Two-Dimensional Golay Complementary Array Sets With Arbitrary Lengths for Omnidirectional MIMO Transmission

You-Qi Zhao (趙宥齊), Cheng-Yu Pai (白承祐), *Graduate Student Member, IEEE*, Zhen-Ming Huang (黃振銘), *Graduate Student Member, IEEE*, Zilong Liu (劉子龍) , *Senior Member, IEEE*, and Chao-Yu Chen (陳昭羽), *Senior Member, IEEE*

***Abstract*—This paper presents a coding approach for achieving omnidirectional transmission of certain common signals in massive multi-input multi-output (MIMO) networks such that the received power at any direction in a cell remains constant for any given distance. Specifically, two-dimensional (2D) Golay complementary array set (GCAS) can be used to design the massive MIMO precoding matrix so as to achieve omnidirectional transmission due to its complementary autocorrelation property. In this paper, novel constructions of new 2D GCASs with arbitrary array lengths are proposed. Our key idea is to carefully truncate the columns of certain larger arrays generated by 2D generalized Boolean functions. Finally, the power radiation patterns and numerical results are provided to verify the omnidirectional property of the GCAS-based precoding. The error performances of the proposed precoding scheme are presented to validate its superiority over the existing alternatives.**

***Index Terms*—Generalized Boolean function (GBF), Golay complementary array pair (GCAP), Golay complementary array set (GCAS), omnidirectional precoding (OP), uniform rectangular array (URA).**

I. Introduction

Complementary pairs/sets of sequences have attracted a sustained research interest owing to their zero aperiodic correlation sums properties. To be specific, a Golay complementary pair (GCP) refers to a pair of equal-length sequences whose summation of aperiodic autocorrelations is zero except at the zero time-shift [1]. Such a concept was extended to Golay complementary set (GCS) with constituent sequences of more than 2 by Tseng and Liu in [2]. Furthermore, a maximum collection of GCSs is called a set of complete complementary code (CCC) [3] if any two different GCSs have zero aperiodic cross-correlation sums for all time-shifts. In the literature, GCSs and CCCs have been widely used for radar sensing [4], channel estimation [5], precoding for massive multi-input multi-output (MIMO) [6], peak-to-average power ratio (PAPR) reduction in orthogonal frequency division multiplexing (OFDM) [7]–[13], interference-free multicarrier code division multiple access [14]–[17], and many other applications [18], [19].

Recently, there is a surge of research attention to study two-dimensional (2D) Golay complementary array sets (GCASs) [18]-[23], each having zero aperiodic autocorrelation sums property for two directions of shifts (compared to conventional GCSs and CCCs with time-shifts only). An important application of the 2D GCASs is for omnidirectional transmission in MIMO communication systems with a uniform rectangular array (URA) configuration [20], [21]. In massive MIMO systems, some common messages (e.g., reference signals, synchronization signals, control signals, etc.) need to be power-uniformly broadcasted to all the angles within the whole cell. In this paper, we consider space-time block code (STBC) for the harvesting of the diversity gain. At the base station (BS), the STBC encoded symbols are assigned to several streams and then mapped onto the antenna arrays in URA by certain 2D GCASs assisted precoding matrices to achieve uniform power radiation at any angle.

On the other hand, since a large number of antennas are considered in massive MIMO systems, a huge pilot overhead may be needed to acquire the channel state information (CSI). As pointed out in [22], this can be alleviated by omnidirectional precoding (OP) based transmission. For uniform linear arrays (ULAs), Zadoff-Chu (ZC) sequences were adopted to satisfy the requirements of the omnidirectional property. However, [22] only considered the omnidirectional transmission in certain directions. Later in [6], GCSs and CCCs based OP matrices were proposed to meet the requirement of omnidirectional transmission across all directions.

In [20], [21], [23], [24], 2D GCASs were employed for precoding matrices in URAs by applying interleaving and Kronecker-product to existing 1D sequences or 2D arrays. As a result, the array sizes of 2D GCASs are only feasible for certain lengths. A construction of 2D GCASs of array size $p^n \times p^m$ was proposed in [25] by using permutation ployno-

The work of Y.-Q. Zhao, C.-Y. Pai, Z.-M. Huang, and C.-Y. Chen was supported in part by the National Science and Technology Council, Taiwan, R.O.C., under Grants NSTC 109–2628–E–006–008–MY3, NSTC 112–2221–E–006–122–MY3, NSTC 111–2917–I–006–010, NSTC 112–2218–E–305–001, and NSTC 112–2927–I–006–503. The work of Z. Liu was supported in part by the UK Engineering and Physical Sciences Research Council under Grants EP/X035352/1 and EP/Y000986/1, by the Royal Society under Grant IEC\R3\223079, and by the Research Council of Norway under Grant 311646/070.

You-Qi Zhao and Cheng-Yu Pai are with the Department of Engineering Science, National Cheng Kung University, Tainan 701, Taiwan, R.O.C. (e-mail: {n96091091, n98081505}@gs.ncku.edu.tw).

Zhen-Ming Huang is with the Department of Engineering Science and the Institute of Computer and Communication Engineering, National Cheng Kung University, Tainan 701, Taiwan, R.O.C. (e-mail: n98101012@gs.ncku.edu.tw).

Zilong Liu is with the School of Computer Science and Electronic Engineering, University of Essex, United Kingdom (e-mail: zilong.liu@essex.ac.uk).

Chao-Yu Chen is with the Department of Electrical Engineering and the Institute of Computer and Communication Engineering, National Cheng Kung University, Tainan 701, Taiwan, R.O.C. (e-mail: super@mail.ncku.edu.tw).

mials (PPs) functions and 2-level autocorrelation sequences, where p is a prime number, m, n are two positive integers, and $p, m, n > 0$. Furthermore, a unifying construction framework for 2D GCASs was developed in [26] by a multivariate polynomial matrix from certain seed para-unitary (PU) matrices. In [27], [28], Pai and Chen proposed direct constructions of 2D Golay complementary array pairs (GCAPs) and GCASs with array size $2^n \times 2^m$ from 2D generliazed Boolean functions (GBFs) [29] where n, m are integers and $n, m \geq 2$. 2D GCAP can be regarded as a case of 2D GCAS when the set size is equal to 2. Moreover, Pai *et al.* [30] proposed a direct construction of 2D CCCs with array size $2^n \times 2^m$, which have ideal autocorrelations and cross-correlations. Later, Liu $et\ al.$ [31] proposed a construction of GCASs with array size $p^n \times p^m$ by using 2D multivariable functions, where p is a prime number, n, m are integers, and $n, m \geq 2$. Based on [27], [32] developed a direct construction of GCASs with set size 4 and array size $2^n \times (2^{m-1} + 2^v)$ by using 2D GBFs, where n, m, v are positive number with $n, m \geq 2,$ and $0 \leq v \leq m-1$.

The aforementioned research efforts are generally driven by the need of highly flexible array sizes of 2D GCASs. Motivated by this, we aim for generating new GCASs with arbitrary array lengths. The main contributions of this work are summarized as follows.

1) We present direct constructions of 2D GCASs with more flexible array sizes based on 2D GBFs. Our key idea is to properly truncate certain columns of larger arrays generated from 2D GBFs. Thus, our constructed GCASs can be applied to URAs with various array sizes. Numerical results indicate that the proposed 2D GCASs are good candidates for precoding matrices to attain omnidirectional transmission.
2) We compare the parameters of existing 2D GCASs with our proposed ones in Table I. It can be observed that by setting proper values of d_α's and v, the array size of the proposed GCASs reduces to the form $2^n \times 2^m$ which is the same as the array sizes of GCASs from [27], [28], [30]. Besides, our proposed GCASs can include the GCASs provided in [32] as a special case. Note that the proposed GCASs can be directly generated from 2D GBFs without the requirements of any specific sequences or tedious sequence operations.

The remainder of this paper is defined as follows. Section II discusses notations, definitions, system models, and the omnidirectional transmission in MIMO systems. Section III describes our proposed constructions of 2D GCASs. Section IV shows the power radiation pattern and bit error rate (BER) performance based on our proposed 2D GCASs precoding. Finally, Section V presents the conclusion.

II. Preliminaries and Definitions

A. Notations

Throughout this paper, we present the notations in the following:

- $(\boldsymbol{a})_i$ refers to the i-th element of the vector $\boldsymbol{a}$.
- $(\boldsymbol{A})_{i,j}$ denotes the (i,j)-th element of the array $\boldsymbol{A}$.
- $(\cdot)^H$ refers to the conjugate transpose.
- diag$(\boldsymbol{A})$ refers to the column vector composed of the main diagonal of $\boldsymbol{A}$.
- $(\cdot)^*$ refers to the complex conjugation of an element.
- $(\cdot)^T$ refers to the transpose.
- vec$(\cdot)$ express stacking one column of the matrix into one another column.
- $\mathbf{1}$ is a vector whose elements are all 1.
- Let $\xi = e^{2\pi\sqrt{-1}/q}$.
- Throughout this paper, q is an even number.

Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be two arrays of size $L_1 \times L_2$. Then $\boldsymbol{X}$ and $\boldsymbol{Y}$ can be stated as

$$\boldsymbol{X} = (X_{g,i}),\ \boldsymbol{Y} = (Y_{g,i}), \tag{1}$$

where $g = 0, 1, \cdots, L_1 - 1$ and $i = 0, 1, \cdots, L_2 - 1$.

