

**STRUCTURAL POUNDING OF AN ADJACENT  
BUILDING UNDER DYNAMIC  
LOADING**

Thesis submitted in accordance with the requirements of  
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by

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## ABSTRACT

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Structural pounding occurs when a building dynamically sways during an earthquake, and collides with an adjacent building in close proximity to it. Observations on past earthquakes have revealed that pounding can be a severe problem during an earthquake. The extent of the significant damage which pounding can produce has never been demonstrated better than the 1985 Mexico earthquake where over 40% of the severely damaged buildings surveyed, experienced pounding.

In analysing pounding two approaches have been used in the past, the finite element method and the displacement compatibility method. The finite element method can be used to analyse possible pounding at any location. In this approach, at all possible pounding locations Lagrange multipliers are used to represent the contact forces. These together with the initial separation will constrain the total potential of the system hence producing an enlarged matrix form of the equations of motion.

The displacement compatibility method has been used with predetermined pounding location. This however has been restricted to aligned floor levels so that the two sets of matrices, non contact and contact, produced are not as large as for the finite element method.

In this study the displacement compatibility method has been developed to analyse pounding contact at interstorey height. This has meant a change in the matrix formulation during the contact stage only. Therefore, the method can be considered to be as versatile as the finite element method. In the development of the model at each stage, the results obtained were verified by comparison with the results obtained with the Oasys DYNA3D computer package.

The main stages in the development of the model were as follows.

- (a) Interstorey pounding of a structure, with axially rigid slabs, against a rigid structure.

- (b) Interstorey pounding of a structure, with axially flexible slabs, against a rigid structure.
- (c) Interstorey pounding of a structure by another structure, with both structures having axially rigid slabs.

In all three cases, pounding was induced by offsetting the structure and then releasing it. For the first case, the pounding of the structure due to an earthquake input motion was also analysed. The effect of damping on the behaviour of the structure was also considered.

The results obtained show

- (a) the magnitude of the contact forces is dependent upon the contact point location.
- (b) the axial flexibility of the slabs has an effect on the magnitude of the contact force and the length of the contact time.
- (c) damping also has an effect on the contact force values.
- (d) the magnitude of the contact forces in some cases suggests the possibility of nonlinear behaviour occurring in the columns. Therefore the results obtained ought to be viewed with some caution.

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## NOTATION

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$\alpha$	generalized mode shapes
$\beta_i$	participation factor = $Q_j^T \underline{m} \underline{r} / M_i$
$\delta_{jl}$	distance between nodes $j$ and $l$
$\delta W_{np}$	work of the nonpotential forces
$\theta$	phase angle
$\lambda_i$	Lagrange multipliers
$\mu_i$	eigenvalues
$\nu$	Poisson's ratio
$\xi$	damping coefficient
$\xi_c$	damping coefficient during contact stage
$\xi_{nc}$	damping coefficient during no contact stage
$\rho$	density
$\Sigma$	summation sign
$\tau$	time
$\phi_i$	geometric constraints
$\omega, \omega_i$	undamped natural frequency
$\omega_c, \omega_{ic}$	undamped natural frequency during contact stage
$\omega_{nc}$	undamped natural frequency during no contact stage
$\omega_d$	damped natural frequency
$a, a, \mathbf{a}$	pounding location that is a distance from the top floor
$A$	cross sectional area
$A_s$	shear area
$b, b$	a distance of height of the column minus distance $a$
$b_s$	static preload force for SDOF system with flexible slab
$\bar{b}$	column vector of static preload forces for MDOF systems with rigid slab
$\underline{b}_s$	column vector of static preload forces for MDOF systems with flexible slab

$\underline{c}, \bar{c}$	state 1 and 2 damping matrices, respectively
$c$	damping constant
$c_c$	critical damping ( $2m\omega$ )
$C_i$	modal damping matrix
$C1, C2$	Rayleigh damping constants
$c_p$	diagonal damping matrix corresponding to the impact elements
$c_{Us}$	the damping coefficient of the dashpot working in parallel to the impact spring.
$c_s$	diagonal damping matrix, with elements the damping constant $c_i$
$dt$	time step
$dt_c$	contact stage duration
$D_i$	a vector including stiffness terms due to the impact elements
$e$	the coefficient of restitution
$E$	Young's Modulus
$EI$	column rigidity
$f_i$	the equivalent lateral forces
$(f_s)_{max}$	the maximum floor force (response spectra)
$Fd(\dot{u})$	the damping force-velocity relationship
$Fs$	spring force
$Fs(u)$	the spring force-displacement relationship
$F3, F2$	top and middle floor acting forces, respectively
$g_i$	generalized coordinates
$G_i$	generalized forces
$h_i$	the height of the floor above the base of a structure
$H$	total height of the building
$i$	number of modes
$I$	identity matrix
$I_{xx}$	torsional constant
$I_{yy}, I_{zz}$	moment of inertia about yy and zz axes, respectively.
$k, k_c$	state 1 and 2 building stiffness, respectively (SDOF) with rigid slab
$\underline{k}, \bar{k}$	state 1 and 2 stiffness matrices, respectively (MDOF) with rigid slab

$k^*, k_c^*$	state 1 and 2 building stiffness, respectively (SDOF) with flexible slab
$\bar{k}^*, \bar{k}_c^*$	state 1 and 2 stiffness matrices, respectively (MDOF) with flexible slab
$k1^*, k2^*$	equivalent top slab stiffness values for chapter 3
$k3, k2, k1$	top, middle and bottom column stiffnesses, respectively
$k3^*, k4^*$	equivalent middle slab stiffness values for chapter 3
$ka^*, kb^*$	equivalent slab stiffness values (MDOF) with flexible slab
$kc^*, kd^*$	equivalent slab stiffness values (MDOF) with flexible slab
$k_{AB}$	expressed in term of a and L
$k_D$	the determinant of the inverse matrix
$kf1^*$	equivalent slab stiffness values due to contact force (SDOF) with flexible slab
$kf2^*$	equivalent slab stiffness values due to axial deformation (SDOF) with flexible slab
$kp$	coefficient of contact force, during contact stage
$ks$	slab stiffness
$k_{Us}$	the stiffness of the impact spring
$K_i$	modal stiffness matrix
$L$	interstorey height
$L_\lambda$	Lagrange function
$\underline{m}$	mass matrices for MDOF system
$m, m_i$	building mass
$m3, m2, m1$	top, middle and bottom masses, respectively
$M_i$	normalised modal mass matrix = $Q_j^T \underline{m} Q_i = 1$
$M_{max}$	the maximum overturning moment at the base (response spectra)
$M_{oi}$	the moment at floor level
$M_{on}$	the total base moment of the structure
$n$	the total number of modes
$N$	total number of generalized coordinates

$p$	the factor relate the 2nd (stiffer) branch to the 1st branch of $F_s$
$P$	contact forces
$P_i$	applied forces that acting on the system
$P^*$	equivalent force on an adjacent building
$P_i^*$	modal force vectors
$P^T$	transpose of $P$
$Q$	modal matrix
$\underline{r}$	Pseudo static vector or Earthquake Influence Coefficient
$R$	the rate of change of the ground acceleration
$R_{\text{eff}}$	the effective force of the slab reactions (SDOF)
$R_i$	a vector of structural resistances, functions of the displacement, $u_i$
$R_k$	slab reactions due to slab flexibility
$R_p$	slab reactions due to point load, $P$
$R_w$	the response modification factor
$R_{1\text{eff}}$	the effective forces of the middle floor reactions (2DOF)
$R_{2\text{eff}}$	the effective forces of the top floor reactions (2DOF)
$\underline{S}$	a stiffness matrix corresponding to the impact elements
$S_d, S_v, S_a$	Pseudo-displacement, Pseudo-velocity, Pseudo-acceleration, respectively
$t, t_i$	time
$t_i$	initial contact time
$t_{\text{IC}}$	time interval between two successive contacts
$T$	undamped natural period
$T$	kinetic energy
$T_c$	undamped natural period during contact stage
$T_d$	damped natural period
$T_{\text{flex}}$	natural period for flexible slab case
$T_{\text{rigid}}$	natural period for rigid slab case
$u_{di}$	flexibility slab displacement at the point of contact
$u_{f2}, u_{f1}$	the right hand side of column displacement in flexible slab case
$\ddot{u}_g, \dot{u}_g, u_g$	ground acceleration, velocity and displacement

$u_i, \dot{u}_i$	displacement at the contact point
$\ddot{u}_i, \dot{u}_i, u_i$	acceleration, velocity and displacement of mode $i$ th
$\dot{u}_1', \dot{u}_2'$	uniform velocities of masses $m_1$ and $m_2$ after impact
$u_{ki}$	axial displacement of the slab due to its flexibility
$U_m$	deflection during contact stage for two flexible buildings
$u_p$	column displacement with stopper at the point of contact
$u_{si}$	the static preload for SDOF system
$u_{st}$	the static displacement at the top of building
$u_{top}, u_{mid}, u_{bott}$	the top, middle and bottom floor displacements, respectively
$u_1, \ddot{u}_1$	first floor displacements
$u_2, \ddot{u}_2$	second floor displacements
$u_{2_{max}}$	maximum second floor displacement
$u_3, \ddot{u}_3$	third floor displacements
$U_s, U_s$	building separation
$U$	potential energy
$U_{s_{min}}$	the minimum separation distance between two buildings
$v_{top}, v_{mid}, v_{bott}$	the top, middle and bottom floor velocities, respectively
$V_j$	an arbitrarily determined modal amplitude
$V_{on}$	the shear at the base of a structure
$V_{max}$	the maximum base shear (response spectra)
$W$	work
$\ddot{y}_i, \dot{y}_i, y_i$	uncoupled acceleration, velocity and displacement of mode $i$ th
$y_{sii}$	the normalized static preload for MDOF systems



# **CHAPTER 1**

## **INTRODUCTION**

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### **1.1 GENERAL**

A structural-dynamic problem differs from a static problem by the fact that the dynamic problem is time varying by nature. Because the load and the response both vary with time, it is evident that a dynamic problem does not have a single solution, as a static problem does; instead a succession of solutions corresponding to all times of interest within a response history is needed which is clearly more complex and time consuming than a static analysis.

Almost any type of structural system may be subjected to one form or another of dynamic loading during its lifetime. There are two types of prescribed loadings, periodic and nonperiodic [1]. The simplest periodic loading is the sinusoidal variation which is termed simple harmonic. Nonperiodic loadings may be either short-duration impulsive loadings (a blast or explosion loading on building) or long duration loadings (an earthquake ).

Although the dynamic loading acting on structural systems may result from any of several different source mechanisms, including wind or wave action and vehicular motions, the type of dynamic input which is of greatest importance to the structural engineer in some parts of the world is undoubtedly that produced by earthquakes. The earthquake loading is unique among other types of dynamic loadings because a great earthquake can cause high casualties and severe damage in major cities.

### **1.2 NATURE OF EARTHQUAKES**

This section introduces the nature of earthquakes. Considering the significance of the earthquakes effect on human societies, scientists have tried to explain why they occur and what is the theory behind it.

### 1.2.1 Causes of Earthquakes

Seismologists have defined two distinct mechanisms which directly result in earthquakes. The major seismic activity is due to *Tectonic* earthquakes which result from the disturbances (wave of distortion) from a rupture or sudden movement along the existing fault in the earth's crust (upper layer of the earth) [1,2,3]. The second one is due to *volcanic* earthquake. It is associated with volcanic eruptions or underground movements of magma.

### 1.2.2 Plate Tectonics theory

According to the plate tectonics theory, the earth's outer layer can be imagined as a lithosphere which consists of several crusts or plates. The lithosphere, the top 100km or so of the earth, is made up of continental and oceanic crust plus rigid mantle rocks beneath, and covers an underlying softer layer. Earthquakes and volcanic activities are believed to be due to the interaction of these plates. Figure 1.1 shows a map of recent earthquakes, volcanic eruptions and the plate boundaries. The various forms of plate boundaries, divergent (spreading), convergent (subduction) and transform (strike-slip) are also denoted.

Almost all major earthquakes and approximately 95% of all earthquakes have occurred in two long and relatively narrow zones;

a) The principal zone, which accounts for about 80% of all earthquakes, is called the *Circum-Pacific Seismic Zone*, which borders the Pacific ocean, running up the west coast of America through the Aleutians, and down the coast of Asia through Japan and the Philippines, to New Zealand (Figure 1.1) . Paralleling this zone, about a hundred or so miles to one side of it, is the chain of active volcanoes making up what has often been referred to as the *Pacific circle of fire*. The closeness of these two zones once led to the mistaken belief that earthquakes are due principally to volcanic activity [2].

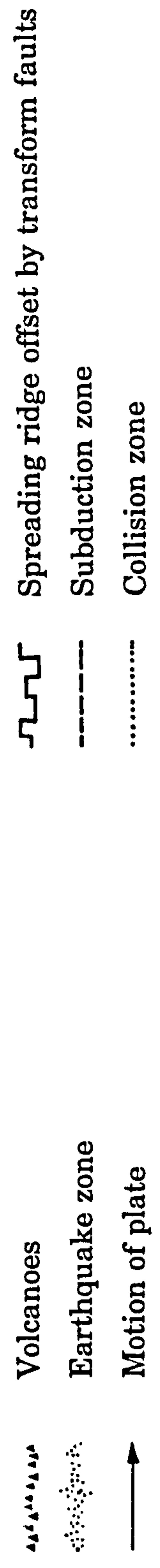
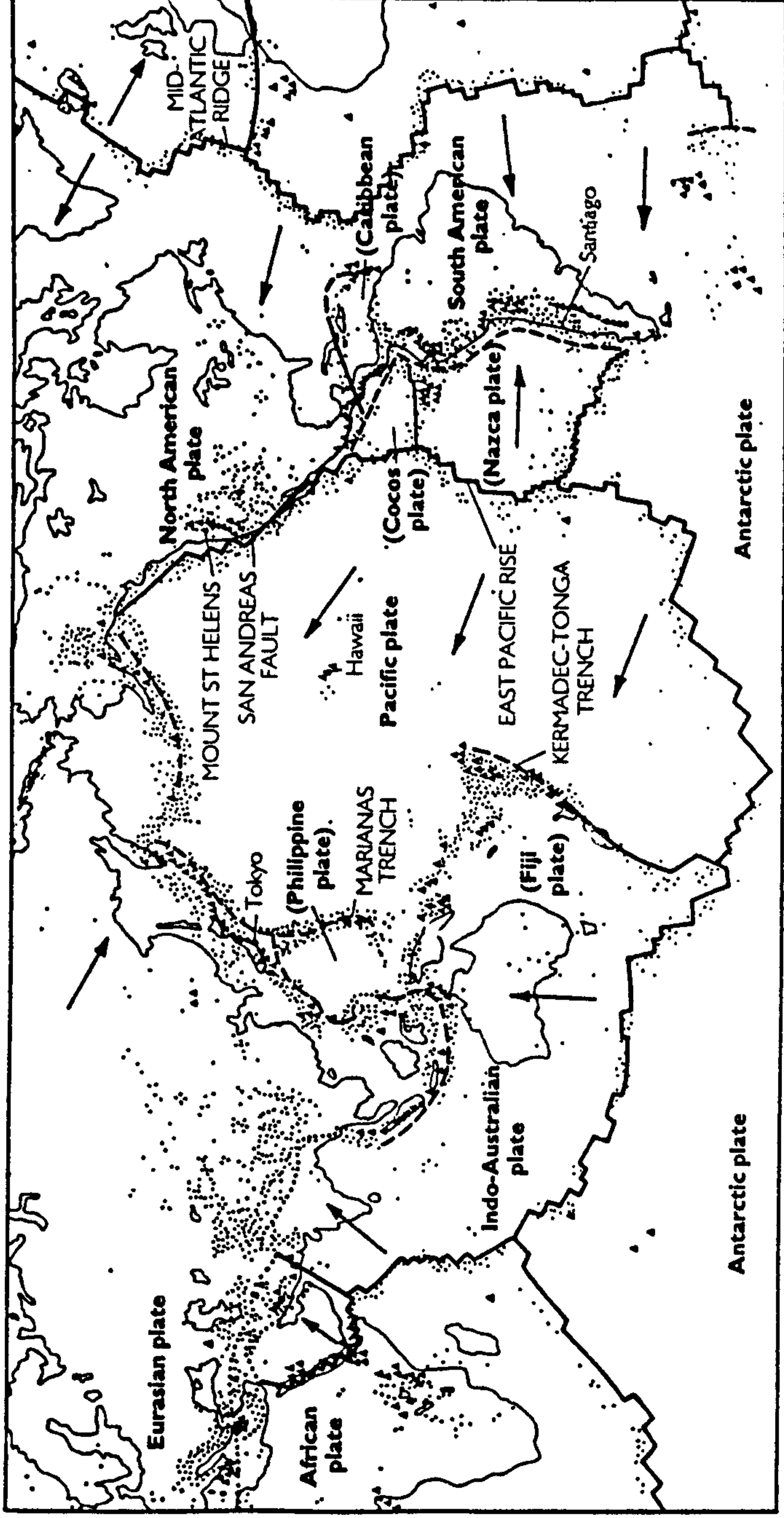


Figure 1.1 : World map showing earthquakes, volcanoes and plates. After Robinson (1993).

b) The second major seismic zone, called the *Alpide Zone*, accounts for about 15% of all earthquakes. This runs in a generally east-west direction across Europe, from Spain, Italy, Greece, Turkey and northern India, turning southeast through Burma and Sumatra to join the *Circum-Pacific belt* in New Guinea.

The remaining 5% of recorded earthquakes, are located in a fairly narrow zone along the midoceanic rifts - particularly those in the Atlantic and Indian oceans.

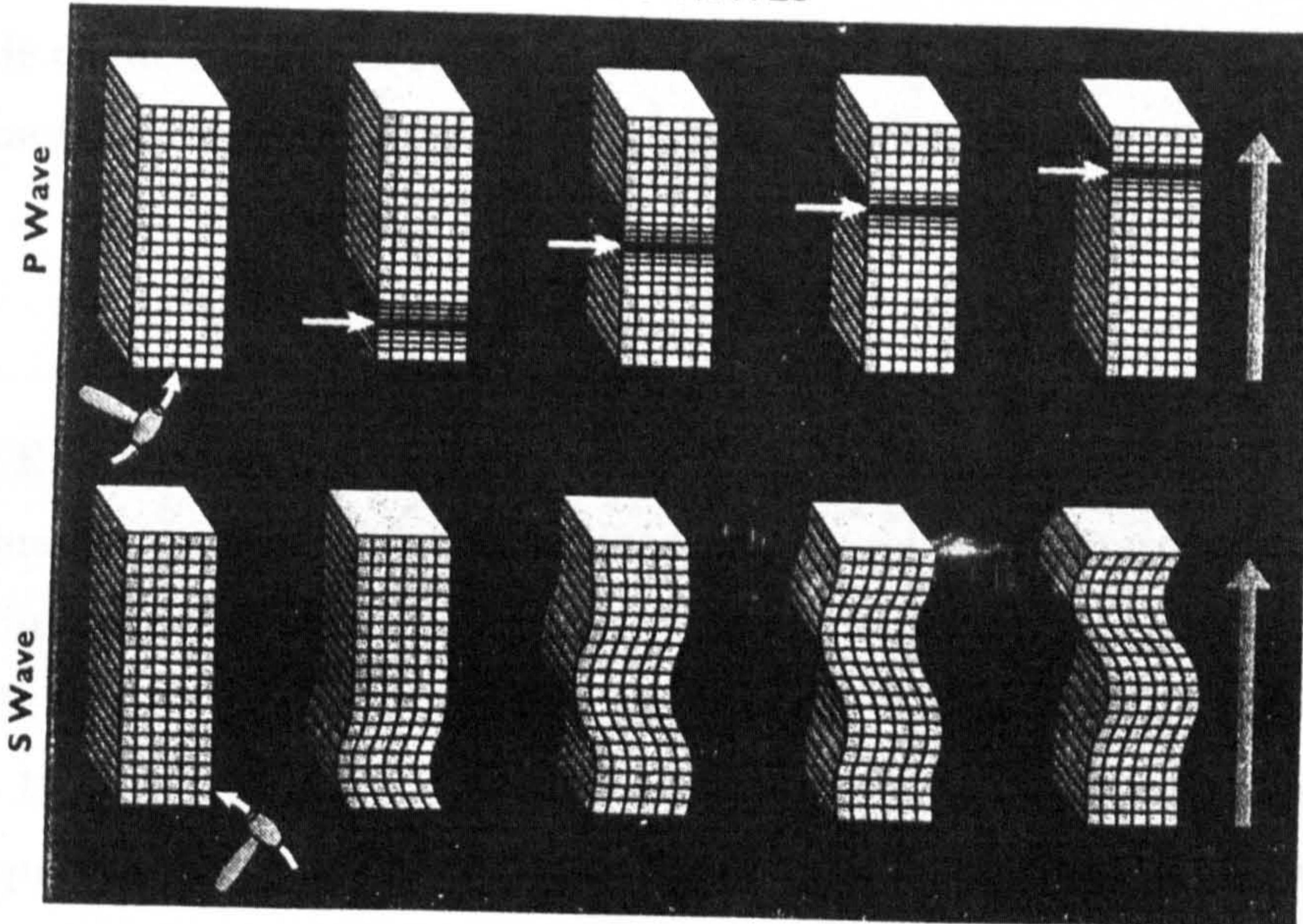
The elastic rebound model is the one most widely employed to explain the mechanism of earthquakes. According to this model, the moving plates of the earth in many places are being slowly displaced producing stress and strain within the rocks at the edge of the plates [1,3]. If the developed stress exceeds its elastic strength, it will fracture along the line of a fault, and the plates regain or rebound to the original shape but in a new position. This sudden movement of the plates which is accompanied by the release of strain energy, produces the earthquakes that caused the damage to buildings.

The point in the earth at which the earthquake occurs is called the *hypocentre* or *focus*. The point immediately above it on the earth's surface is called the *epicentre*. Earthquakes are associated with two types of waves, body waves and surface waves. The body waves are transmitted from the earthquake hypocentre to the surface and consist of P (primary) waves and S (secondary waves).

The surface waves are produced by the transformation of the body waves once they reach the surface (Figure 1.2). Since P waves travel faster than S waves, earthquakes are usually felt in two successive shocks. The first one, a light jolt, indicates the arrival of P waves; the second one, a heavy rocking shock, is the result of the arrival of S waves. The passage of P waves is like repeated hammer blows on the blocks shown in Figure (1.2a) which generates a vertical motion.

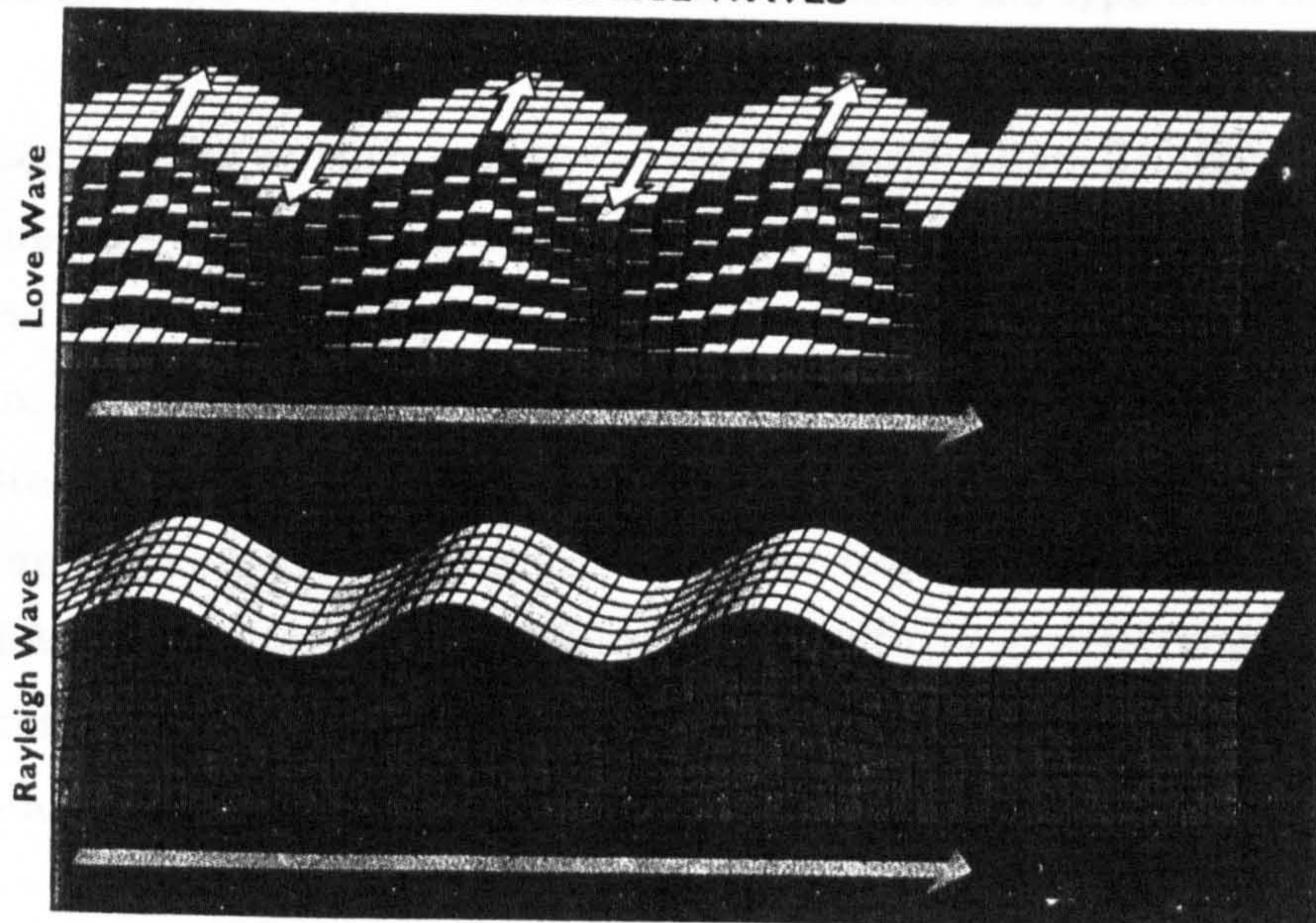
The two types of surface wave are called Love waves and Rayleigh waves. These waves move as shown in Figure 1.2b, causing the ground to move both vertically and horizontally, which in turn, damage the buildings.

BODY WAVES



a)

SURFACE WAVES



b)

Figure 1.2 : a) Body waves consist of P wave and S wave b) Surface waves consist of Love wave and Rayleigh wave. *After Robinson (1993).*

The effect of earthquakes on structures depends not only on their magnitude at the focus, but also on the distance of the structures from the focal distance. The size of an earthquake is calculated from the magnitude scale set up by C.Ritcher (1935). Richter expressed the magnitude of an earthquake (M) as the base 10 logarithm of the maximum amplitude in micrometers (  $1 \mu\text{m} = 10^{-4} \text{ cm}$ ). Since the record varies with distance, it is corrected for a standard distance of a 100 km. The magnitude of earthquake is related to the amount of strain energy released at the source. The following empirical formula shows the relationship

$$\text{LOG } E = 11.8 + 1.5 M$$

in which E is the energy in ergs and M is the magnitude on Richter scale. The above equation indicates a 32-fold increase in released energy for every unit increase in magnitude [1]. The largest earthquakes recorded have had a Richter magnitude of 8.9. One on the Columbia-Ecuador border in 1906 and another in Japan in 1933. There are eight scales of magnitude in the Richter classification. An earthquake of magnitude 8 releases approximately  $10^{25}$  ergs of energy, which is equivalent to the energy of 10,000 atomic bombs of the type used in Hiroshima.

While magnitude is a measure of the energy released, intensity, depending on both magnitude and distance, is a measure of the destructiveness of an earthquake. Therefore, an earthquake has one magnitude, but its intensity varies from station to station. Intensity represents the severity of the ground motions and decreases with distance from the source. Observations of the effects of earthquakes on natural and man-made structures have been used to determine intensity. The Modified Mercalli intensity (MM scale) scale, with 12 grades ranging from I (not felt by man) to XII (total destructiveness), has been the standard intensity measure for many years.

As a conclusion, it may be stated that the basic knowledge about the nature of earthquakes is important in order to relate the waves motion within the earth to the wave on the earth surface.

### **1.3 AREA OF INVESTIGATION**

During an earthquake, buildings will sway and lateral collisions can occur between adjacent structures. This phenomenon is commonly referred to as structural pounding.

The phenomenon of pounding between closely spaced buildings during major earthquakes has only recently been fully appreciated by the engineering community. The interest in this problem is motivated by the fact that it is now considered to be responsible for a large amount of the damage caused by seismic events in the last 20 years. The magnitude of the pounding problem has never been better demonstrated than during the 1985 Mexico City earthquake. Of the buildings classified as severely damaged, 40% experienced pounding during this seismic event [4,5].

Structural pounding occurs mainly between adjacent structures which exhibit significant differences in mass, stiffness, and (or) strength. The different dynamic characteristics of the buildings will usually induce out-of-phase lateral vibrations under earthquake ground motions. Pounding will then occur if the spacing between the buildings is not sufficient to allow them to vibrate freely. Each time a collision occurs, the building is subjected to short lateral impact forces not specifically accounted for in the conventional design process. These impact forces produce high-amplitude, short-duration local accelerations which can induce damage to structural members or non-structural members of the building, depending on the position of the contact elevation. Furthermore, earthquake pounding can amplify the overall dynamic response of the building.

Most analytical studies on pounding idealize the buildings as single degree of freedom (SDOF) system and (or) multi degree of freedom (MDOF) systems with degrees-of-freedom and masses concentrated at each floor level. It is also assumed that the floor levels of adjacent structures are aligned and pounding will occur at one of the floor level. The present study is aimed at investigating structural pounding at interstorey locations, when the floor of adjacent buildings are unaligned.

So far, the existing analysis of pounding has been derived by neglecting the inplane or axial floor deformation. In this study, the effects of the axial floor deformation are also investigated.

#### **1.4 STRUCTURE OF THESIS**

The development of the two main approaches to structural pounding theory is presented in Chapter Two. Methods for formulating the equations of motion of structural pounding are also presented.

Numerical examples for MDOF undamped free vibration pounding response for both aligned and unaligned floor elevations are discussed in Chapter Three where the analysis is based on the displacement compatibility method. This chapter begins with a brief introduction to the Mode Superposition technique, followed by the direct integration method to obtain the response of the system. The model assumes that the floors are rigid and strikes a rigid adjacent building.

A MDOF damped free vibration pounding and undamped forced vibration pounding are considered at the end of this chapter. The forced vibration pounding is induced by the ground motion of the 1985, Mexico Earthquake.

The assumption that a floor is always rigid will ease the analysis but this may not be a valid assumption in many cases. The effects of inplane or axial slab flexibility are investigated through parametric studies in terms of floor displacements and the contact forces in Chapter Four.

The relative stiffness of the adjacent building is also considered in this study. The SDOF and MDOF systems are adopted and again, the aligned and unaligned floor elevation, which is governed by the position of the point of contact along the height of the top column are described in Chapter Five.

Finally, all the analytical studies in Chapter Three to Five are verified using finite element packages namely, PATRAN and OASYS DYNA3D. The brief description



of the model set-up as well as data input are described in Chapter Six.

The discussion of all the results is presented in Chapter Seven, and finally this chapter draws conclusions and makes recommendations for further studies.

# **CHAPTER 2**

## **LITERATURE REVIEW**

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### **2.1 INTRODUCTION**

This chapter reviews the literature on methods of solving the system of equations arising from the pounding problem and several parameters that affect structural pounding such as separation distances, pounding locations and ground motion characteristics. This chapter also includes an overview of existing studies on structural pounding prevention.

### **2.2 STRUCTURAL POUNDING PERFORMANCE**

Pounding has been noted regularly by earthquake investigators over the past several decades [6]. In the 1964 Alaska earthquake, the 14-storey Anchorage Westward Hotel pounded against its low rise ballroom and an adjoining six-storey wing. The pounding was severe enough to dislocate some of the hotel metal floor decking from its steel beam supports. In the 1972 Managua earthquake, the five-storey Grand Hotel suffered a complete collapse of its third floor when battered by the roof level of the adjacent two-storey building.

This problem of earthquake induced pounding was initially studied but not to a great extent, in an analytical investigation (a case study) of the pounding between the Olive View Hospital main structure and its stairway tower in the United States that led to collapse in the 1971 San Fernando earthquake [6,7]. Since then very little research was carried out in the area until two major earthquakes occurred; i.e the 1985 Mexico City earthquake and the 1989 Loma Prieta earthquake.

### 2.2.1 The 1985 Mexico City Earthquake.

The extent of the significant damage which pounding can produce has never been demonstrated better than during the 1985 Mexico City earthquake. It has generated the current widespread attention on pounding hazards during seismic events. In this earthquake, over 40% of the 330 collapsed or severely damaged buildings surveyed experienced pounding. Furthermore, 15% of all the collapsed buildings were caused by severe pounding [5]. This was due to the long duration of shaking and the relative flexibility of the heavy reinforced-concrete frame buildings that were the major damage victims. Figure 2.1 illustrates pounding failure, Hotel de Carlo at Mexico City, in 1985 earthquake.



Figure 2.1 : Pounding Failure, Hotel de Carlo, Mexico City, 1985.

