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Probabilistic Approach to Buckling Analysis of Thin Panels Subject to Combined Loads

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This paper gives the procedure for the reliability calculation of mechanical components and structures. The stability (buckling) of a tin panel subjected to in-plane combined (normal and shearing) loads is considered in particular. The attention is focused on the buckling analysis of the typical thin-walled aircraft structure such as the skin of wings or fuselage. In this analysis the variables are treated as stochastic: loads, thickness and width of the panel. The numerical example illustrates the procedure. The probability of buckling is calculated for the wing skin of a light trainer aircraft. The effect of the panel thickness (t) and the load factor (j) on the buckling probability is shown. The dimensional tolerance levels affect the critical stress calculation as well as the buckling probability. The variation of the parameters (especially the thickness effect on the buckling) is facilitated by a software program for reliability calculation.

Key words: **structure load, panel, panel structure, skin, fuselage, aircraaft wing, structure stability, stability analysis, stochastic variable, probabilistic analysis.**

Introduction

FOR many years, a significant amount of research has been directed towards experimental modelling of thin-walled plates and shells, as well as towards the development of analytical and numerical methods to improve their design against buckling. The sensitivity of buckling load to structural parameters and imperfections has long been investigated in structural design, assessing that probabilistic considerations are unavoidable when stability problems are of concern. In this context, the reliability theory [11] is a powerful tool for the rational treatment of uncertainties and for the evaluation of structural safety with respect to the buckling limit state.

This paper presents the methodologies for the probabilistic buckling analysis and the reliability assessment of such structural components and examines the link between probabilistic and deterministic studies.

Structural stability is a significant part of the analyses of elements as well as of complete structures. The large bars, panels and tubes are sensitive to the axial compressing loads. Usually, the loads are combined causing normal and shearing stresses. Increasing the loads above the critical stress level produces plastic material deformations, or buckling. For the mass saving (lower thicknesses of the panel), the buckling is allowed below the maximum load factor ($j_b<j=1$). The designing requirements prescribe the buckling load factor (j_b) and the level of buckling deformation. For calculation and structural optimization, it is useful to have a practical procedure for the variation of parameters and their effect on reliability.

During the last 30 years, significant development took place in probabilistic modelling and the structural reliability theory. Such methods were first used in the early 1980s to illustrate how new designs could be compared in a reliability framework and how the assessment of existing structures could benefit by introducing time-dependent performance criteria, also based on probabilistic concepts. These early studies invariably concentrated on introducing the new methodologies, but were somewhat lacking in terms of the mathematical models used for strength prediction. However, as probabilistic modelling was progressively made more detailed and specific to thin-walled structures, the quantitative treatment of uncertain parameters which have an important influence on buckling response became an important issue in reliability studies. Thus, strength formulations with explicit dependence on manufacturing parameters were required for reliability purposes in order to allow random variability in imperfections and residual stresses to be taken into account. The aim of this paper is to introduce the application of probabilistic methods and the reliability analysis for thin-walled plates and shells, with emphasis on how decisions on codified rules and on design selection have been aided by the introduction of these methods. A set of design equations and its application for the assessment of buckling strength of ring and stringer stiffened shells related to marine structures are described in Ref [8]. A method for reliability assessment of the post-buckling compressive strength of laminated composite plates and stiffened panels under axial compression is presented in paper [9].

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An efficient computation procedure for the reliability analysis of frame structures with respect to the buckling limit state is proposed under the assumption that no imperfections are present and that the elastic parameters are uncertain and modeled as random variables [10].

The paper [12] gives the probabilistic procedure for the analysis of thin panel buckling under single normal stress. The panel geometry (thickness t and wide b) were taken as stochastic variable. In this paper the more general case is considered as typical wing or fuselage tin panel under combined loads (axial and shearing forces).

In this paper, the procedure for a probabilistic approach to the buckling analysis is presented. A typical wing or fuselage thin panel under combined loads (axial and shearing forces) is considered. A software program for buckling calculation is developed. The procedure is illustrated by a numerical example. The buckling of thin wing panel under normal and shearing stresses is calculated.

Structural stability analyses

In one of the earliest probabilistic studies specifically aimed at plate buckling, Ivanov and Rousev [6], by using a strength formula with explicit dependence on imperfection amplitude, developed the expressions for the statistical parameters of the allowable axial load, in a manner similar to Bolotin [7].

A thin skin panel, Fig.1, compressed by increased combined load (normal and shearing) begins to lose stability after the critical stress level is surpassed.