Definition 1: Given two arrays $\boldsymbol{X}$ and $\boldsymbol{Y}$ of size $L_1 \times L_2$, the *2D aperiodic cross-correlation function* (AACF) is defined by

$$\rho(\mathbf{X}, \mathbf{Y}; u_1, u_2) = \begin{cases} \sum\limits_{g=0}^{L_1-1-u_1} \sum\limits_{i=0}^{L_2-1-u_2} Y_{g+u_1,i+u_2} X^*_{g,i}, 0 \leq u_1 < L_1, \\ 0 \leq u_2 < L_2; \\ \sum\limits_{g=0}^{L_1-1-u_1} \sum\limits_{i=0}^{L_2-1-u_2} Y_{g+u_1,i} X^*_{g,i-u_2}, 0 < u_1 < L_1, \\ -L_2 < u_2 < 0; \\ \sum\limits_{g=0}^{L_1-1-u_1} \sum\limits_{i=0}^{L_2-1-u_2} Y_{g,i} X^*_{g-u_1,i-u_2}, -L_1 < u_1 < 0, \\ -L_2 < u_2 < 0; \\ \sum\limits_{g=0}^{L_1-1+u_1} \sum\limits_{i=0}^{L_2-1-u_2} Y_{g,i+u_2} X^*_{g-u_1,i}, -L_1 < u_1 < 0, \\ 0 < u_2 < L_2. \end{cases} \tag{2}$$

When $\boldsymbol{X} = \boldsymbol{Y}$, then it is called *2D aperiodic autocorrelation function* (AACF) and denoted by $\rho(\boldsymbol{X}; u_1, u_2)$. If taking $L_1 = 1$, two 2D arrays $\boldsymbol{X}$ and $\boldsymbol{Y}$ are degraded as a 1-D sequence $\boldsymbol{X} = X_i$ for $i = 0, 1, \cdots, L_2 - 1$ and $\boldsymbol{Y} = Y_i$ for $i = 0, 1, \cdots, L_2 - 1$, respectively. Then the 1-D AACF of 1-D sequence $\boldsymbol{X}$ is related by

$$\rho(\boldsymbol{X}; u) = \begin{cases} \sum\limits_{i=0}^{L_2-1-u} X_{i+u} X_i^*, & 0 \leq u \leq L_2 - 1; \\ \sum\limits_{i=0}^{L_2-1+u} X_i X^*_{i-u}, & -L_2 + 1 \leq u < 0. \end{cases} \tag{3}$$

In this paper, q-PSK modulation is employed. Thus, $\boldsymbol{x}$ and $\boldsymbol{y}$ denote q-ary arrays and (1) is expressed as

$$\begin{aligned} \boldsymbol{X} &= (X_{g,i}) = (\xi^{x_{g,i}}) = \xi^{\boldsymbol{x}}; \\ \boldsymbol{Y} &= (Y_{g,i}) = (\xi^{y_{g,i}}) = \xi^{\boldsymbol{y}}, \end{aligned} \tag{4}$$

where $\boldsymbol{x} = (x_{g,i}), \boldsymbol{y} = (y_{g,i})$, and $x_{g,i},\ y_{g,i} \in \mathbb{Z}_q = \{0, 1, \cdots, q-1\}$ for $0 \leq g < L_1,\ 0 \leq i < L_2$.

Definition 2: Let the array set $G = \{\boldsymbol{X}_0, \boldsymbol{X}_1, \cdots, \boldsymbol{X}_{N-1}\}$ where each array in set G is of size $L_1 \times L_2$. If the array set G satisfies

$$\sum_{k=0}^{N-1} \rho(\boldsymbol{X}_k; u_1, u_2) = \begin{cases} NL_1L_2, & u_1 = u_2 = 0; \\ 0, & u_1 \neq 0 \text{ or } u_2 \neq 0, \end{cases} \tag{5}$$

TABLE I
A COMPARISON OF CONSTRUCTIONS FOR 2D GCASS

Construction	Parameters	Approaches
[26, Th. 5]	$(N, N^n, N^m), N, n, m > 0$	Seed PU matrices
[26, Th. 7]	$(2^k, 2^{kn}, 2^{km}), n, m, k > 0$	
[25, Th. 4]	(p, p^n, p^m), prime $p, n, m > 0$	PPs and 2-level autocorrelation sequences
[25, Th. 6]	(p^k, p^{kn}, p^{km}), prime $p, k, n, m > 0$	
[31, Th. 1]	$(p_1^{k_1} p_2^{k_2}, p_1^n, p_2^m)$, primes p_1, p_2	2D multivariable functions
[31, Th. 2]	(p^k, p^n, p^m), prime p, $n + m \geq k > 0$	
[27], [28], [30]	$(2^k, 2^n, 2^m)$, $n, m \geq k > 0$, and $k > 0$	2D GBFs
[32]	$(4, 2^n, 2^{m-1} + 2^v)$, $n, m \geq 2$, and $k > 0$	
Th. 1	$(2^{k+1}, 2^n, 2^{m-1} + \sum_{\alpha=1}^{k-1} d_\alpha 2^{m-k+\alpha-1} + d_0 2^v)$, $k < m,\ 0 \leq v \leq m - k,\ d_\alpha \in \{0, 1\}$	
Th. 2	$(2^{k+1}, 2^n, 2^{m-1} + \sum_{\alpha=1}^{k-1} d_\alpha 2^{\pi_1(m-k+\alpha)-1} + d_0 2^v)$, $k < m,\ 0 \leq v \leq m - k,\ d_\alpha \in \{0, 1\}$	

the set G is called the *Golay complementary array set* of set size N denoted by (N, L_1, L_2)-GCAS where L_2 is defined as the length of the GCAS. When $N = 2$, the 2D GCAS G is reduced to a 2D *Golay complementary array pair* (GCAP).

B. Generalized Boolean Functions

A 2D generalized Boolean function (GBF) f in $n+m$ binary variables $y_1, y_2, \cdots, y_n$, $x_1, x_2, \cdots, x_m$, is a function mapping: $\mathbb{Z}_2^n \times \mathbb{Z}_2^m \to \mathbb{Z}_q$, where $x_i, y_g \in \{0, 1\}$ for $i = 1, 2, \cdots, m$ and $g = 1, 2, \cdots, n$. A monomial of degree r is given by any product of r distinct variables among $y_1, y_2, \cdots, y_n, x_1, x_2, \cdots, x_m$. For instance, $x_1 x_3 y_1 y_2$ is a monomial of degree 4. Next, the variables $z_1, z_2, \cdots, z_{n+m}$ are defined as

$$z_l = \begin{cases} y_l & \text{if } 1 \leq l \leq n; \\ x_{l-n} & \text{if } n < l \leq m + n, \end{cases} \tag{6}$$

which are useful for our proposed constructions. For a 2D GBF with $n + m$ variables, the 2D $\mathbb{Z}_q$-valued array

$$\boldsymbol{f} = \begin{pmatrix} f_{0,0} & f_{0,1} & \cdots & f_{0,2^m-1} \\ f_{1,0} & f_{1,1} & \cdots & f_{1,2^m-1} \\ \vdots & \vdots & \ddots & \vdots \\ f_{2^n-1,0} & f_{2^n-1,1} & \cdots & f_{2^n-1,2^m-1} \end{pmatrix} \tag{7}$$

of size $2^n \times 2^m$ is given by letting $f_{g,i} = f((g_1, g_2, \cdots, g_n), (i_1, i_2, \cdots, i_m))$, where $(g_1, g_2, \cdots, g_n)$ and $(i_1, i_2, \cdots, i_m)$ are binary vector representations of integers $g = \sum_{h=1}^{n} g_h 2^{h-1}$ and $i = \sum_{j=1}^{n} i_j 2^{j-1}$, respectively.

Example 1: Taking $q = 4,\ n = 2$, and $m = 3$ for example, the 2D GBF is given as $f = 3z_5 z_4 + z_2 z_3 + 2z_2$. Then the array $\boldsymbol{f}$ of size 4×8 corresponding to f can be obtained, i.e.,

$$\boldsymbol{f} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 & 1 & 1 & 3 & 3 \\ 2 & 3 & 2 & 3 & 2 & 3 & 1 & 2 \\ 2 & 3 & 2 & 3 & 2 & 3 & 1 & 2 \end{pmatrix}. \tag{8}$$

The GBF f can be rewritten as $f = 3x_3 x_2 + y_2 x_1 + 2y_2$. In this paper, we consider the array size $\neq 2^n \times 2^m$. Hence, we define the truncated array $\boldsymbol{f}^{(L)}$ corresponding to the 2D GBF f by ignoring the last $2^m - L$ columns of the corresponding array $\boldsymbol{f}$.

Example 2: Following the same notations given in Example 1, the truncated array $\boldsymbol{f}^{(6)}$ is given by

$$\boldsymbol{f}^{(6)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 2 & 3 & 2 & 3 & 2 & 3 \\ 2 & 3 & 2 & 3 & 2 & 3 \end{pmatrix}. \tag{9}$$

For simplicity, we use $\boldsymbol{f}$ to stand for $\boldsymbol{f}^{(L)}$ when L is known.

C. System Model

Considering downlink transmission from a BS to UEs where each has one single antenna, we suppose that the number of antennas at the BS is $M = L_1 \times L_2$, i.e., the URA consists of L_1 rows and L_2 columns. Fig. 1 illustrates the diagram of data downlink transmission. For an $L_1 \times L_2$ URA, the steering matrix $\boldsymbol{A}(\varphi, \theta)$ at the direction (φ, θ) with the (g, i)-th entry can be expressed as

$$\begin{aligned} (\boldsymbol{A}(\varphi, \theta))_{g,i} &= e^{-j\frac{2\pi}{\lambda} g d_y \sin\varphi \sin\theta - j\frac{2\pi}{\lambda} i d_x \sin\varphi \cos\theta}, \\ &\text{for } g = 0, 1, \ldots, L_1 - 1,\ \ i = 0, 1, \ldots, L_2 - 1, \\ &\quad \theta \in [0, 2\pi],\ \varphi \in [0, \pi/2], \end{aligned} \tag{10}$$

where d_x and d_y denote the vertical antenna and horizontal antenna inter-element spacings of the URA, respectively, and λ denotes the carrier wavelength. To enhance the spatial diversity and communication reliability, the STBC signal transmission scheme is used. The $N \times M$ STBC is given by

$$\boldsymbol{S} \triangleq \begin{pmatrix} s_0(0) & s_0(1) & \cdots & s_0(M-1) \\ s_1(0) & s_1(1) & \cdots & s_1(M-1) \\ \vdots & \vdots & \ddots & \vdots \\ s_{N-1}(0) & s_{N-1}(1) & \cdots & s_{N-1}(M-1) \end{pmatrix} \in \mathbb{C}^{N \times M} \tag{11}$$

where $\mathbb{C}^{N \times M}$ refers to the N-by-M complex space and $s_n(t)$ denotes the (n, t)-th element of the STBC at time instant t for $t = 0, 1, \cdots, M - 1$. We define the precoding matrix $\boldsymbol{W}_n$ of size $L_1 \times L_2$. The encoded symbols is given by

$$\begin{aligned} \boldsymbol{x}(t) &= (x_0(t), x_1(t), \cdots, x_{L_1 L_2 - 1}(t))^T \\ &= \text{vec}\left(\sum_{n=0}^{N-1} \boldsymbol{W}_n \cdot s_n(t) \right),\ \text{for } t = 0, 1, \cdots, M - 1, \end{aligned} \tag{12}$$