*After Arnold (1989).*

This has created an awareness to evaluate existing building configurations and to provide guidance in future building designs to avoid such spectacular damage.

### 2.2.2 The 1989 Loma Prieta Earthquake.

In 1989, the Loma Prieta earthquake, once again indicated the importance of understanding the characteristic behaviour and also the extent of seismic hazard caused by structural pounding. Like several other previous earthquakes, it occurred in a highly populated metropolitan city which includes both San Francisco and Oakland. Even though this event caused mainly architectural (i.e. cosmetic/appearance) and/or minor structural damage such as cracks of building exteriors above the pounding level (Figure 2.2), there were some cases where major structural damage caused by pounding was reported to have occurred [4].

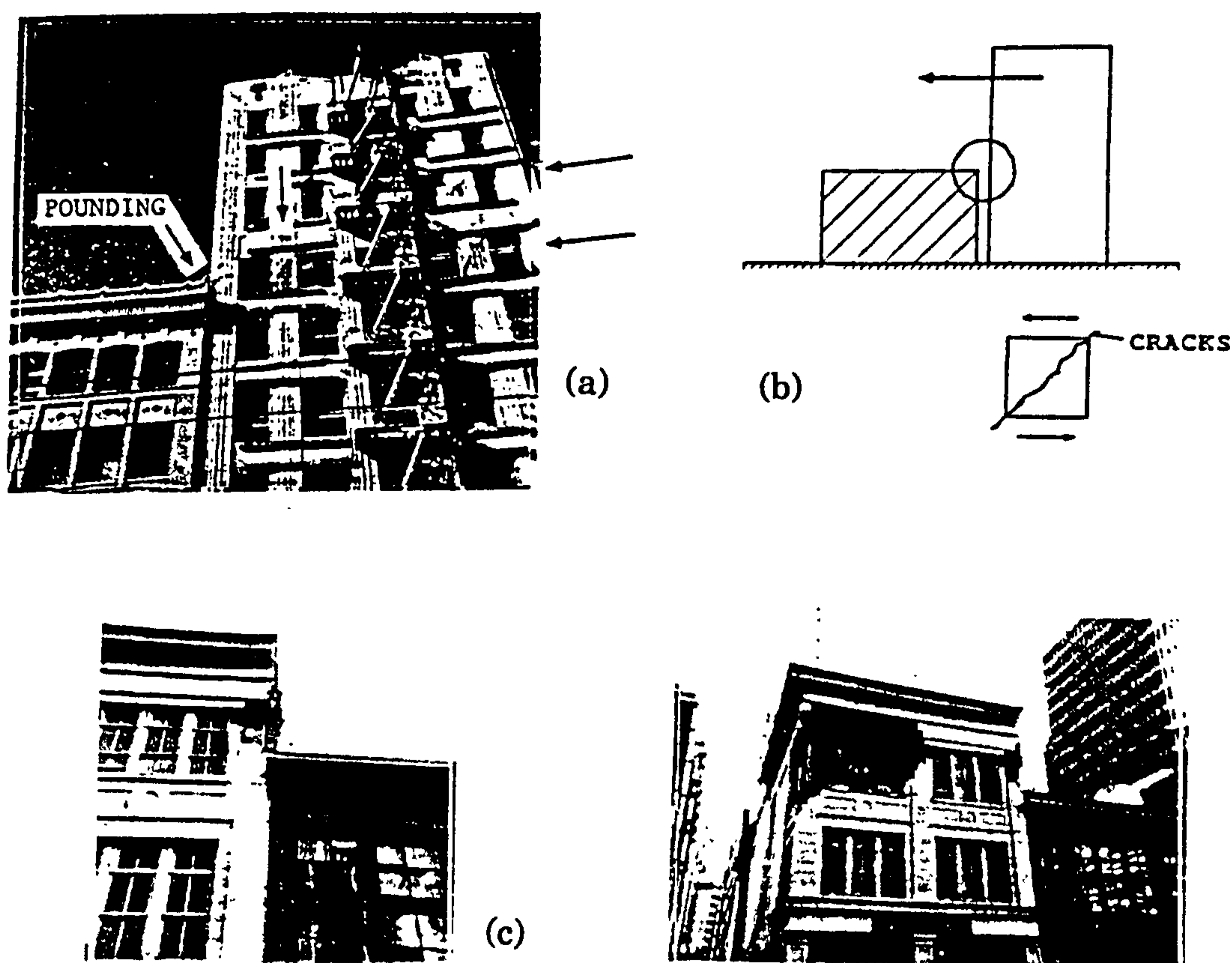


Figure 2.2: (a) Structural damage due to pounding (b) increased shear above pounding level, causing large crack development (c) structural damage above pounding level of unreinforced masonry building during San Francisco, 1989 Loma Prieta Earthquake. *After Kasai, et al. (1990).*

## 2.3 TYPE OF STRUCTURAL POUNDING ANALYSIS

A number of analytical studies have been conducted recently on the effect of structural pounding during earthquakes [4,7-19,22,24,28,30]. In general, these investigations can be classified into three main categories; studies on single degree of freedom systems (SDOF), multi degrees of freedom (MDOF) systems with contact occurs at floor to floor level (aligned floor case) and MDOF systems with contact occurs between two floor levels (unaligned floor case). This section summarizes the main assumptions and results of these various studies.

### 2.3.1 SDOF systems

Most of the past analytical studies on pounding idealize the buildings as a set of single degree of freedom (SDOF) systems [7-10]. One of the first investigations of earthquake pounding involved the seismic response of two adjacent SDOF systems as shown in Figure 2.3 [8]. In this study, the buildings were modelled as linear or bilinear SDOF systems with equivalent stiffness and concentrated mass. The pounding structure is presented by a mass,  $m$  and adjacent structure is represented by an impact spring-dashpot mechanism.

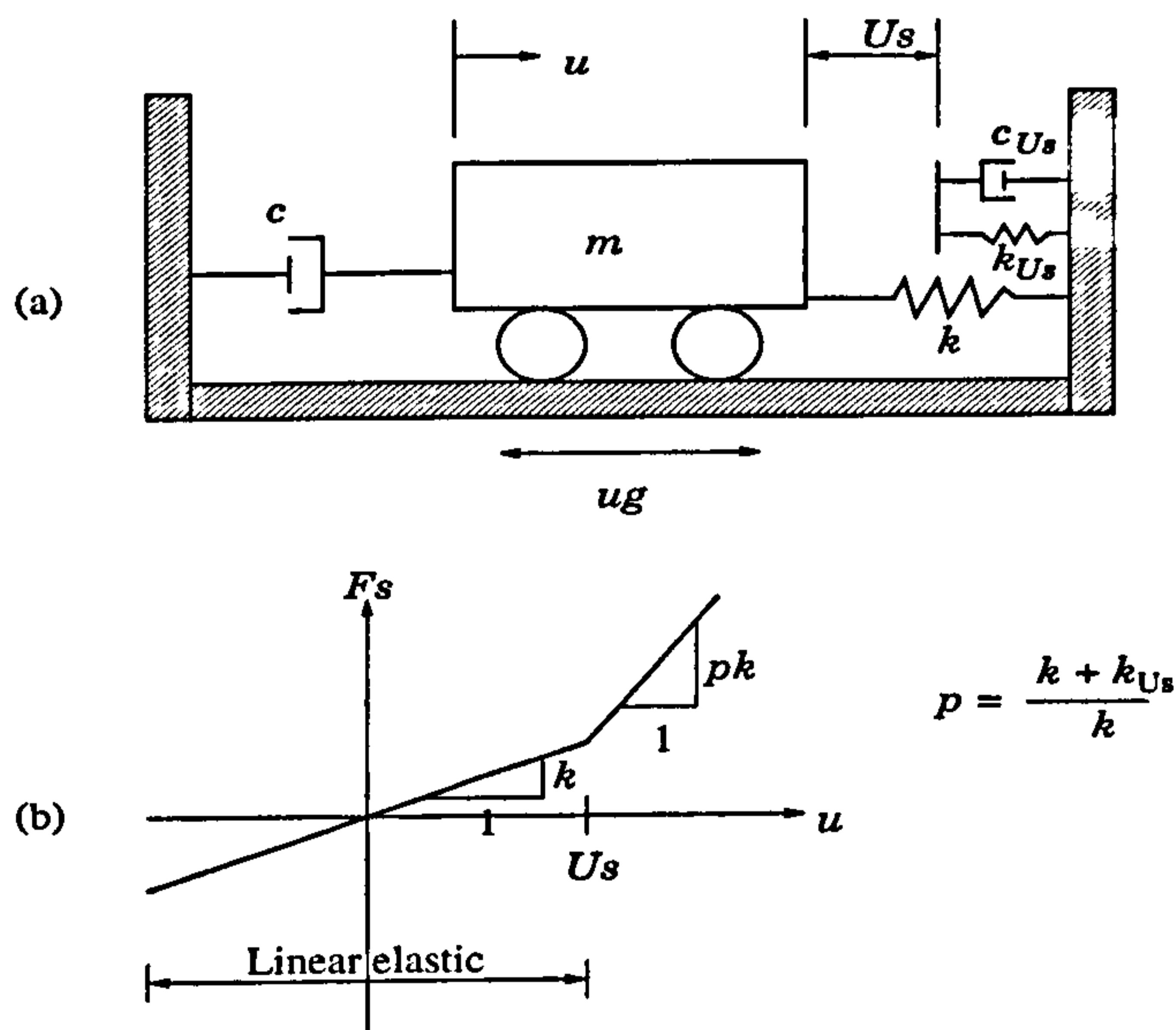


Figure 2.3: a) One-side impact of SDOF model, b) Force-displacement relationship of a spring, *After Wolf, et al. (1980)*.

A linear compressive spring  $F_s(u) = k u$ , was activated between the systems when the relative displacement,  $u$  of the mass is smaller than the initial separation,  $U_s$ .

$$u < U_s \quad (2.1)$$

The spring stiffness,  $k$  was established based on the axial stiffness of a typical composite steel-concrete floor. The damping force,  $F_d(\dot{u})$  is defined analogously, merely by replacing  $k$  and  $k_{U_s}$  by  $c$  and  $c_{U_s}$  and  $u$  by  $\dot{u}$ , where within linearly-elastic limits is given by  $F_d(\dot{u}) = c \dot{u}$ . Where  $k_{U_s}$  is the stiffness of the impact spring and  $c_{U_s}$  is the damping coefficient of the dashpot, working parallel to the impact spring.

For  $u > U_s$ , a nonlinear hardening-spring behaviour (Figure 2.3b) with a relationship of  $F_s(u) = (k + k_{U_s}) u$ . The stiffness for this range is related to the first (elastic) range by the factor  $p = (k + k_{U_s}) / k$ . That is  $F_s(u) = p k u$ .

The displacement of the SDOF system mass was denoted by  $u$  relative to the ground displacement,  $u_g$  and used dots to indicate differentiation with respect to time, then the equation of motion were obtained using Newton's law in a form of

$$m \ddot{u} + F_d(\dot{u}) + F_s(u) = - m \ddot{u}_g \quad (2.2)$$

where  $m$  is the mass of the SDOF system,

$F_s(u)$  is the spring force from the relationship as in Figure 2.3b

$F_d(\dot{u})$  is damping force-velocity

The numerical integration was performed to find the displacement response spectrum. It was found that, for systems with significant differences in mass, the displacement amplification of the lighter system was greater than that experienced by the one with the larger mass. This could explain the collapse of an elevator shaft connected, by an expansion joint, to the main building of the Olive View Hospital during the 1971 San Fernando earthquake [11]. It was also shown that, for systems with significant differences in stiffness, the maximum displacement of the stiffer system is increased substantially when pounding occurs. This increase in displacement, coupled with gravity loads, enhances the possibility of a building overturning.

Multiple collisions can occur between a series of adjacent buildings located on the same city block. To investigate this phenomenon, the study described above was expanded to a series of adjacent SDOF system [7], where pounding was simulated by means of linear viscoelastic impact elements known as spring-dashpots as shown in Figure 2.4.

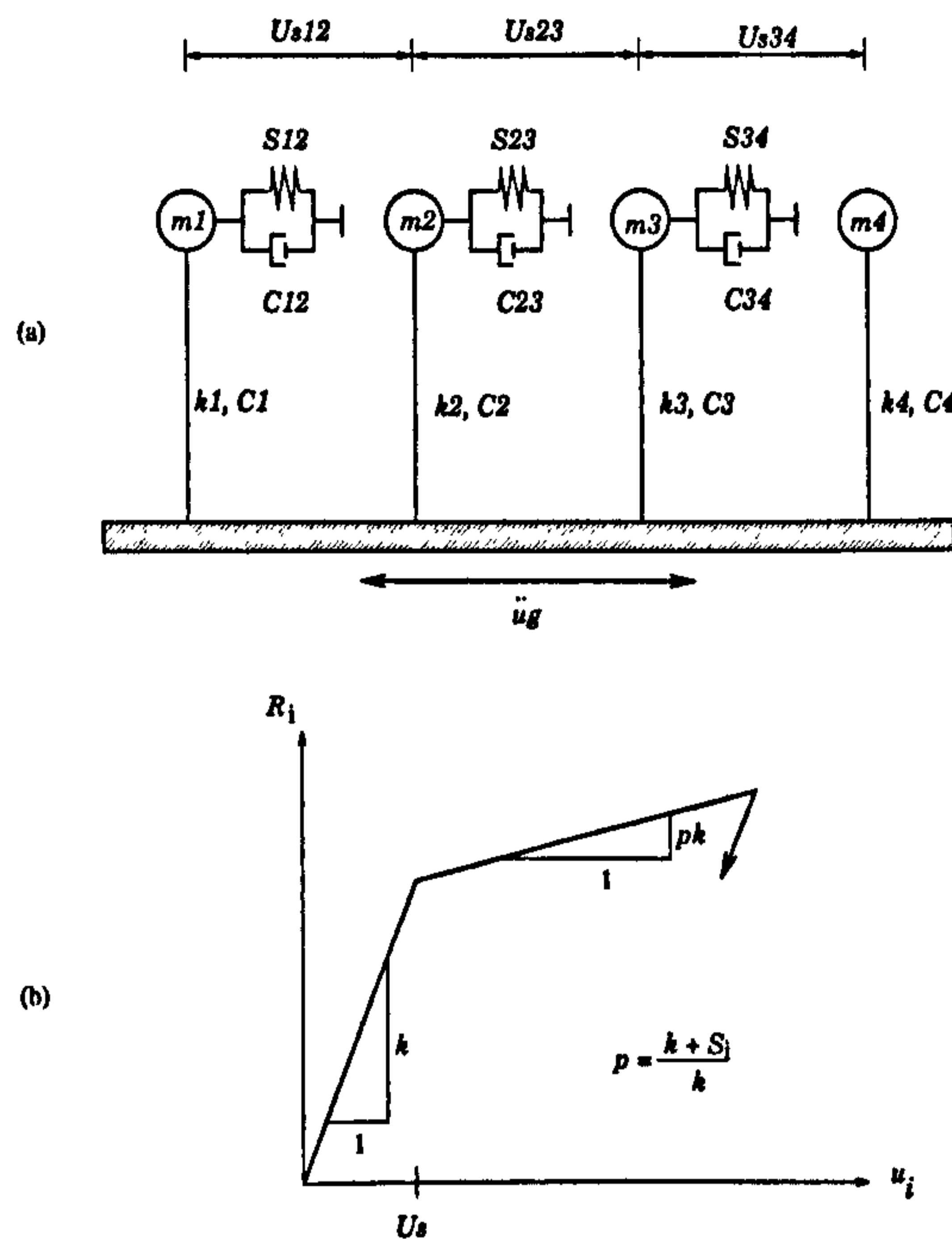


Figure 2.4: a) Lumped mass model of several adjacent SDOF systems, b) Bilinear structural resistances  $R_i$ , functions of displacements  $u_i$ .  
After Anagnostopoulos, (1988).

The modelling of the collision was enhanced by considering the energy dissipation through equivalent viscous damping at the moment impact between two adjacent masses. The dashpot constant,  $c_p$  of the impact elements determined the amount of energy dissipated by relating it to the coefficient of restitution,  $e$  ( $e=1.0$  is elastic impact and  $e=0.0$  is plastic impact) as given in reference [7]. The equation of motion, for the  $i$  th modes, were expressed as :

$$\underline{m} \ddot{u}_i + \underline{c} \dot{u}_i + R_i + S u_i + D_i = -m_i \ddot{u}_g \quad (2.3)$$

where  $\underline{m}$  is a diagonal mass matrix, with elements the masses  $m_i$

$\underline{c} = (c_s + c_p)$  is a damping matrix,

$c_s$  is a diagonal damping matrix, with elements the damping constant  $c_i$

$c_p$  is a diagonal damping matrix corresponding to the impact elements

$R_i$  is a vector of structural resistances, functions of the displacement  $u_i$

$\underline{S}$  is a stiffness matrix corresponding to the impact elements

$D_i$  is a vector including stiffness terms due to the impact elements.

$\ddot{u}_g$  is the ground acceleration.

In these matrices,  $c_p$  and  $\underline{S}$  are equal to zero if the masses  $m_i$  and  $m_{i+1}$  are not at impact. i.e  $(u_i - u_{i+1}) \leq U_{s_i}$ .

These equation were solved numerically by direct integration using central differences method and a linear acceleration of the ground motion. Results were expressed in terms of displacement amplification factors i.e as ratio of peak displacements of pounding to the peak displacement of no-pounding case.

It was found that the interior buildings are subjected to collisions from both sides. It was found that the dynamic response of interior systems can decrease or increase, depending on the properties of the adjacent systems and the characteristics of the ground motion. Exterior buildings, however, are subjected to collisions from one side only. These exterior systems experienced a significant increase in their dynamic response (i.e. displacement) when pounding occurred and this could explain the high percentages of corner buildings collapse during earthquakes. Again, it was observed that the effects of pounding were more damaging for light systems than for heavy ones.

In reality, the impact phenomenon between adjacent buildings is highly nonlinear and mostly approximated by the use of linear springs and viscous dashpots. During an impact, the contact surfaces increase with the impact force. It was proposed to use Hertz's contact law to improve the modelling of the collision by an impact oscillator [12]. The contact force,  $P$ , is proportional to the power 3/2 of the relative displacement.

$$P = k (u_{i+1} - u_i - U_{s_{i,i+1}})^{3/2} \quad (2.4)$$

where  $u_i$  and  $u_{i+1}$  are the displacements of masses  $m_i$  and  $m_{i+1}$  respectively.  $U_{s_{i+1}}$  is the initial separation between  $m_i$  and  $m_{i+1}$ , and  $k$  is the spring stiffness of the impact element. Later, it was noted that the exponent in the above equation could deviate from 3/2 in real pounding situations [13].



The above studies were based on the assumption that the adjacent buildings have different dynamic characteristics (natural periods, damping), since otherwise they would oscillate in phase and no pounding or other type of interaction would occur. However, in actual practical situation, pounding during an earthquake may also occur between adjacent buildings with similar dynamic characteristics. This has been studied by assuming the starting time of excitation was not the same for all adjacent structures, due to a phase difference which depends mainly on the propagation velocity of surface seismic waves, such as Rayleigh waves, as well as on the dimensions of the building layout [14]. Again, the analysis indicated an increased response of the end structures, as well as of the most rigid structures in the series.

### 2.3.2 MDOF systems (aligned floor)

In these studies, each building was considered as a MDOF system with degrees-of-freedom and masses concentrated at each floor. This modelling considers the higher mode response of the structures and the collisions at various floor levels. One important limitation of this approach, however, is that all the buildings must have identical floor elevations. The first study on simplified MDOF systems considered a 15 storey flexible building, colliding elastically with an adjacent rigid building at a specified floor level, i.e. the upper floor level of the lower structure as shown in Figure 2.5 [15].

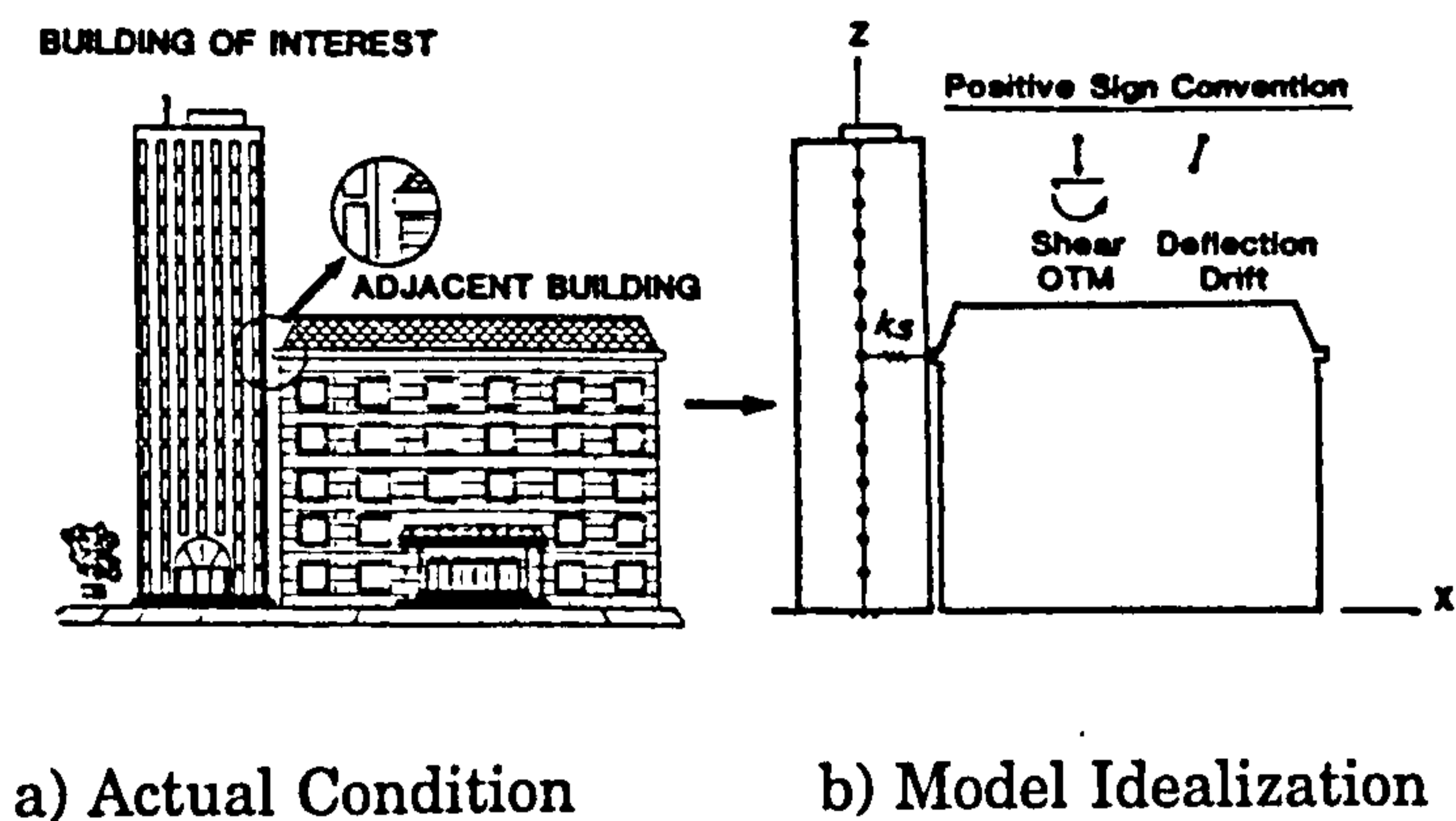


Figure 2.5: Pounding problem a) Actual condition and b) Model idealization  
*After Maison and Kasai 1990.*

In this study, the pounding problem was considered as two uncoupled linear problems, depending if contact (state 2) is present or not (state 1). The equations of motion for the MDOF system in state 1 and 2 were expressed as:

$$\begin{aligned} \underline{m} \ddot{u}_i + \underline{c} \dot{u}_i + \underline{k} u_i &= - \underline{m} \underline{r} \ddot{u}_g \quad (\text{no contact}) \\ \underline{m} \ddot{u}_i + \bar{c} \dot{u}_i + \bar{k} u_i &= - \underline{m} \underline{r} \ddot{u}_g + \bar{b} \quad (\text{contact}) \end{aligned} \quad (2.5)$$

where  $u_i$ ,  $\dot{u}_i$ ,  $\ddot{u}_i$  is the floor displacement, velocity and acceleration at time  $t$  for  $i$ th mode shapes.

$\underline{m}$  is the building diagonal mass matrix

$\underline{c}$ ,  $\bar{c}$  are damping matrix assumed to be linear combination of the mass and stiffness matrix for state 1 (no contact) and state 2 (contact) respectively.

$\underline{k}$ ,  $\bar{k}$  are building stiffness for state 1 (no contact) and state 2 (contact) respectively.

$\underline{r}$  is the Pseudo static vector or earthquake influence coefficient.

$\bar{b}$  is a column vector of static preload forces.

The static preload forces in the contact stage are use the local flexibility spring stiffness,  $k_s$  (Figure 2.5b) based on the in-plane axial stiffness of the concrete floor system at the pounding level only. Other floors were assumed very rigid in compared to the column which simplified into lumped mass model. The change from state 1 to state 2 and so on is based on the displacement,  $u_i$  and velocity,  $\dot{u}_i$  at the floor of contact point given by:

$$\begin{aligned} u_i = U_s \quad \text{but} \quad \dot{u}_i < 0 \quad (\text{no contact}) \\ u_i = U_s \quad \text{but} \quad \dot{u}_i > 0 \quad (\text{contact}) \end{aligned} \quad (2.6)$$

The solution is based on displacement compatibility method (chapter 3) using a modal superposition technique. Then, the procedure is programmed into a microcomputer to perform the pounding analysis of the building. The results of this study show that large shear forces are developed below and above the contact level.

The levels above the contact point experience large amplifications of interstorey drifts (relative displacement between two adjacent floors), shear forces, and overturning moments.

The maximum responses of levels below the contact points are reduced, however, compared with the maximum responses when pounding does not occur. It is found that, the maximum base shear exists when pounding occurs at roof level.

Later they improved the capabilities of their model by considering two flexible buildings colliding [16]. Again, it was shown that the effect of pounding is critical in a light building colliding against a heavy one and the point of contact is assumed at the floor to floor levels.

Just a few pounding analyses involving a series of nonlinear MDOF systems have been performed [17]. In this study, the same bilinear structural resistance characteristics as in Figure (2.4b) was adopted. It was demonstrated that exterior buildings are more vulnerable to damage from pounding than interior ones. For buildings of different heights, the largest increase in ductility demand occurs in the taller building above the roof of the smaller one.

An experimental study has been conducted recently on two storey frames [18]. The experimental results confirmed that pounding increases the response (maximum displacement) of a stiffer building when colliding against a flexible one.

### **2.3.3 MDOF systems (Unaligned floor)**

The study on the response of two or more adjacent buildings during earthquake for both aligned floor and the unaligned floor pounding has been studied recently [19]. The structures are modelled as MDOF systems with rigid slab response at each floor. The floors are assumed to have a frictionless contact without sliding. (Figure 2.6).

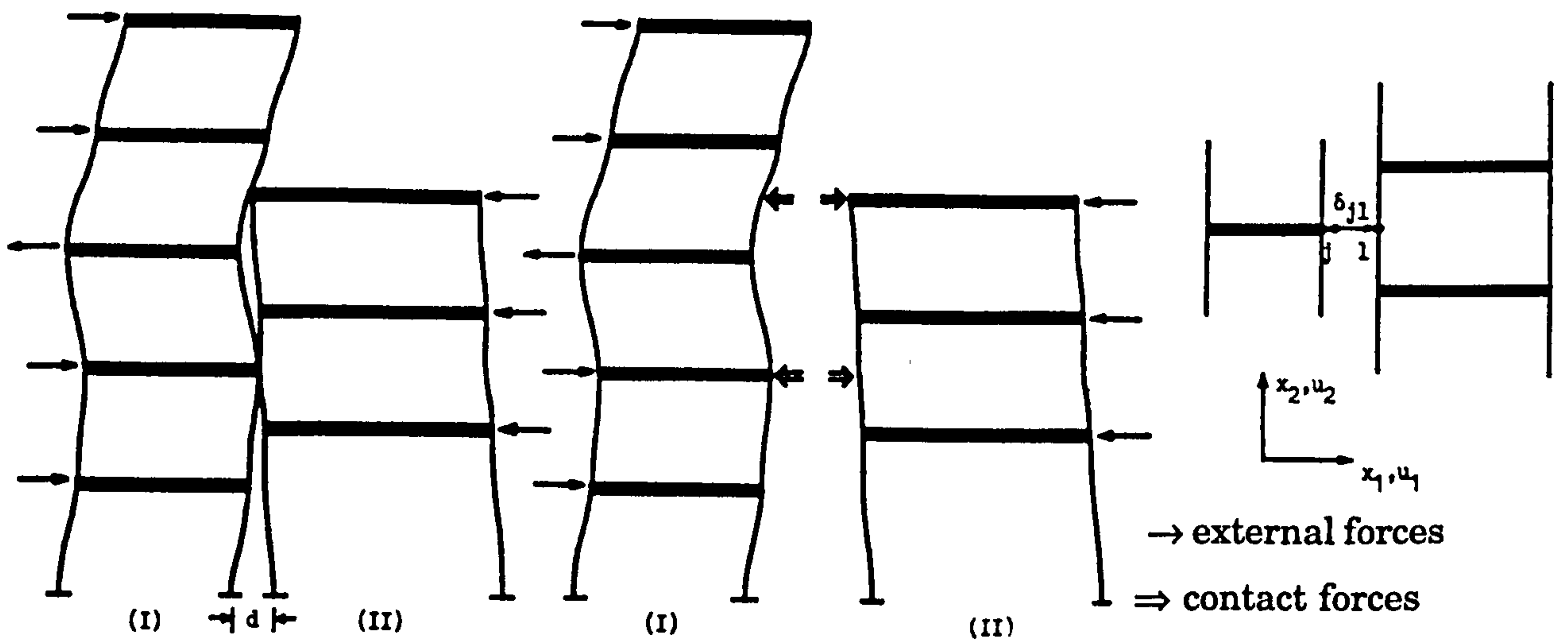


Figure 2.6 : Schematic representation of the contact problem.

*After Papadrakakis, et al. (1991)*

The solution method was based on the Lagrange Multiplier approach which formulated from the scalar quantities of kinetic energy,  $T$ , potential energy,  $U$  and work,  $W$ . In this study, the Lagrange's equations are in terms of constraint equations of the two nodes  $j$  and  $l$  in contact (Figure 2.6) is written as

$$u_1^{(j)} - u_1^{(l)} = \delta_{jl} \quad (2.7)$$

where  $u_1^{(j)}$  and  $u_1^{(l)}$  are the nodal displacements in direction  $x_1$  of nodes  $j$  and  $l$  respectively. The Lagrange's equations are differential equations of motion expressed in terms of  $N$  generalized coordinates,  $g_i$ . ( $i = 1, \dots, N$ ). For a system subjected to nonpotential force, such as damping, the differential of the total energy (sum of the kinetic and potential energies) is

$$d(T + U) = \delta W_{np} \quad (2.8)$$

where  $\delta W_{np}$  is the work of the nonpotential forces and can be expressed in terms of the generalized coordinates,  $g_i$ , thus

$$\delta W_{np} = \sum_{i=1}^N G_i \delta g_i \quad (2.9)$$

The quantities  $G_i$  are known as the generalized forces and  $G_i \delta g_i$  has the unit of work and when the system is influenced by viscous damping, it is equal to the negative change of  $G_i \delta g_i$ , that is

$$G_i = - \sum_{i=1}^N c_i g_i \quad (2.10)$$

The Lagrange's equation including damping forces is given by:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{g}_i} \right) - \frac{\partial T}{\partial g_i} + \frac{\partial L_\lambda}{\partial g_i} = G_i \quad (i=1, \dots, N) \quad (2.11)$$

The kinetic energy is the function of  $\dot{g}_i$  and not a function of  $g_i$ . In this study,  $g_1$  becomes  $u$  and  $g_2$  becomes  $\lambda_i$

$$T = \frac{1}{2} \sum_{i=1}^N \dot{u}_i^T m_i \dot{u}_i \quad (2.12)$$

The partial derivatives of  $T$  are

$$\begin{aligned} \frac{\partial T}{\partial \dot{u}_i} &= m_i \dot{u}_i \quad \therefore \quad \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{u}_i} \right) = m_i \ddot{u}_i \\ \frac{\partial T}{\partial u} &= 0 \end{aligned} \quad (2.13)$$

And the total potential energy,  $U$  subject to the geometric constraints,  $\phi_i$  is transformed into Lagrangian function [20],  $L_\lambda$  is given by

$$L_\lambda = U + \sum_{j=1}^M \lambda_j \phi_j \quad (j=1, \dots, M) \quad (2.14)$$

where  $U =$  total potential function of the system  $= 1/2 u_i^T k_i u_i - P_i^T u_i$

$\lambda_j =$  Lagrange multipliers

$\phi_j =$  all the geometric constraints i.e.  $k_{\lambda_i} u_i = \delta_i$

The partial derivatives of  $L_\lambda$ ,  $\partial L_\lambda / \partial g_i = 0$ . where  $g_1$  becomes  $u$  and  $g_2$  becomes  $\lambda_i$  give

$$\begin{aligned} \frac{\partial L}{\partial u} &= k_i u_i - P_i + k_{\lambda i}^T \lambda_i = 0 \\ \frac{\partial L}{\partial \lambda} &= k_{\lambda i} u_i - \delta_i = 0 \end{aligned} \quad (2.15)$$

When  $g_1$  becomes  $u$ , the generalized force,  $G_i$  from Equation (2.10) becomes

$$G_i = -c_i \dot{u}_i \quad (2.16)$$

Substituting all derivatives from Equations (2.13) and (2.15) into the Lagrange's equation in Equation (2.11) produces the equations of motion such as

$$\begin{aligned} m_i \ddot{u}_i - 0 + k_i u_i - P_i + k_{\lambda i}^T \lambda_i &= -c_i \dot{u}_i \\ k_{\lambda i} u_i - \delta_i &= 0 \end{aligned} \quad (2.17)$$

Or in matrix form of

$$\begin{bmatrix} m_i & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_i \\ 0 \end{Bmatrix} + \begin{bmatrix} c_i & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_i \\ 0 \end{Bmatrix} + \begin{bmatrix} k_i & k_{\lambda i}^T \\ k_{\lambda i} & 0 \end{bmatrix} \begin{Bmatrix} u_i \\ \lambda_i \end{Bmatrix} = \begin{Bmatrix} P_i \\ \delta_i \end{Bmatrix} \quad (2.18)$$

The matrix equations were first condensed by Lagrange Function,  $L_\lambda$  and followed by solution of the equations using numerical iteration process (e.g. Newmark's constant acceleration method ( $\beta = 1/4$ ) which mainly involving large computational works). Compared to a typical dynamic problem of equilibrium equations of motion

$$m_i \ddot{u}_i + c_i \dot{u}_i + k_i u_i = P_i \quad (2.19)$$

The size of the above equation is increased to Equation (2.18) by a presence of Lagrange Multipliers,  $\lambda_i$ .