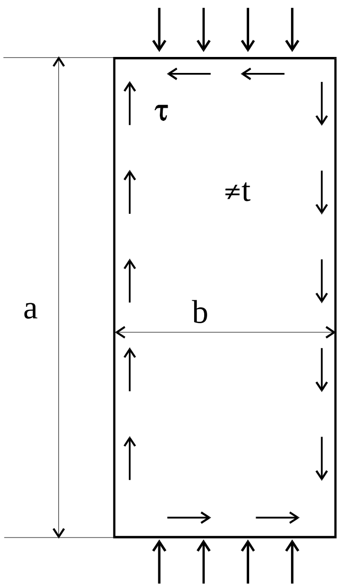

Figure 1. Compressed loads applied to the skin area

Within the deterministic calculation of aircraft skin stability, the safety factor is determined by [13]:

$$RF = \frac{2}{R_L + \sqrt{R_L^2 + 4 \cdot R_F^2}} \qquad (1)$$

where

$$R_L = \frac{\sigma}{\sigma_{CR}}$$

$$R_F = \frac{\tau}{\tau_{CR}}$$

The critical normal and shearing stresses are determined by

$$\sigma_{CR} = k \cdot E \cdot \left(\frac{t}{b}\right)^2$$

$$\tau_{CR} = \frac{k_S \cdot \pi^2 \cdot E}{12 \cdot \left(1 - \mu^2\right)} \cdot \left(\frac{t}{b}\right)^2$$

Where RF - safety factor, σ, τ - load stress, normal and shearing, respectively; σ_{CR}, τ_{CR} - critical stress level (normal and shearing); k, k_S - constants; E - elasticity module; t, b - geometric values: thickness and width, respectively; μ - elastic Poisson’s ratio.

Eq.(1) represents the parabola

$$\left(RF \cdot R_F\right)^2 + \left(RF \cdot R_L\right) = 1 \qquad (2)$$

For $RF = 1$ the condition is satisfied and the instability of the skin begins. The relationship between normal and shear stresses is

$$\left(\frac{\tau}{\tau_{CR}}\right)^2 + \left(\frac{\sigma}{\sigma_{CR}}\right) = 1 \qquad (3)$$

For a probabilistic approach, the stochastic variables have to be considered:

- geometry - $\left(\bar{t}, s_t\right), \left(\bar{b}, s_b\right)$,
- load - $\left(\bar{\sigma}, s_\sigma\right), \left(\bar{\tau}, s_\tau\right)$,
- characteristics of material - $\left(\bar{E}, s_E\right)$.

Taking t as the only geometry stochastic variable gives

$$\overline{\left(t^2\right)} = \bar{t}^2 + s_t^2 \qquad (4)$$

$$s_{t^2} = \sqrt{4 \cdot \bar{t}^2 \cdot s_t^2 + 2 \cdot s_t^4}$$

and the normal stresses

$$\overline{\sigma_{CR}} \approx \frac{k \cdot E}{b^2} \cdot \overline{\left(t^2\right)}, \qquad s_{\sigma CR} \approx \frac{k \cdot E}{b^2} \cdot s_{t^2} \qquad (5)$$

For simplification, the shearing stresses might be approximated by the dominant thickness random variable

$$\left(\overline{\tau_{CR}}, s_{\tau CR}\right) = k_\tau \cdot \left(\overline{t^2}, s_{t^2}\right) \qquad (6)$$

where the constant is

$$k_\tau = \frac{k_S \cdot \pi^2 \cdot E}{12 \cdot (1 - \mu) \cdot b^2} \qquad (7)$$

From parabola (2), it is possible to determine the mean values and the standard deviation, as

$$\left(\frac{\bar{\tau}}{\overline{\tau_{CR}}}\right)^2 + \left(\frac{\bar{\sigma}}{\overline{\sigma_{CR}}}\right) = 1 \qquad (8)$$

$$\left(\frac{\bar{\tau} + s_\tau}{\overline{\tau_{CR}} \pm s_{\tau CR}}\right)^2 + \left(\frac{\bar{\sigma} + s_\sigma}{\overline{\sigma_{CR}} \pm s_{\sigma CR}}\right) = 1 \qquad (9)$$