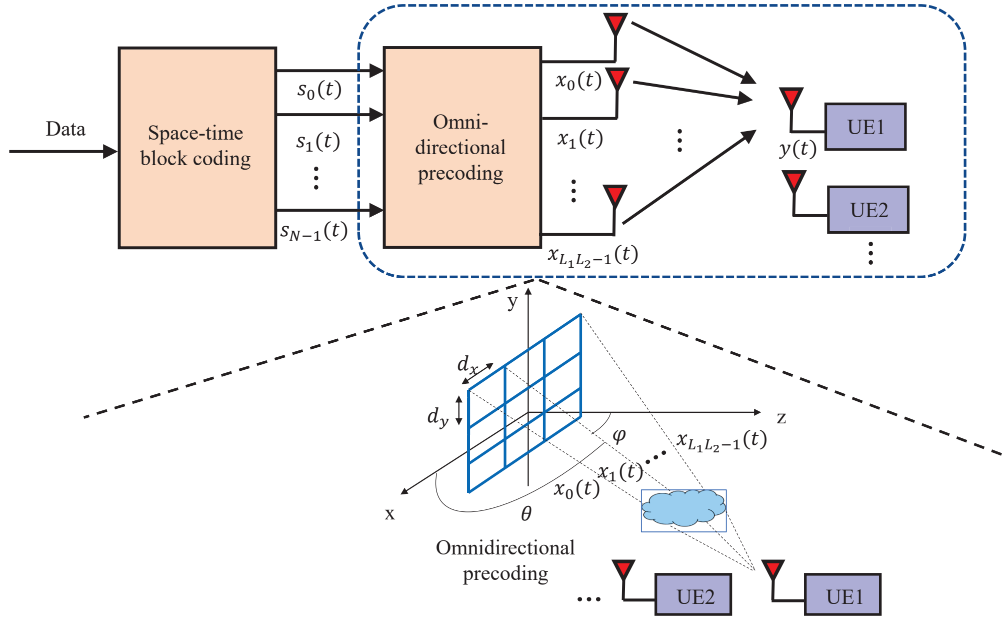


Fig. 1. Diagram of data transmission through STBC encoding and omnidirectional precoding.

which are transmitted by the L_1L_2 antennas of the URA. In the light-of-sight (LOS) channel without multipaths, the received signal at the direction (φ,θ) can be written as

$$y(t)=\sum_{n=0}^{N-1}\left(\text{vec}(\boldsymbol{A}(\varphi,\theta))^T\text{vec}(\boldsymbol{W}_n)\right)\cdot s_n(t)+\eta(t),\quad t=0,\ldots,M-1, \tag{13}$$

where $\eta(t)$ is the additive Gaussian white noise (AWGN) with zero mean and variance σ^2 at time instant t.

D. Omnidirectional Precoding Matrices Based on 2D Arrays

In this subsection, we list two necessary requirements for the design of OP matrices. Then, we will connect these two requirements with the conditions of 2D arrays.

Requirement 1 (R1): Omnidirectional transmission.

We consider the MIMO system with URA. Following (13), the received power E at the angle (φ,θ) is represented as

$$E=\sum_{n=0}^{N-1}\left|\left[\text{vec}(\boldsymbol{A}(\varphi,\theta))^T\text{vec}(\boldsymbol{W}_n)\right]\right|^2. \tag{14}$$

Therefore, to satisfy the omnidirectional transmission in the whole cell, (14) must be constant for all φ and θ.

Requirement 2 (R2): Equal average power on each antenna.

To enhance the efficiency of the power amplifier, the average transmission power on all $L_1\times L_2$ antennas is required to be equal. We define

$$\boldsymbol{W}=\left(\text{vec}(\boldsymbol{W}_0),\text{vec}(\boldsymbol{W}_1),\cdots,\text{vec}(\boldsymbol{W}_{N-1})\right), \tag{15}$$

where the array size of $\boldsymbol{W}$ is $L_1L_2\times N$. Hence, (12) can be rewritten as

$$\boldsymbol{X}=\left(\boldsymbol{x}(0),\boldsymbol{x}(1),\cdots,\boldsymbol{x}(M-1)\right)=\boldsymbol{W}\boldsymbol{S}. \tag{16}$$

Let $\boldsymbol{s}(t)$ be the t-th column of $\boldsymbol{S}$. Throughout this paper, we assume $\mathbb{E}\left[\boldsymbol{s}(t)\boldsymbol{s}(t)^H\right]=\boldsymbol{I}_N$. The transmitted signal on the (l_1,l_2)-th antenna is $(\boldsymbol{W}\boldsymbol{s})_{l_2L_1+l_1}$. The average power on the (l_1,l_2)-th antenna can be expressed as

$$\begin{aligned}\mathbb{E}\left[|(\boldsymbol{W}\boldsymbol{s})_{l_2L_1+l_1}|^2\right]&=\left(\boldsymbol{W}\mathbb{E}\left[\boldsymbol{s}(t)\boldsymbol{s}(t)^H\right]\boldsymbol{W}^H\right)_{l_2L_1+l_1,l_2L_1+l_1}\\&=(\boldsymbol{W}\boldsymbol{W}^H)_{l_2L_1+l_1,l_2L_1+l_1}.\end{aligned} \tag{17}$$

Therefore, the condition to guarantee equal power on each antenna is equivalent to

$$\text{diag}(\boldsymbol{W}\boldsymbol{W}^H)=N\mathbf{1}. \tag{18}$$

Next, we will derive two sufficient conditions on the precoding matrices to fulfill requirements R1 and R2.

Lemma 1: [21] For an $L_1\times L_2$ URA, if the precoding matrices $\boldsymbol{W}_0,\boldsymbol{W}_1,\cdots,\boldsymbol{W}_{N-1}$ of size $L_1\times L_2$ form an (N,L_1,L_2)-GCAS, then the omnidirectional transmission is achieved.

Lemma 2: For an $L_1\times L_2$ URA, if the precoding matrices $\boldsymbol{W}_0,\boldsymbol{W}_1,\cdots,\boldsymbol{W}_{N-1}$ of size $L_1\times L_2$ are unimodular, then the average power on each antenna is equal.

Proof: In order to meet the requirement for equal average power on each antenna, the precoding matrix $\boldsymbol{W}$ must satisfy (18). We let $\boldsymbol{w}_i=\text{vec}(\boldsymbol{W}_i)$, for $i=0,1,\cdots,N-1$. Then,

$$\begin{aligned}&\text{diag}\left(\boldsymbol{W}\boldsymbol{W}^H\right)\\&=\left(\sum_{i=0}^{N-1}|(\boldsymbol{w}_i)_0|^2,\sum_{i=0}^{N-1}|(\boldsymbol{w}_i)_1|^2,\cdots,\sum_{i=0}^{N-1}|(\boldsymbol{w}_i)_{L_1L_2-1}|^2\right)^T\\&=N\mathbf{1}\end{aligned} \tag{19}$$

since we have $|(\boldsymbol{w}_i)_n|^2=1$ for $i=0,1,\cdots,N-1$ and $n=0,1,\cdots,L_1L_2-1$. According to (18), the requirement (R2) is fulfilled. ∎

In the sequel, the design of OP matrices $\boldsymbol{W}_0, \boldsymbol{W}_1, \cdots, \boldsymbol{W}_{N-1}$ are based on Lemma 1 and Lemma 2. That is, our goal is to construct unimodular GCASs with flexible sizes.

III. GCASs With Flexible Array Size

In this section, two constructions of 2D GCASs with arbitrary array lengths based on 2D GBFs will be proposed. By recalling the function mapping in (6), we present our first theorem in the following.

Theorem 1: For any integers q, m, $n \geq 2$, and $k < m$, v is an integer satisfies $0 \leq v \leq m-k$ and let π be a permutation of $\{1, 2, \cdots m+n-k\}$ satisfying $\{z_{\pi(1)}, z_{\pi(2)}, \cdots, z_{\pi(v+n)}\} = \{z_1, z_2, \cdots, z_{v+n}\}$. The 2D generalized Boolean function can be written as

$$f = \frac{q}{2}\left(\sum_{l=1}^{m+n-k-1} z_{\pi(l)} z_{\pi(l+1)}\right) + \sum_{s=1}^{m+n} p_s z_s + p_0 \quad (20)$$

where $p_s \in \mathbb{Z}_q$. The array set

$$G = \left\{\boldsymbol{f} + \frac{q}{2}\sum_{\alpha=1}^{k} \lambda_\alpha \boldsymbol{z}_{m+n-k+\alpha} + \frac{q}{2}\lambda_{k+1}\boldsymbol{z}_{\pi(1)} : \lambda_\alpha \in \{0,1\}\right\} \quad (21)$$

is a q-ary $(2^{k+1}, 2^n, 2^{m-1} + \sum_{\alpha=1}^{k-1} d_\alpha 2^{m-k+\alpha-1} + d_0 2^v)$-GCAS where $d_\alpha \in \{0,1\}$.

Proof: Please see the proof in Appendix A. ■

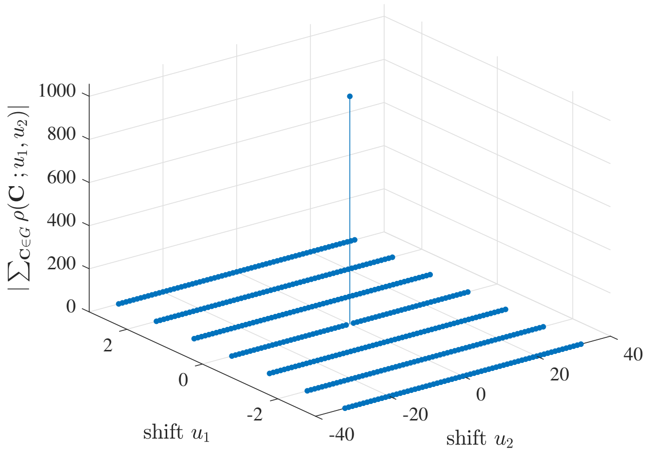


Fig. 2. The summation of autocorrelations of constituent arrays in the GCAS in Example 3.

Remark 1: The parameter $2^{m-1} + \sum_{\alpha=1}^{k-1} d_\alpha 2^{m-k+\alpha-1} + d_0 2^v$ of the proposed GCASs in Theorem 1 can be any arbitrary length since m, k, v are flexible and $d_\alpha \in \{0,1\}$. Besides, by setting $v = m-k$ and $d_\alpha = 1$ for $\alpha = 0, 1 \ldots, k-1$ in Theorem 1, the constructed $(2^{k+1}, 2^n, 2^m)$-GCASs have the same array sizes of GCASs from [28].

Example 3: Taking $q = 2$, $m = 6$, $n = 2$, $k = 1$, and $v = 0$, we let $\pi = (1,2,3,4,5,6,7)$. The generalized Boolean function is $f = z_1z_2 + z_2z_3 + z_3z_4 + z_4z_5 + z_5z_6 + z_6z_7 = x_1x_2 + x_2x_3 + x_3x_4 + x_4x_5 + y_1y_2 + y_2x_1$ by setting $p_k = 0$ for $k = 0, 1, \ldots, m+n$. The array set $G = \{\boldsymbol{f}, \boldsymbol{f} + \boldsymbol{x}_8, \boldsymbol{f} + \boldsymbol{y}_1, \boldsymbol{f} + \boldsymbol{x}_8 + \boldsymbol{y}_1\}$ is a GCAS of size 4 and the array size is 4×33. We let $G = \{\boldsymbol{c}_0, \boldsymbol{c}_1, \boldsymbol{c}_2, \boldsymbol{c}_3\}$ and list the constituent arrays in Table II as given in Appendix C. Fig. 2 shows the AACF sum of set G is zero at shift $u_1 \neq 0$ or $u_2 \neq 0$. Thus, we can find that array set G is a $(4, 4, 33)$-GCAS.