The dynamic aspects of post-contact conditions must fulfil the energy balance condition by introducing the coefficient of restitution  $e$ . This is usually defined in theory of impact [21] as

$$-e (\dot{u}_2 - \dot{u}_1) = (\dot{u}'_2 - \dot{u}'_1) \quad (2.20)$$

where  $\dot{u}_1, \dot{u}_2$  are uniform velocities of two masses ( $m_1, m_2$ ) before impact.

$\dot{u}'_1, \dot{u}'_2$  are uniform velocities of two masses ( $m_1, m_2$ ) after impact.

Details of this method can be found in Reference [19]. The only drawback of this approach is that the size of the system equations (Equation 2.19 become Equation 2.18) is increased because of the Lagrangian Multipliers.

Recently, pounding between adjacent three and eight storey buildings has been studied using a shaking table [22]. Impact elements were represented by hammers and receivers were placed at first three floors of each building (Figure 2.7). It was found that the acceleration for floor to column pounding (unaligned floor) was very large compared to floor to floor pounding (aligned floor). No direct measurement of acceleration (hence, impact force) was taken at the mid-height of the column where contact occurred.

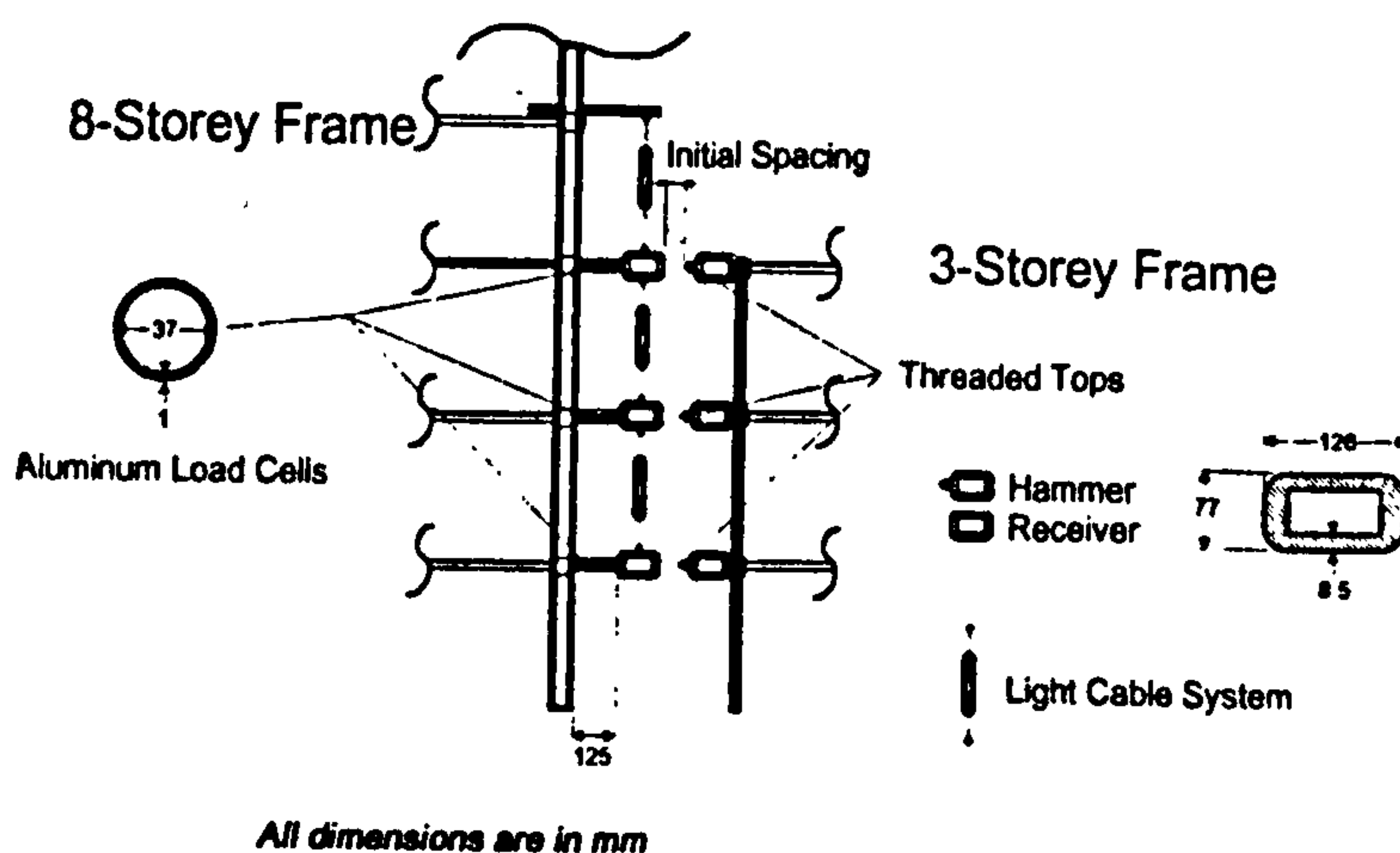


Figure 2.7 : Impact elements. After Filiatrault, et al. (1995).

## **2.4 PARAMETRIC STUDIES**

Some parameters of pounding that affect the response behaviours of buildings have been studied by researchers mentioned in the previous section. These parametric studies are summarized into four main groups and discussed briefly in this section. They are, the relative building mass, the at-rest building separation, the building stiffness and the local slab stiffness.

### **2.4.1 Relative Mass**

In general, the greater the magnitude of the mass that pounds against on the lighter structure, the more pronounced is the effect of pounding to the latter structure [7,14].

### **2.4.2 Building Separation (gap size)**

As the gap size increases, the number of impacts decrease and typically the structural response decreases [7,16]. It was shown that the peak response of the building above the pounding level increases significantly with a decreasing gap size.

### **2.4.3 Building Stiffness**

Most of the studies carried out in section (2.3) have concluded that the effect of pounding is greatest on the more flexible of the two structures [7,14,16,18].

### **2.4.4 Local slab stiffness**

The local slab stiffness was based on the axial stiffness of slab floor. Two cases of slab stiffness of 3539370 kN/m (200,000 kips/in) and 1751969 kN/m (10,000 kips/in) have been studied [15]. The stiffer slab results in larger peak responses.



The twentyfold increase in the slab stiffness has relatively minor effects on the peak displacements, interstorey drifts and overturning moments. However, it has considerable effect on the shear force value.

#### **2.4.5 Other parameters**

1) Dynamic characteristic. The effect of pounding is smaller when the adjacent structures have similar dynamic characteristics [14]. Pounding becomes more critical when the phase difference in the starting time of excitation of the first and the last structure in a row is relatively large.

2) The pounding location elevation. The study assumed that the pounding occurs at different elevations and concluded that the peak roof deflection progressively decreases as the pounding location elevation increases [15].

3) Initial displacement amplitude. It is found that the peak responses are directly proportional to the initial condition amplitude. i.e scaling the initial displacement pattern by two leads to about doubling of the peak responses [15].

## **2.5 SOLUTION METHODS FOR STRUCTURAL POUNDING**

There are two main approaches to formulating the pounding equations of motion. They are the displacement compatibility method and finite element method based on Lagrange Multiplier Approach.

### **2.5.1 The Displacement Compatibility Method**

This is the most common method used in the analysis of dynamic problems, particularly in structural pounding [15,16,17].

The structural pounding equations of motion are derived and then transformed into a matrix form and these are solved using direct integration technique [23].

### **2.5.2 The Finite Element and Lagrange Multipliers Approach**

In Finite Element applications, the contact problem was solved by the Lagrange Multipliers approach [19]. In this approach, pounding is viewed as a contact-impact problem. The geometric compatibility conditions due to contact is enforced with the presence of the Lagrange Multipliers.

The conservation of momentum after and before contact is preserved by introducing the coefficient of restitution and this method is briefly mentioned in section (2.3.3). For the Lagrange Multipliers method, the size of the system equations is increased due to the Lagrangian Multipliers being used to represent the contact forces. This is considered as a major drawback of this approach.

## **2.6 POUNDING PREVENTION**

Prevention or mitigation of the pounding potential between adjacent structures can be achieved by either allowing an adequate separation between buildings or linking adjacent buildings to force inphase vibrations. Each of the above is discussed below.

### **2.6.1 Separation between buildings**

The most obvious way to eliminate pounding is to allow sufficient separation between adjacent buildings. Although building codes prescribe a minimum separation between adjacent structures, nontechnical issues often govern the choice of distances. The reluctance from owners, engineers and architects to respect code requirements arises mainly from high land costs and limited land sizes. Owner responsibilities in the case of pounding damage have yet to be clearly defined [24].

For protection against pounding, some code of practice, such as Uniform Building Code - 1988 (UBC) specifies the required minimum separation distance between two buildings,  $U_{s_{min}} = \Sigma [ ( 3 R_w / 8 ) u_{st} ]$

where,  $R_w$  = the response modification factor, which represents the ductility of the structural system. For example, for the special steel moment frame,  $R_w = 12$  and for the concrete shear walls,  $R_w = 8$ . The values of  $R_w$  for various structural system can be found in Table 4.5 in Reference [25]. The  $u_{st}$  is the static displacements at the top of building as defined in the CEB Model Code [14].

Stricter requirements have been proposed for Eurocode No 8, where the minimum separation distance between two buildings [26] as shown in Figure 2.8,  $U_{s_{min}}$  should not be less than

$U_{s_{min}} = 50 \text{ mm} + H / 200$ , where  $H$  = the total building height (mm).

A number of comparative studies on pounding based on the code of practice have concluded that both codes are sufficient to reduce the effect of pounding [14,17].

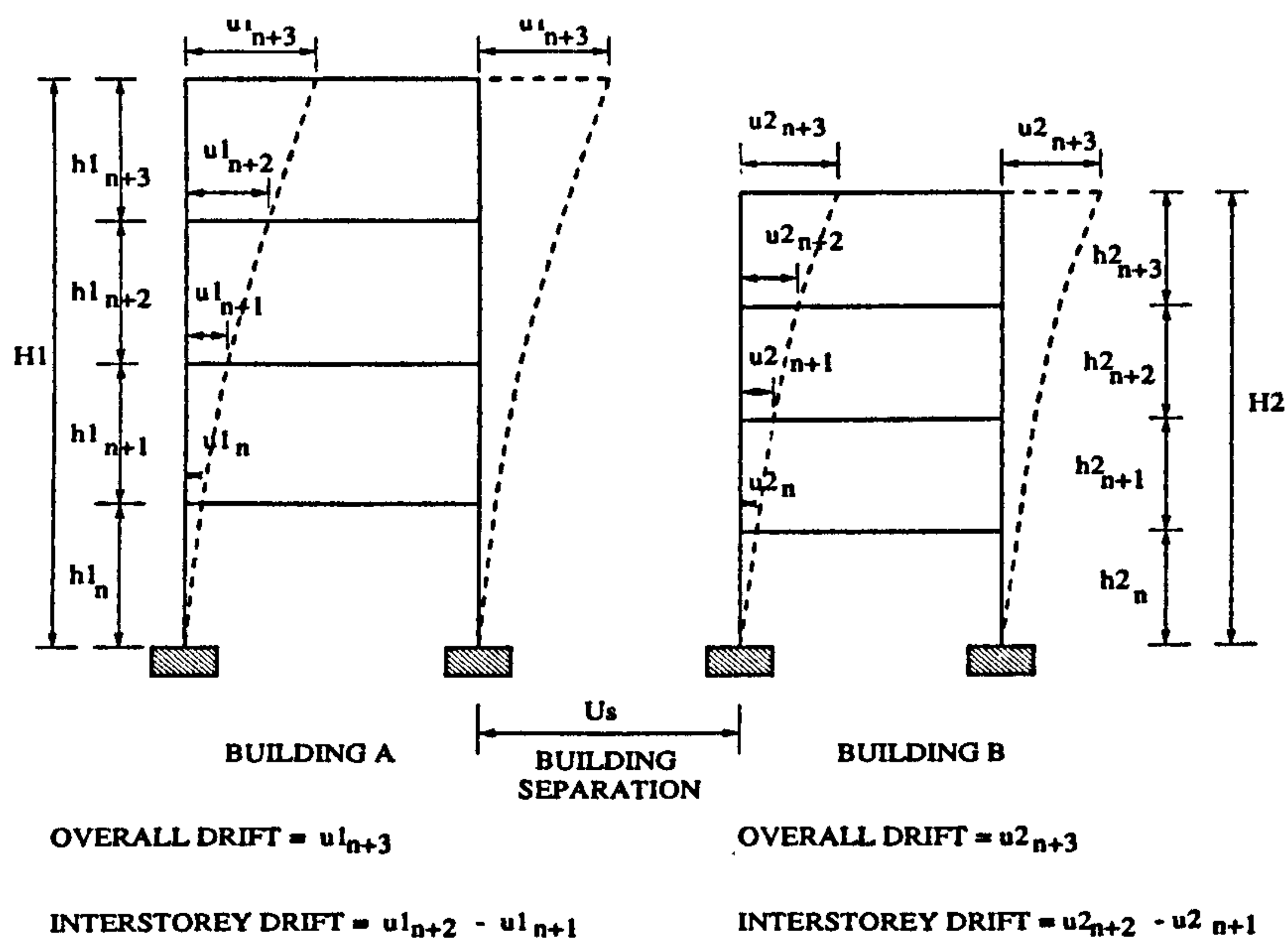


Figure 2.8 : Definition of drift and building separation.

### 2.6.2 Linkage of adjacent buildings

One of the first method to reduce the pounding effects was introduced, as in Reference [27]. In this approach, an energy-absorbing material was placed between the buildings, such as soft timber, where the contact could happened during pounding. Figure 2.9 illustrates a solution of this type.

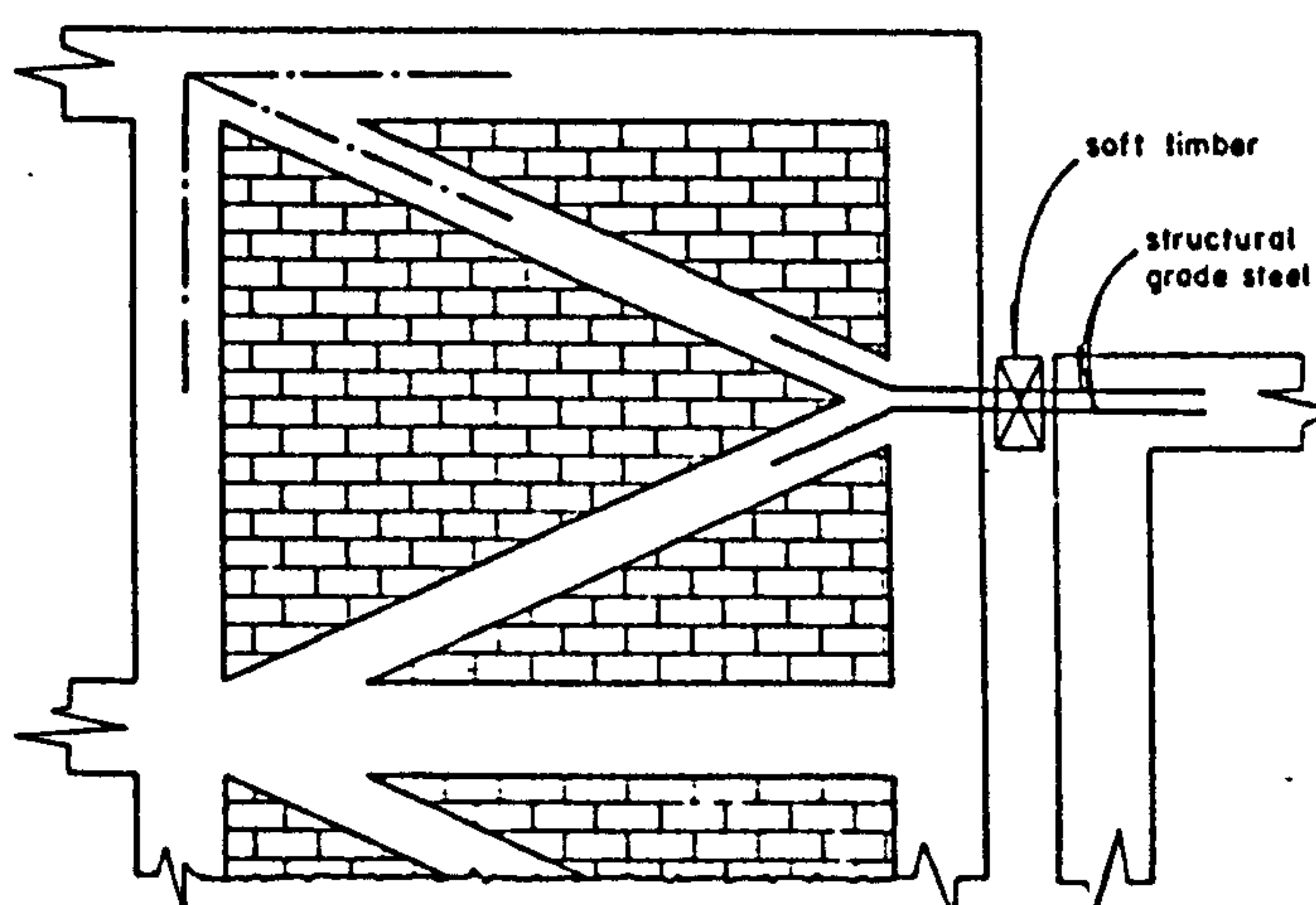


Figure 2.9 : Protection against pounding. After Rosenblueth and Esteva (1962).

When adjoining structures have approximately the same natural modes and periods of vibration, it might be advisable to tie them together so as to force them to vibrate in phase.

The elastic vibrational response of coupled buildings was investigated recently [28]. The structures are connected by a link and beam system which transmits the connection forces to the floors of the structures (Figure 2.10).

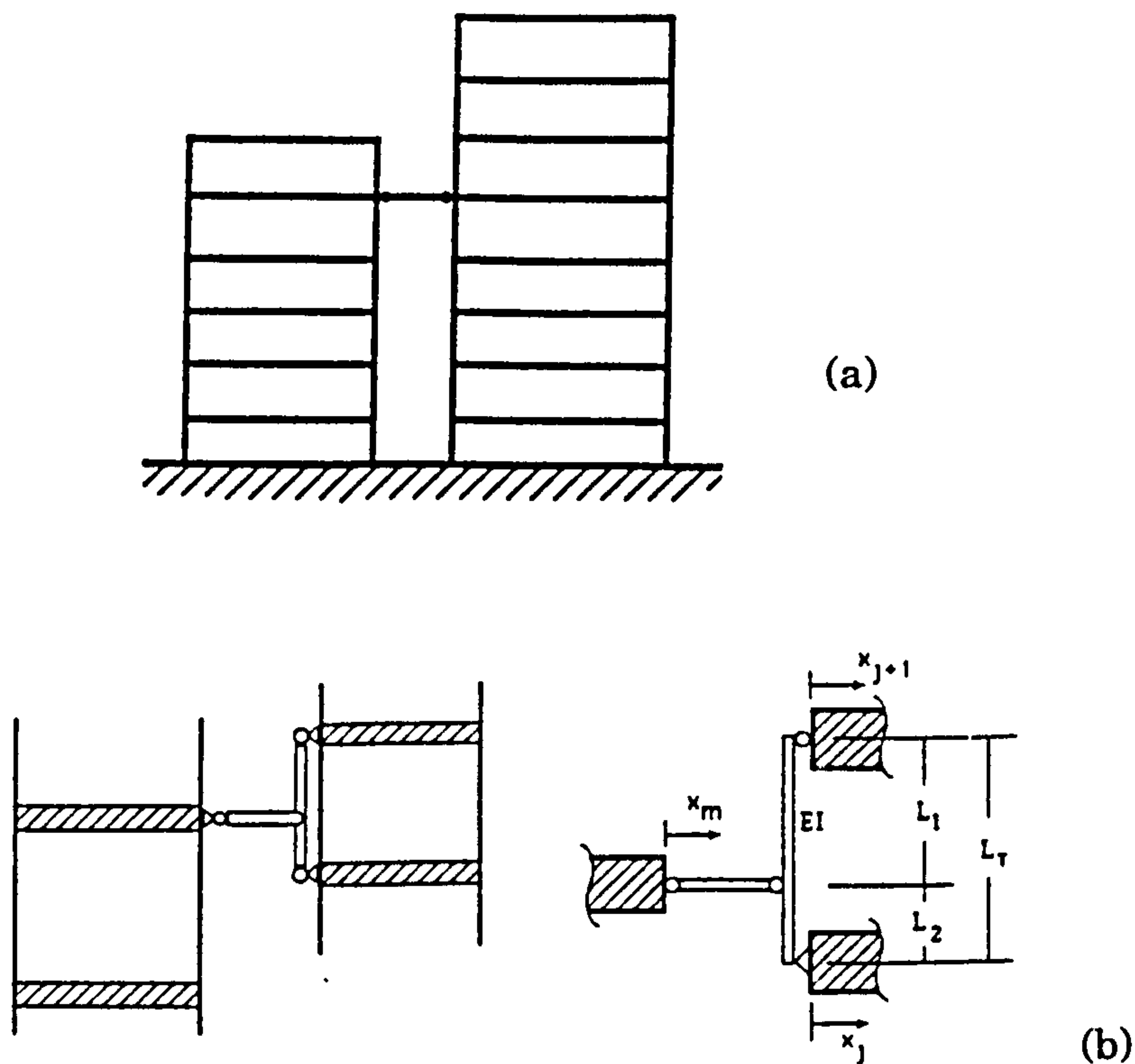


Figure 2.10 : (a) Schematic for the rigid interconnection of structures to eliminate pounding (b) Simply supported beam linkage schematic for connecting unaligned floors. *After Westermo (1989).*

It was found that structural coupling reduces the pounding potential, but also increases the seismic forces on the structure which had the smallest base shear when uncoupled.

## **2.7 CONCLUSION**

As stated in this brief literature review, considerable analytical and numerical studies have been carried out on 3 main types of structural pounding (section 2.3). Most of these studies, however were based on the displacement compatibility method for the aligned floor case and the Finite Element and Lagrange Multiplier method for the unaligned floor case.

In this study, a further application of the displacement compatibility method is applied to the unaligned floor case with a rigid slab and both aligned and unaligned floor cases with a flexible slab. This method has advantages over the Finite Element and Lagrange Multiplier methods in terms of its matrix simplicity.

## **CHAPTER 3**

# **POUNDING OF ADJACENT RIGID BUILDING WITH RIGID SLAB**

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### **3.1 INTRODUCTION**

The importance of pounding during dynamic loading has only recently begun to be studied thoroughly. Observations of past earthquakes have revealed that pounding can be a severe problem [29,30]. The extent of pounding damage in Mexico City in 1985 and during the 1989, Loma Prieta earthquake, has confirmed this as a major problem.

In this chapter, the analysis of pounding is presented by deriving the differential equations of motion of Single Degree of Freedom (SDOF) systems and Multi Degree of Freedom (MDOF) systems. The chapter covers the pounding of buildings where the slabs are assumed to be rigid so that the axial deformations of the slab can be neglected.

The initial section studies the free vibration response using the Mode Superposition techniques. Then the analysis of the pounding of adjacent rigid buildings is presented. This analysis is based on the displacement compatibility method which is an extension of the formulation method used in References [15,16]. The numerical examples are based on both undamped and damped free vibrations by means of a snap back analysis. Later the earthquake response analysis is presented and this analysis is illustrated by numerical examples at the end of the chapter.

The usual analysis procedure is to use either an earthquake response spectrum or time history accelerograms as the basis of design. In either case, the Mode Superposition technique (including modal analysis ) has usually been adopted to solve the problem [31,32,33]. The time history can be conducted by direct integration of the general equation of motion. In this study, the response quantities of interest are the displacement, velocity and contact force time history. The

derivations of the general equations of motion for the case where pounding occurred between the adjacent building is explained in section (3.4). All these equation of motions are solved using the Mode Superposition technique where the symmetrical properties of the matrices are employed.

Figure 3.1 below shows the summary of the procedure in solving the equation of motion for earthquake response analysis.

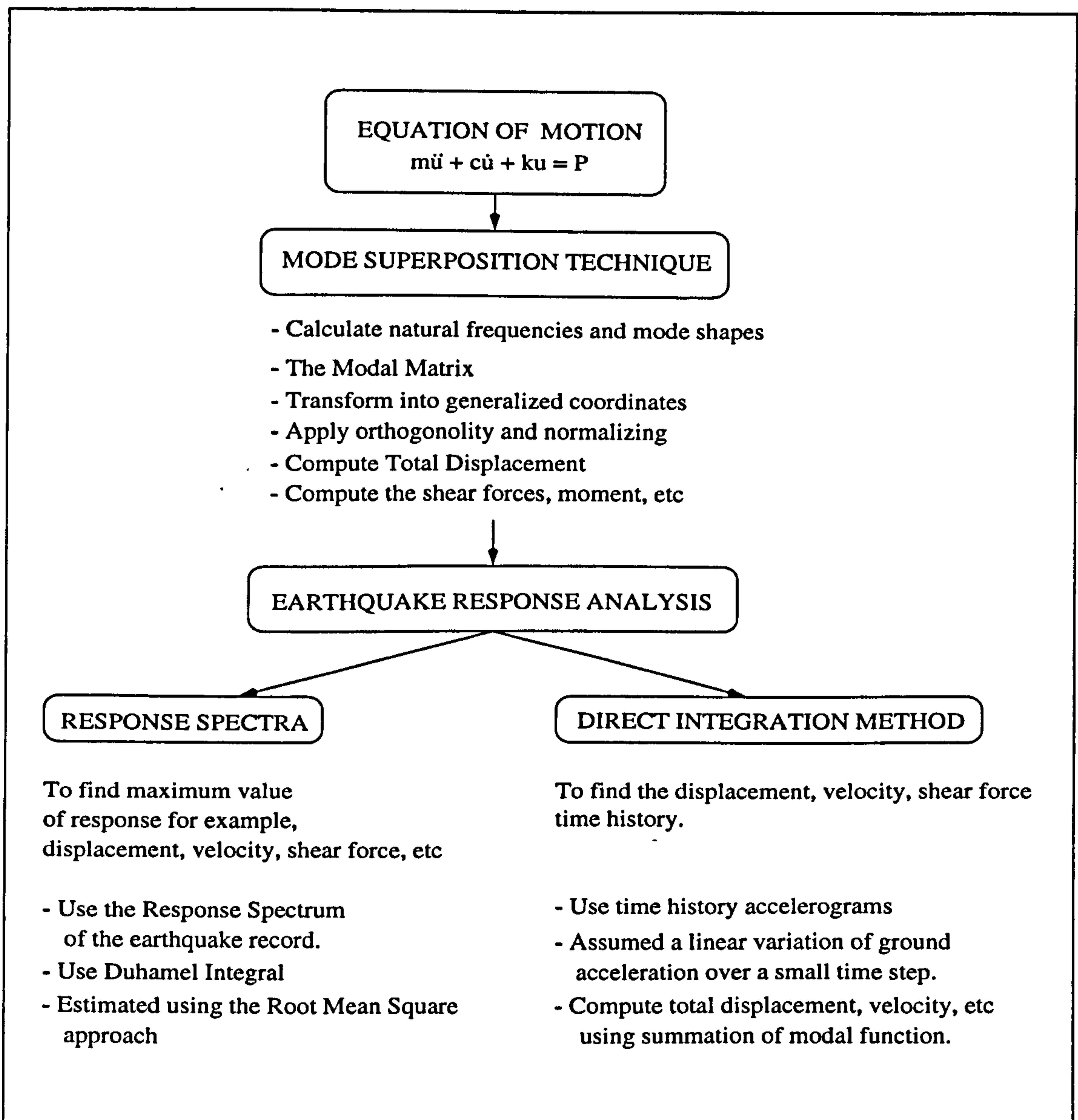


Figure 3.1 : Schematic diagram for solving the equation of motion

This chapter introduces the basic concept of the mathematical formulation of the equation of motion using the direct integration method via Mode Superposition



technique. This method has the advantage over the response spectrum method as it gives the full structural response time history (for example displacement) for the specified time of analysis. This concept is important in order to develop a further understanding of the structural pounding analysis throughout this study.

## **3.2 MODE SUPERPOSITION TECHNIQUES**

The dynamic response of any linear structure can be obtained after its vibration mode shapes and frequencies have been determined. One way of obtaining those results using mode superposition techniques. The use of the normal (modal) coordinates serves to transform the equations of motion from a set of  $n$  simultaneous differential equations, which are coupled by the off-diagonal terms in the mass and stiffness matrices, to a set of  $n$  independent normal coordinate equations. The dynamic response therefore can be obtained by solving separately for the response of each modal coordinate and then superimposing these into the generalized displacement coordinates. This section introduces the Mode Superposition techniques in the following order (section 3.2.1 - 3.2.6).

### **3.2.1 Undamped Free Vibration**

The first step in a mode superposition techniques is to obtain the natural frequencies and mode shapes of the system. Consider the undamped free vibration of MDOF systems where the damping matrix and applied load vectors are omitted, hence the equation of motion becomes

$$\underline{m} \ddot{\underline{u}}_i + \underline{k} \underline{u}_i = 0 \quad (3.1)$$

where  $\underline{m}$  is the mass matrix,  $\underline{k}$  is the stiffness matrix and  $\ddot{\underline{u}}_i$ ,  $\underline{u}_i$  is the acceleration and displacement vectors for the  $i$ th number of modes, respectively. Free vibration occurs when for example, the mass is displaced and then suddenly released (snap back analysis).

By analogy with the behaviour of SDOF system, it is assumed that the free-vibration motion is simple harmonic, which can also be expressed for a MDOF systems as

$$u_i = \alpha_i \sin ( \omega_i t + \theta ) \quad (3.2)$$

where  $u_i$  = displacement vectors  
 $\alpha_i$  = amplitude (mode shape) vectors  
 $\omega_i$  = free vibration frequencies  
 $\theta$  = phase angle

Second derivative of Equation (3.2) gives the acceleration vector as

$$\begin{aligned} \ddot{u}_i &= -\omega_i^2 \alpha_i \sin ( \omega_i t + \theta ) \\ &= -\omega_i^2 u_i \end{aligned} \quad (3.3)$$

Substituting Equation (3.3) and (3.2) into Equation (3.1) gives

$$-\omega_i^2 \underline{m} \alpha_i \sin ( \omega_i t + \theta ) + \underline{k} \alpha_i \sin(\omega_i t + \theta) = 0 \quad (3.4)$$

Since the sine term is arbitrary and it may be omitted from Equation (3.4) which can be written as

$$[ \underline{k} - \omega_i^2 \underline{m} ] \alpha_i = 0 \quad (3.5)$$

Hence for a nontrivial solution, the determinant of the left hand side of Equation (3.5) can be obtained only under condition:

$$| \underline{k} - \omega_i^2 \underline{m} | = 0 \quad (3.6)$$

Equation (3.6) is called the *frequency equation* of the system and this is represent the eigenvalue problem, and can be written in the form:

$$[ \underline{k} - \mu_i \underline{m} ] \alpha_i = 0 \quad (3.7)$$

where the eigenvalues,  $\mu_i = \omega_i^2$ . The values of  $\omega_i$  correspond to the natural frequencies of the structure. The eigenvectors  $\alpha_i$  corresponding to the eigenvalues  $\mu_i$ , which can be obtained by the back substitution of  $\mu_i$  into Equation (3.7), to obtain the mode shapes. In practice only the lowest frequencies and corresponding mode shapes are necessary to give actual deflection.