So, the actual critical value of the combined normal and shearing stresses, when the undulation starts to be created, is

$$\overline{\Theta_{CR}} = \sqrt{\overline{\sigma}^2 + \overline{\tau}^2} \tag{10}$$

$$s_{\Theta CR} = \left(\frac{\overline{\sigma}^2 \cdot s_\sigma^2 + \overline{\tau}^2 \cdot s_\tau^2}{\overline{\sigma}^2 + \overline{\tau}^2} \right)^{1/2} \tag{11}$$

The ratio between normal and shearing stresses determines the angle of undulation, Fig.2.

$$tg(\alpha) = r = \frac{\overline{\sigma_L}}{\overline{\tau_L}} \tag{12}$$

The load stress is represented by

$$\overline{\Theta_L} = \sqrt{\overline{\sigma_L}^2 + \overline{\tau_L}^2} \tag{13}$$

$$s_{\Theta L} = \left(\frac{\overline{\sigma_L}^2 \cdot s_{\sigma L}^2 + \overline{\tau_L}^2 \cdot s_{\tau L}^2}{\overline{\sigma_L}^2 + \overline{\tau_L}^2} \right)^{1/2} \tag{14}$$

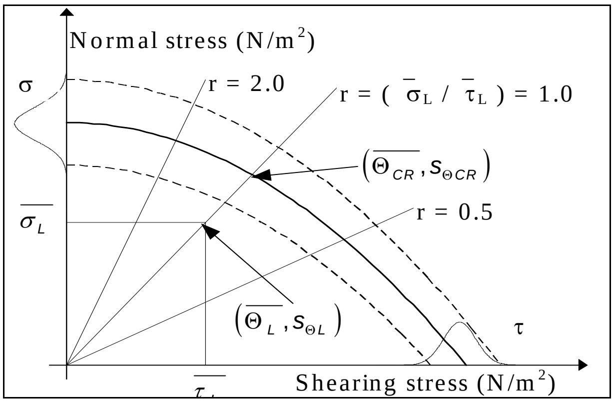

Figure 2. Representation of normal and shearing stresses

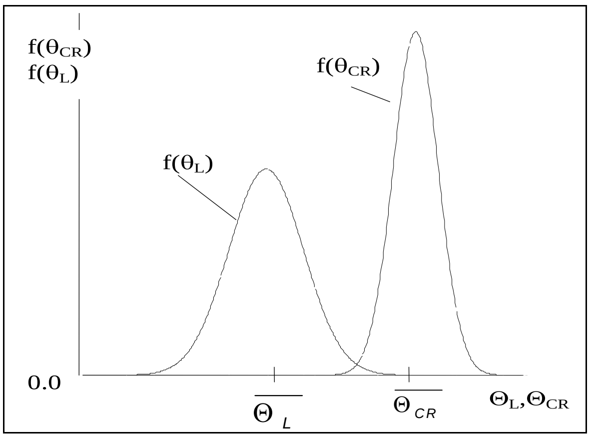

Figure 3. Distributions of load stresses $\left(\overline{\Theta_L}, s_{\Theta L}\right)$ and critical buckling stresses $\left(\overline{\Theta_{CR}}, s_{\Theta CR}\right)$

The probability of buckling is determined from the distributions of load stresses $\left(\overline{\Theta_L}, s_{\Theta L}\right)$ and critical buckling stresses $\left(\overline{\Theta_{CR}}, s_{\Theta CR}\right)$, Fig.3 [14]. In a general form, the probability of buckling is determined from the expression

$$P_b = P(\Theta_L \geq \Theta_{CR}) = 1 - R = \\ = 1 - \int_{-\infty}^{+\infty} f(\Theta_{CR}) \left[\int_{-\infty}^{\Theta_{CR}} f(\Theta_L) d\Theta_L \right] d\Theta_{CR} \tag{15}$$

The probability integer might be solved in a general case for normal distributions. Numerical methods are available for the reliability calculation whit non-normal distribution functions.

Numerical example

To illustrate the probabilistic approach to the buckling behavior of a thin panel under combined loads, the aircraft wing skin is presented as a numerical example.