Next, we introduce a lemma which illustrates a construction of $(4, 2^n, 2^{m-1} + 2^v)$-GCAS from 2D GBFs.

Lemma 3: [32, Th. 1] For nonnegative integers m, n, and v with $0 \leq v < m-1$, let π_1 be a permutation of $\{1, 2, \cdots, m-1\}$ and π_2 be a permutation of $\{1, 2, \cdots, n\}$. The 2D GBF is given by

$$\begin{aligned} f =& \frac{q}{2}\Big(\sum_{k=1}^{m-2} x_{\pi_1(k)} x_{\pi_1(k+1)} + \sum_{k=1}^{n-1} y_{\pi_2(k)} y_{\pi_2(k+1)} \\ &+ x_{\pi_1(m-1)} x_m + x_m y_{\pi_2(1)}\Big) + \sum_{l=1}^{m} p_l x_l + \sum_{s=1}^{n} \kappa_s y_s + p_0 \end{aligned} \quad (22)$$

where $p_l,\ \kappa_s \in \mathbb{Z}_q$. Then the array set

$$G = \left\{\boldsymbol{f}, \boldsymbol{f} + \frac{q}{2}\boldsymbol{x}_{\pi_1(1)}, \boldsymbol{f} + \frac{q}{2}\boldsymbol{y}_{\pi_2(n)}, \boldsymbol{f} + \frac{q}{2}\boldsymbol{x}_{\pi_1(1)} + \frac{q}{2}\boldsymbol{y}_{\pi_2(n)}\right\}$$

is a $(4, 2^n, 2^{m-1} + 2^v)$-GCAS.

Since the set size of the GCAS from Lemma 3 is limited to 4, we propose a general construction of 2D GCASs with more flexible array sizes and set sizes which can include Lemma 3 as a special case.

Theorem 2: For any integers q, m, $n \geq 2$, and $k < m$, v is an integer satisfies $0 \leq v \leq m-k$. Assume that π_1 is a permutation of $\{1, 2, \cdots m\}$ and π_2 is a permutation of $\{1, 2, \cdots n\}$. The 2D generalized Boolean function can be written as

$$\begin{aligned} f =& \frac{q}{2}\left(\sum_{l=1}^{m-k-1} x_{\pi_1(l)} x_{\pi_1(l+1)} + \sum_{s=1}^{n-1} y_{\pi_2(s)} y_{\pi_2(s+1)} \right. \\ &\left. + x_{\pi_1(m)} y_{\pi_2(n)}\right) + \sum_{l=1}^{m-k} \mu_l x_{\pi_1(l)} x_{\pi_1(m)} + \sum_{l=1}^{m} p_l x_k \\ &+ \sum_{s=1}^{n} \kappa_s y_s + p_0 \end{aligned} \quad (23)$$

where $\mu_l,\ p_l,\ \kappa_s, \in \mathbb{Z}_q$. The array set

$$\begin{aligned} G = \Big\{ & \boldsymbol{f} + \frac{q}{2}\sum_{\alpha=1}^{k-1} \lambda_\alpha \boldsymbol{x}_{\pi_1(m-k+\alpha)} \\ &+ \frac{q}{2}\lambda_k \boldsymbol{y}_{\pi_2(1)} + \frac{q}{2}\lambda_{k+1}\boldsymbol{x}_{\pi_1(1)} : \lambda_\alpha \in \{0,1\}\Big\} \end{aligned} \quad (24)$$

is a q-ary $(2^{k+1}, 2^n, 2^{m-1} + \sum_{\alpha=1}^{k-1} d_\alpha 2^{\pi_1(m-k+\alpha)-1} + d_0 2^v)$-GCAS where $d_\alpha \in \{0,1\}$ if the following three conditions hold.

(C1) $\{\pi_1(1), \pi_1(2), \cdots, \pi_1(v)\} = \{1, 2, \cdots, v\}$ if $v > 0$;
(C2) $\pi_1(m-k+\alpha) < \pi_1(m-k+\alpha+1)$ for $1 \leq \alpha \leq k-1$ where $\pi_1(m) = m$;
(C3) For $1 \leq \alpha \leq k-1$ and $2 \leq \beta \leq m-k$, if $\pi_1(\beta) < \pi_1(m-k+\alpha)$, then $\pi_1(\beta-1) < \pi_1(m-k+\alpha)$.

Proof: The proof is given in Appendix B. ■

Remark 2: Taking $\sigma_2(l) = \pi_2(n-l+1)$ for $l = 1, 2, \ldots, n$ and $\pi_1(m-k+\alpha) = m-k+\alpha$ for $\alpha = 1, 2, \ldots, k$ in Theorem 2, (22) can be represented as

$$\begin{aligned} f =& \frac{q}{2}\Bigg(\sum_{k=1}^{m-k-1} x_{\pi_1(k)} x_{\pi_1(k+1)} + \sum_{k=1}^{n-1} y_{\sigma_2(k)} y_{\sigma_2(k+1)} \\ &+ x_m y_{\sigma_2(1)} \Bigg) + \sum_{l=1}^{m-k} \mu_l \boldsymbol{x}_{\pi_1(l)} \boldsymbol{x}_m + \sum_{l=1}^{m} p_l x_l \\ &+ \sum_{s=1}^{n} \kappa_s y_s + p_0 \end{aligned} \tag{25}$$

where $p_l,\ \kappa_s \in \mathbb{Z}_q$. We can find that the result of Lemma 3 is a special case of Theorem 2 by simply setting $k = 1$, $\mu_{m-1} = \frac{q}{2}$, and $\mu_l = 0$ for $l = 1, \cdots, m-2$.

Example 4: Taking $q = 2$, $m = 5$, $n = 2$, $k = 2$, and $v = 0$, we let $\pi_1 = (1, 2, 4, 3, 5)$ and $\pi_2 = (1, 2)$. The generalized Boolean function is $f = x_1x_2 + x_2x_4 + y_1y_2 + x_5y_1$ by setting $p_l, \kappa_s = 0$. The array set G is a GCAS of size 8 when the truncated size $L_1 = 4$ $L_2 = 21$. We let $G = \{\boldsymbol{c}_0, \boldsymbol{c}_1, \cdots, \boldsymbol{c}_7\}$ and list the constituent arrays in Table III as provided in Appendix C. Also, their AACF sum is shown as Fig. 3.

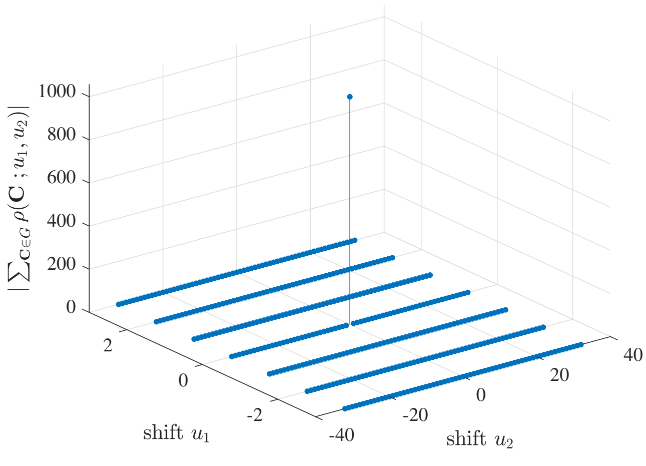
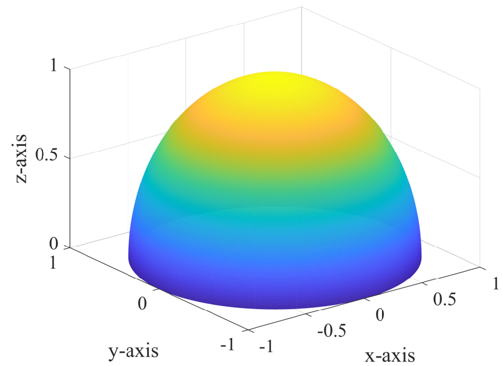
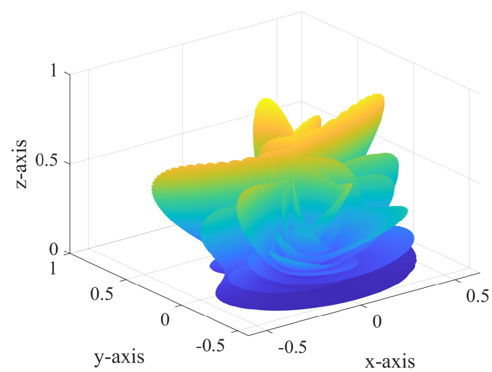


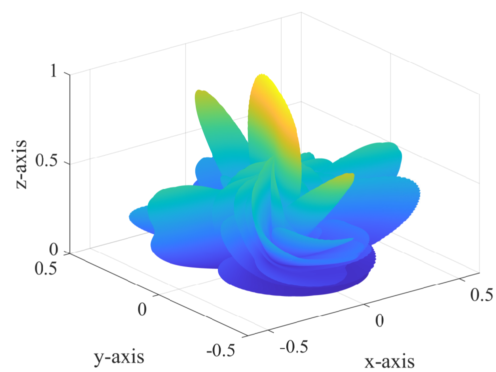
Fig. 3. The summation of autocorrelations of constituent arrays in the GCAS in Example 4.



(a) GCAS-based precoding.



(b) ZC-based precoding.



(c) Random-matrix-based precoding.

Fig. 4. Power radiation pattern with 4×33 URA and 4×4 STBC.

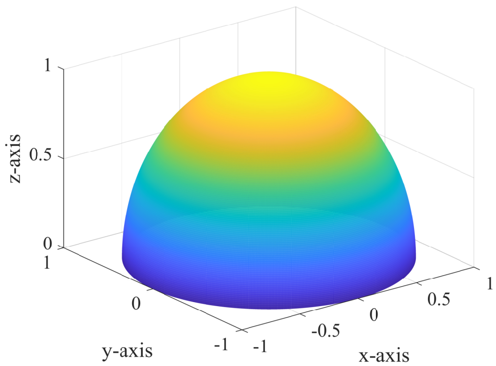
IV. Simulation Results

In this section, we present the numerical results including the power radiation pattern and BER performance by using our proposed 2D GCASs for massive MIMO systems with URA.