### 3.2.2 The Modal Matrix

As mentioned in section (3.1), the dynamic-response analysis of linear systems can be obtained by using normal coordinates where the displaced position is defined by the  $i$  components of the displacement vector  $u_i$ . Figure 3.2 for example illustrates the cantilever column for which the deflected shape is defined by translational displacement coordinates at three levels. Any displacement vector  $u_i$  for this structure can be developed by superimposing the amplitudes of the three modes of vibration, as shown in Figure 3.2. For any modal component  $\alpha_i$ , the displacements are given by the mode-shape vectors  $\alpha_i$  multiplied by the modal amplitudes  $y_i$ . Thus, the total displacements,  $u_i$  is equal to the sum of the modal components.

$$\begin{aligned}
 u_i &= \alpha_1 y_1 + \alpha_2 y_2 + \dots + \alpha_n y_n \\
 &= [ \alpha_i \dots \alpha_n ] y_i \quad (i=1\dots n) \\
 &= \sum_{i=1}^n \alpha_i y_i
 \end{aligned} \tag{3.8}$$

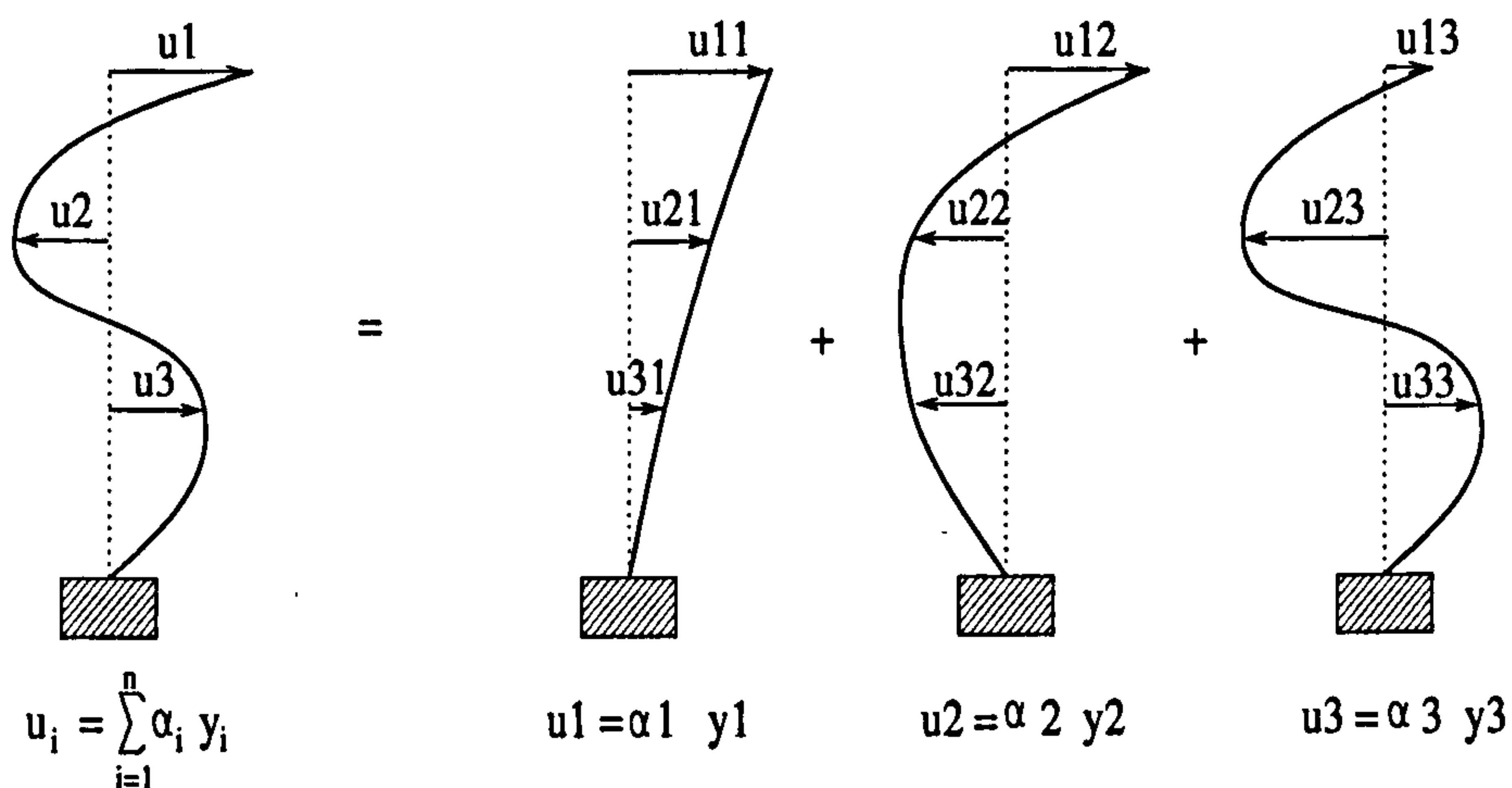


Figure 3.2 : Representing deflections as sum of the modal components.

or in matrix form,

$$u_i = \sum_{i=1}^n Q_i y_i \quad (i=1\dots n) \quad (3.9)$$

where  $u_i$  = actual displacements (function of time)  
 $Q_i$  = modal matrix =  $[\alpha_1, \alpha_2, \dots, \alpha_n]$   
 $y_i$  = modal displacement vectors (function of time)

The matrix  $Q_i$ , the columns of which are the mode shape vectors  $\alpha_1, \alpha_2, \dots, \alpha_n$ , is called the modal matrix.

### 3.2.3 Forced Vibration

The general equation of motion for a damped - forced vibration of MDOF systems is given by:

$$\underline{m} \ddot{u}_i + \underline{c} \dot{u}_i + \underline{k} u_i = \underline{P}_i \quad (3.10)$$

where  $\underline{m}$  is the mass matrix,  $\underline{c}$  is a damping matrix,  $\underline{k}$  is the stiffness matrix and  $\underline{P}_i$  is any applied force that may be acting on the systems.  $\underline{P}_i$  is a function of the time  $t$  and usually in the form of simple harmonic motion or any other type of dynamic loading.  $\ddot{u}_i$ ,  $\dot{u}_i$  and  $u_i$  is the acceleration, velocity and displacement vectors for the  $i$ th number of modes respectively.

These set of Equations (3.10) are coupled i.e. each equation contains more than one unknown, which means the set of Equations (3.10) cannot be easily solved directly. Hence, in the following step of Mode Superposition technique (modal analysis), these set of  $n$  coupled equations are transformed into a set of  $n$  uncoupled equations. Therefore, each transformed equation would contain only one unknown.

For modal analysis, the first and second derivative of Equation (3.9) are in the form:

$$\begin{aligned}
u_i &= \sum_{i=1}^n Q_i y_i \\
\dot{u}_i &= \sum_{i=1}^n Q_i \dot{y}_i \quad (i=1\dots n) \\
\ddot{u}_i &= \sum_{i=1}^n Q_i \ddot{y}_i
\end{aligned} \tag{3.11}$$

Substituting Equation (3.11) into Equation (3.10) gives:

$$\underline{m} \sum_{i=1}^n Q_i \ddot{y}_i + \underline{c} \sum_{i=1}^n Q_i \dot{y}_i + \underline{k} \sum_{i=1}^n Q_i y_i = \underline{P}_i \tag{3.12}$$

The modal matrix makes it possible to include the orthogonality condition by premultiplying each term by the transpose of  $Q_j$ , which is  $Q_j^T$ ,

$$Q_j^T \underline{m} Q_i \ddot{y}_i + Q_j^T \underline{c} Q_i \dot{y}_i + Q_j^T \underline{k} Q_i y_i = Q_j^T \underline{P}_i \tag{3.13}$$

Since the stiffness matrix,  $\underline{k}$  and the mass matrix,  $\underline{m}$  are both symmetrical, the orthogonality condition can be used to simplify the above equation as below:

$$M_i \ddot{y}_i + C_i \dot{y}_i + K_i y_i = P_i^* \tag{3.14}$$

where

$$\begin{aligned}
M_i &= Q_j^T \underline{m} Q_i = \text{modal mass matrix} \\
C_i &= Q_j^T \underline{c} Q_i = \text{modal damping matrix} \quad (i=1\dots n) \\
K_i &= Q_j^T \underline{k} Q_i = \text{modal stiffness matrix} \\
P_i^* &= Q_j^T \underline{P}_i = \text{modal force vectors}
\end{aligned} \tag{3.15}$$

The orthogonality conditions are:

$$Q_j^T \underline{m} Q_i = Q_j^T \underline{k} Q_i = 0 \quad \text{if } i \neq j \tag{3.16}$$

Therefore, when  $i = j$

$$Q_j^T \underline{m} Q_i = Q_j^T \underline{k} Q_i = 1 \quad \text{if } i = j \tag{3.17}$$

Dividing throughout by

$$M_i = Q_j^T m Q_i = 1 \quad \text{if } i = j \quad (3.18)$$

and let

$$K_i = \omega_i^2 M_i \quad \text{and} \quad C_i = 2 \xi_i \omega_i M_i \quad (3.19)$$

produces,

$$\ddot{y}_i + [2 \xi_i \omega_i] \dot{y}_i + \omega_i^2 y_i = \frac{P_i^*}{M_i} \quad (3.20)$$

where  $P_i^* / M_i$  is called the participation factor of the  $i$ th mode, where

$$\frac{P_i^*}{M_i} = \frac{Q_j^T P_i}{Q_j^T m Q_i = 1} \quad (3.21)$$

and  $\xi_i$  is the damping coefficient for  $i$ th mode. Normally, a damping coefficient,  $\xi_i$  for each mode has to be assumed and for most of practical structures, the amount of damping is small, falling in the range of 0.01 to 0.10. The damping coefficient for the steel frame is usually assumed to be between 1 % to 2 %.

The modal damping matrix can also be expressed in term of mass and stiffness as

$$C_i = C1 M_i + C2 K_i \quad (3.22)$$

where  $C1$  and  $C2$  are constants. This is usually termed Rayleigh damping. If  $C_i$  is proportional to  $M_i$  and  $K_i$  which are symmetrical and diagonal matrix, therefore the modal damping matrix,  $C_i$  can also be diagonal.

Substituting the Rayleigh damping into Equation (3.13) and also using orthogonality properties of Equation (3.17) produces :

$$M_i \ddot{y}_i + [C1 M_i + C2 K_i] \dot{y}_i + K_i y_i = P_i^* \quad (3.23)$$

Dividing throughout by Equation (3.18) and also using Equation (3.19) for  $K_i$  gives

$$\ddot{y}_i + [C1 + C2 \omega_i] \dot{y}_i + \omega_i^2 y_i = \frac{P_i^*}{M_i} \quad (3.24)$$

Comparing the coefficient of  $\dot{y}$  in the above equation with Equation (3.20), the modal damping can be defined by the equation:

$$\begin{aligned} 2 \xi_i \omega_i &= C1 + C2 \omega_i^2 \\ \therefore \xi_i &= \frac{1}{2} \left[ \frac{C1}{\omega_i} + C2 \omega_i \right] \end{aligned} \quad (3.25)$$

This definition of  $\xi_i$  will further simplify Equation (3.24) to the Equation (3.20). Thus, instead of  $n$  coupled equations (Equation 3.10), now Equations (3.20) have  $n$  uncoupled equations similar to that of a SDOF system. This equation can be solved using the Duhamel's integral response spectrum (section 3.3.1) or Direct Integration method (section 3.3.2).

### 3.2.4 The total displacement, velocity and acceleration

After the determination of  $y_i$  for each mode (section 3.3), the total displacement, velocity and acceleration is given by Equation (3.11) respectively. For example the total displacement,  $u_i$  is the sum of the  $\alpha_i y_i$  (the contribution from the 1st mode) to  $\alpha_n y_n$  (the contribution from the  $n$  th mode).

### 3.2.5 The Lateral Forces , Base Shear And Moments

For a particular mode, the equivalent lateral forces,  $f_i$  can be calculated using

$$\begin{aligned} f_i &= k u_i \\ &= k \sum_{i=1}^n Q_i y_i \\ &= \omega_i^2 m_i \sum_{i=1}^n \alpha_i y_i \quad (i=1..n) \end{aligned} \quad (3.26)$$

The shear at the base of a structure,  $V_{on}$  , is simply obtained by addition of all the lateral forces at floor levels,

$$V_{on} = \sum_{i=1}^n f_i \quad (3.27)$$

If the height of the floor above the base of a structure is known ( $h_i$ ), then, the moment at floor level,  $M_{oi}$  is calculated by :

$$\begin{aligned} M_{oi} &= \sum_{i=1}^n f_i h_i \\ M_{on} &= \sum_{i=1}^n M_{oi} \quad (i=1\dots n) \end{aligned} \quad (3.28)$$

where  $M_{on}$  is the total base moment of the structure.

### 3.2.6 Normalizing (Scaling)

It was noted earlier that the vibration mode amplitude will satisfy the basic frequency Equation (3.5), and only the resulting shapes are uniquely defined. This is called normalizing the mode shapes with respect to the specified reference.

It is frequently adopted in many computer programs that the shapes are normalized to the maximum displacement value in each mode rather than with respect to any particular coordinate. Thus the maximum value in each modal vector is unity, which provides convenient numbers for use in subsequent calculations. However, the normalizing procedure most often used in computer programs for structural-vibration analysis, involves adjusting each modal amplitude  $\alpha_i$  which satisfies the condition ;

$$Q_j^T m Q_i = M_i = 1 \quad (i=1\dots n) \quad \text{if } i=j \quad (3.29)$$

This can be accomplished by computing the scalar factor :

$$V_j^T m V_j = 1 \quad (j=1\dots n) \quad (3.30)$$

where  $V_j$  represent an arbitrarily determined modal amplitude, Let consider a structure with three masses ( $m_1, m_2, m_3$ ) and three mode shapes ( $\alpha_1 \alpha_2 \alpha_3$ ). Substituting these values into Equation (3.29) produces



$$(\alpha_1 \alpha_2 \alpha_3) \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = M_i \quad (3.31)$$

Or in summation form of

$$M_i = \sum_{i=1}^n m_i \alpha_i^2 \quad (3.32)$$

Consider first mode, where  $i=1$ , thus Equation (3.32) becomes

$$M_i = m_1 \alpha_1^2 + m_2 \alpha_2^2 + m_3 \alpha_3^2 \quad (3.33)$$

Then computing the normalized mode shapes as follows :

$$(V_1 V_2 V_3) \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 1 \quad (3.34)$$

To satisfies the condition of Equation (3.29)

$$1 = m_1 V_1^2 + m_2 V_2^2 + m_3 V_3^2 \quad (3.35)$$

Multiply Equation (3.35) by  $M_1$ , hence

$$M_1 = M_1 m_1 V_1^2 + M_1 m_2 V_2^2 + M_1 m_3 V_3^2 \quad (3.36)$$

Subtract Equation (3.33) from Equation (3.36) gives

$$\begin{aligned} 0 &= M_1 m_1 V_1^2 - m_1 \alpha_1^2 \\ 0 &= M_1 m_2 V_2^2 - m_2 \alpha_2^2 \\ 0 &= M_1 m_3 V_3^2 - m_3 \alpha_3^2 \end{aligned} \quad (3.37)$$

Rearrange above Equation ;

$$\begin{aligned} 0 &= (M_1 V_1^2 - \alpha_1^2) m_1 \\ 0 &= (M_1 V_2^2 - \alpha_2^2) m_2 \\ 0 &= (M_1 V_3^2 - \alpha_3^2) m_3 \end{aligned} \quad (3.38)$$

The solution is when

$$\begin{aligned}
(M_1 V_1^2 - \alpha_1^2) = 0 & \Rightarrow V_1 = \frac{\alpha_1}{\sqrt{M_1}} \\
(M_1 V_2^2 - \alpha_2^2) = 0 & \Rightarrow V_2 = \frac{\alpha_2}{\sqrt{M_1}} \\
(M_1 V_3^2 - \alpha_3^2) = 0 & \Rightarrow V_3 = \frac{\alpha_3}{\sqrt{M_1}}
\end{aligned} \tag{3.39}$$

This is normalizing for the first mode shape only. For the second and third mode shapes, the same procedure applies but with the multiplication of Equation (3.29) by  $M_2$  and  $M_3$  respectively. In general, this is in the form of

$$V_j = \frac{Q_j}{\sqrt{M_j}} \quad (j=1\dots n) \tag{3.40}$$

where  $Q_j$  is the modal matrix. A consequence of this type of normalizing, together with the modal orthogonality relationships relative to mass matrix is that ,

$$V_j^T m V_j = I \tag{3.41}$$

where  $I$  is the identity matrix.

### **3.3 EARTHQUAKE RESPONSE ANALYSIS**

The uncoupled equations of motion (Equation 3.20) can be analyzed using the time history analysis of the ground acceleration motion or response spectrum for a given earthquake record. Both methods use the mode superposition technique as mentioned in section (3.2).

#### **3.3.1 Response Spectra**

The problem that is frequently encountered in preliminary design, is to know the maximum absolute displacement or maximum stress, when the system is subjected to a given dynamic loading. Response spectra have been employed for this purpose.

Basically it consists a series of plots of maximum "response" (for example displacement, velocity and acceleration) of SDOF systems to a given input versus some system parameter, generally the undamped natural frequency.

The SDOF example below is illustrates the method of Response Spectra in finding the displacement, velocity and acceleration earthquake spectra. Or in other words, the Pseudo - displacement,  $S_d$ , the Pseudo - velocity,  $S_v$  and the Pseudo - acceleration,  $S_a$  .

##### ***3.3.2.1 Single Degree of Freedom System***

Consider a SDOF system as an example for the earthquake response illustrated in Figure 3.3 below;

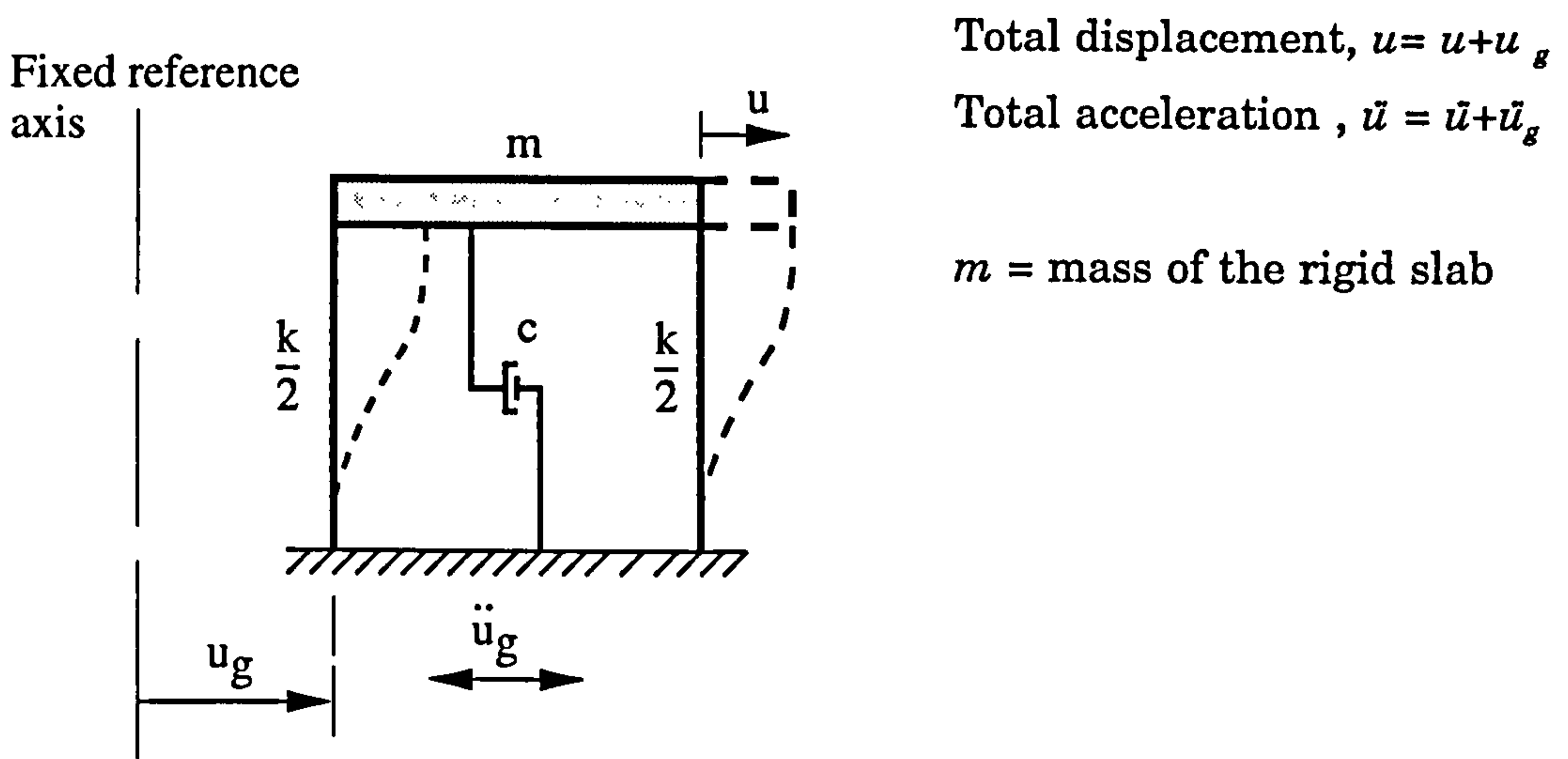


Figure 3.3 : Influence of support excitation on SDOF system.

It is frequently desirable to formulate the equation of motion in terms of the relative displacement ( $u = u - u_g$ ) since the forces applied to the mass are directly related to this relative displacement. Therefore, in this section, all the symbols of  $u$ ,  $\dot{u}$ , and  $\ddot{u}$ , represent the relative displacement, velocity and acceleration respectively.

The general equation of motion for earthquake response is written in the form of

$$m \ddot{u} + c \dot{u} + k u = - m \ddot{u}_g \quad (3.42)$$

where :

$k/2$  = massless column with elastic stiffness.

$c$  = the damper provides a velocity-proportional resistance to the motion.

$u_g$  = known ground motion displacement

$\ddot{u}_g$  = known ground acceleration

$u$  = the relative displacement of mass  $m$ , move only in simple translation.

$\ddot{u}$  = the relative acceleration of mass  $m$

Equation (3.42) can be treated in the same way as in section(3.2.3) where the right hand side is equal to a known earthquake force  $P$ .

$$- m \ddot{u}_g = P \quad (3.43)$$

Assuming that the system is initially at rest, then the Duhamel's Integral solution

is of the form:

$$\begin{aligned}
 u(t) &= \frac{1}{m \omega_d} \int_0^t P(\tau) e^{\xi \omega (t-\tau)} \sin \omega_d (t-\tau) d\tau \\
 &= \frac{1}{m \omega_d} \int_0^t -m \ddot{u}_g(\tau) e^{\xi \omega (t-\tau)} \sin \omega_d (t-\tau) d\tau \quad (3.44) \\
 &= \frac{1}{\omega_d} \int_0^t -\ddot{u}_g(\tau) e^{\xi \omega (t-\tau)} \sin \omega_d (t-\tau) d\tau
 \end{aligned}$$

where

$$\omega_d = \omega \sqrt{1 - \xi^2} \quad \text{and} \quad \omega = \frac{2\pi}{T} \quad (3.45)$$

That is  $\omega_d$  is the damped natural frequency  
 $\omega$  is the undamped natural frequency  
 $T$  is the undamped natural period.

It is further assumed that the  $\omega_d \approx \omega$  when the damping coefficient  $\xi$  is very much smaller than 1 % .

$$u(t) = -\frac{1}{\omega} \int_0^t \ddot{u}_g(\tau) e^{\xi \omega (t-\tau)} \sin \omega (t-\tau) d\tau \quad (3.46)$$

For a complete earthquake record of displacement as shown in Figure 3.4 below, the maximum value of absolute displacement can be found.

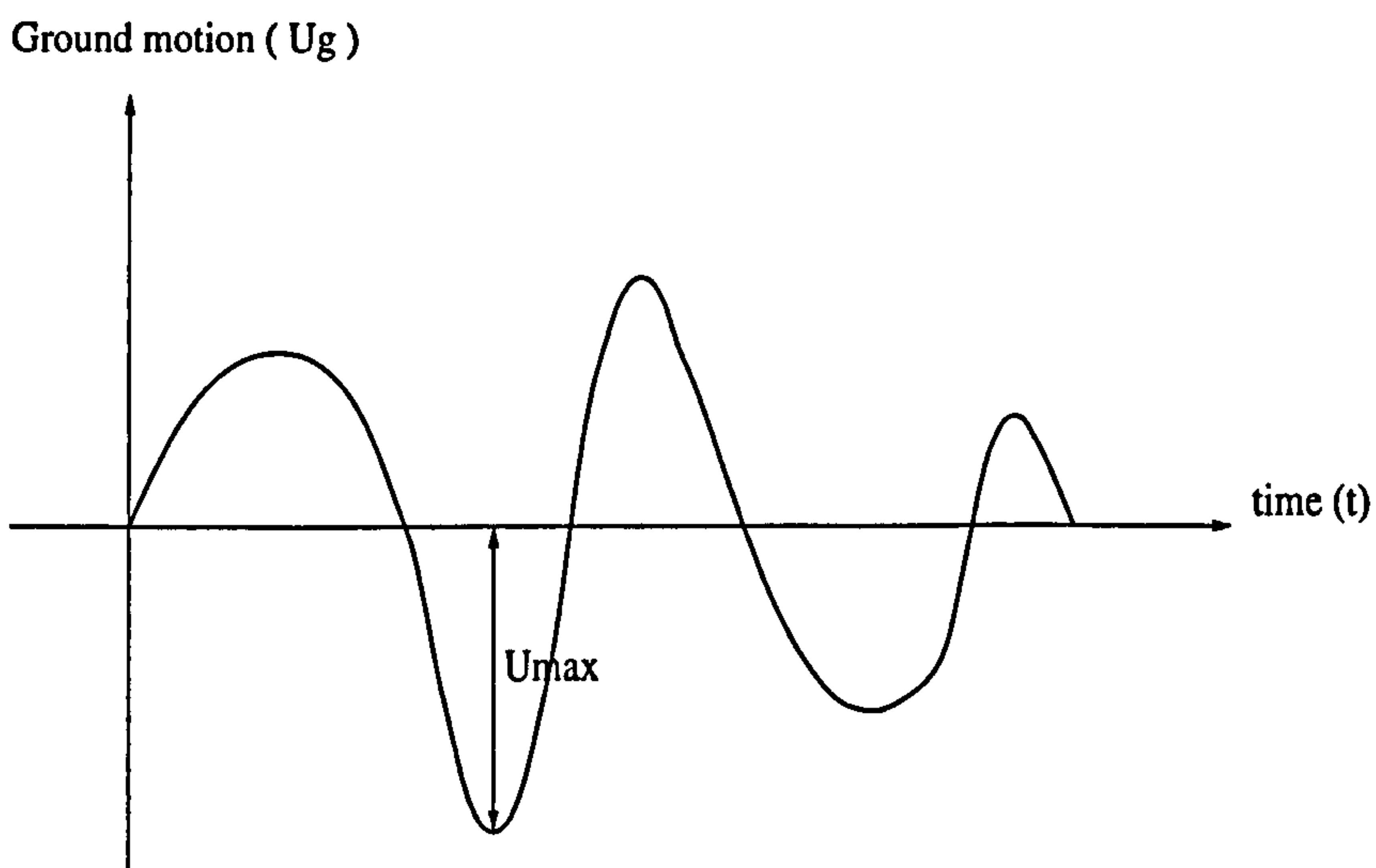


Figure 3.4 : Complete earthquake record

The maximum absolute displacement ,  $u_{\max}$  is usually designated as  $Sd$  . Recall Equation (3.46) and this will gives

$$u_{\max} = \frac{1}{\omega} \left| \int_0^t \ddot{u}_g (\tau) e^{\xi \omega (t - \tau)} \sin \omega ( t - \tau ) d\tau \right| \quad (3.47)$$

By definition

$$Sv = \left| \int_0^t \ddot{u}_g (\tau) e^{\xi \omega (t - \tau)} \sin \omega ( t - \tau ) d\tau \right| \quad (3.48)$$

Thus, Pseudo-velocity,  $Sv$  :

$$\begin{aligned} u_{\max} &= Sd = \frac{1}{\omega} Sv \\ Sv &= \omega Sd \end{aligned} \quad (3.49)$$

The Pseudo-acceleration ,  $Sa$  is defined by

$$\begin{aligned} Sa &= \omega Sv \\ &= \omega ( \omega Sd ) \\ &= \omega^2 Sd \end{aligned} \quad (3.50)$$

In other words,  $Sd$  is  $u_{\max}$  i.e maximum absolute displacement  
 $Sv$  is the maximum absolute velocity  
 $Sa$  is the maximum total acceleration.

Figure 3.5a shows the spectrum of the El-Centro May 18, 1940 record plotted against maximum velocity. The sharp peaks and valleys in Figure 3.5a are result of local resonances and antiresonances of the ground motion. For design purposes, these irregularities can be smoothed out as shown in Figure 3.5b by averaging the response as proposed by Housner in Reference [32].

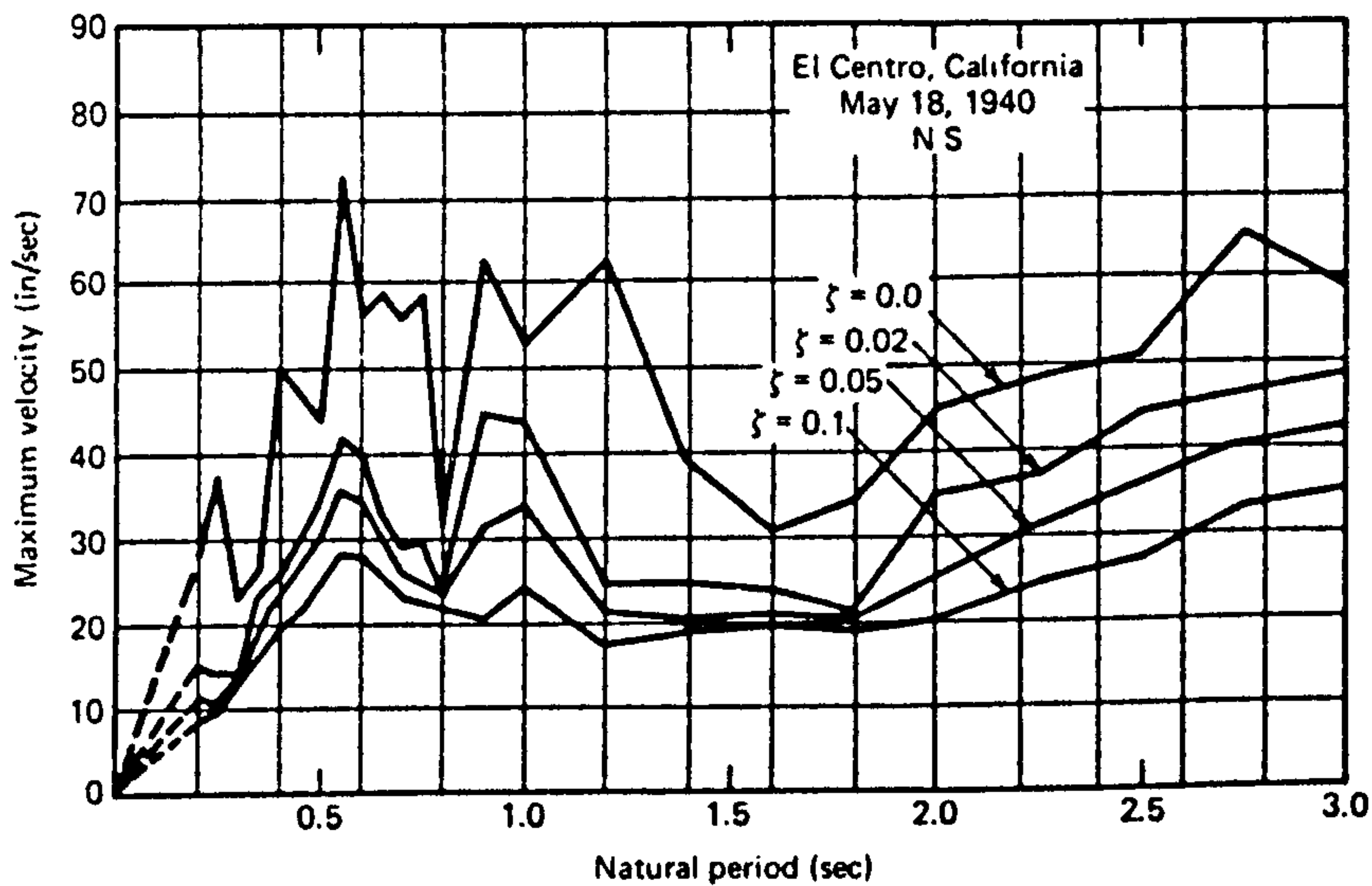


Figure 3.5a : Pseudo-velocity response spectra for the N-S component of the 1940 El Centro earthquake, California. (After Housner , 1970)

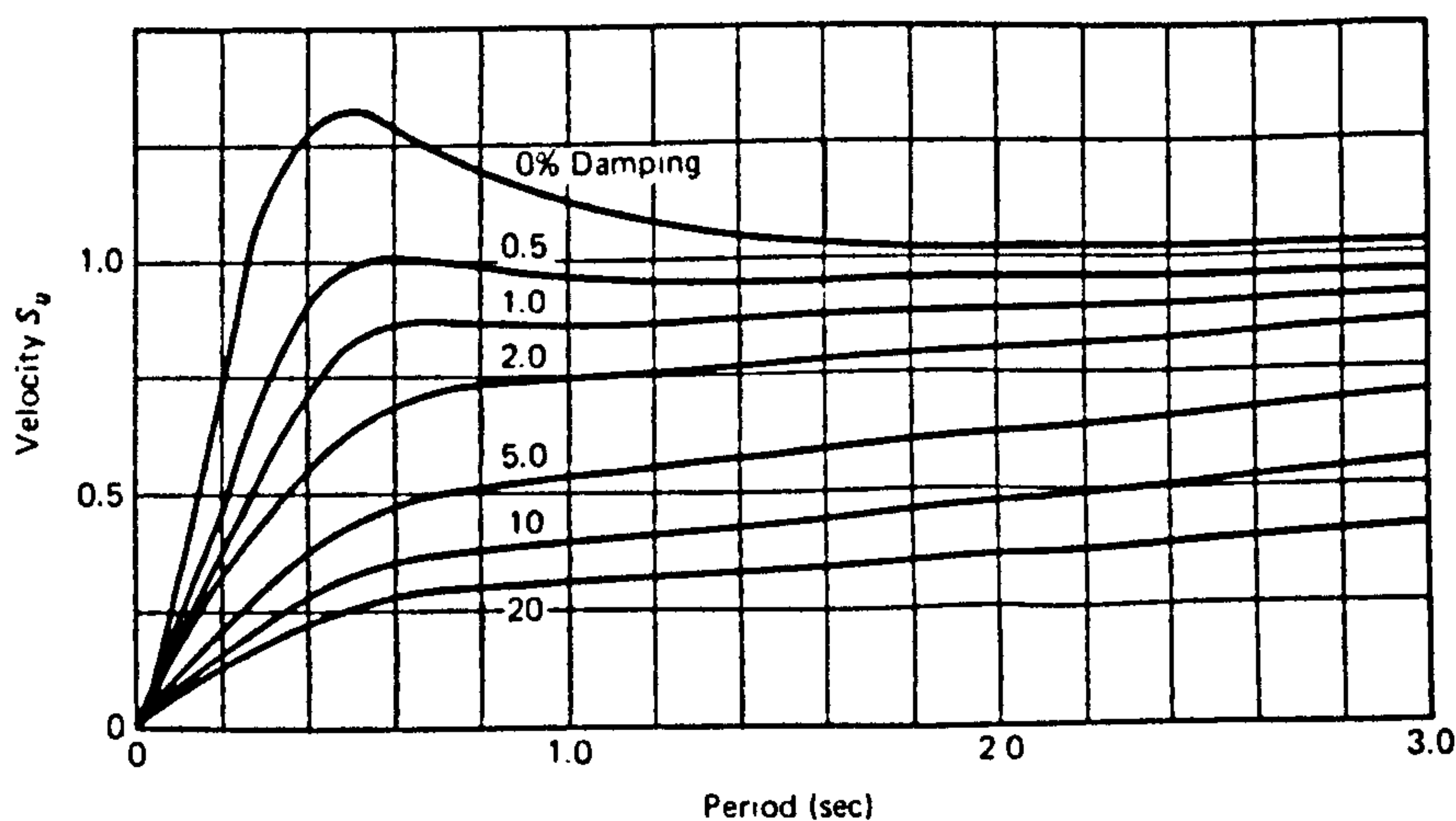


Figure 3.5b : Averaged velocity response spectrum, 1940 El Centro Intensity. (After Housner , 1970)

Because of the simple relationships between the three response spectra (displacement, velocity and acceleration) it is possible to present them all in a single graph plotted with log scales on each axis as shown in Figure 3.5c.