The skin is made of 3.1354T3 aluminum alloy, $t = 1.2\,\text{mm}$ thick and $b = 128\,\text{mm}$ wide The tolerance level for thickness is determined by LN 9073, and depends on the sheet width, Table 1

Table 1. The tolerance level for thickness is determined by LN 9073

	Tolerance level (LN 9073) thickness ≠1.2 mm		
	sheet < 500mm	**sheet < 1000mm**	**sheet < 1250 mm**
Tolerance (mm)	±0.04	±0.07	±0.08
s_t	0.0133	0.023	0.026
$\overline{(t^2)}$	1.4402	1.4405	1.4407
s_{t^2}	0.032	0.056	0.064
	$b = 128\text{mm}$ Tolerance by ISO 2768-1973		
Tolerance (mm)	±0.2	±0.5	±1.2

The data for the panel, loads and material are:

$$k = 4$$

$$E = 74000 \frac{N}{mm^2}$$

$$k_S = 6$$

$$\mu = 0.3$$

$$\left(\overline{\sigma_L}, s_{\sigma L}\right) = \left(124 \frac{N}{mm^2}, 6 \frac{N}{mm^2}\right), \quad \text{for } (j = 1.5)$$

$$\left(\overline{\tau_L}, s_{\tau L}\right) = \left(15.6 \frac{N}{mm^2}, 0.75 \frac{N}{mm^2}\right), \; for (j = 1.5)$$

The effect of thickness tolerances to the probability of buckling, for different load factors, can be seen in Fig.4.

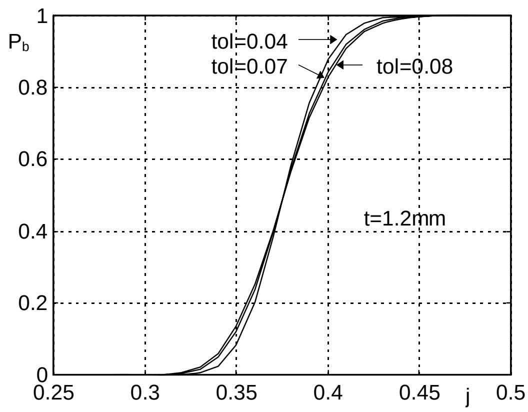

Figure 4. Thickness tolerance effect on the probability of buckling

The probability of buckling is calculated for different panel thicknesses and different load factors, Fig.5.

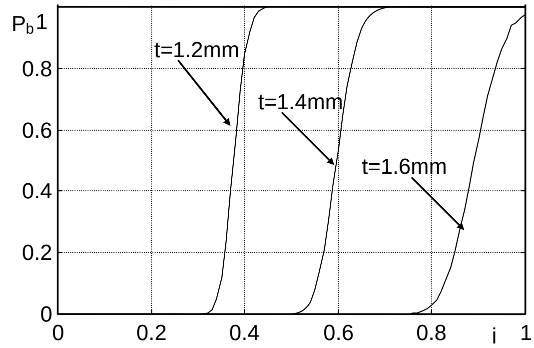

Figure 5. Probability of buckling as a function of the load factor for different thicknesses

The required load factor for buckling is j_b=0.6 (with 50% probability). The thickness to satisfy the requirements is thus $t > 1.4$ mm.

For real wing design, the mass minimization is very important. Therefore, the effect of different parameters variation on the structure mass has to be analyzed (in the probabilistic approach), similarly to the reliability as a function of mass for different load factors, Fig.6.

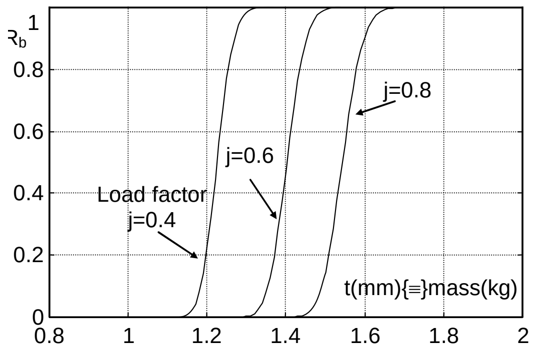

Figure 6. Reliability as a function of thickness for different load factors

The typical shape of the probability curve, as a function of mass (equivalent to skin thicknesses), is presented in Fig.7.

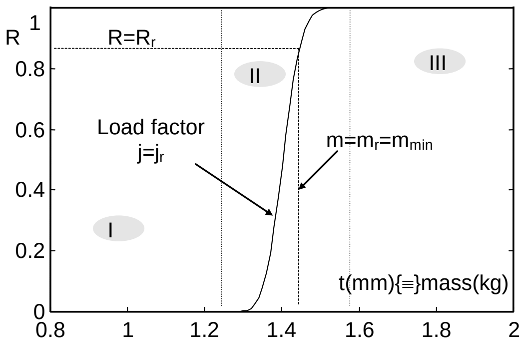

Figure 7. Typical shape of the reliability curve as a function of mass (thicknesses)

Tree different regions characterize the reliability-mass curve. At the beginning, in region I, there is negligible reliability (large probability of buckling) increase with large mass increase, and reliability is very small ($R \approx 0$). In region II, the reliability and mass increase significantly. However, after certain mass value, in region III, reliability of buckling does not increase regardless of large mass increase, but the reliability values are very high ($R \approx 1$).