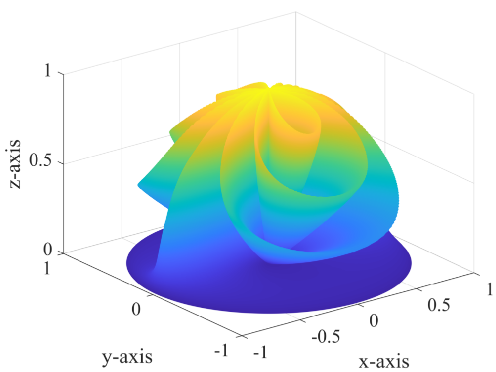
A. Power Radiation Pattern

According to (13), the power radiation pattern $\sum_{n=0}^{N-1} \left| [\text{vec}(\boldsymbol{A}(\varphi, \theta))^T \text{vec}(\boldsymbol{W}_n)] \right|^2$ can be obtained. We first consider the massive MIMO system equipped with a URA of size 4×33, i.e., $L_1 = 4$ and $L_2 = 33$. We take the GCAS $G = \{\boldsymbol{c}_0, \boldsymbol{c}_1, \boldsymbol{c}_2, \boldsymbol{c}_3\}$ listed in Table II to generate the precoding matrices $\{\boldsymbol{W}_0, \boldsymbol{W}_1, \boldsymbol{W}_2, \boldsymbol{W}_3\} = \{(-1)^{\boldsymbol{c}_0}, (-1)^{\boldsymbol{c}_1}, (-1)^{\boldsymbol{c}_2}, (-1)^{\boldsymbol{c}_3}\}$ with the omnidirectional property. The power radiation pattern of the GCAS-based scheme with array size 4×33 is perfectly omnidirectional as illustrated in Fig 4(a).

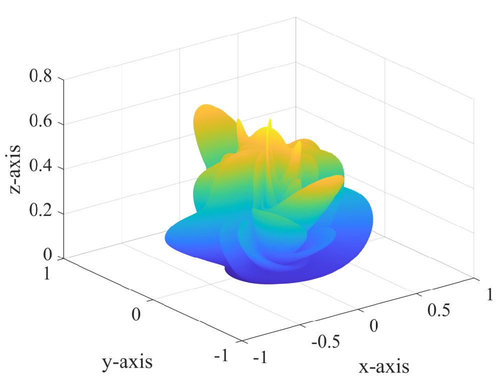
For the purpose of comparison, we also show the power radiation patterns of the precoding matrices based on Zadoff-Chu sequences and random-matrices whose elements are randomly generated from "$+1$" and "-1". The ZC-based precoder consists of four 4×33 precoding matrices, which are obtained based on a ZC sequence of length 4 and a ZC sequence of length 33 [21]. Fig. 4(b) illustrates the power radiation pattern of the ZC-based precoder. We can find that its power radiation pattern is not omnidirectional. The random-matrix-



(a) GCAS-based precoding.



(b) ZC-based precoding.



(c) Random-matrix-based precoding.

Fig. 5. Power radiation pattern with 4×21 URA and 8×8 STBC.

based precoder consists of four 4×33 precoding matrices. The elements in the random-matrix-based precoding matrices are generated by selecting the elements from $\{1, -1\}$ with equal probability. Fig. 4(c) describes the power radiation pattern of the random matrix-based precoder. We can observe that the power radiation pattern is not omnidirectional.

Next, we consider the massive MIMO system equipped with a URA of size 4×21, i.e., $L_1 = 4$ and $L_2 = 21$. We use the GCAS $G = \{\boldsymbol{c}_0, \boldsymbol{c}_1, \cdots, \boldsymbol{c}_7\}$ listed in Table III for the precoding matrix $\{\boldsymbol{W}_0, \boldsymbol{W}_1, \cdots, \boldsymbol{W}_7\} = \{(-1)^{\boldsymbol{c}_0}, (-1)^{\boldsymbol{c}_1}, \cdots, (-1)^{\boldsymbol{c}_7}\}$. The power radiation pattern of the GCAS-based scheme with array size 4×21 is described in Fig. 5(a). The perfect omnidirectional property can be observed. We also see that the power radiation patterns of the ZC-based precoder and the random-matrix precoder shown in Fig. 5(b) and Fig. 5(c) are not omnidirectional. The ZC-based precoding matrices are obtained by a ZC sequence of length 4 and ZC sequence of 21 [21].

B. Bit Error Rate Performance

In this subsection, we present the BER performance of our proposed 2D GCAS-based schemes. We first consider the massive MIMO system equipped with a URA of size 4×33. We let $N = 4$ and then the 4×4 orthogonal real STBC be presented as

$$\boldsymbol{S} = \begin{pmatrix} s_0 & -s_1 & -s_2 & -s_3 \\ s_1 & s_0 & s_3 & -s_2 \\ s_2 & -s_3 & s_0 & s_1 \\ s_3 & s_2 & -s_1 & s_0 \end{pmatrix}, \tag{26}$$

where s_0, s_1, s_2, s_3 are binary phase shift keying (BPSK) modulated symbols. According to (13), the signal-to-noise ratio (SNR) is given by

$$\text{SNR} = \frac{\mathbb{E}\left[\sum_{n=0}^{N-1} \left|\text{vec}(\boldsymbol{A}(\varphi, \theta))^T \text{vec}(\boldsymbol{W}_n)\right|^2\right]}{\sigma^2}, \tag{27}$$

where σ^2 is the variance of the AWGN and the maximum likelihood (ML) decoding is employed here. For each realization, the elevation and the azimuth angles are uniformly distributed at random between $[0, \pi/2]$ and $[0, 2\pi]$, respectively. For comparison, the ZC-based precoder and random-matrix-based precoder are the same as mentioned in Section IV-A. The BER performances of three different schemes are depicted in Fig. 6. We can find that the 2D GCAS-based scheme outperform the others. At BER of 10^{-4}, there are 1.6 dB and 3.6 dB gains over the ZC-based scheme and the random-matrix-based scheme, respectively.

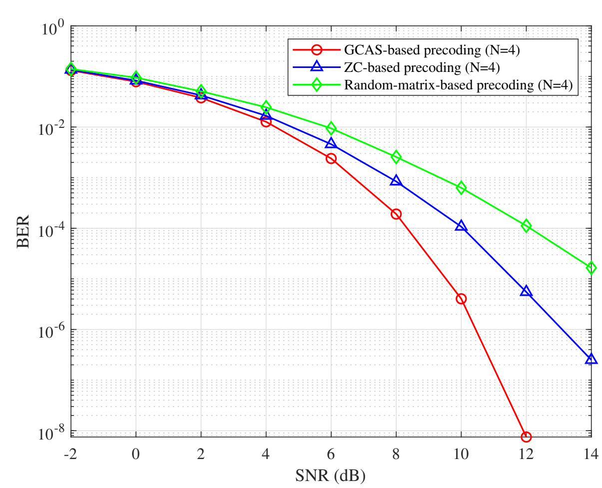


Fig. 6. BER performance of the different schemes for a 4×33 URA.

Next, we consider the massive MIMO system equipped with a URA of size 4×21. We consider 8×8 STBC and the 8×8 orthogonal real STBC is given by

$$\boldsymbol{S}=\begin{pmatrix} s_0 & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7\\ -s_1 & s_0 & s_3 & -s_2 & s_5 & -s_4 & -s_7 & s_6\\ -s_2 & -s_3 & s_0 & s_1 & s_6 & s_7 & -s_4 & -s_5\\ -s_3 & s_2 & -s_1 & s_0 & s_7 & -s_6 & s_5 & -s_4\\ -s_4 & -s_5 & -s_6 & -s_7 & s_0 & s_1 & s_2 & s_3\\ -s_5 & s_4 & -s_7 & s_6 & -s_1 & s_0 & -s_3 & s_2\\ -s_6 & s_7 & s_4 & -s_5 & -s_2 & s_3 & s_0 & -s_1\\ -s_7 & -s_6 & s_5 & -s_4 & -s_3 & s_2 & s_1 & s_0 \end{pmatrix} \tag{28}$$

where $s_0, s_1, \cdots, s_7$ are BPSK modulated symbols. We also take the ZC-based precoding and random-matrix-based precoding for comparison. The BER performance comparison for these three different schemes is depicted in Fig. 7. At BER of 10^{-4}, there are 0.2 dB and 1.8 dB gains over the ZC-based scheme and the random-matrix-based scheme, respectively. Besides, the elements of the GCAS-based precoding matrices in the simulation are all binary values, as demonstrated in Tables II and III in Appendix C, whereas the elements of the ZC-based precoding matrices are complex values. Therefore, the computational complexity in precoding based on our proposed GCASs can be reduced. As a result, the 2D GCASs are good candidates as precoding matrices for omnidirectional transmission in massive MIMO systems.

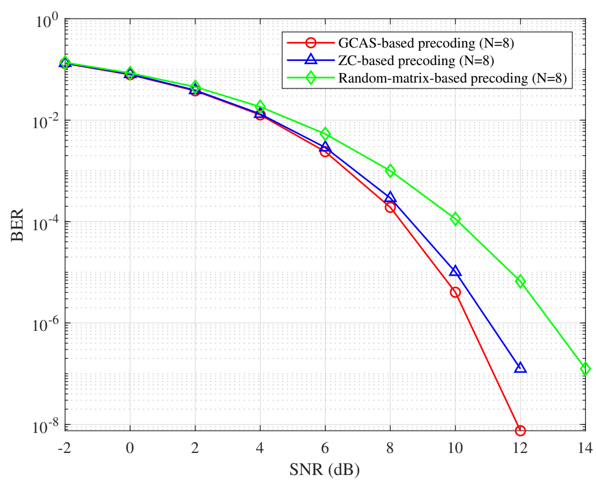


Fig. 7. BER performance of the different schemes for a 4×21 URA.

V. Conclusion

In this paper, constructions of 2D GCASs with flexible array sizes have been proposed in Theorems 1 and 2. Our constructions can be obtained directly from 2D GBFs without the aid of special sequences. Besides, our proposed GCASs have flexible array sizes which can fit more antenna configuration. Furthermore, Theorem 2 can include the results in [32] as a special case. Simulation results showed that the omnidirectional transmission can be achieved when the precoding matrices are based on the proposed GCASs. The BER performance due to their omnidirectional power radiation patterns, the ZC-based scheme and random-matix-based have inferior performances because their power radiation patterns both are not ideally omnidirectional. Although Theorems 1 and 2 can provide direct constructions of 2D GCASs, the first dimension has size L_1 limited to 2^n. Therefore, the future work includes the extension of constructions of 2D GCASs of which both dimensions have non-power-of-two sizes.