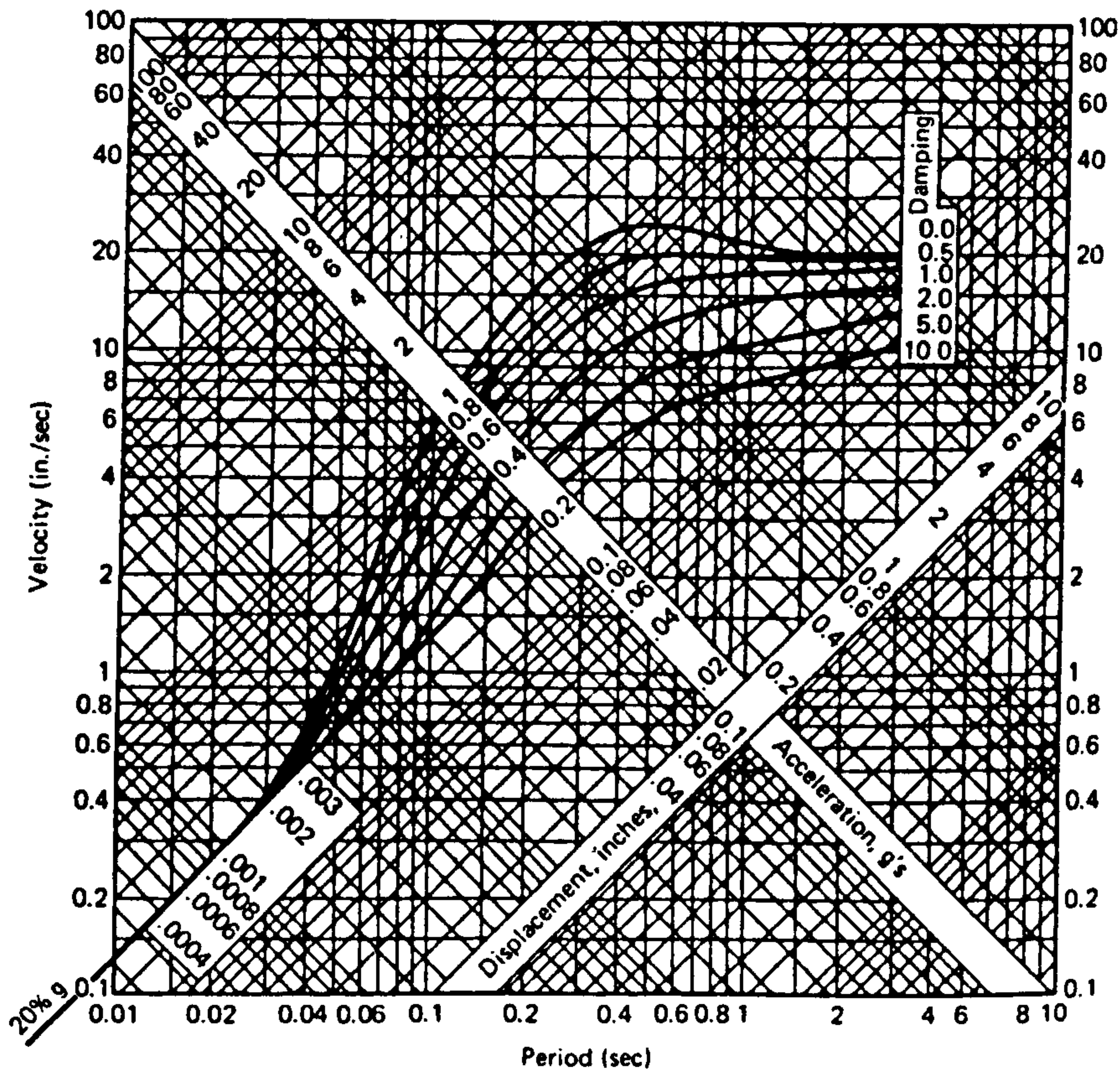


Figure 3.5c : Tripartite plot of design spectrum subjected to 1940, El Centro earthquake (N-S Component). (After Housner, 1970).

This plots show values for 0, 0.5, 1.0, 2.0, 5.0 and 10.0% of damping coefficient. This indicates the large reduction in the response resulting from damping. As can be seen, even the small amount of 2% of critical damping produces a significant reduction in the level of the response.

It is of interest to determine the maximum base shear,  $V_{max}$  and the maximum overturning moment,  $M_{max}$ , due to earthquake. The maximum base shear can be obtained by multiplying the stiffness of the column,  $k$ , by the spectral pseudo-displacement, where :

$$\begin{aligned} V_{max} &= k u_{max} \\ &= k S_d \end{aligned} \quad (3.51)$$



This leads to the fact that the maximum floor force,  $f_s$  is also given by  $k Sd$  and thus :

$$\begin{aligned}(f_s) \max &= k Sd \\ &= \frac{k}{\omega^2} Sa \\ &= m Sa\end{aligned}\tag{3.52}$$

that is the maximum floor force which is obtained by multiplying the pseudo-acceleration,  $Sa$  by the mass,  $m$ . Hence, the maximum overturning moment,  $M_{max}$  at the base can be determined by :

$$\begin{aligned}M_{max} &= (f_s) \max * L \\ &= m Sa L\end{aligned}\tag{3.53}$$

where  $L$  is the interstorey height.

### 3.3.1.2 Multi Degree Of Freedom Systems

In the previous chapter, although SDOF models may adequately describe the dynamic behaviour of some systems, in most cases, it is necessary to employ more "realistic" models, for example MDOF model. To illustrate this, consider a three story shear building subjected to earthquake motion, as shown in Figure 3.6 below. The motion of the building is to be simplified using the lumped mass methods of discretization as shown in Figure 3.6 (b).

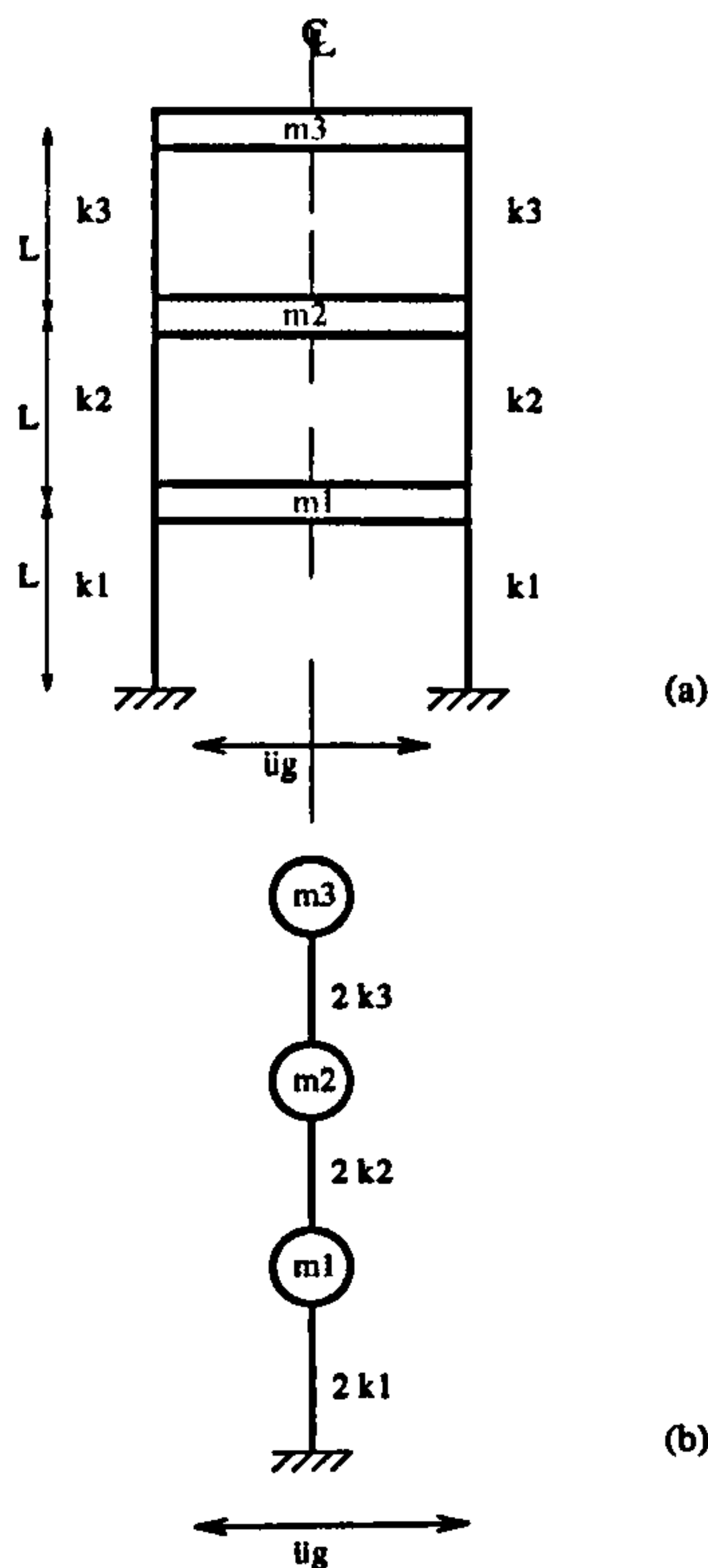


Figure 3.6 : (a) Multistorey building subjected to earthquake excitation (b) Lumped masses idealization

The response of MDOF systems can be determined by using mode superposition technique or direct integration of the equations of motion . However, some of the simplifications made for SDOF systems, as described in a previous section by the introduction of a response spectra, can be extended to MDOF systems. The general equation for a MDOF system is in the form,

$$m \ddot{u}_i + c \dot{u}_i + k u_i = - m r \ddot{u}_g \quad (3.54)$$

where  $\ddot{u}_g$  is the ground acceleration.  
 $r$  is Pseudo static vector .  
 $m r \ddot{u}_g$  is the earthquake force, or termed as  $P_i$   
 $i$ th number of modes

The Pseudo static vector or the Earthquake Influence coefficient, is a vector composed of displacement produced by unit rigid translation in earthquake direction. Figure 3.7 shows how these static vectors are derived for a particular structure.

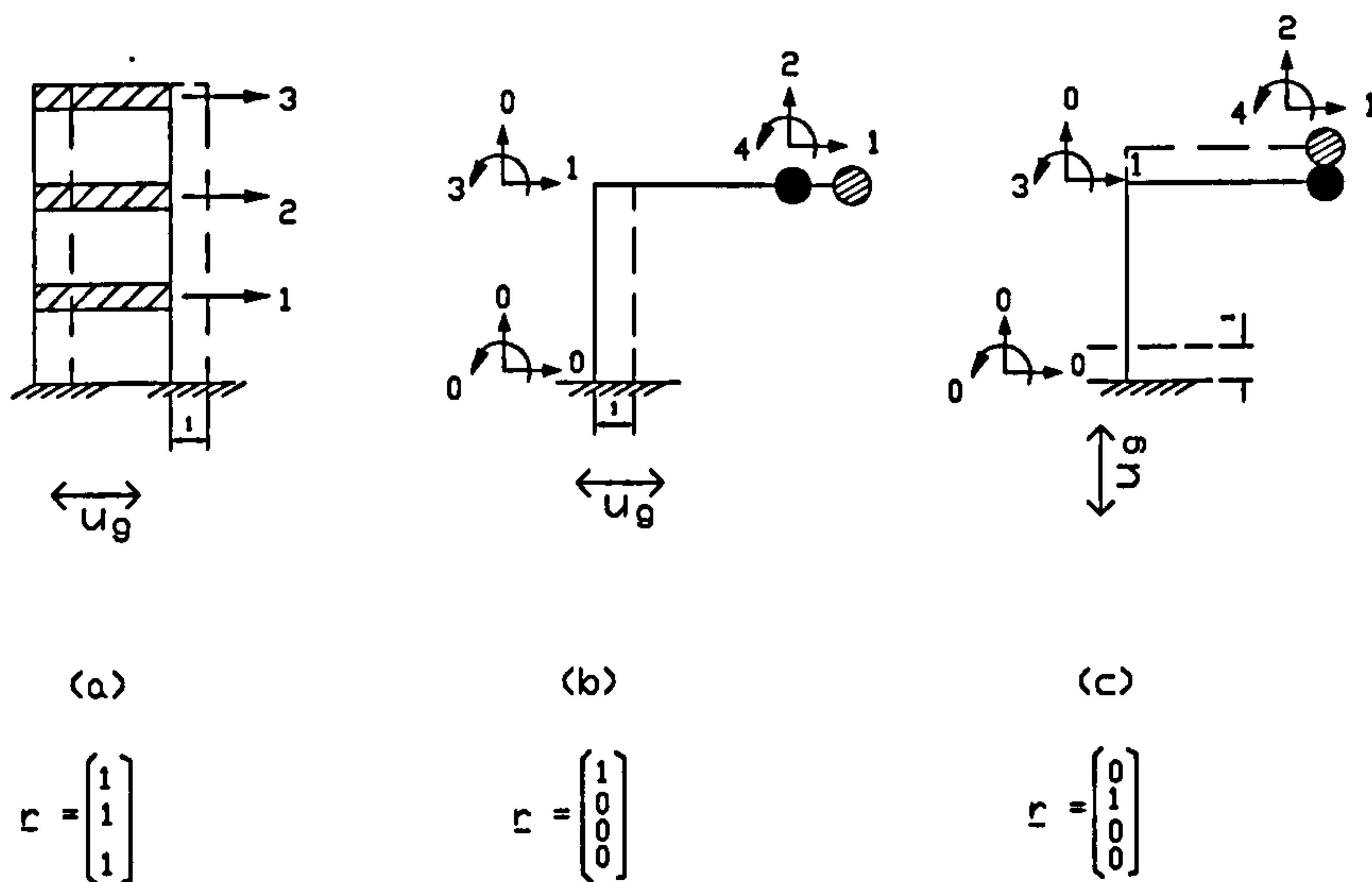


Figure 3.7 : Pseudo Static vector.

Solving Equation (3.54) using the mode superposition technique as described in previous section of this chapter, gives

$$\ddot{y}_i + [2 \xi_i \omega_i] \dot{y}_i + \omega_i^2 y_i = \frac{Q_j^T m r \ddot{u}_g}{M_i} \quad (3.55)$$

Let the right hand side of Equation (3.55) equal to

$$\beta_i = \frac{Q_j^T m r}{M_i} \quad (3.56)$$

The term  $\beta_i$  is called the Participation Factor. In most computer programs for structural vibrations, the value of  $M_i$  is made equal to unity using a normalizing procedure which is explained in section (3.2.6). Substitute Equation (3.56) into Equation (3.55) gives;

$$\ddot{y}_i + [ 2 \xi_i \omega_i ] \dot{y}_i + \omega_i^2 y_i = \beta_i \ddot{u}_g \quad (3.57)$$

This can be solve using Duhamel's Integral solution and the maximum response can be found using the Response Spectra for a complete earthquake record. The procedure is the same as for SDOF except the term  $\beta_i$ , the Participation Factor is associated with MDOF system.

Therefore, the modified solutions as in Equation (3.47) is in the form of:

$$y_{i\max} = \frac{\beta_i}{\omega_i} \left| \int_0^t \ddot{u}_g(\tau) e^{\xi_i \omega_i (t - \tau)} \sin \omega_i (t - \tau) d\tau \right| \quad (3.58)$$

Thus, the Pseudo-displacement,  $Sd_i$ , Pseudo-velocity,  $Sv_i$  and Pseudo-acceleration,  $Sa_i$ , are :

$$\begin{aligned} y_{i\max} &= \beta_i Sd_i \\ &= \frac{\beta_i}{\omega_i} Sv_i \\ &= \frac{\beta_i}{\omega_i^2} Sa_i \end{aligned} \quad (3.59)$$

The total displacement, velocity and acceleration is given by Equation (3.11). As not all the modes maximised at the same time, the overall value is estimated using the Root Mean Square approach or sometimes called the Square Root of Sum of the Squares (SRSS) method.

$$u_{i\max} = \sqrt{\sum_{i=1}^n (\alpha_i y_{i\max})^2} \quad (i=1\dots n) \quad (3.60)$$

### 3.3.1.3 Summary on Response Spectra for SDOF and MDOF systems.

#### SDOF SYSTEM

Governing Equation:

$$m \ddot{u} + c \dot{u} + k u = - m \ddot{u}_g$$

Rearrange :

$$\ddot{u} + 2 \xi \omega \dot{u} + \omega^2 u = \ddot{u}_g$$

displacement:  $u$

natural frequency:  $\omega$

damping coefficient:  $\xi$

participation factor: 1

Maximum displacement:

$$\begin{aligned} u_{\max} &= Sd(\xi, T) \\ &= \frac{Sv}{\omega} \\ &= \frac{Sa}{\omega^2} \end{aligned}$$

#### MDOF SYSTEMS

Governing Equation:

$$m \ddot{u}_i + c \dot{u}_i + k u_i = - m r \ddot{u}_g$$

Transform to uncoupled equation:

$$\ddot{y}_i + 2 \xi_i \omega_i \dot{y}_i + \omega_i^2 y_i = \beta_i \ddot{u}_g$$

$y_i$

$\omega_i$

$\xi_i$

$\beta_i$

Maximum displacement for  $i$ th mode:

$$\begin{aligned} y_{i\max} &= \beta_i Sd(\xi_i, T_i) \\ &= \beta_i Sd_i \\ &= \frac{Sv_i}{\omega_i} \\ &= \frac{Sa_i}{\omega_i^2} \end{aligned}$$

Total maximum displacement:

$$u_{i\max} = \sqrt{\sum_{i=1}^n (\alpha_i y_{i\max})^2} \quad (i=1\dots n)$$

### **3.3.2 Direct Integration Method by Time History Analysis**

It has been mentioned previously, that the response of structures can be determined by using either the mode superposition technique which leads to the Duhamel's Integral in conjunction with the response spectrum, or by direct integration of the equations of motion, for a specified ground motion history.

In the case of pounding under dynamic ground motion, the direct integration method by means of the time history analysis is most commonly used. The maximum response such as displacement or contact force of the structure for a very short period of "contact" stage during pounding can be calculated by specifying a small time step so that maximum response will not be missed. Two basic assumptions of this method are:

1. The ground acceleration record must be subdivided into small time steps, with the maximum of 1/100 th of the basic period of the ground motion.
2. During this time interval, an approximation can be made about the variation of the structures acceleration. It is assumed that the time step, (dt) is very small and a linear variation of acceleration can be assumed.

Knowing this variation and the initial conditions of displacement, velocity and acceleration at the beginning of the time step, the values of velocity and displacement can be obtained at the end of the time step.

A computation method has been developed, which based on the exact solution to the governing differential equation for the successive linear variations of the ground acceleration [23]. This solution is to compute the response at equal discrete time steps in a purely arithmetical way.

First, consider a SDOF system as in section (3.3.2.1). Then, a MDOF system can be developed by analogy to SDOF system.

### 3.3.2.1 Formulation for SDOF system

The general equation of motion for the SDOF system is given in Equation (3.42).

Rearranging this equation gives :

$$\ddot{u} + 2 \xi \omega \dot{u} + \omega^2 u = - \beta \ddot{u}_g \quad (3.61)$$

where  $\beta$  is the participation factor, and equal to 1 for SDOF system as described in Equation (3.56). Assumed the variation of the ground motion is linear over a small time step, as shown in Figure 3.8 below:

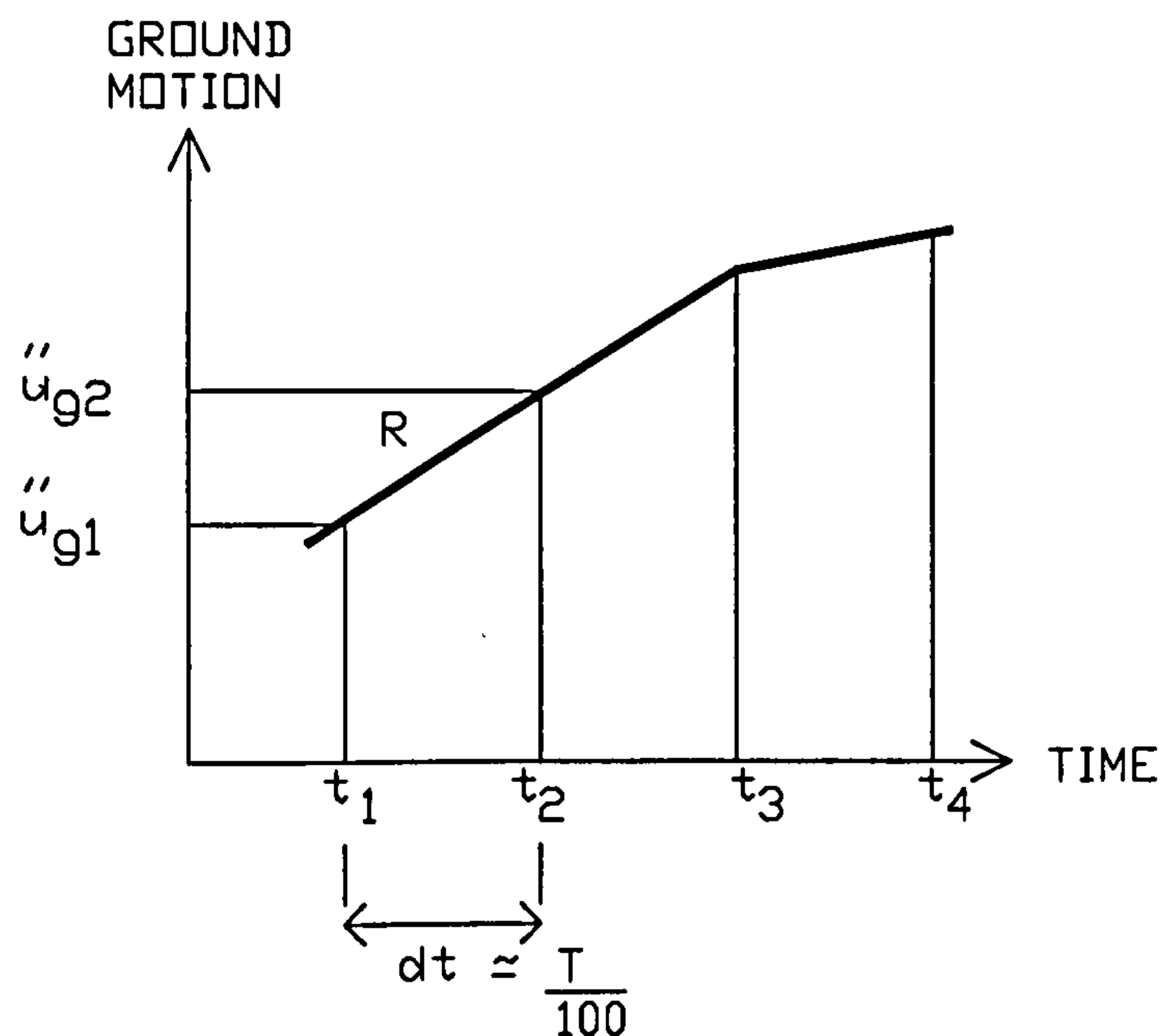


Figure 3.8 : Idealized ground acceleration.

where  $R$  is the rate of change of the ground acceleration.

$$\begin{aligned} R &= \frac{\ddot{u}_{g2} - \ddot{u}_{g1}}{t_2 - t_1} \\ &= \frac{\ddot{u}_{g2} - \ddot{u}_{g1}}{dt} \end{aligned} \quad (3.62)$$

Rearrange Equation (3.62) :

$$\ddot{u}_{g2} = \ddot{u}_{g1} + R dt \quad (3.63)$$

Substituting Equation (3.63) into Equation (3.61):

$$\ddot{u} + 2 \xi \omega \dot{u} + \omega^2 u = - \beta ( \ddot{u}_{g1} + R dt ) \quad (3.64)$$

The solution for the equation (3.64), consists of the Complementary Function (C.F) and the Particular Integral (P.I). The general solution = C.F + P.I. The derivation for the solution is shown fully in Appendix A.

The general equation for displacement at the end of the time step is

$$\begin{aligned} u_2 = & e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} - \frac{(2 \xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt \\ & + e^{-\xi \omega dt} \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2 \xi R)}{\omega^3} \right] \cos \omega_d dt \\ & - \beta \frac{R dt}{\omega^2} - \beta \frac{\ddot{u}_g}{\omega^2} + \beta \frac{(2 \xi)}{\omega^3} R \end{aligned} \quad (3.65)$$

The general equation of velocity at the end of time step is given by:

$$\begin{aligned} \dot{u}_2 = & e^{-\xi \omega dt} \left[ \dot{u}_1 + \beta \frac{R}{\omega^2} \right] \cos \omega_d dt + e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ -\omega^2 u_1 \right. \\ & \left. - \beta \ddot{u}_{g1} - \xi \omega \dot{u}_1 + \frac{\beta \xi R}{\omega} \right] \sin \omega_d dt \\ & - \beta \frac{R}{\omega^2} \end{aligned} \quad (3.66)$$

And the solution for the acceleration is given by Equation (3.64)

### 3.3.2.2 Formulation for MDOF systems

The equation of motion for a MDOF systems is given by Equation (3.54). These coupled equations are then transformed into a set of  $i$ th uncoupled equations which contains only one unknown. Each equation can then be treated in the same way a SDOF system. Recall Equation (3.57)

$$\ddot{y}_i + [ 2 \xi_i \omega_i ] \dot{y}_i + \omega_i^2 y_i = \beta_i \ddot{u}_g$$

where  $\beta_i$  is the Participation Factor. The linear variation of the ground motion over a small time step, as shown in Figure 3.8 is also assumed for the MDOF systems. The equations of displacement and velocity, at the end of the time step are based



on Equations (3.65) and (3.66) respectively, where  $u$  term is replaced by  $y$  term.

$$\begin{aligned}
 y_{i2} = & e^{-\xi_i \omega_i dt} \frac{1}{\omega_{id}} \left[ \dot{y}_{i1} + \xi_i \omega_i y_{i1} + \xi_i \beta_i \frac{\ddot{u}_{g1}}{\omega_i} - \frac{(2\xi_i^2 - 1)}{\omega_i^2} \beta_i R \right] \sin \omega_{id} dt \\
 & + e^{-\xi_i \omega_i dt} \left[ y_{i1} + \beta_i \frac{\ddot{u}_{g1}}{\omega_i^2} - \beta_i \frac{(2\xi_i R)}{\omega_i^3} \right] \cos \omega_{id} dt \\
 & - \beta_i \frac{R dt}{\omega_i^2} - \beta \frac{\ddot{u}_g}{\omega_i^2} + \beta_i \frac{(2\xi_i)}{\omega_i^3} R
 \end{aligned} \tag{3.67}$$

and velocity,

$$\begin{aligned}
 \dot{y}_{i1} = & e^{-\xi_i \omega_i dt} \left[ \dot{y}_{i1} + \beta_i \frac{R}{\omega_i^2} \right] \cos \omega_{id} dt + e^{-\xi_i \omega_i dt} \frac{1}{\omega_{id}} \left[ -\omega_i^2 y_{i1} \right. \\
 & \left. - \beta_i \ddot{u}_{g1} - \xi_i \omega_i \dot{y}_{i1} + \frac{\beta_i \xi_i R}{\omega_i} \right] \sin \omega_{id} dt \\
 & - \beta_i \frac{R}{\omega_i^2}
 \end{aligned} \tag{3.68}$$

Also the acceleration is

$$\ddot{y}_{i2} = -2\xi_i \omega_i \dot{y}_{i2} - \omega_i^2 y_{i2} - \beta_i \ddot{u}_g \tag{3.69}$$

The total displacement, velocity and acceleration at the end of the time step is the sum of these response of each mode which is given by Equation (3.11).

The application of these equations (3.67 - 3.69) using the mode superposition technique and direct integration method will be shown in various analysis of pounding for the next section and chapters.

### 3.4 POUNDING OF ADJACENT RIGID BUILDING WITH RIGID SLAB

The analysis of the pounding is considered to be an impact (contact) problem. It can be idealized as having two states, buildings not in contact and buildings in contact. A complete response (for example displacement) time history can be achieved as the response continues to change from one state to another. Three cases of pounding are studied in this section, i.e. undamped free vibration, damped free vibration and undamped forced-vibration. The slab is assumed rigid in all cases.

First, a study of pounding occurring at a single floor level (aligned floor case) is presented. The displacement compatibility method is used to formulate the equation of motion. Then this method is extended to a study of floor to column pounding (unaligned floor case), occurring at any location on a laterally unsupported building element such as column.

#### 3.4.1 Undamped Free Vibration

In this section, the free vibration analysis is performed using a snap back analysis. Consider a shear frame as shown in Figure 3.9.

