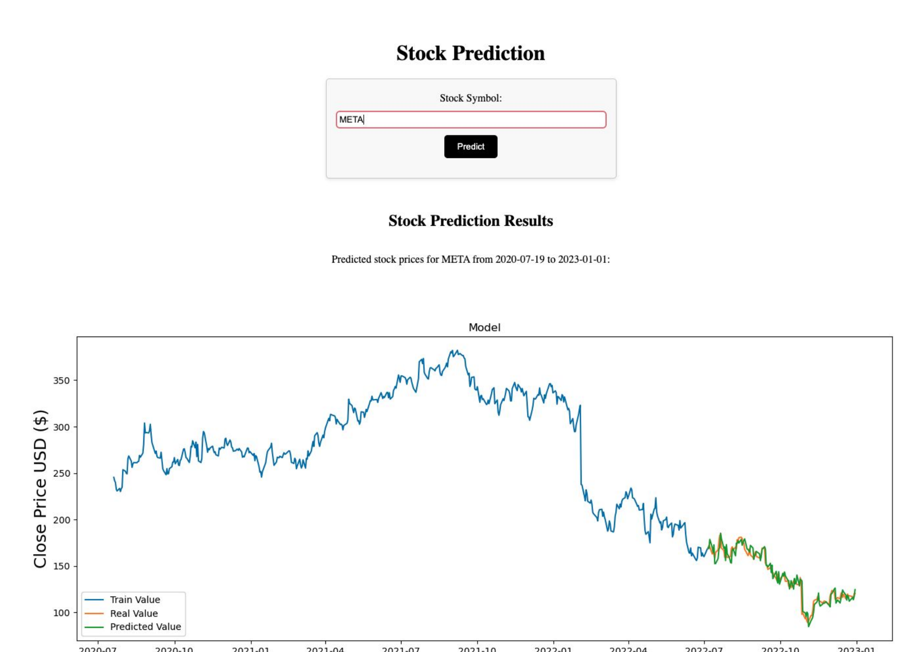



| Algorithms | RMSE | MAPE (%) |
|---|---|---|
| ARIMA | 166.09 | 3.35 |
| XGBoost | 64.75 | 1.33 |
| Brownian Motion | 63.42 | 1.23 |

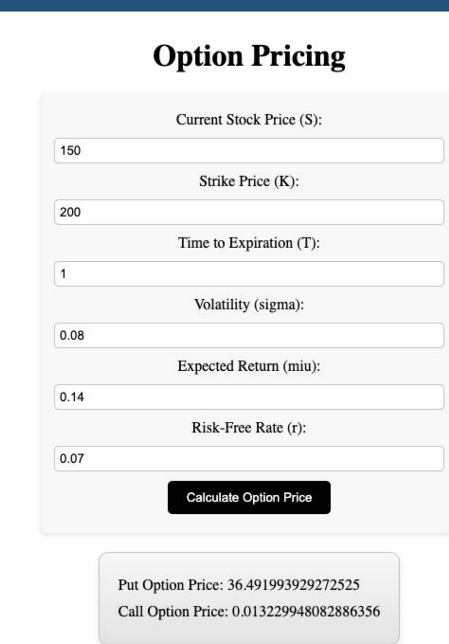
Table 1: RMSE and MAPE Metrics

- We employed the XGBoost Regressor with 100 estimators as our chosen model
- We generated 30 lagged variables for the stock price to capture historical patterns and dependencies in the time series data.
- We trained and tested our model on S&P 500 index closing prices.
- Brownian Motion emerged as **the most accurate method**.
- The superior performance of Brownian Motion suggests its efficacy in modeling the stochastic nature of stock price movements, making it a promising approach for stock prediction.

## Application Overview




Stock Prediction




Option Pricing

## BLACK SCHOLES PDE

*Black-Scholes PDE:* $$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} = rV$$

*Dynamics of an option's value over time in a financial market V(t) is explained by the option's value with respect to changes in time, changes in price and price convexity*

**Analytical solution**
Call *C(t)* and Put *P(t)* option are defined as

$$C(t) = SN(d_1) - Ke^{-rT}N(d_2))$$
$$P(t) = -SN(-d_1) + Ke^{-rT}N(-d_2)$$

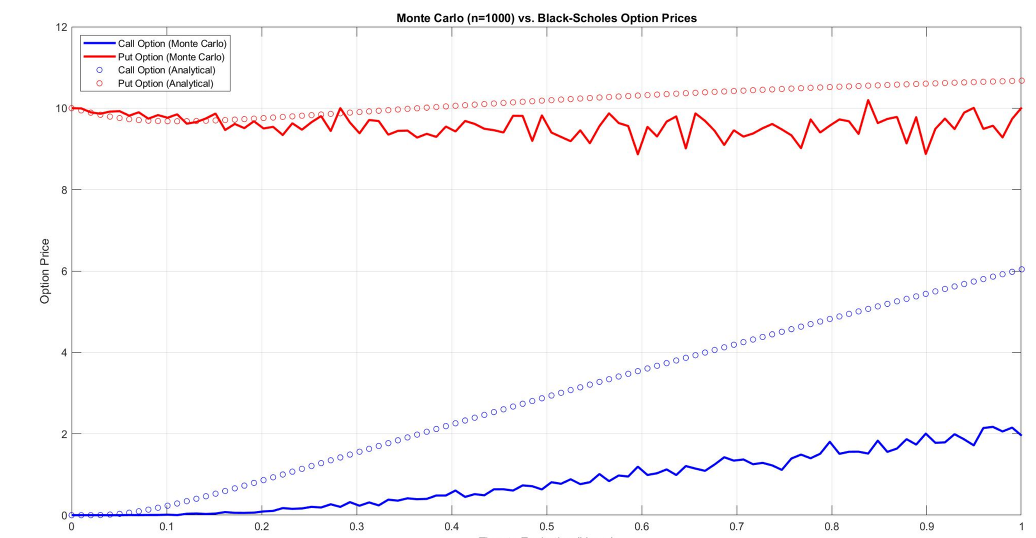
where *N(d)* is the cumulative normal distribution of the function d, with

$$d_1 = \frac{ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = \frac{ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}.$$

S: Underlying stock price
K: Strike price
T: Time to expiration
r: Risk-free interest rate
σ: volatility

**Numerical Approximation**

As the number of iterations increases, the approximation converges.




➢ Monte Carlo: By using random sampling and statistical techniques, it can approximate solutions for a wide range of mathematical and physical systems without requiring explicit formulae or grid-based discretization; slow convergence rate O(√n).

➢ Finite Difference (explicit scheme): Requires grid-based discretization; fast convergence rate O(Δt) and O($\Delta S^2$).

| Stock Price | BS Call | Finite Call | Finite Error |
|---|---|---|---|
| 50 | 1.623738 | 1.620865 | 0.002872 |
| 50.25 | 1.696605 | 1.696837 | 0.002322 |
| 50.5 | 1.771619 | 1.772808 | 0.001189 |

Table 2: European Call option pricing, where X=50, r = 5%, T = 1, $\sigma$ = 0.2

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