Appendix A
Proof of Theorem 1

Proof: Without loss of generality, we consider $L_1=2^n$ and $L_2=2^{m-1}+\sum_{\alpha=1}^{k-1}2^{m-k+\alpha-1}+\ 2^v$. We need to show that

$$\sum_{\boldsymbol{c}\in G}\sum_{g=0}^{L_1-1-u_1}\sum_{i=0}^{L_2-1-u_2}\left(\xi^{c_{g+u_1,i+u_2}-c_{g,i}}\right)=0 \tag{29}$$

for $0\le u_1<2^n,\ 0\le u_2<2^{m-1}+\sum_{\alpha=1}^{k-1}2^{m-k+\alpha-1}+\ 2^v$ and $(u_1,u_2)\neq(0,0)$. Then we let $h=g+u_1$ and $j=i+u_2$ for any integers g and i. We also let $(g_1,g_2,\cdots,g_n)$, $(i_1,i_2,\cdots,i_m)$, $(h_1,h_2,\cdots,h_n)$, and $(j_1,j_2,\cdots,j_m)$ be the binary representations of $g,i,h,$ and j, respectively. For the ease of presentation, we denote

$$\begin{aligned} a_l&=\begin{cases} g_l & \text{for } 1\le l\le n;\\ i_{l-n} & \text{for } n<l\le n+m;\end{cases}\\ b_l&=\begin{cases} h_l & \text{for } 1\le l\le n;\\ j_{l-n} & \text{for } n<l\le n+m;\end{cases}\end{aligned} \tag{30}$$

In what follows, we consider four cases to show that the above formula holds.

Case 1: If $a_{\pi(1)}\neq b_{\pi(1)}$, we can find that $\boldsymbol{c}'=\boldsymbol{c}+(q/2)\boldsymbol{z}_{\pi(1)}$ for any array$\boldsymbol{c}\in G$ satisfying

$$c_{h,j}-c_{g,i}-c'_{h,j}+c'_{g,i}=\frac{q}{2}(a_{\pi(1)}-b_{\pi(1)})\equiv\frac{q}{2}\pmod q. \tag{31}$$

Therefore, we have

$$\xi^{c_{h,j}-c_{g,i}}+\xi^{c'_{h,j}-c'_{g,i}}=0. \tag{32}$$

Case 2: If $a_{m+n-k+\alpha}\neq b_{m+n-k+\alpha}$, we can find that $\boldsymbol{c}'=\boldsymbol{c}+(q/2)\boldsymbol{z}_{m+n-k+\alpha}$ for any array $\boldsymbol{c}\in G$. Similar to Case 1, we have $\xi^{c_{h,j}-c_{g,i}}+\xi^{c'_{h,j}-c'_{g,i}}=0$.

Case 3: If $a_{\pi(1)}=b_{\pi(1)}$ and $a_{m+n-k+\alpha}=b_{m+n-k+\alpha}$ for $\alpha=1,2,\cdots,k$. Suppose that α' is the largest integer satisfying $a_{m+n-k+\alpha'}=b_{m+n-k+\alpha'}=0$ for $\alpha'\le k$. Then we assume β is the smallest integer which satisfies $a_{\pi(\beta)}\neq b_{\pi(\beta)}$. Let a' and b' be integers distinct from a and b, respectively, only in one position $\pi(\beta-1)$. In other words, $a'_{\pi(\beta-1)}=1-a_{\pi(\beta-1)}$ and $b'_{\pi(\beta-1)}=1-b_{\pi(\beta-1)}$. If $1\le\pi(\beta-1)\le n$, by using the above definition, we have

$$\begin{aligned} &c_{g',i}-c_{g,i}\\ &=\frac{q}{2}\Big(a_{\pi(\beta-2)}g'_{\pi(\beta-1)}-a_{\pi(\beta-2)}g_{\pi(\beta-1)}+g'_{\pi(\beta-1)}a_{\pi(\beta)}\\ &-g_{\pi(\beta-1)}a_{\pi(\beta)}\Big)+p_{\pi(\beta-1)}g'_{\pi_2(\beta-1)}-p_{\pi(\beta-1)}g_{\pi(\beta-1)}\\ &\equiv\frac{q}{2}(a_{\pi(\beta-2)}+a_{\pi(\beta)})+p_{\pi(\beta-1)}(1-2g_{\pi(\beta-1)})\pmod q. \end{aligned} \tag{33}$$

where $a'_{\pi(\beta-1)} = g'_{\pi(\beta-1)}$ and $a_{\pi(\beta-1)} = g_{\pi(\beta-1)}$. Since $a_{\pi(\beta-2)} = b_{\pi(\beta-2)}$ and $a_{\pi(\beta-1)} = b_{\pi(\beta-1)}$, we have

$$\begin{aligned} & c_{h,j} - c_{g,i} - c_{h',j} + c_{g',i} \\ & \equiv \frac{q}{2}(a_{\pi(\beta-2)} - b_{\pi(\beta-2)} + a_{\pi(\beta)} - b_{\pi(\beta)}) \\ & \quad + p_{\pi(\beta-1)}(2h_{\pi(\beta-1)} - 2g_{\pi(\beta-1)}) \\ & \equiv \frac{q}{2}(a_{\pi(\beta)} - b_{\pi(\beta)}) \equiv \frac{q}{2} \pmod q \end{aligned} \tag{34}$$

implying $\xi^{c_{h,j}-c_{g,i}}/\xi^{c_{h',j}-c_{g',i}} = -1$. We can also obtain

$$\xi^{c_{h,j}-c_{g,i}} + \xi^{c_{h',j}-c_{g',i}} = 0. \tag{35}$$

If $n < \pi(\beta-1) \leq n+m$, note that $a'_{\pi(\beta-1)} = i'_{\pi(\beta-1)-n}$ and $a_{\pi(\beta-1)} = i_\pi(\beta-1)-n$ according to (30). Following the similar argument as given above, we can get $\xi^{c_{h,j}-c_{g,i}} + \xi^{c_{h,j'}-c_{g,i'}} = 0$.

Case 4: If $a_{\pi(1)} = b_{\pi(1)}$ and $a_{m+n-k+\alpha} = b_{m+n-k+\alpha} = 1$ for $\alpha = 1, 2, \cdots, k$. We assume β is the smallest integer such that $a_{\pi(\beta)} \neq b_{\pi(\beta)}$. Since $a_s = b_s = 0$ for $s = v+n+1, v+n+2, \cdots, m+n-k$, we can obtain $\pi(\beta) \leq v+n$ implying $\pi(\beta-1) \leq v+n$. If $1 \leq \pi(\beta-1) \leq n$, by following the similar argument as given above, we have $\xi^{c_{h,j}-c_{g,i}} + \xi^{c_{h',j}-c_{g',i}} = 0$. If $n < \pi(\beta-1) \leq v+n$, we have $\xi^{c_{h,j}-c_{g,i}} + \xi^{c_{h,j'}-c_{g,i'}} = 0$. From Cases 1 to 4, the theorem can be proved. ■

APPENDIX B
PROOF OF THEOREM 2

Proof: Similarly, we consider $L_1 = 2^n$ and $L_2 = 2^{m-1} + \sum_{\alpha=1}^{k-1} 2^{\pi_1(m-k+\alpha)-1} + 2^v$. Then we would like to prove that

$$\begin{aligned} \sum_{\boldsymbol{C}} \rho(\boldsymbol{C}; u_1, u_2) &= \sum_{\boldsymbol{c}\in G} \sum_{g=0}^{L_1-1-u_1} \sum_{i=0}^{L_2-1-u_2} \left(\xi^{c_{g+u_1,i+u_2}-c_{g,i}}\right) \\ &= 0 \end{aligned} \tag{36}$$

for $0 \leq u_1 < 2^n$, $0 \leq u_2 < 2^{m-1} + \sum_{\alpha=1}^{k-1} 2^{\pi_1(m-k+\alpha)-1} + 2^v$ and $(u_1, u_2) \neq (0, 0)$. From (4) we can find that

$$\begin{aligned} \boldsymbol{c} = \frac{q}{2}\Bigg(& \sum_{l=1}^{m-k-1} \boldsymbol{x}_{\pi_1(l)}\boldsymbol{x}_{\pi_1(l+1)} + \sum_{s=1}^{n-1} \boldsymbol{y}_{\pi_2(s)}\boldsymbol{y}_{\pi_2(s+1)} \\ & + \boldsymbol{x}_{\pi_1(m)}\boldsymbol{y}_{\pi_2(n)}\Bigg) + \sum_{l=1}^{m-k} \mu_l \boldsymbol{x}_{\pi_1(l)}\boldsymbol{x}_{\pi_1(m)} + \sum_{l=1}^{m} p_l \boldsymbol{x}_l \\ & + \sum_{s=1}^{n} \kappa_s \boldsymbol{y}_s + p_0 \cdot \mathbf{1}. \end{aligned} \tag{37}$$

Then we let $h = g + u_1$ and $j = i + u_2$ for any integers g and i. Next, we discuss seven cases to complete the proof.

Case 1: Assuming $u_1 > 0$, $u_2 \geq 0$, and $g_{\pi_2(1)} \neq h_{\pi_2(1)}$, we can find an array $\boldsymbol{c}' = \boldsymbol{c} + (q/2)\boldsymbol{y}_{\pi_2(1)} \in G$ for any array $\boldsymbol{c} \in G$. Therefore, we can obtain

$$c_{h,j} - c_{g,i} - c'_{h,j} + c'_{g,i} = \frac{q}{2}(g_{\pi_2(1)} - h_{\pi_2(1)}) \equiv \frac{q}{2} \pmod q \tag{38}$$

Since $g_{\pi_2(1)} \neq h_{\pi_2(1)}$, we have

$$\xi^{c_{h,j}-c_{g,i}}/\xi^{c'_{h,j}-c'_{g,i}} = \xi^{\frac{q}{2}} = -1. \tag{39}$$

Thus,

$$\xi^{c_{h,j}-c_{g,i}} + \xi^{c'_{h,j}-c'_{g,i}} = 0. \tag{40}$$

Case 2: If $u_1 > 0$, $u_2 \geq 0$, and $g_{\pi_2(1)} = h_{\pi_2(1)}$. Let β be the smallest integer such that $g_{\pi_2(\beta)} \neq h_{\pi_2(\beta)}$. We define g' and h' are two integers which are distinct from g and h only in one position $\pi_2(\beta-1)$, respectively. Then, similar to Case 2 of Theorem 1, we have

$$\xi^{c_{h,j}-c_{g,i}} + \xi^{c_{h',j}-c_{g',i}} = 0. \tag{41}$$

Case 3: We suppose $i_m \neq j_m$, $u_1 = 0$ and $u_2 > 0$. We let g' be an integer distinct from i only in one position, i.e., $g'_{\pi_2(n)} = 1 - g_{\pi_2(n)}$. Similar to Case 3 of Theorem 1, we have $\xi^{c_{g,j}-c_{g,i}} + \xi^{c_{g',j}-c_{g',i}} = 0$.