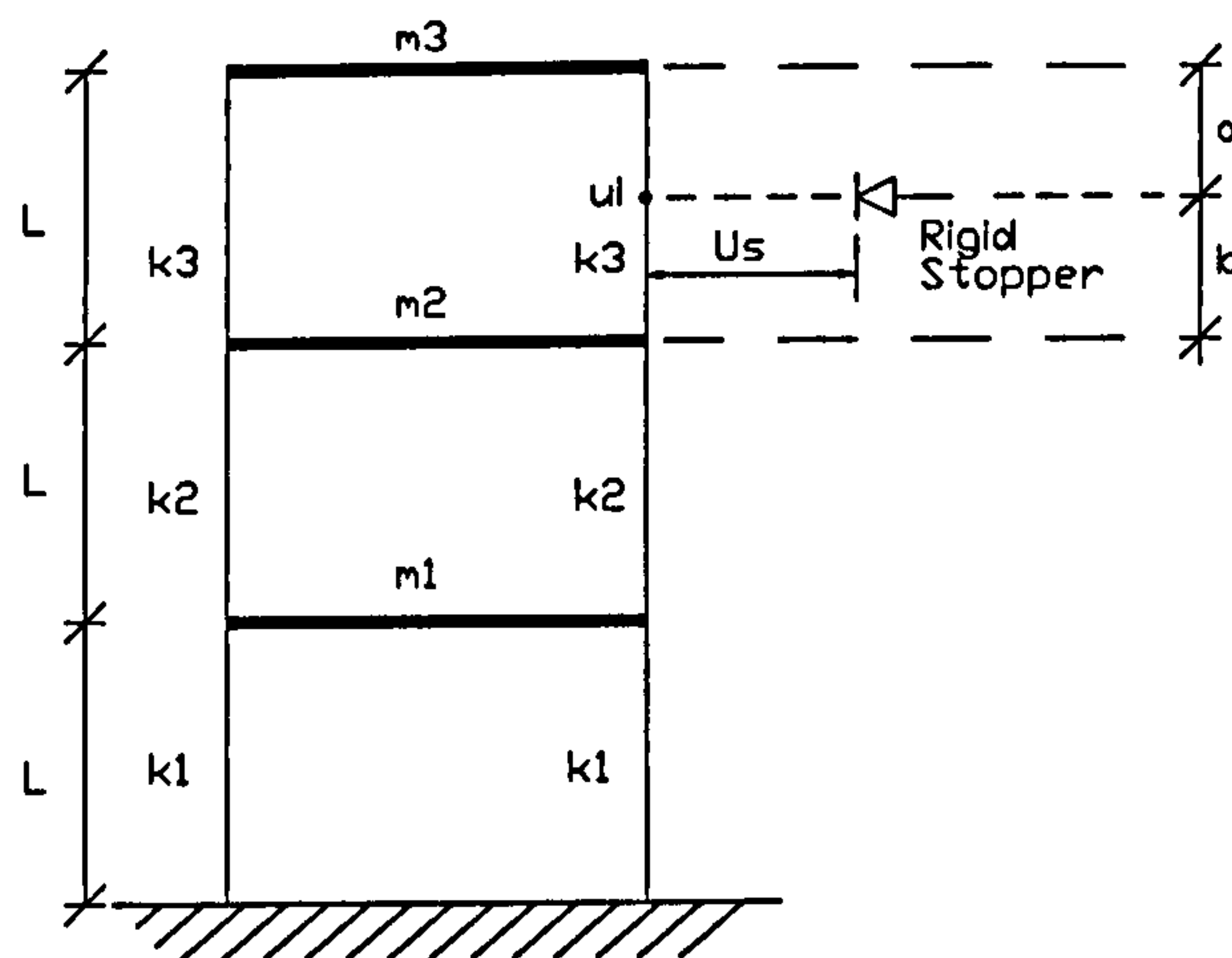


Figure 3.9 : Shear frame model idealization.

Two states of contact problem; no contact and contact are presented. The adjacent rigid building is represented by the rigid stopper at a distance  $U_s$  from the building of interest and at the distance  $\alpha$  from the top of the building.

### 3.4.1.1 State 1 (No Contact)

The equations of motion for undamped free vibration of MDOF systems as in Equation (3.54) can be written as

$$\begin{vmatrix} m3 & 0 & 0 \\ 0 & m2 & 0 \\ 0 & 0 & m1 \end{vmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} 2k3 & -2k3 & 0 \\ -2k3 & 2k3 + 2k2 & -2k2 \\ 0 & -2k2 & 2k2 + 2k1 \end{vmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} = 0 \quad (3.70a)$$

Or in matrix form

$$m \ddot{u}_i + k u_i = 0 \quad (3.70b)$$

These equations are uncoupled using the mode superposition technique (section 3.2) produces

$$\ddot{y}_i + \omega_i^2 y_i = 0 \quad (3.71)$$

The solutions for the generalized displacement and velocity at the end of time step are given by Equations (3.67 - 3.68) respectively. For the undamped free vibration, these equations are simplified as

$$\begin{aligned} y_i &= \frac{\dot{y}_{it}}{\omega_i} \sin \omega_i dt + y_{it} \cos \omega_i dt \\ \dot{y}_i &= \dot{y}_{it} \cos \omega_i dt - \omega_i y_{it} \sin \omega_i dt \end{aligned} \quad (3.72)$$

The solution for the generalized acceleration is given by rearranging Equation (3.71) produces

$$\ddot{y}_i = -\omega_i^2 y_i \quad (3.73a)$$

If the initial total displacement and velocity vectors are  $u_i$  and  $\dot{u}_i$ , then the modal initial displacement ( $y_{it}$ ) and velocity ( $\dot{y}_{it}$ ) vectors at time ( $t=0$ ) can be normalized as described in Equation (3.40), that is

$$\begin{aligned} y_{i0} &= Q_j^T m u_i \\ \dot{y}_{i0} &= Q_j^T m \dot{u}_i \end{aligned} \quad (M_i = 1) \quad (3.73b)$$

The solution process can be carried out by taking a series of time steps using these equations, where the generalized displacement, velocity and ground acceleration at the end of current time step become the starting values for the next time step, and so forth. The total displacement, velocity and acceleration responses are the sum of the modal contributions as in Equation (3.11).

The displacement and velocity at the point of contact, are given by

$$\begin{aligned} u_i &= (1 - k_{AB}) u_3 + k_{AB} u_2 \\ \dot{u}_i &= (1 - k_{AB}) \dot{u}_3 + k_{AB} \dot{u}_2 \end{aligned} \quad (3.74)$$

where  $k_{AB} = 3 \left(\frac{a}{L}\right)^2 - 2 \left(\frac{a}{L}\right)^3 = \frac{a^2}{L^3} [ a + 3b ]$

The basic derivation of  $u_i$  and  $k_{AB}$  can be seen in Appendix B

### 3.4.1.2 State 2 ( Contact )

The use of displacement compatibility method in deriving the MDOF equations of motion is illustrated in Figure 3.10.

In general, there are 3 basic stages in performing the overall time history analysis for the contact problem. In Figure 3.10a , the lateral displacements of the frame are  $u_3$ ,  $u_2$  and  $u_1$  for the top, middle and bottom floors respectively.

When the building hits the rigid stopper at a distance  $a$  from the top of the floor, the  $u_3$  and  $u_2$  on the right hand side of the top column are now termed  $us3$  and  $us2$  (Figure 3.10b). The  $us3$  and  $us2$  values are constant throughout a particular contact stage.

The only parameters that vary throughout the contact stage are the  $ud3$  and  $ud2$  terms which are called flexibility displacements. These are shown in Figure (3.10c).

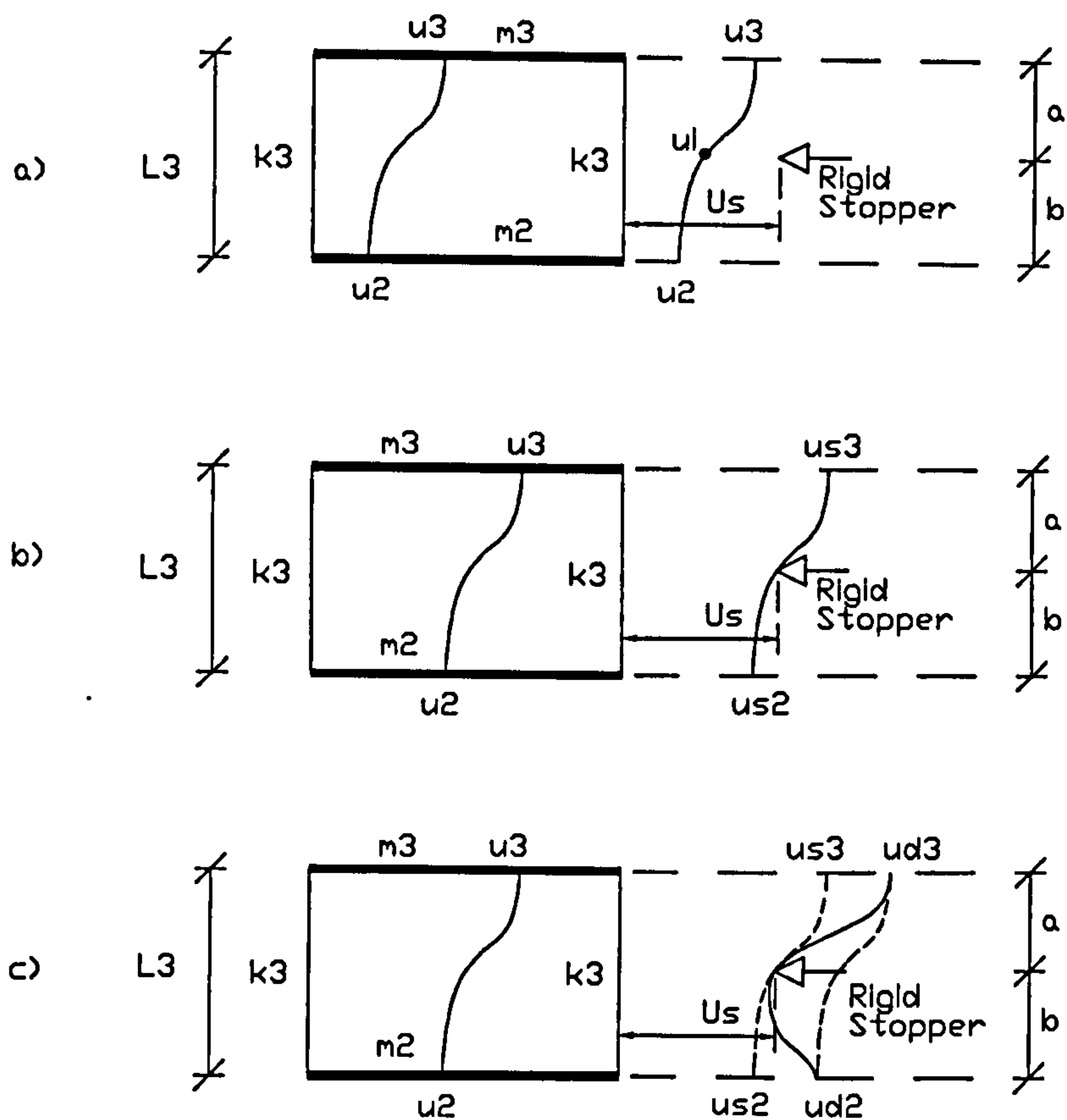


Figure 3.10 : Displacement Compatibility Stages a) no contact, b) contact just starting and c) into contact.

Assuming that the floor slab is rigid compared to the column, then

$$\begin{aligned} u_3 &= us3 + ud3 \\ u_2 &= us2 + ud2 \end{aligned} \quad (3.75)$$

Now consider the right hand side of the column behaviour, which hits the stopper. For simplicity, it can be split into two parts (Figure 3.11) where the displacement of the column without the presence of the rigid stopper, termed  $udi$  (Figure 3.11(i)) and the column displacement at the contact point,  $up$  due to the point load  $P$  created during contact (Figure 3.11(ii)) can be expressed in the following equations:

$$\begin{aligned} ui &= (1 - k_{AB}) u_3 + k_{AB} u_2 \\ udi &= (1 - k_{AB}) ud3 + k_{AB} ud2 \\ up &= - \frac{P b^3 a^3}{3 L^3 E I} \end{aligned} \quad (3.76)$$

where

$$k_{AB} = \frac{\alpha^2}{L^3} [a + 3b] \quad (3.77)$$

The  $udi$  is defined by analogy to  $ui$ . The basic derivation of  $udi$  and  $up$  can be seen in Appendix B.

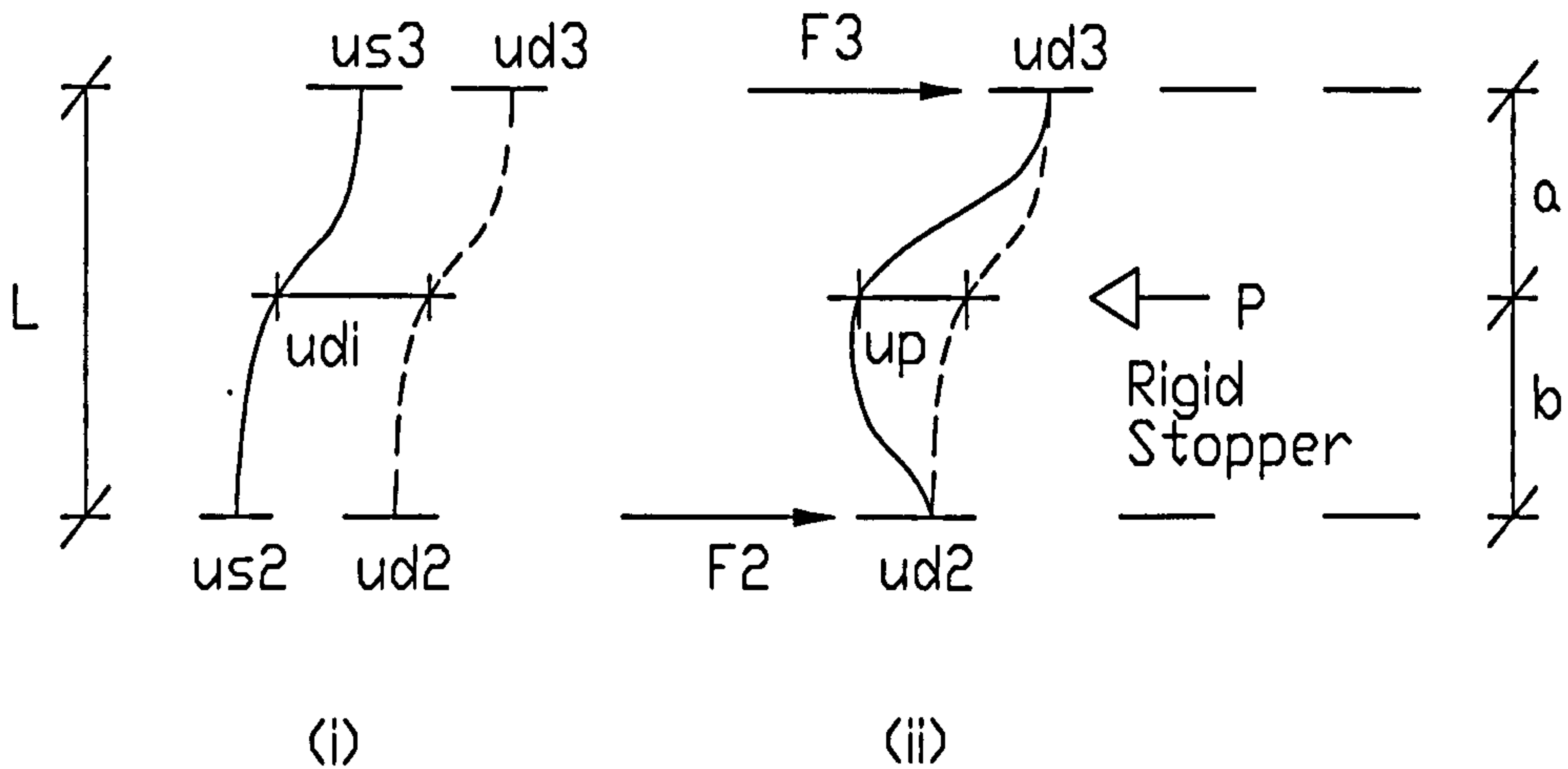


Figure 3.11 : Contact stage on the right hand side column behaviour

The contact forces,  $P$  acting at the contact point, is derived from the displacement compatibility where :

$$udi + up = 0 \quad (3.78)$$

Hence,

$$P = \frac{3 L^3 E I}{b^3 \alpha^3} [ ( 1 - k_{AB} ) ud3 + k_{AB} ud2 ] \quad (3.79)$$

Let the column rigidity,  $k_p = 3L^3 EI / b^3 \alpha^3$ . The equivalent forces acting on the top and middle floor, termed  $F3$  and  $F2$  respectively are expressed in term of  $P$  where;

$$\begin{aligned} F3 &= \frac{P b}{L} + \frac{(M3 + M2)}{L} \\ &= \frac{P b}{L} + \frac{P a b^2}{L^3} - \frac{P a^2 b}{L^3} \\ &= \frac{P b^2}{L^3} ( 3 a + b ) \\ &= P ( 1 - k_{AB} ) \end{aligned} \quad (3.80)$$

Using the assumption in Equation (3.75) and direct substitution of the value  $P$ , from Equation (3.79) into above expression of  $F3$  will gives:

$$F3 = k1^* ( u3 - us3 ) + k2^* ( u2 - us2 ) \quad (3.81)$$

Also,

$$\begin{aligned} F2 &= \frac{P a}{L} - \frac{(M3 + M2)}{L} \\ &= \frac{P a}{L} - \frac{P a b^2}{L^3} + \frac{P a^2 b}{L^3} \\ &= \frac{P a^2}{L^3} ( a + 3 b ) \\ &= P (k_{AB}) \end{aligned} \quad (3.82)$$

And,

$$F2 = k3^* ( u3 - us3 ) + k4^* ( u2 - us2 ) \quad (3.83)$$

where,

$$\begin{aligned} k1^* &= \frac{3 E I}{b a^3} ( 3 a + b ) ( 1 - k_{AB} ) \\ &= \frac{3 E I b^2}{L^3 b a^3} ( 3 a + b )^2 \\ k2^* &= \frac{3 E I}{b a^3} ( 3 a + b ) k_{AB} \\ &= \frac{3 E I}{L^3 b a} ( 3 a + b ) ( a + 3 b ) \\ k3^* &= \frac{3 E I}{b^3 a} ( a + 3 b ) ( 1 - k_{AB} ) \\ &= \frac{3 E I}{L^3 b a} ( a + 3 b ) ( 3 a + b ) \\ k4^* &= \frac{3 E I}{b^3 a} ( a + 3 b ) k_{AB} \\ &= \frac{3 E I a^2}{L^3 b^3 a} ( a + 3 b )^2 \end{aligned} \quad (3.84)$$

The shear forces diagram on the frame are illustrated in Figure 3.12 below. It is performed using the snap back analysis ( $\ddot{u}_{gt} = 0$ ) and no damping case.

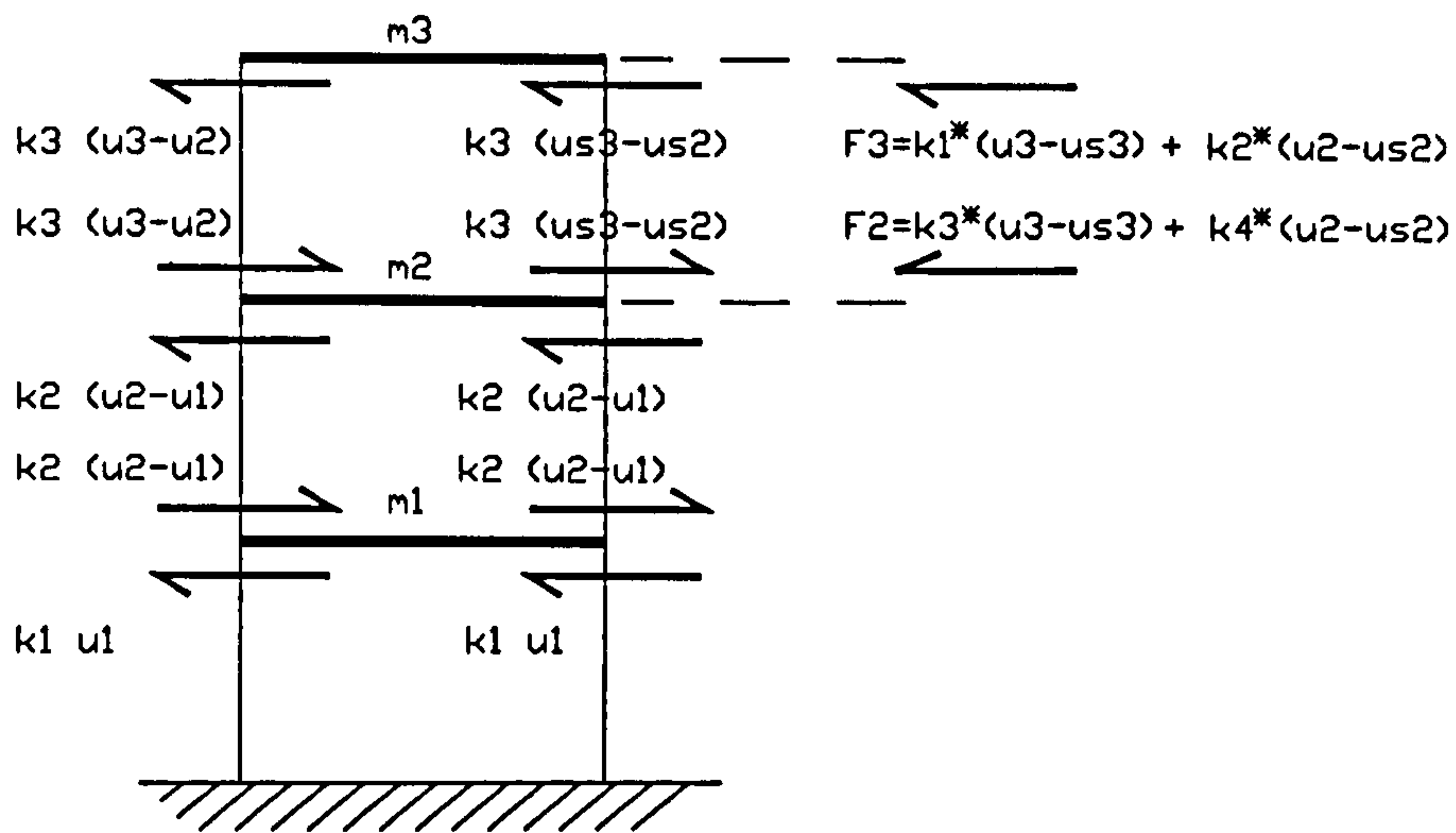


Figure 3.12 : Shear forces diagram

The equations of motion for a contact stage can be written in the matrix form as:

$$\begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} k_3 + k_1^* & -k_3 + k_2^* & 0 \\ -k_3 + k_3^* & k_3 + k_4^* + 2k_2 & -2k_2 \\ 0 & -2k_2 & 2k_2 + 2k_1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} = \begin{bmatrix} -k_3 + k_1^* & k_3 + k_2^* & 0 \\ k_3 + k_3^* & -k_3 + k_4^* & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_{s3} \\ u_{s2} \\ u_{s1} \end{Bmatrix} \quad (3.85)$$

These matrices are always symmetrical at any contact position ( $a = 0$  to  $L$ ) along the height of the column, as  $k_2^* = k_3^*$ . In matrix form

$$\underline{m} \ddot{u}_i + \bar{k} u_i = \bar{b} \quad (3.86)$$

where  $\underline{m}$  is the mass matrix as in State 1;  $\bar{k}$ , is the stiffness matrix and  $\bar{b}$  is a column vector of static preload forces. These equations are uncoupled using the modal analysis.



Since the stiffness matrix,  $\bar{k}$ , is symmetrical, similarly, the mass matrix  $\underline{m}$  is diagonal, the orthogonality properties (if  $i = j$   $Q_j^T \underline{m} Q_i = 1$ ) described in the mode superposition technique are employed (section 3.2). Hence the uncoupled equations are

$$\ddot{y}_i + \omega_i^2 y_i = Q_j^T \bar{b} \quad (3.87)$$

The solution for displacement and velocity at the initial time ( $t$ ) are:

$$y_i = A \sin \omega_{ic} t + B \cos \omega_{ic} t + \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \quad (3.88)$$

$$\dot{y}_i = A \omega_{ic} \cos \omega_{ic} t - B \omega_{ic} \sin \omega_{ic} t$$

where  $\omega_{ic}$  is the natural frequencies during the contact stage. The constant A and B can be found by knowing the modal initial conditions,  $y_i$  and  $\dot{y}_i$  at time ( $t$ ):

$$\text{at } t = 0 \quad y_i = y_{it} \quad \dot{y}_{it} = \dot{y}_{it}$$

$$\therefore A = \frac{\dot{y}_{it}}{\omega_{ic}} \quad B = \left( y_{it} - \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \right) \quad (3.89)$$

Substituting Equation (3.89) into Equation (3.88) produces

$$y_i = \frac{\dot{y}_{it}}{\omega_{ic}} \sin \omega_{ic} t + \left( y_{it} - \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \right) \cos \omega_{ic} t + \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \quad (3.90)$$

$$\dot{y}_i = \dot{y}_{it} \cos \omega_{ic} t - \left( y_{it} - \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \right) \omega_{ic} \sin \omega_{ic} t$$

For simplicity, let

$$ysii = \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \quad (3.91)$$

Therefore, at the end of the time step ( $t = dt$ ), the generalized displacement and velocity equations become

$$y_i = \frac{\dot{y}_{it}}{\omega_{ic}} \sin \omega_{ic} dt + (y_{it} - ysii) \cos \omega_{ic} dt + ysii \quad (3.92)$$

$$\dot{y}_i = \dot{y}_{it} \cos \omega_{ic} dt - \omega_{ic} (y_{it} - ysii) \sin \omega_{ic} dt$$

The total displacement and velocity responses are the sum of the modal contributions defined in Equations (3.11).

### **3.4.1.3 Change of State Rules**

The process of changing from **state 1** (no contact) to **state 2** (contact) and so on is continued until the required time of analysis is reached. Often, the solution difficulties are associated with contact problems which are due to the large change in stiffness upon impact or contact, resulting in large forces which caused a problem to the dynamic equilibrium. This can be overcome by the automatic adjustment of the time step, thereby negating any adverse effects on dynamic equilibrium caused by over or under shooting.

The no contact stage starts when the displacement and velocity at the point of contact are such:

$$u_i = U_s \quad \text{but} \quad \dot{u}_i \geq 0 \quad (3.93a)$$

And the contact stage is finished when the displacement and velocity at the point of contact are such:

$$u_i = U_s \quad \text{but} \quad \dot{u}_i \leq 0 \quad (3.93b)$$

#### ***3.4.1.4 Fortran Program***

The program called "CAL89" (Computer Assisted Learning of Structural Analysis) was used to evaluate the natural frequencies with the corresponding mode shapes.

The FORTRAN program which is summarised in the following flow chart (Figure 3.13) was used to evaluate the displacement and the contact forces. All results are presented in plot form using the ready made software namely "UNIGRAPH 2000" version 1.4 on the UNIX - Sun workstation in the Civil Engineering department of Liverpool University.

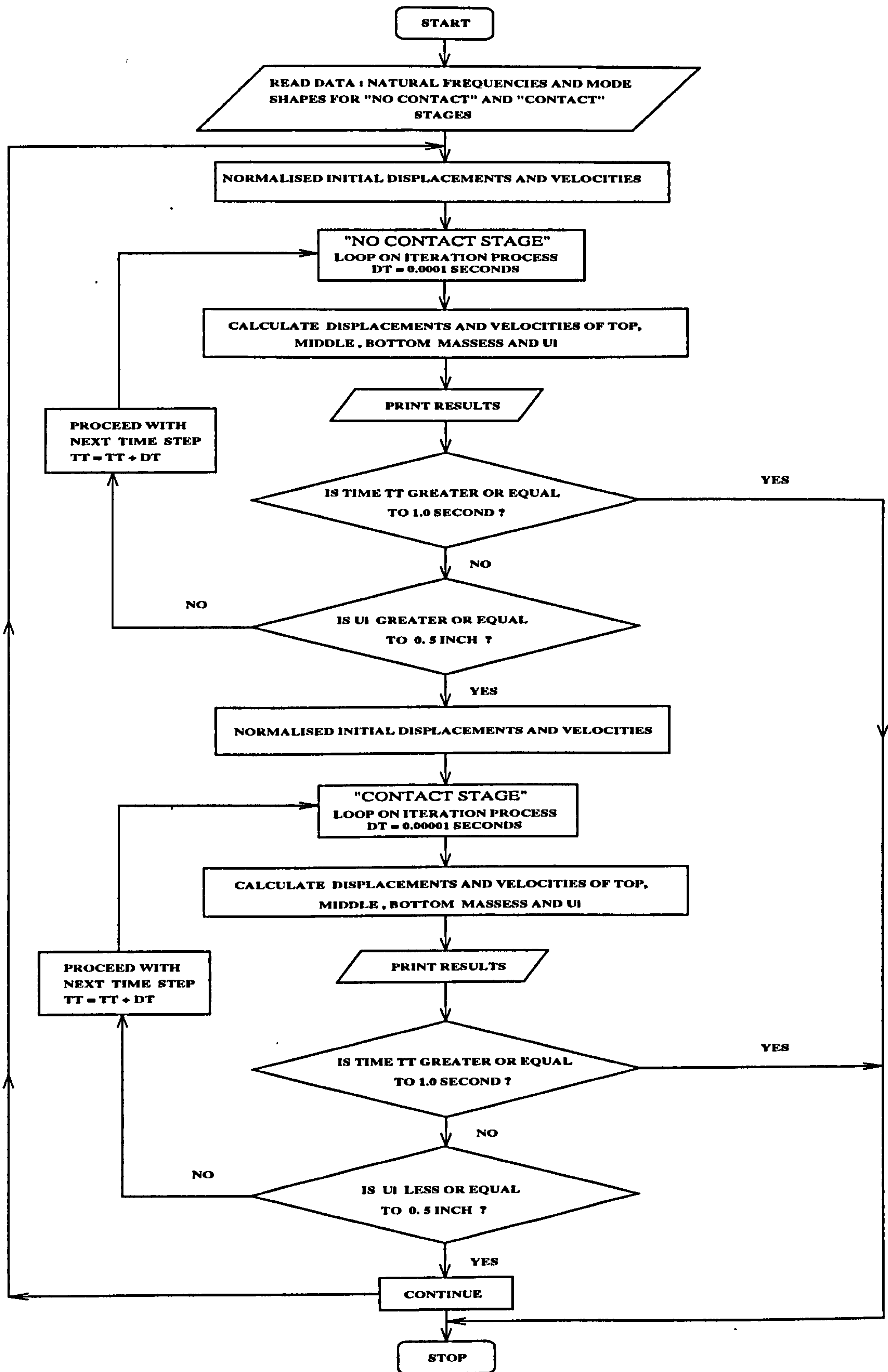


Figure 3.13: Flow chart of Snap Back Analysis Using Iteration Process

### 3.4.1.5 Numerical Examples

#### 3.4.1.5.1 SDOF system, contact at floor level (aligned floor case)

In this section, the SDOF undamped free vibration pounding response induced by a snap back analysis is presented. The purpose is to illustrate the basic problem characteristics, and it is important because the results can be extended to treat MDOF system in later section of this chapter. Consider a frame shown in Figure 3.14 below, having the physical properties in Table 3.1 [15].

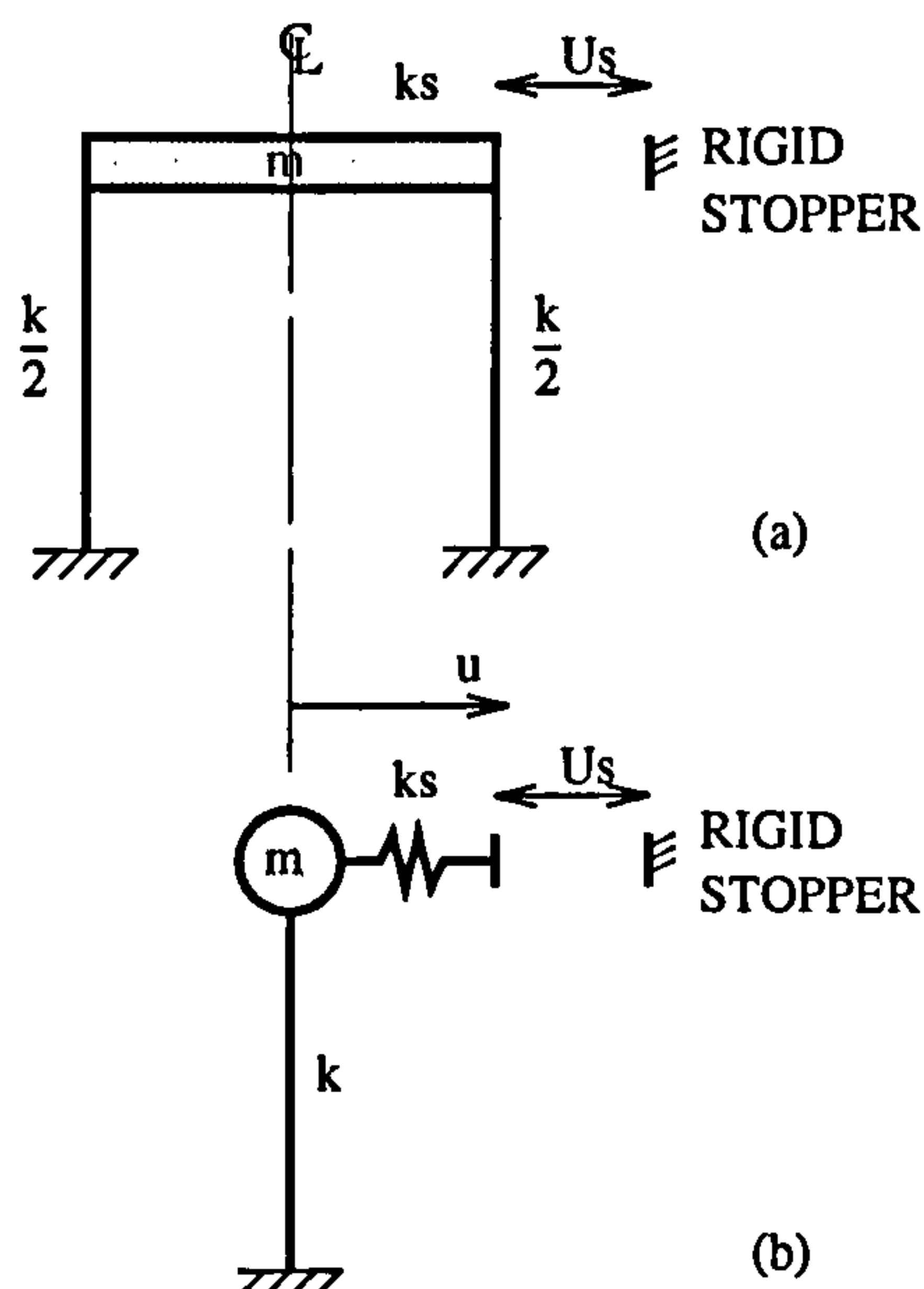


Figure 3.14 : (a) Pounding of SDOF problem and (b) Model idealization.