Conclusion

In this paper, the procedure for the probabilistic calculation of buckling is presented. Thin panels, like wing or fuselage sections, under combined loads are considered. A software program is developed to improve the reliability calculation and the variation of parameters.

The procedure is illustrated by a numerical example. The thin panel of a light trainer aircraft wing is calculated. The thickness of the panel is determined for the required load factor and the probability of buckling. The procedure is found to be effective for the variation of parameters such as tolerances, thickness (or mass), load factors, material characteristics and different constants.

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Probabilistički proračun gubitka stabilnosti tankih panela pod dejstvom kombinovanog opterećenja

Ovaj rad izlaže postupak za proračun pouzdanosti mehaničkih komponenati i struktura. Razmatran je problem gubitka stabilnosti tankog panela pod dejstvom kombinovanog opterećenja (normalni i smičući naponi). Posebno je izložen slučaj gubitka stabilnosti tipičnog tankog panela avionske strukture kao što je oplata trupa ili krila. U analizi su uzete kao stohastičke promenljive: opterećenje, debljina i širina polja oplate. Numerički primer ilustruje probabilistički pristup proračuna gubitka stabilnosti. Proračunata je verovatnoća pojave gubitka stabilnosti. Tolerancije dimenzija imaju uticaja na proračun kritičnih napona i pojavu gubitka stabilnosti. Softverski program omogućuje lakšu varijaciju parametara (posebno efekat debljine oplate).

Ključne reči: **opterećenje konstrukcije, panel, panelna konstrukcija, oplata, trup aviona, krilo aviona, stabilnost strukture, analiza stabilnosti, stohastička promenljiva, probabilistička analiza.**

Вероятные и приблизительные расчёты потери устойчивости тонких панелей под действием комбинированных нагрузок

В настоящей работе показана методика проведения для расчёта надёжности механических комплектующих частей и структур. Здесь рассматривана проблема потери устойчивости тонкой панели под воздействием комбинированной нагрузки (нормальные и срезывающие напряжения). Особо представлен случай потери устойчивости типичной тонкой панели структуры самолёта каковы оболочка фюзеляжа или крыла. В анализе охвачены основные математические переменные: нагрузка, толщина и ширина поля оболочки. Цифровой пример иллюстрирует вероятные и приблизительные подходы к расчёту потери устойчивости. Тоже расчитана вероятность явления потери устойчивости. Допустимые отклонения от стандартных размеров оказывают влияние на расчёт критических напряжений и на явление потери устойчивости. Программное обеспечение компьютера обеспечивает лёгкую добычу разновидности параметров (особо эффекта толщины оболочки).

Ключевые слова: **нагрузка конструкции, панель, панельная конструкция, оболочка, фюзеляж самолёта, крыло самолёта, устойчивость структуры, анализ устойчивости, основная математическая переменная, вероятный и приблизительный анализ.**

Le calcul probabiliste de la perte de stabilité chez les panneaux minces sous l'action de la charge combinée

Ce travail présente le procédé pour le calcul fiable des composantes et structures mécaniques. On a considéré le problème de la perte de stabilité du panneau mince sous l'action de la charge combinée (normale et de tension de cisaillement). On a exposé en particulier le cas de la perte de stabilité du panneau mince typique chez la structure de l'avion comme le revêtement du fuselage ou l'aile. Au cours de l'analyse on a étudié les variables stochastiques : charge, épaisseur et largeur du champ de revêtement. L'exemple numérique illustre l'approche probabiliste du calcul de la perte de stabilité. On a calculé la probabilité de l'apparition de la perte de stabilité. Les tolérances des dimensions ont influence sur le calcul des tensions critiques ainsi que sur l'apparition de la perte de stabilité. Le programme logiciel facilite la variation des paramètres (notamment l'effet de l'épaisseur de revêtement).

Mots clé: **charge de construction, panneau, construction en panneau, revêtement, fuselage, aile, stabilité de la structure, analyse de la stabilité, variable stochastique, analyse probabiliste.**