Case 4: If $u_1 = 0$, $u_2 > 0$, and $i_{\pi_1(1)} \neq j_{\pi_1(1)}$ or $i_{\pi_1(m-k+\alpha)} \neq j_{\pi_1(m-k+\alpha)}$, we can find an array $\boldsymbol{c}' = \boldsymbol{c} + (q/2)\boldsymbol{x}_{\pi_1(1)} \in G$ or $\boldsymbol{c}' = \boldsymbol{c} + (q/2)\boldsymbol{x}_{\pi_1(m-k+\alpha)}$ for any array $\boldsymbol{c} \in G$. Similar to Case 1, we can obtain $\xi^{c_{g,j}-c_{g,i}} + \xi^{c'_{g,j}-c'_{g,i}} = 0$.

Case 5: Suppose $u_1 = 0$, $u_2 > 0$, $i_{\pi_1(1)} = j_{\pi_1(1)}$, and $i_{\pi_1(m-k+\alpha)} = j_{\pi_1(m-k+\alpha)}$ for all $\alpha = 1, 2, \cdots, k$. Suppose that α' is the largest non-negative integer satisfying $i_{\pi_1(m-k+\alpha')} = j_{\pi_1(m-k+\alpha')} = 0$. Then we assume β is the smallest integer which satisfies $i_{\pi_1(\beta)} \neq j_{\pi_1(\beta)}$. Here, we have $i_s = j_s = 0$ for $s = \pi_1(m-k+\alpha')+1, \pi_1(m-k+\alpha')+2, \ldots, m-1$, and $s \neq \pi_1(m-k+\alpha)$ for $\alpha = \alpha'+1, \alpha'+2, \ldots, k$. Hence, it implies $\pi_1(\beta) < \pi_1(m-k+\alpha')$ and $\pi_1(\beta-1) < \pi_1(m-k+\alpha')$ according to the condition (C-3). Let i' and j' be integers that differ from i and j, respectively, in the position $\pi_1(\beta-1)$. Similar to Case 2, we have

$$\xi^{c_{g,j}-c_{g,i}} + \xi^{c_{g,j'}-c_{g,i'}} = 0. \tag{42}$$

Case 6: Suppose $u_1 = 0$, $u_2 > 0$, $i_{\pi_1(1)} = j_{\pi_1(1)}$, and $i_{\pi_1(m-k+\alpha)} = j_{\pi_1(m-k+\alpha)} = 1$ for all $\alpha = 1, 2, \cdots, k$. Then we assume β is the smallest integer which satisfies $i_{\pi_1(\beta)} \neq j_{\pi_1(\beta)}$. Since $i_s = j_s = 0$ for $s = v+1, v+2, \cdots, m-k$ and $s \neq \pi_1(m-k+\alpha)$ for $\alpha = 1, 2, \ldots, k-1$, we can obtain $\pi_1(\beta) \leq v$ implying $\pi_1(\beta-1) \leq v$. Similar to Case 2, we have

$$\xi^{c_{g,j}-c_{g,i}} + \xi^{c_{g,j'}-c_{g,i'}} = 0. \tag{43}$$

From Cases 1 to 6, the theorem can be proved. ■

APPENDIX C
CONSTRUCTED GCASS IN EXAMPLES 3 AND 4

The constructed GCASs in Example 3 and Example 4 are presented in Table II and Table III, respectively.

ACKNOWLEDGMENT

The authors would like to thank the Editor Prof. Angeliki Alexiou and the anonymous reviewers for their valuable comments that have greatly improved the quality of this paper.

TABLE II
THE CONSTRUCTED $(4, 4, 33)$-GCAS IN EXAMPLE 3

$c_0 =$	0	1	1	1	1	0	1	1	1	0	0	0	1	0	1	1	1	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	1
	1	0	0	0	0	1	0	0	0	1	1	1	0	1	0	0	0	1	1	1	1	0	1	1	0	1	1	1	0	1	0	0	1
	1	0	0	0	0	1	0	0	0	1	1	1	0	1	0	0	0	1	1	1	1	0	1	1	0	1	1	1	0	1	0	0	0
	1	0	0	0	0	1	0	0	0	1	1	1	0	1	0	0	0	1	1	1	1	0	1	1	0	1	1	1	0	1	0	0	1
$c_1 =$	0	0	1	0	1	1	1	0	1	1	0	1	1	1	1	0	1	1	0	1	0	0	0	1	1	1	0	1	1	1	1	0	1
	1	1	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	1	1	1	0	0	0	1	0	0	0	0	1	1
	1	1	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	1	1	1	0	0	0	1	0	0	0	0	1	0
	1	1	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	1	1	1	0	0	0	1	0	0	0	0	1	1
$c_2 =$	0	1	1	1	1	0	1	1	1	0	0	0	1	0	1	1	1	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	1
	1	0	0	0	0	1	0	0	0	1	1	1	0	1	0	0	0	1	1	1	1	0	1	1	0	1	1	1	0	1	0	0	1
	0	1	1	1	1	0	1	1	1	0	0	0	1	0	1	1	1	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	1
	0	1	1	1	1	0	1	1	1	0	0	0	1	0	1	1	1	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0
$c_3 =$	0	0	1	0	1	1	1	0	1	1	0	1	1	1	1	0	1	1	0	1	0	0	0	1	1	1	0	1	1	1	1	0	1
	1	1	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	1	1	1	0	0	0	1	0	0	0	0	1	1
	0	0	1	0	1	1	1	0	1	1	0	1	1	1	1	0	1	1	0	1	0	0	0	1	1	1	0	1	1	1	1	0	1
	0	0	1	0	1	1	1	0	1	1	0	1	1	1	1	0	1	1	0	1	0	0	0	1	1	1	0	1	1	1	1	0	0

TABLE III
THE CONSTRUCTED $(8, 4, 21)$-GCAS IN EXAMPLE 4

$c_0 =$	0	0	0	1	0	1	1	1	0	0	1	0	1	0	1	1	0	1	1	1	0
	0	0	0	1	0	1	1	1	0	0	1	0	1	0	1	1	0	1	1	1	0
	0	1	0	0	0	0	1	0	0	1	1	1	1	1	1	0	0	0	1	0	0
	1	0	1	1	1	1	0	1	1	0	0	0	0	0	0	1	1	1	0	1	1
$c_1 =$	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	0	1	1	1	1
	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	0	1	1	1	1
	0	1	0	0	1	1	0	1	0	1	1	1	0	0	0	1	0	0	1	0	1
	1	0	1	1	0	0	1	0	1	0	0	0	1	1	1	0	1	1	0	1	0
$c_2 =$	0	0	0	1	0	1	1	1	0	0	1	0	1	0	1	1	1	0	0	0	1
	0	0	0	1	0	1	1	1	0	0	1	0	1	0	1	1	1	0	0	0	1
	0	1	0	0	0	0	1	0	0	1	1	1	1	1	1	0	1	1	0	1	1
	1	0	1	1	1	1	0	1	1	0	0	0	0	0	0	1	0	0	1	0	0
$c_3 =$	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0
	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0
	0	1	0	0	1	1	0	1	0	1	1	1	0	0	0	1	1	1	0	1	0
	1	0	1	1	0	0	1	0	1	0	0	0	1	1	1	0	0	0	1	0	1
$c_4 =$	0	0	0	1	0	1	1	1	0	0	1	0	1	0	1	1	0	1	1	1	0
	1	1	1	0	1	0	0	0	1	1	0	1	0	1	0	0	1	0	0	0	1
	0	1	0	0	0	0	1	0	0	1	1	1	1	1	1	0	0	0	1	0	0
	0	1	0	0	0	0	1	0	0	1	1	1	1	1	1	0	0	0	1	0	0
$c_5 =$	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	0	1	1	1	1
	1	1	1	0	0	1	1	1	1	1	0	1	1	0	1	1	1	0	0	0	0
	0	1	0	0	1	1	0	1	0	1	1	1	0	0	0	1	0	0	1	0	1
	0	1	0	0	1	1	0	1	0	1	1	1	0	0	0	1	0	0	1	0	1
$c_6 =$	0	0	0	1	0	1	1	1	0	0	1	0	1	0	1	1	1	0	0	0	1
	1	1	1	0	1	0	0	0	1	1	0	1	0	1	0	0	0	1	1	1	0
	0	1	0	0	0	0	1	0	0	1	1	1	1	1	1	0	1	1	0	1	1
	0	1	0	0	0	0	1	0	0	1	1	1	1	1	1	0	1	1	0	1	1
$c_7 =$	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0
	1	1	1	0	0	1	1	1	1	1	0	1	1	0	1	1	0	1	1	1	1
	0	1	0	0	1	1	0	1	0	1	1	1	0	0	0	1	1	1	0	1	0
	0	1	0	0	1	1	0	1	0	1	1	1	0	0	0	1	1	1	0	1	0

REFERENCES

[1] M. J. E. Golay, “Complementary series,” *IRE Trans. Inf. Theory*, vol. IT-7, pp. 82–87, Apr. 1961.

[2] C.-C. Tseng and C. L. Liu, “Complementary sets of sequences,” *IEEE Trans. Inf. Theory*, vol. IT-18, no. 5, pp. 644–652, Sep. 1972.

[3] N. Suehiro and M. Hatori, “N-shift cross-orthogonal sequences,” *IEEE Trans. Inf. Theory*, vol. 34, no. 1, pp. 143–146, Jan. 1988.

[4] A. Pezeshki, A. R. Calderbank, W. Moran, and S. D. Howard, “Doppler resilient Golay complementary waveforms,” *IEEE Trans. Inf. Theory*, vol. 54, no. 9, pp. 4254–4266, Sep. 2008.

[5] P. Spasojevic and C. N. Georghiades, “Complementary sequences for ISI channel estimation,” *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 1145–1152, Mar. 2001.

[6] D. Su, Y. Jiang, X. Wang, and X. Gao, “Omnidirectional precoding for massive MIMO with uniform rectangular array—part I: complementary codes-based schemes,” *IEEE Trans. Signal Process.*, vol. 67, no. 18, pp. 4761–4771, Sep. 2019.

[7] Z. Liu, Y. Li, and Y. L. Guan, “New constructions of general QAM Golay complementary sequences,” *IEEE Trans. Inf. Theory*, vol. 59, no. 11, pp. 7684–7692, Nov. 2013.

[8] R. van Nee, “OFDM codes for peak-to-average power reduction and error correction,” in *Proc. IEEE Global Telecommun. Conf.*, London, U.K., Nov. 1996, pp. 740–744.

[9] J. A. Davis and J. Jedwab, “Peak-to-mean power control in OFDM, Golay complementary sequences, and Reed-Muller codes,” *IEEE Trans. Inf. Theory*, vol. 45, no. 7, pp. 2397–2417, Nov. 1999.

[10] K. G. Paterson, “Generalized Reed-Muller codes and power control in OFDM modulation,” *IEEE Trans. Inf. Theory*, vol. 46, no. 1, pp. 104–120, Jan. 2000.