$m$ = building mass of 39 kips-sec <sup>2</sup> /in. (6.83*10 <sup>3</sup> kN-sec <sup>2</sup> /m)
$k$ = building stiffness of 1200 kips/in. (210.24*10 <sup>3</sup> kN/m)
$ks$ = slab stiffness of 50,000 kips/in. ( 8.76*10 <sup>6</sup> kN/m)
$Us$ = building separation of 0.4 in. (0.01016 m)
Initial displacement = - 0.81 in. (-0.020574 m)
Ground acceleration, ( $\ddot{u}_{gt} = 0$ )
Undamped free vibration, $c = 0$

Table 3.1 : Physical properties of SDOF building.

### ***a) State 1 - No Contact Stage***

The equation of the motion is represented by Equation (3.1) i.e.

$$m \ddot{u} = -k u$$

Rearranged and it gives

$$\begin{aligned} m \ddot{u} + k u &= 0 \\ \ddot{u} + \omega^2 u &= 0 \end{aligned}$$

The value of free vibration frequency,  $\omega$  and free vibration period,  $T$  are :

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{39}} = 5.547 \text{ rad/sec} \\ T &= \frac{2\pi}{\omega} = 1.133 \text{ sec} \end{aligned}$$

The solution for displacement and velocity are :

$$\begin{aligned} u &= A \sin \omega t + B \cos \omega t \\ \dot{u} &= A \omega \cos \omega t - B \omega \sin \omega t \end{aligned}$$

The boundary conditions are:

$$\begin{aligned} \text{at } t = 0 \quad u &= -0.81'' \quad \therefore B = -0.81'' \\ \text{at } t = 0 \quad \dot{u} &= 0 \quad \therefore A = 0 \end{aligned}$$

Hence, the displacement and velocity of no contact stage are given by:

$$\begin{aligned} u &= -0.81 \cos \omega t \\ \dot{u} &= 0.81 \omega \sin \omega t \end{aligned}$$

These equations also represent the no pounding case.

### ***b) State 2 - Contact Stage***

The contact stage starts when the change of state rules (3.4.1.3) are satisfied, i.e

$$u = 0.4'' \quad \text{but} \quad \dot{u} \geq 0$$

The shear force diagram is illustrated in Figure 3.15 below:

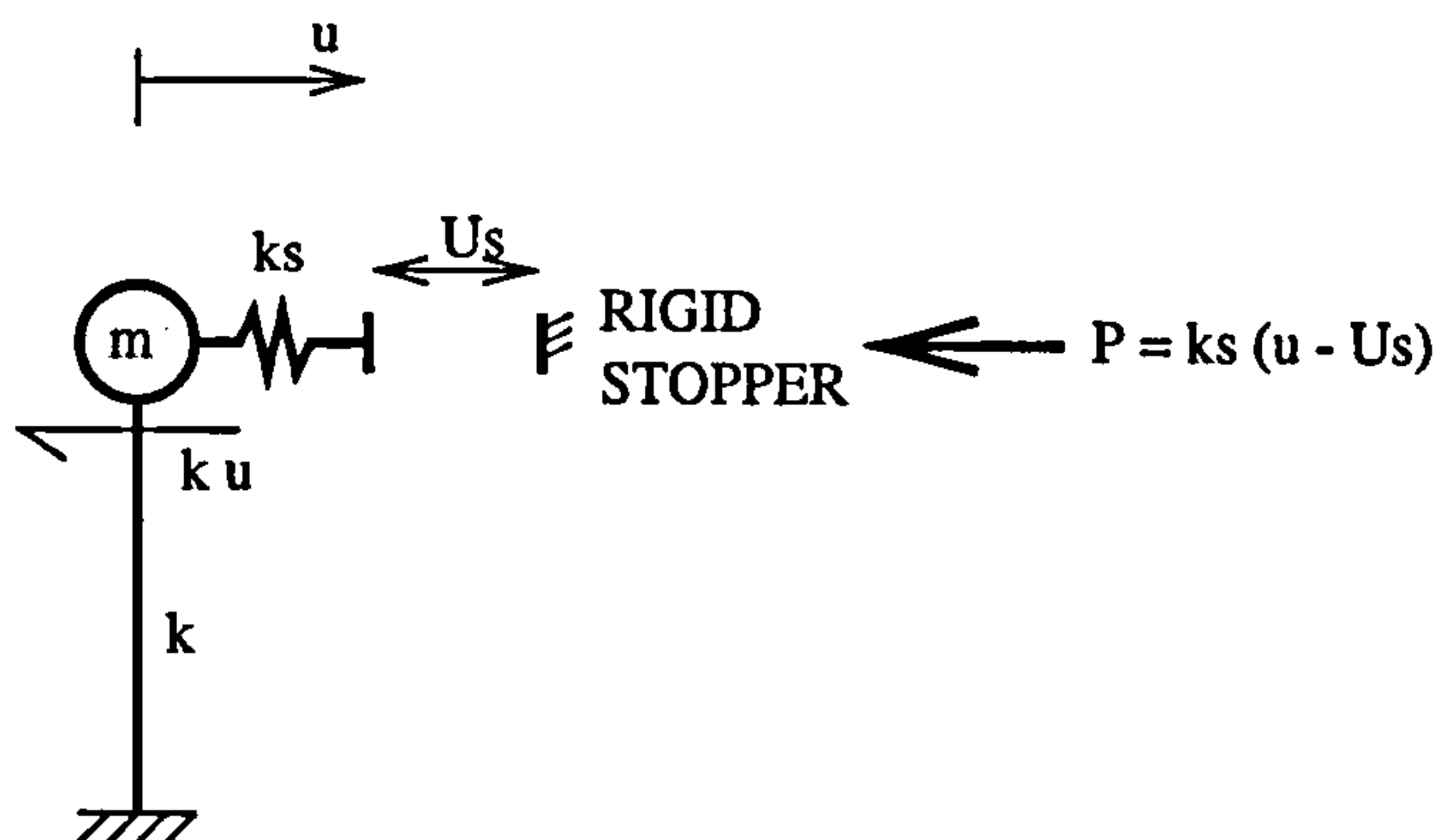


Figure 3.15 : Shear force diagram of SDOF system (contact stage).

The equation of motion becomes:

$$m \ddot{u} = -k u - ks (u - Us)$$

Rearranged

$$\begin{aligned} m \ddot{u} + (k + ks) u &= ks Us \\ m \ddot{u} + k_c u &= b_s \\ \ddot{u} + \frac{k_c}{m} u &= \frac{b_s}{m} \\ \ddot{u} + \omega_c^2 u &= \frac{b_s}{m} \end{aligned}$$

where  $\omega_c^2$  is the contact stage vibration frequency,  $b_s$  is the static preload force and the vibration period  $T_c$  are:

$$\begin{aligned} k_c &= (k + ks) \\ b_s &= ks Us \\ \omega_c &= \sqrt{\frac{k_c}{m}} = \sqrt{\frac{51200}{39}} = 36.23 \text{ rad/sec} \\ T_c &= \frac{2\pi}{\omega_c} = 0.1734 \text{ sec} \end{aligned}$$

The solution for displacement and velocity at time (t) is given by:

$$u = A \sin \omega_c t + B \cos \omega_c t + \frac{b_s}{\omega_c^2 m}$$

$$\dot{u} = A \omega_c \cos \omega_c t - B \omega_c \sin \omega_c t$$

at  $t = 0$       $u = u_t$       $\dot{u}_t = \dot{u}_t$

$$\therefore A = \frac{\dot{u}_t}{\omega_c} \quad B = \left( u_t - \frac{b_s}{\omega_c^2 m} \right)$$

where for simplicity, let :

$$usi = \frac{b_s}{\omega_c^2 m}$$

at the end of the time step ( $t = dt$ ),

$$u = \frac{\dot{u}_t}{\omega_c} \sin \omega_c dt + (u_t - usi) \cos \omega_c dt + usi$$

$$\dot{u} = \dot{u}_t \cos \omega_c dt - \omega_c (u_t - usi) \sin \omega_c dt$$

The acceleration, is given by

$$\ddot{u} = -\omega_c^2 u + \frac{b_s}{m}$$

$$= -\frac{(k + ks) u}{m} + \frac{b_s}{m}$$

$$= -\frac{ku}{m} - \frac{ks(u - Us)}{m}$$

These equations are valid as long as the condition  $u = Us$  and  $\dot{u} \geq 0$  are satisfied and when  $\dot{u} \leq 0$ , the equation of state 1 (section b) is used. The changing of states from no contact to contact and so on is continue until the required time is reached.



**3.4.1.5.2 MDOF systems, contact at floor level (aligned floor case)**

In this section, the MDOF undamped free vibration pounding response induced by a snap back analysis is presented. There are two examples for the aligned floor case of pounding which are when it hits at the top floor ( $a = 0$ ) and when it hits at the second floor level ( $a = L$ ). Consider a shear frame (Figure 3.9) having the physical properties as in Table 3.2 below :

$m_3, m_2, m_1$ = building masses of 1.0, 1.5, 2.0 kips-sec <sup>2</sup> /in. = 175197, 262795 , and 350394 kg (N-sec <sup>2</sup> /m)
$2k_3, 2k_2, 2k_1$ = building stiffness of 600, 1200, 1800 kips/in. = $105.12 \cdot 10^3$ , $210.24 \cdot 10^3$ , and $315.35 \cdot 10^3$ (kN/m)
$k_s$ = slab stiffness of 50,000 kips/in. ( $8.76 \cdot 10^6$ kN/m)
Initial displacement for top, middle and bottom floor = -1.0 in. (-0.0254 m), -0.505 in. (-0.0128m) and - 0.144 in. ( -0.00366m) respectively.
Building separation, $U_s = 0.5$ in. (0.0127 m)
Ground acceleration, ( $\ddot{u}_{gt} = 0$ )
Undamped free vibration, $c = 0$
Height between floors, $L = 72$ in. (1.8288 m)
Young Modulus of steel, $E = 30 \cdot 10^3$ kip/in <sup>2</sup> ( $205 \cdot 10^6$ kN/m <sup>2</sup> )

Table 3.2 : Physical properties of the building. (\*\*1 kip = 4450N, 1 in = 0.0254m)

**1) Example 1 ( $a = 0$ )**

The first case where pounding occurs at the top floor level is illustrated in Figure 3.16a below. The displacement of the contact point,  $u_i$  is equal to the top displacement,  $u_3$ . The shear forces diagram are shown in Figure 3.16b.

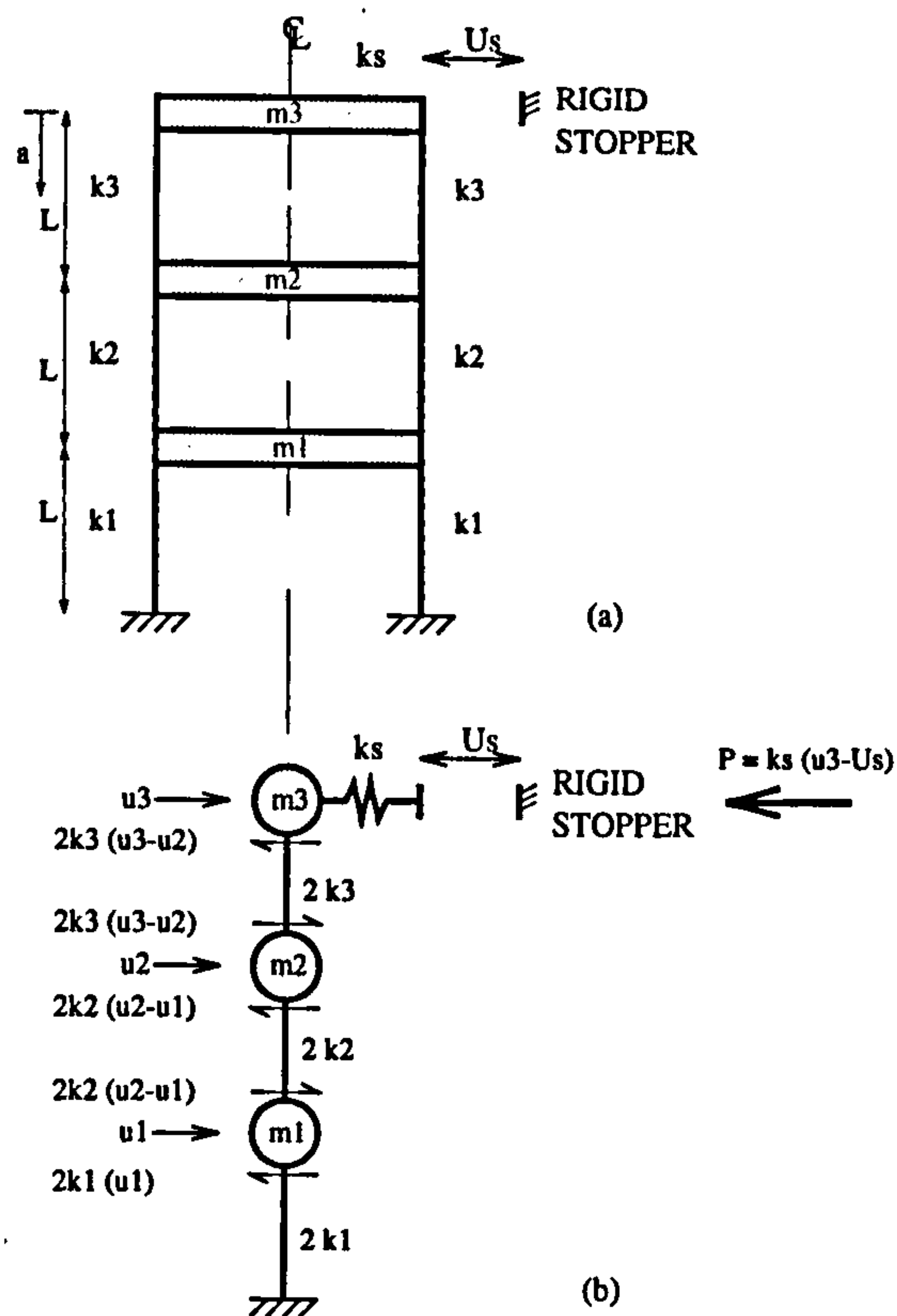


Figure 3.16 : (a) Pounding of MDOF system and (b) The shear forces diagram.

*a) No Contact Stage*

Substituting the physical properties given in Table 3.2 into Equation (3.70a) gives

$$\begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} 600 & -600 & 0 \\ -600 & 1800 & -1200 \\ 0 & -1200 & 3000 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} = 0$$

This can be represented by Equation (3.70b) with the value of free vibration frequencies

$$\begin{aligned} \omega_1 &= 14.522 \text{ rad/sec} \\ \omega_2 &= 31.048 \text{ rad/sec} \\ \omega_3 &= 46.099 \text{ rad/sec} \end{aligned}$$

with the associate normalized mode shapes of

1st modes	2nd modes	3rd modes
0.74265	-0.63577	0.21037
0.48164	0.38566	-0.53475
0.22417	0.43168	0.51323

The displacement and velocity solutions are defined by Equation (3.72). The total displacement and velocity responses are the sum of the modal contributions defined in Equation (3.11).

*b) Contact Stage.*

When the condition of Equation (3.93a) is satisfied, then the change from no contact stage to contact stage takes place. The equation of motions are such

$$\begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} 2k_3 + k_s & -2k_3 & 0 \\ -2k_3 & 2k_3 + 2k_2 & -2k_2 \\ 0 & -2k_2 & 2k_2 + 2k_1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} = \begin{bmatrix} k_s \\ 0 \\ 0 \end{bmatrix} U_s$$

Substituting the values from Table 3.2 gives

$$\begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} 50600 & -600 & 0 \\ -600 & 1800 & -1200 \\ 0 & -1200 & 3000 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} = \begin{bmatrix} 50000 \\ 0 \\ 0 \end{bmatrix} \quad (0.5)$$

The general matrix equation is in the form of Equation (3.86). The natural frequencies during contact stage are given by

$$\begin{aligned} \omega_{1c} &= 25.263 \text{ rad/sec} \\ \omega_{2c} &= 45.353 \text{ rad/sec} \\ \omega_{3c} &= 224.96 \text{ rad/sec} \end{aligned}$$

with the associate normalized mode shapes of

1st modes	2nd modes	3rd modes
0.0076419	-0.0063228	0.99995
0.63633	-0.51155	-0.0080976
0.44303	0.55111	0.000098942

The general matrix equation is in Equation (3.86) and the displacement and velocity solutions are defined by Equation (3.88) where the value of

$$y_{sii} = \frac{Q_j^T \bar{b}}{\omega_{ic}^2} = \frac{\alpha_{1i}^T \bar{b}}{\omega_{ic}^2}$$

as defined in Equation (3.91). Again, the total displacement and velocity responses are the sum of the modal contributions given in Equation (3.11). The total contact force,  $P$  is given by

$$\begin{aligned} P &= ks (u_3 - U_s) \\ &= 50,000 (u_3 - 0.5) \text{ kips} \end{aligned}$$

## 2) Example 2 ( $a = L$ )

In this section, the contact point occurs at the second floor level. The displacement at the contact point,  $u_i$  is equal to  $u_2$ . The same physical properties of the three multistorey shear frames as in Example 1 is adopted. This is illustrated in Figure 3.17 below.

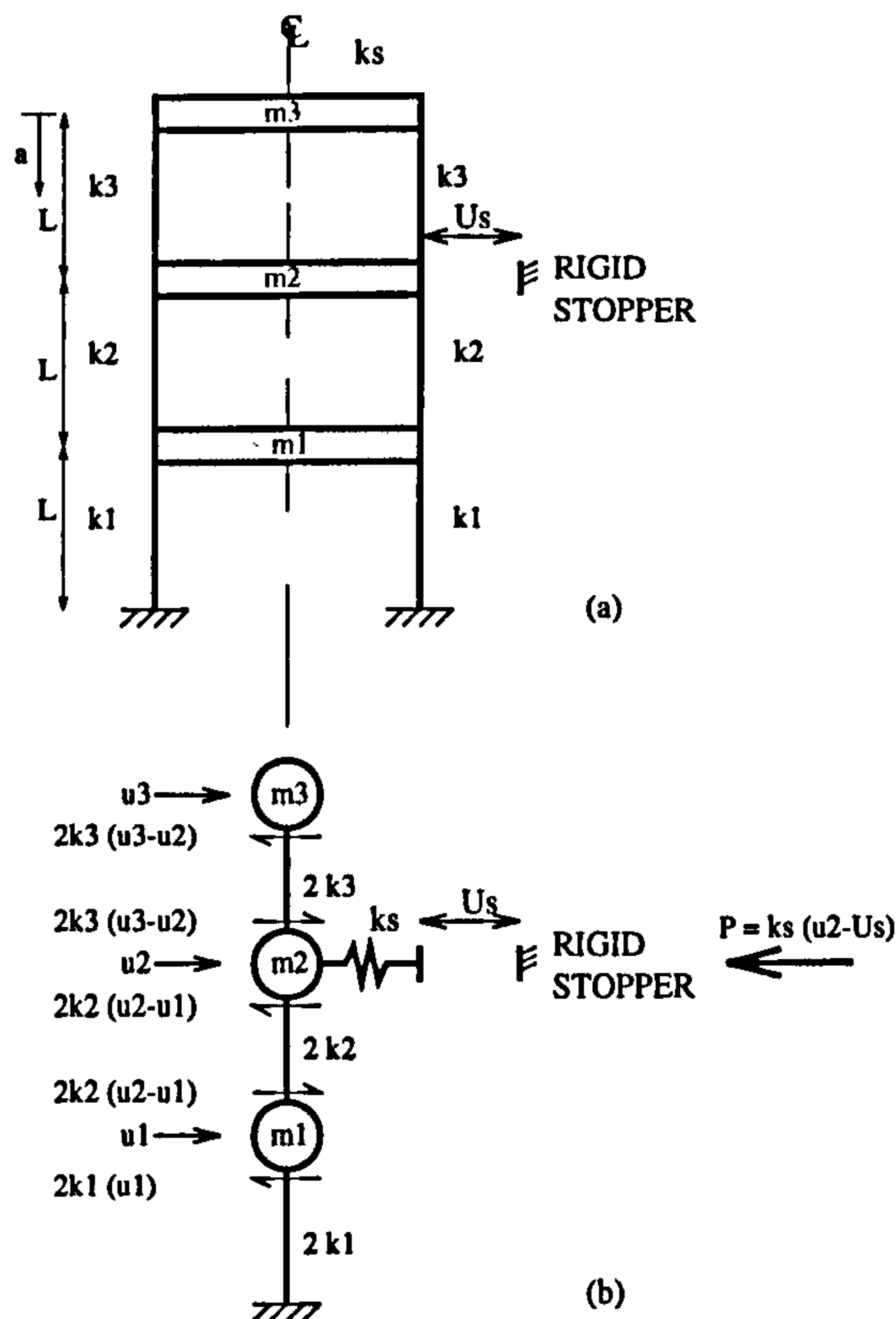


Figure 3.17 : (a) Pounding of MDOF system and (b) The shear forces diagram.

*a) No Contact Stage*

The derivation of the equations of motion are the same as in Example 1 (  $a = 0$  ) and therefore, the natural frequencies, the associate normalized mode shapes and the solutions for the equations of motion are similar to Example 1. The only difference is in the contact stage as described below.

*b) Contact Stage.*

The contact stage starts when the condition of Equation (3.93a) is satisfied, then the change from no contact stage to contact stage takes place. The equation of motions are such

$$\begin{vmatrix} m3 & 0 & 0 \\ 0 & m2 & 0 \\ 0 & 0 & m1 \end{vmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} 2k3 & -2k3 & 0 \\ -2k3 & 2k3 + 2k2 + ks & -2k2 \\ 0 & -2k2 & 2k2 + 2k1 \end{vmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} = \begin{vmatrix} 0 \\ ks \\ 0 \end{vmatrix} Us$$

Substituting the values from Table 3.2 gives :

$$\begin{vmatrix} 1.0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.0 \end{vmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} 600 & -600 & 0 \\ -600 & 51800 & -1200 \\ 0 & -1200 & 3000 \end{vmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} = \begin{vmatrix} 0 \\ 50000 \\ 0 \end{vmatrix} (0.5)$$

The general matrix equation is in the form of Equation (3.86). The natural frequencies during contact stage are given by

$$\begin{aligned} \omega_{1c} &= 24.348 \text{ rad/sec} \\ \omega_{2c} &= 38.543 \text{ rad/sec} \\ \omega_{3c} &= 185.89 \text{ rad/sec} \end{aligned}$$

with the associate normalized mode shapes of

<i>1st modes</i>	<i>2nd modes</i>	<i>3rd modes</i>
0.99983	-0.011500	-0.014423
0.011970	0.016973	0.81623
0.0079168	0.70691	-0.014816

Thus the solution are the same as described in Equation (3.92) except the term  $ysii$  is expressed as:

$$y_{sii} = \frac{Q_j^T \bar{b}}{\omega_{ic}^2} = \frac{\alpha_{2i}^T \bar{b}}{\omega_{ic}^2}$$

The total displacement and velocity response are given by Equation (3.11). The total contact force,  $P$  is given by

$$\begin{aligned} P &= ks (u_2 - U_s) \\ &= 50,000 (u_2 - 0.5) \text{ kips} \end{aligned}$$

### 3.4.1.5.3 MDOF systems, contact between floor levels (unaligned floor case)

As mentioned earlier, previous studies of pounding assumed that the point of contact occurred at a floor level. Pounding occurring at any location on a column of height,  $(0 < a < L)$  is presented in this section. The same shear frame model of Figure 3.9 is adopted.

There are four different values of  $a$  are considered in this study, they are  $a=L/4$ ,  $L/3$ ,  $L/2$  and  $3L/4$ . Only two cases ( $a=L/3$  and  $a=L/2$ ) are shown as numerical examples in this section. The summary of the parameters calculated for other values of  $a$  are listed in Table 3.3.

It is convenient to idealise the model as a lumped mass model as shown in Figure 3.18 below:

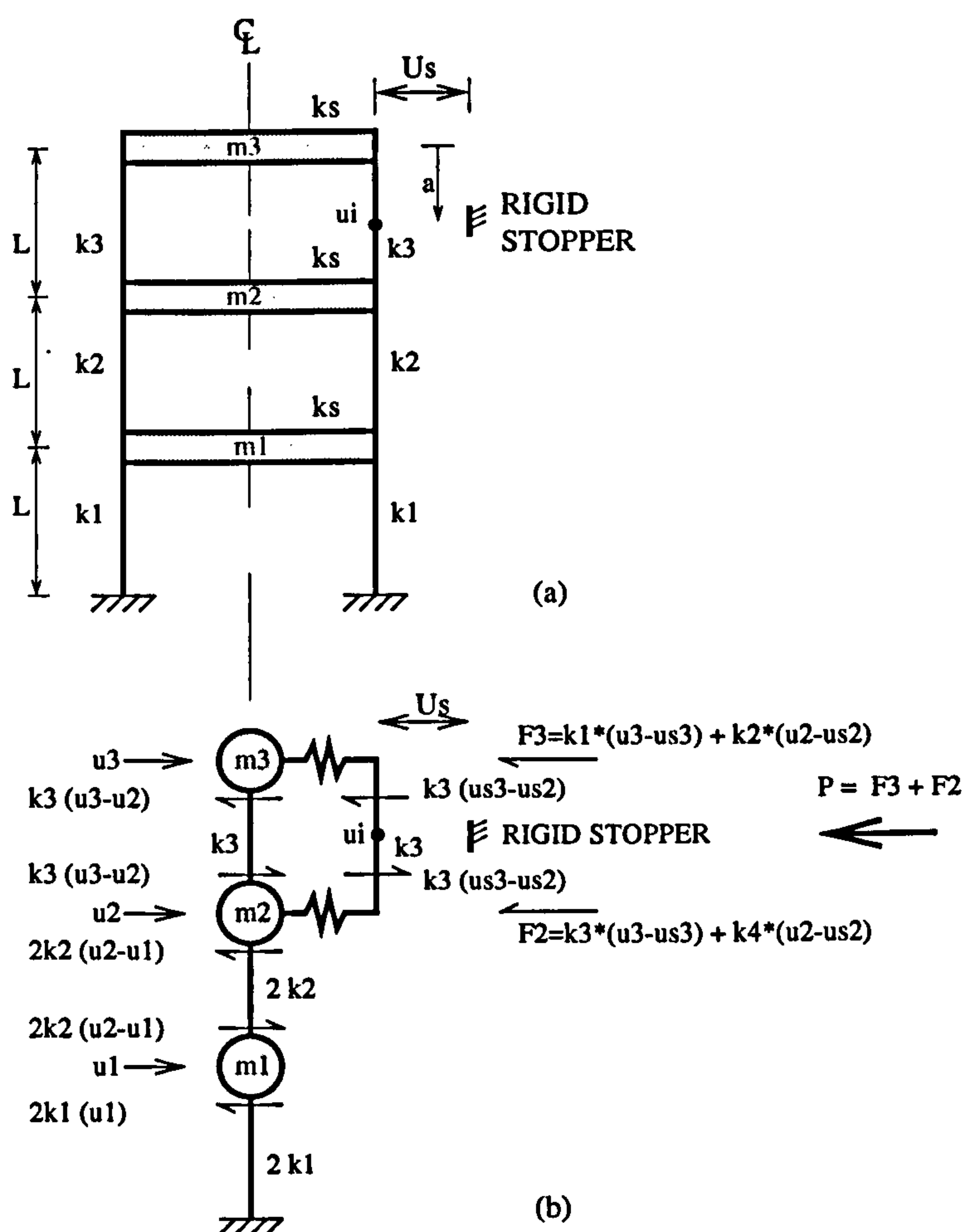


Figure 3.18 : (a) Pounding of MDOF system and (b) The shear forces diagram.

### 1) Example 1 ( $a = L / 3$ )

For this case, the  $k_{AB}$  term from Equation (3.77) and the column displacement at the contact point ( $a = L/3$ ) are expressed in Equation (3.76) :

$$\begin{aligned}k_{AB} &= \frac{7}{27} & (1 - k_{AB}) &= \frac{20}{27} \\ \therefore u_i &= \frac{20}{27} u_3 + \frac{7}{27} u_2 \\ u_{di} &= \frac{20}{27} u_{d3} + \frac{7}{27} u_{d2}\end{aligned}$$

The lateral stiffness of the top column is given by

$$\begin{aligned}\frac{12EI}{L^3} &= 300 \text{ kips/in.} \\ \therefore EI &= 9331200 \text{ kipsin.}^2\end{aligned}$$

#### a) No Contact Stage.

Again, the equations of motion for the no contact stage for unaligned floor case are the same as for aligned floor case in section (3.4.1.5.2). The main concern of this study is the contact stage.

#### b) Contact Stage.

The boundary condition for this stage is given by Equation (3.93a) that is when the total displacement at the point of contact,  $u_i = U_s$  where

$$\begin{aligned}u_i &= \frac{20}{27} u_3 + \frac{7}{27} u_2 = U_s \\ \text{and } \dot{u}_i &\geq 0\end{aligned}$$

The displacements of  $u_3$  and  $u_2$ , when it just into the contact stage are:

$$u_3 = u_{s3} \quad u_2 = u_{s2}$$

Different contact stages, will have different values of  $u_{s3}$  and  $u_{s2}$ . During the contact stage the only parameters that vary throughout the contact stage are the  $u_{d3}$  and  $u_{d2}$  terms. The relationship between the  $u_{s3}$ ,  $u_{s2}$  and the  $u_{d3}$ ,  $u_{d2}$  are



defined by Equation (3.75). Substituting values of  $k_{AB}$ ,  $a=L/3$ ,  $b=2L/3$  and  $EI$  into Equation (3.84) produces

$$k1^* = \frac{150EI}{L^3} = 3750 \text{ kips/in.}$$

$$k2^* = k3^* = \frac{105EI}{2L^3} = 31312.5 \text{ kips/in.}$$

$$k4^* = \frac{147EI}{8L^3} = 459.375 \text{ kips/in.}$$

Substituting into Equation (3.85) gives

$$\begin{vmatrix} 1.0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.0 \end{vmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} 4050 & 1012.5 & 0 \\ 1012.5 & 1959.375 & -1200 \\ 0 & -1200 & 3000 \end{vmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix}$$

$$= \begin{vmatrix} 3450 & 1612.5 & 0 \\ 1612.5 & 159.375 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{Bmatrix} us3 \\ us2 \\ us1 \end{Bmatrix}$$

The natural frequencies are

$$\omega_{1c} = 24.179 \text{ rad/sec}$$

$$\omega_{2c} = 44.484 \text{ rad/sec}$$

$$\omega_{3c} = 65.520 \text{ rad/sec}$$

The general equation are in the matrix form as in Equation (3.86). Thus the same solutions are applied as in Equation (3.92) where the term  $ysii$  is expressed as :

$$ysii = \frac{Q_j^T \bar{b}}{\omega_{ic}^2}$$

$$= \frac{Q_j^T [(3450us3 + 1612.5us2) + (1612.5us3 + 159.375us2)]}{\omega_{ic}^2}$$

$$= \frac{\alpha_{1i}^T (3450us3 + 1612.5us2) + \alpha_{2i}^T (1612.5us3 + 159.375us2)}{\omega_{ic}^2}$$

The total contact forces,  $P$  can be calculated by adding up the floor forces ,  $F3$  and  $F2$  or direct substitution into Equation (3.79) where

$$P = 6834.375 \left[ \frac{20}{27} ud3 + \frac{7}{27} ud2 \right] \text{ kips}$$

## 2) Example 2 ( $a = L / 2$ )

For this case, the  $k_{AB}$  term from Equation (3.77) and the column displacement at the contact point ( $a = L/2$ ) are defined in Equation (3.76) becomes

$$\begin{aligned}k_{AB} &= \frac{1}{2} & (1 - k_{AB}) &= \frac{1}{2} \\ \therefore u_i &= \frac{u_3}{2} + \frac{u_2}{2} \\ u_{di} &= \frac{ud3}{2} + \frac{ud2}{2}\end{aligned}$$

The lateral stiffness of the top column is the same as in Example 1 ( $a=L/3$ ), that is  $EI = 9331200$  kips in<sup>2</sup>.