[11] K.-U. Schmidt, “Complementary sets, generalized Reed-Muller codes, and power control for OFDM,” *IEEE Trans. Inf. Theory*, vol. 53, no. 2, pp. 808–814, Feb. 2007.

[12] C.-Y. Chen, C.-H. Wang, and C.-C. Chao, “Complementary sets and Reed-Muller codes for peak-to-average power ratio reduction in OFDM,” in *Proc. 16th Int. Symp. AAECC, LNCS 3857*, Las Vegas, NV, Feb. 2006, pp. 317–327.

[13] C.-Y. Chen, “Complementary sets of non-power-of-two length for peak-to-average power ratio reduction in OFDM,” *IEEE Trans. Inf. Theory*, vol. 62, no. 12, pp. 7538–7545, Dec. 2016.

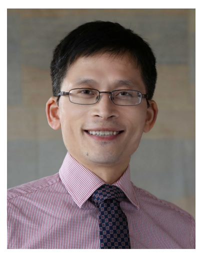
[14] Z. Liu, Y. L. Guan, and H. H. Chen, “Fractional-delay-resilient receiver design for interference-free MC-CDMA communications based on complete complementary codes,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 3, pp. 1226–1236, Mar. 2015.

[15] H.-H. Chen, J.-F. Yeh, and N. Suehiro, "A multicarrier CDMA architecture based on orthogonal complete complementary codes for new generations of wideband wireless communications," *IEEE Commun. Mag.*, vol. 39, pp. 126–134, Oct. 2001.
[16] N. Suehiro, "A signal design without co-channel interference for approximately synchronized CDMA systems," *IEEE J. Sel. Areas Commun.*, vol. 12, pp. 837–841, Jun. 1994.
[17] S.-M. Tseng and M. R. Bell, "Asynchronous multicarrier DS-CDMA using mutually orthogonal complementary sets of sequences," *IEEE Trans. Commun.*, vol. 48, no. 1, pp. 53–59, Jan. 2000.
[18] J. M. Groenewald and B. T. Maharaj, "MIMO channel synchronization using Golay complementary pairs," in *Proc. AFRICON 2007*, Windhoek, South Africa, Sep. 2007, pp. 1–5.
[19] S. Boyd, "Multitone signals with low crest factor," *IEEE Trans. Circuits Syst.*, vol. CAS-33, no. 10, pp. 1018–1022, Oct. 1986.
[20] A.-A. Lu, X. Gao, X. Meng, and X.-G. Xia, "Omnidirectional precoding for 3D massive MIMO with uniform planar arrays," *IEEE Trans. Wireless Commun.*, vol. 19, no. 4, pp. 2628–2642, Apr. 2020.
[21] F. Li, Y. Jiang, C. Du, and X. Wang, "Construction of Golay complementary matrices and its applications to MIMO omnidirectional transmission," *IEEE Trans. Signal Process.*, vol. 69, pp. 2100–2113, Mar. 2021.
[22] X. Meng, X. Gao, and X.-G. Xia, "Omnidirectional precoding based transmission in massive MIMO systems," *IEEE Trans. Commun.*, vol. 64, no. 1, pp. 174–186, Jan. 2016.
[23] Y. Jiang, F. Li, X. Wang, and J. Li, "Autocorrelation complementary matrices," in *Proc. 53rd Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, Nov. 2019, pp. 1596–1600.
[24] S. Matsufuji, R. Shigemitsu, Y. Tanada, and N. Kuroyanagi, "Construction of complementary arrays," in *Proc. Joint 1ST Workshop on Mobile Future Symp. Trends Commun. (SympoTIC)*, Bratislave, Slovakia, Oct. 2004, pp. 78–81.
[25] Z. Wang and G. Gong, "Constructions of complementary sequence sets and complete complementary codes by 2-level autocorrelation sequences and permutation polynomials," May 2020. [Online]. Available: https://arxiv.org/abs/2005.05825
[26] Z. Wang, D. Ma, G. Gong, and E. Xue, "New construction of complementary sequence (or array) sets and complete complementary codes," *IEEE Trans. Inf. Theory*, vol. 67, no. 7, pp. 4902–4928, Jul. 2021.
[27] C.-Y. Pai and C.-Y. Chen, "Constructions of two-dimensional Golay complementary array pairs based on generalized Boolean functions," in *Proc. IEEE Int. Symp. Inf. Theory*, Los Angeles, California, USA, Jun. 2020, pp. 2931–2935.
[28] C.-Y. Pai and C.-Y. Chen, "Two-dimensional Golay complementary array pairs/sets with bounded row and column sequence PAPRs," *IEEE Trans. Commun.*, vol. 70, no. 6, pp. 3695–3707, Jun. 2022.
[29] Z. Liu, Y. L. Guan, and U. Parampalli, "New complete complementary codes for peak-to-mean power control in multi-carrier CDMA," *IEEE Trans. Commun.*, vol. 62, pp. 1105–1113, Mar. 2014.
[30] C.-Y. Pai, Z. Liu, Y.-Q. Zhao, Z.-M. Hunag, and C.-Y. Chen, "Designing two-dimensional complete complementary codes for omnidirectional transmission in massive MIMO systems," in *Proc. IEEE Int. Symp. Inf. Theory*, Espoo, Finland, Jun. 2022, pp. 1699–1704.
[31] T. Liu, X. Men, Y. Li, and X. Chen, "Constructions of 2-D Golay complementary array sets for MIMO omnidirectional transmission," *IEEE Commun. Lett.*, pp. 1459 – 1463, Jul. 2022.
[32] B. Shen, Y. Yang, and R. Ren, "Three constructions of Golay complementary array sets," *Adv. Math. Commun.*, Oct. 2022.

You-Qi Zhao received his B.S. degree in mechanical and electro-mechanical engineering from the National Sun Yat-sen University (NSYSU), Kaohsiung, in 2020 and M.S. degree in engineering science from the National Cheng Kung University (NCKU), Tainan, Taiwan in 2022. Currently, he is an engineer at the Wistron NeWeb Corp. in Taiwan.

Cheng-Yu Pai (Graduate Student Member, IEEE) received the B.S. degree in engineering science from the National Cheng Kung University (NCKU), Tainan, Taiwan in 2018, where he is currently pursuing the Ph.D. degree in engineering science. From July 2022 to June 2023, he was a Visiting Ph.D. Student with the University of Essex, Colchester, UK (with Prof. Zilong Liu). His research interest includes sequence design and its applications in communications. He was a recipient of the 2nd Hon Hai Technology Award administered by Hon Hai Education Foundation, Taiwan, in 2022.

Zhen-Ming Huang (Graduate Student Member, IEEE) received the B.S. degree in electrical engineering from the National Kaohsiung University of Science and Technology (NKUST), Kaohsiung, Taiwan, in 2019, and the M.S. degree in engineering science from the National Cheng Kung University (NCKU), Tainan, Taiwan, in 2021, where he is currently pursuing the Ph.D. degree in engineering science and communications engineering. His research interests include sequences and their applications.



Zilong Liu (Senior Member, IEEE) has been with the School of Computer Science and Electronic Engineering, University of Essex, since December 2019, first as a Lecturer and then a Senior Lecturer since October 2023. He received his PhD (2014) from School of Electrical and Electronic Engineering, Nanyang Technological University (NTU, Singapore), Master Degree (2007) in the Department of Electronic Engineering from Tsinghua University (China), and Bachelor Degree (2004) in the School of Electronics and Information Engineering from Huazhong University of Science and Technology (HUST, China). He was a Visiting PhD student to the University of Melbourne and the Hong Kong University of Science and Technology. From Jan. 2018 to Nov. 2019, he was a Senior Research Fellow at the Institute for Communication Systems (ICS), Home of the 5G Innovation Centre (5GIC), University of Surrey, during which he studied the air-interface design of 5G communication networks (e.g., machine-type communications, V2X communications, 5G New Radio). Prior to his career in the UK, he spent nine and half years in NTU, first as a Research Associate (Jul. 2008 to Oct. 2014) and then a Research Fellow (Nov. 2014 to Dec. 2017). His PhD thesis "Perfect- and Quasi- Complementary Sequences", focusing on fundamental limits, algebraic constructions, and applications of complementary sequences in wireless communications, has settled a few long-standing open problems in the field. His research lies in the interplay of coding, signal processing, and communications, with a major objective of bridging theory and practice as much as possible. Recently, he has developed an interest in advanced 6G V2X communication technologies for future connected autonomous vehicles as well as machine learning for enhanced communications and networking. He is a Senior Member of IEEE and an Associate Editor of IEEE Transactions on Vehicular Technology, IEEE Transactions on Neural Networks and Learning Systems, IEEE Wireless Communications Letters, IEEE Access, Frontiers in Communications and Networks, Frontiers in Signal Processing, and Advances in Mathematics of Communications. He was a Track Co-Chair on Networking and MAC in IEEE PIMRC'2023. He was the Hosting General Co-Chair of the 10th IEEE International Workshop on Signal Design and its Applications in Communications (IWSDA'2022) and a TPC Co-Chair of the 2020 IEEE International Conference on Advanced Networks and Telecommunications Systems (ANTS'2020). He was a tutorial speaker of VTC-Fall'2021 and APCC'2021 on code-domain NOMA. He was a Consultant to the Japanese government on 6G assisted autonomous driving in 2023. Details of his research can be found at: https://sites.google.com/site/zilongliu2357.

Chao-Yu Chen (Senior Member, IEEE) received the B.S. degree in electrical engineering and the M.S. and Ph.D. degrees in communications engineering from the National Tsing Hua University (NTHU), Hsinchu, in 2000, 2002 and 2009, respectively, under the supervision of Prof. Chi-chao Chao.

He was a Visiting Ph.D. Student with the University of California, Davis, CA, USA, from 2008 to 2009 (with Prof. Shu Lin). From 2009 to 2016, he was a Technical Manager with the Communication System Design Division, Mediatek Inc., Hsinchu, Taiwan. From July 2018 to August 2018, he was with the University of California at Davis, as a Visiting Scholar (with Prof. Shu Lin). Since February 2016, he has been a Faculty Member with the National Cheng Kung University (NCKU), Tainan, Taiwan, where he is currently an Associate Professor with the Department of Electrical Engineering and the director of the Institute of Computer and Communication Engineering. His current research interests include sequence design, error-correcting codes, digital communications, and wireless networks.

Dr. Chen was a recipient of the 15th and 20th Y. Z. Hsu Science Paper Award administered by Far Eastern Y. Z. Hsu Science and Technology Memorial Foundation, Taiwan, in 2017 and 2022, and the Best Paper Award for Young Scholars by the IEEE Information Theory Society Taipei/Tainan Chapter and the IEEE Communications Society Taipei/Tainan Chapter in 2018. Since January 2019, he has been serving as the Vice Chair for the IEEE Information Theory Society Tainan Chapter.