### a) No Contact Stage.

The equations of motion for the no contact stage is the same as in section (3.4.1.5.2). The total displacement and velocity at the point of contact are given by Equation (3.74).

### b) Contact Stage.

The contact stage starts when the boundary conditions in Equation (3.93a) are satisfied.

$$\begin{aligned}u_i &= \frac{u_3}{2} + \frac{u_2}{2} = U_s \\ u_i &\geq 0\end{aligned}$$

During the contact stage the  $ud3$  and  $ud2$  vary as in Equation (3.75)

$$\begin{aligned}u_3 &= us3 + ud3 \\ u_2 &= us2 + ud2\end{aligned}$$

Substituting values of  $k_{AB}$ ,  $a=b=L/2$  and  $EI$  into Equation (3.84) produces

$$k1^* = k2^* = k3^* = k4^* = \frac{48EI}{L^3} = 1200 \text{ kips/in.}$$

Substituting into Equation (3.85) gives

$$\begin{vmatrix} 1.0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.0 \end{vmatrix} \begin{Bmatrix} \ddot{u}_3 \\ \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} 1500 & 900 & 0 \\ 900 & 2700 & -1200 \\ 0 & -1200 & 3000 \end{vmatrix} \begin{Bmatrix} u_3 \\ u_2 \\ u_1 \end{Bmatrix} \\ = \begin{vmatrix} 900 & 1500 & 0 \\ 1500 & 900 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{Bmatrix} us3 \\ us2 \\ us1 \end{Bmatrix}$$

The natural frequencies during the contact stage are

$$\begin{aligned} \omega_{1c} &= 25.079 \text{ rad/sec} \\ \omega_{2c} &= 38.730 \text{ rad/sec} \\ \omega_{3c} &= 51.682 \text{ rad/sec} \end{aligned}$$

Since the equations of motion are also in the matrix form:

$$m \ddot{u}_i + \bar{k} u_i = \bar{b}$$

the same solutions are applied as in Equation (3.92) except the term  $ysii$  is expressed as :

$$\begin{aligned} y_{sii} &= \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \\ &= \frac{Q_j^T [ ( 900us3 + 1500us2 ) + ( 1500us3 + 900us2 ) ]}{\omega_{ic}^2} \\ &= \frac{\alpha_{1i}^T ( 900us3 + 1500us2 ) + \alpha_{2i}^T ( 1500us3 + 900us2 )}{\omega_{ic}^2} \end{aligned}$$

The total displacement and velocity responses are the sum of the sum of the modal contributions defined in Equation (3.11). The total contact forces,  $P$  can be calculated using Equation (3.79) where

$$P = 4800 \left[ \frac{1}{2} ud3 + \frac{1}{2} ud2 \right] \text{ kips}$$

The summary of the parameters calculated for all six cases of  $a$  (0,  $L/4$ ,  $L/3$ ,  $L/2$ ,  $3L/4$  and  $L$ ) are listed in Table 3.3. The FORTRAN programs called MDOF1.FOR, MDOF2.FOR, MDOF3.FOR, MDOF4.FOR, MDOF5.FOR and MDOF6.FOR for  $a=0$ ,  $L/4$ ,  $L/3$ ,  $L/2$ ,  $3L/4$  and  $L$  respectively, are written in Appendix C.

### **3.4.1.6 Results and discussions**

A free vibration is performed by a snap back analysis. The displacement and velocity time history curves are shown in Figure 3.19a and 3.19b respectively. This is also known as no pounding case where the displacement response curves are moving in-phase and dominated by the fundamental modes. These can then be compared with the pounding case.

An analysis for various value of  $a$  (0,  $L/4$ ,  $L/3$ ,  $L/2$ ,  $3L/4$  and  $L$ ) are performed and shown in Figure 3.20 - Figure 3.25. The displacement and velocity of the top floor ( $u_{top}$ ,  $v_{top}$ ), middle floor ( $u_{mid}$ ,  $v_{mid}$ ), bottom floor ( $u_{bott}$ ,  $v_{bott}$ ) and the point of contact ( $u_i$ ) are presented.

It shows that the maximum top floor displacement increases as the pounding location elevation increases from  $a=0$  to  $a=L$ . The velocity curves, during the contact stage suddenly decreases until it reaches a zero velocity (maximum  $u_i$  displacement) and then becomes negative within this contact region. This is one of the characteristic of a contact problem where the velocity at the contact point is positive when it is going into the contact stage and negative when it is leaving the contact stage.

Figure 3.26a) shows the  $u_i$  displacement curves for various value of  $a$ . As one would expect, these curves do not exceed 0.5 in.(0.0127m) which is the building separation.

Overall results of the contact forces and the contact times are tabulated in Table 3.4 and shown in Figure 3.27a) and b) respectively. Figure 3.27a) shows that the maximum first contact forces,  $P$  decrease from  $a = 0$  to  $L/2$  and then increase

again until  $a = L$ . This is because of the different value of slab flexibility,  $k_s$  for aligned floor case and both  $k_s$  and column rigidity,  $k_p$  for unaligned floor case (Table 3.3). Thus, the contact forces,  $P$  for the aligned floor are always higher than the unaligned floor due to high value of  $k_s$ , which is assumed rigid. The lower value of  $P$  for unaligned floor case at  $a=L/2$  are due to the lower value of column rigidity,  $k_p$  compared to other values of  $a=L/3, L/4$  and  $3L/4$ .

In general, the second stage contact forces are less than the first stage contact forces. They follow the same pattern as in the first contact for  $L/2 < a < L$ . Figure 3.27a shows a sudden decrease of  $P$  at  $a = L/4$ .

This is also associated with a short duration of second contact at  $a=L/4$  as shown in Figure (3.27b). As expected, the contact duration for  $a=L/2$  is longer than other values of  $a$  due to the minimum value of column rigidity,  $k_p$  (Table 3.3).

All the iteration processes are solved using computer language programme (FORTRAN). The results are verified using standard dynamic software OASYS DYNA3D as shown in Figure 3.21b - Figure 3.26b. The results from theoretical analysis (FORTRAN) and DYNA3D agree very well.

PARAMETERS (UNITS)	ALIGNED FLOOR	UNALIGNED FLOOR				ALIGNED FLOOR
	CASE a=0	CASE a=L/4	CASE a=L/3	CASE a=L/2	CASE a=3L/4	CASE a=L
$k_{AB}$ , $(1 - k_{AB})$ (dimensionless)	0 , 1	5/32 , 27/32	7/27 , 20/27	1/2 , 1/2	27/32 , 5/32	1 , 0
$ka^*$ (kips/in)	-	8100.0	3750.0	1200.0	277.78	-
$kb^* = kc^*$ (kips/in)	-	1500.0	1312.5	1200.0	1500.0	-
$kd^*$ (kips/in)	-	277.78	459.375	1200.0	8100.0	-
$\omega_{1c}$ (rad/sec)	25.263	23.557	24.179	25.079	20.125	24.348
$\omega_{2c}$ (rad/sec)	45.353	44.703	44.484	38.730	37.703	38.543
$\omega_{3c}$ (rad/sec)	224.96	92.368	65.520	51.682	81.555	185.89
Contact Force, P (kips)	P=ks (u3-Us)	$P = 3L^3EI / a^3b^3 [ (1-k_{AB}) ud3 + k_{AB} ud2 ] = kp [ (1-k_{AB}) ud3 + k_{AB} ud2 ]$				P=ks (u2-Us)
$ks$ , $kp$ (kips/in)	P=50,000 (u3-Us)	$kp = 11377.78$	$kp = 6834.375$	$kp = 4800.0$	$kp = 11377.78$	P=50,000 (u2-Us)

Table 3.3 : Summary of parameters during contact stage of MDOF systems for case a= 0, L/4, L/3, L/2, 3L/4 and L.

DISTANCE "a"		1st CONTACT		2nd CONTACT		3rd CONTACT	
		FORTRAN	DYNA3D	FORTRAN	DYNA3D	FORTRAN	DYNA3D
0	Contact time (sec)	0.14790-0.16122 (0.01332)	0.14918-0.16242 (0.01324)	0.65271-0.66574 (0.01303)	0.65549-0.66808 (0.01259)	0.96949-0.98295 (0.01346)	0.97029-0.98224 (0.01195)
	Contact forces (kN)	5,495.750	5,355.030	4,567.925	4,931.060	3,275.200	4,293.01
L/4	Contact time (sec)	0.15340-0.18428 (0.03088)	0.15420-0.18861 (0.03441)	0.54616-0.56459 (0.01843)	0.55218-0.57278 (0.02060)	-	-
	Contact forces (kN)	2,804.995	2,646.090	565.597	519.641	-	-
L/3	Contact time (sec)	0.15660-0.19876 (0.04216)	0.15730-0.20291 (0.04561)	0.53823-0.57256 (0.03433)	0.54408-0.58159 (0.03751)	-	-
	Contact forces (kN)	2,334.376	2,229.490	1,610.164	1,425.150	-	-
L/2	Contact time (sec)	0.16290-0.21717 (0.05427)	0.16390-0.21981 (0.05591)	0.54444-0.59692 (0.05248)	0.55008-0.60369 (0.05361)	0.92958-0.97994 (0.05036)	0.94505-0.99506 (0.05001)
	Contact forces (kN)	2,306.666	2,364.380	2,185.769	2,079.050	2,018.093	1,532.160
3L/4	Contact time (sec)	0.16940-0.20491 (0.03551)	0.16900-0.20800 (0.03900)	0.55188-0.58450 (0.03262)	0.55499-0.58999 (0.03500)	0.94296-0.97319 (0.03023)	0.94499-0.97699 (0.03200)
	Contact forces (kN)	4,137.108	3,750.120	4,034.892	3,644.500	3,605.843	3,184.590
L	Contact time (sec)	0.17170-0.18772 (0.01602)	0.17275-0.18858 (0.01583)	0.54872-0.56345 (0.01473)	0.55281-0.56670 (0.01389)	0.94552-0.95709 (0.01157)	0.95157-0.96158 (0.01001)
	Contact forces (kN)	9,057.975	9,022.520	7,765.250	7,970.940	3,043.800	3,847.110

Table 3.4 : Overall results for the contact stages of 3DOF systems.

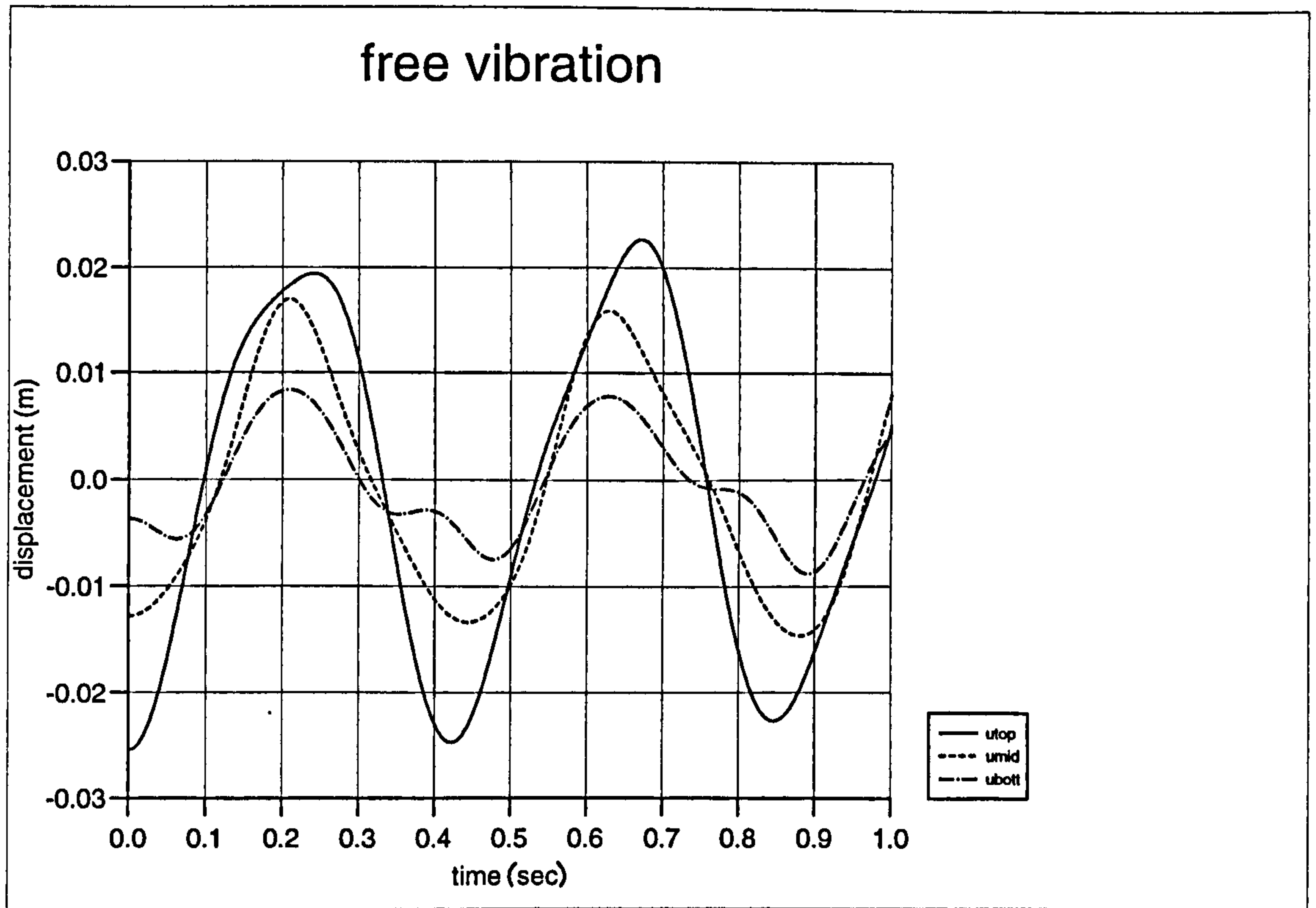


Figure 3.19a) : Displacement curves for no pounding free vibration (FORTRAN)

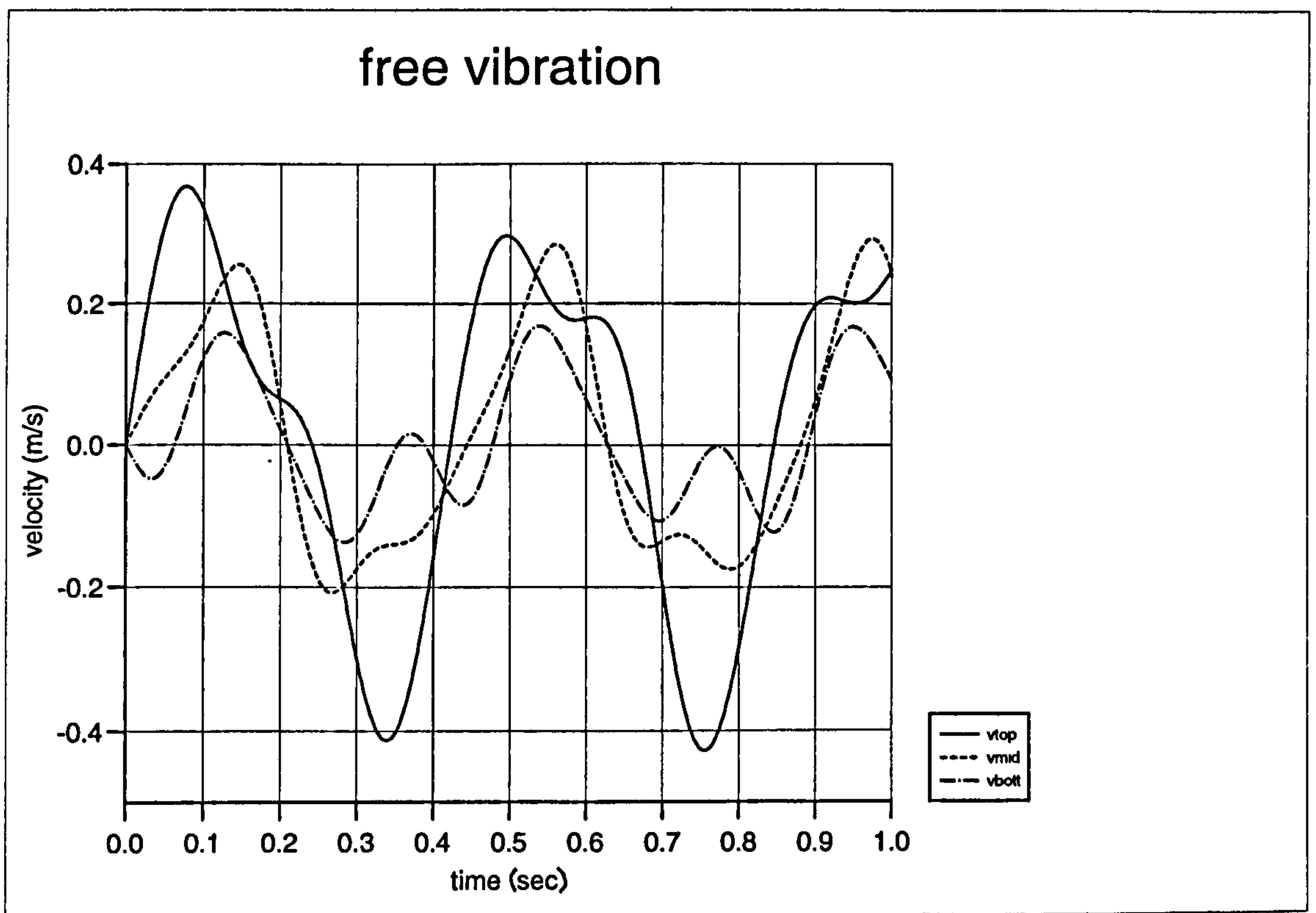


Figure 3.19b) : Velocity curves for no pounding free vibration (FORTRAN)



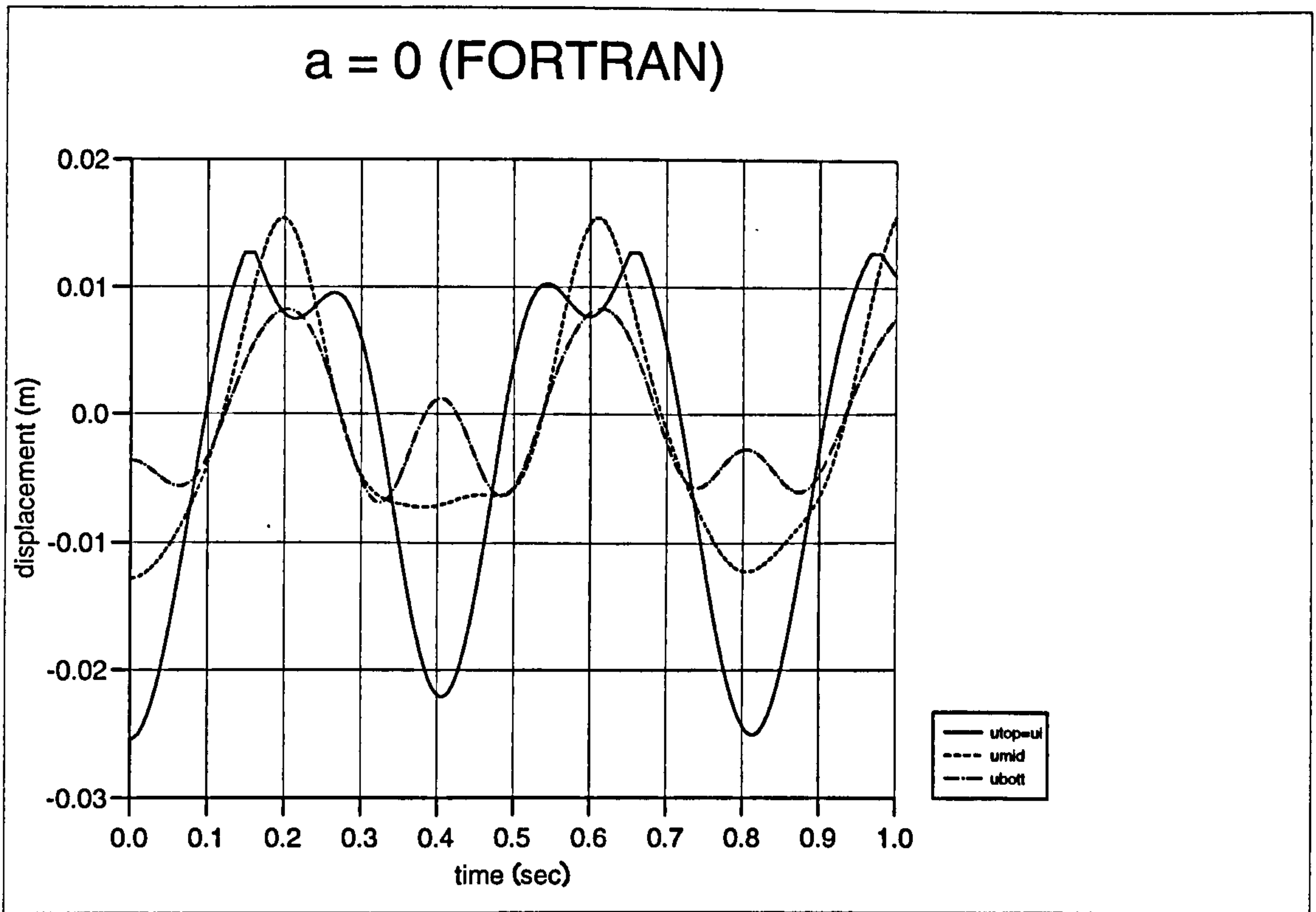


Figure 3.20a) : Displacement curves for pounding case at a = 0 (FORTRAN)

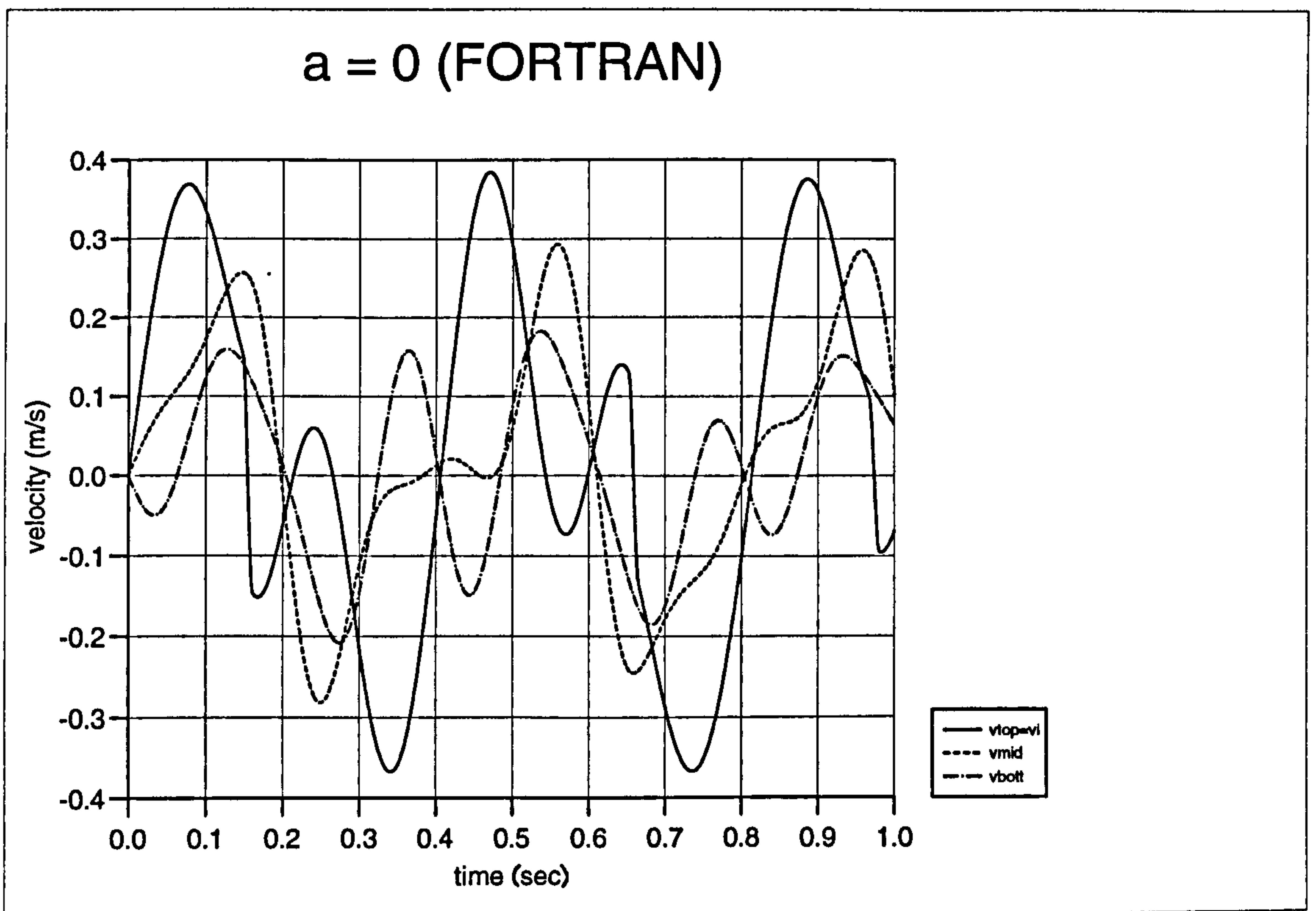


Figure 3.20b) : Velocity curves for pounding case at a= 0 (FORTRAN)

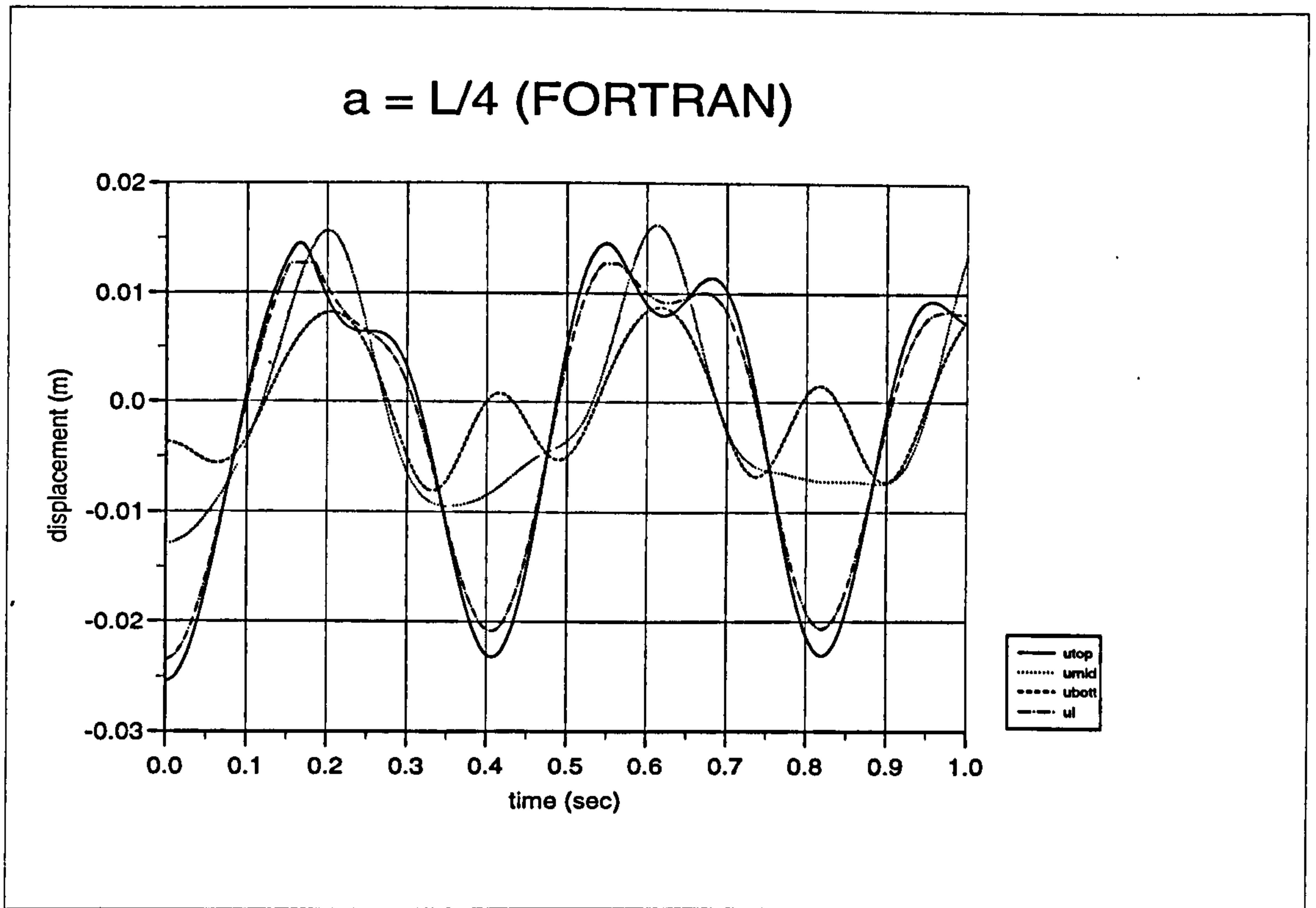


Figure 3.21a) : Displacement curves for pounding case at  $a = L/4$  (FORTRAN)

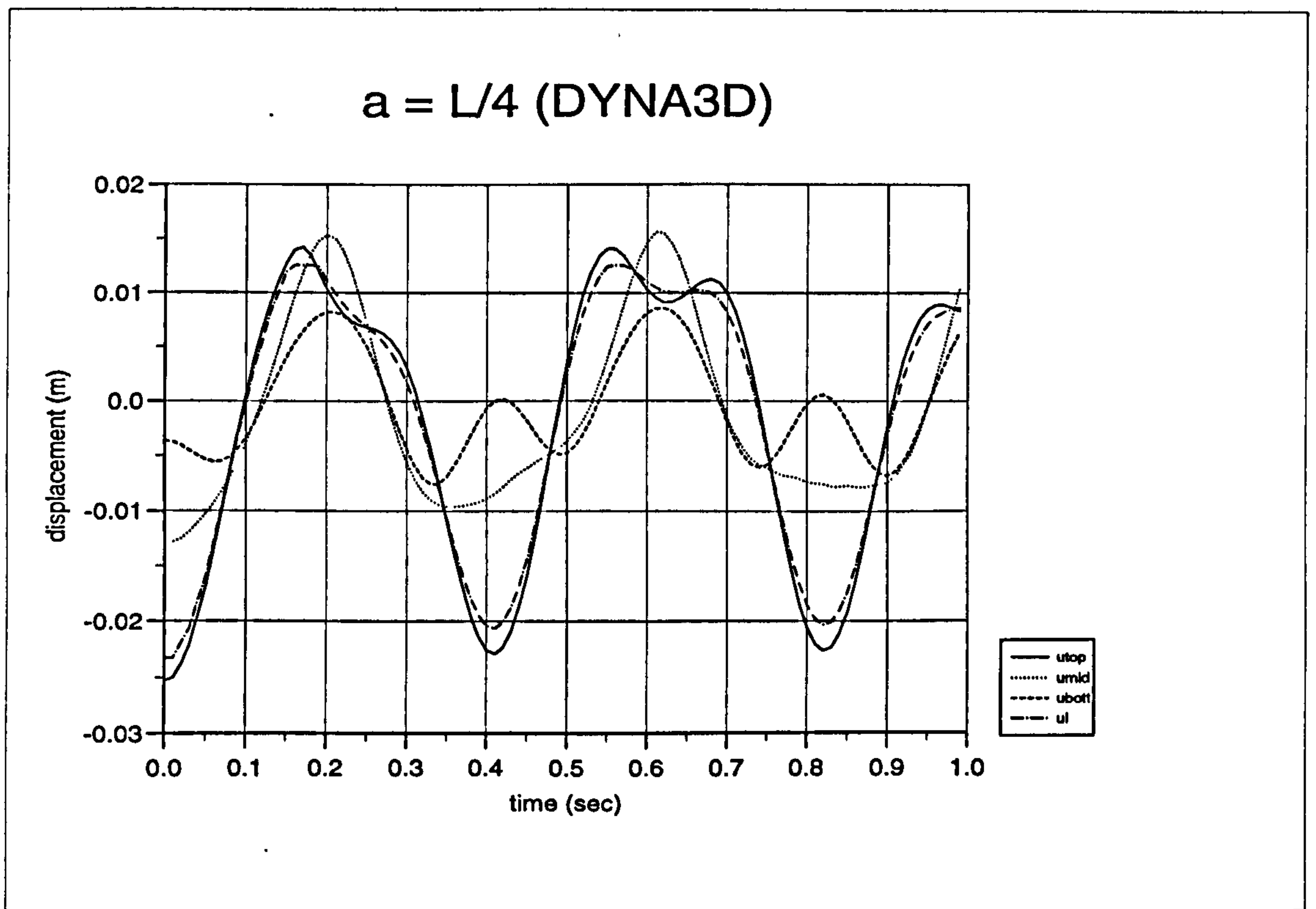


Figure 3.21b) : Displacement curves for pounding case at  $a = L/4$  (DYNA3D)

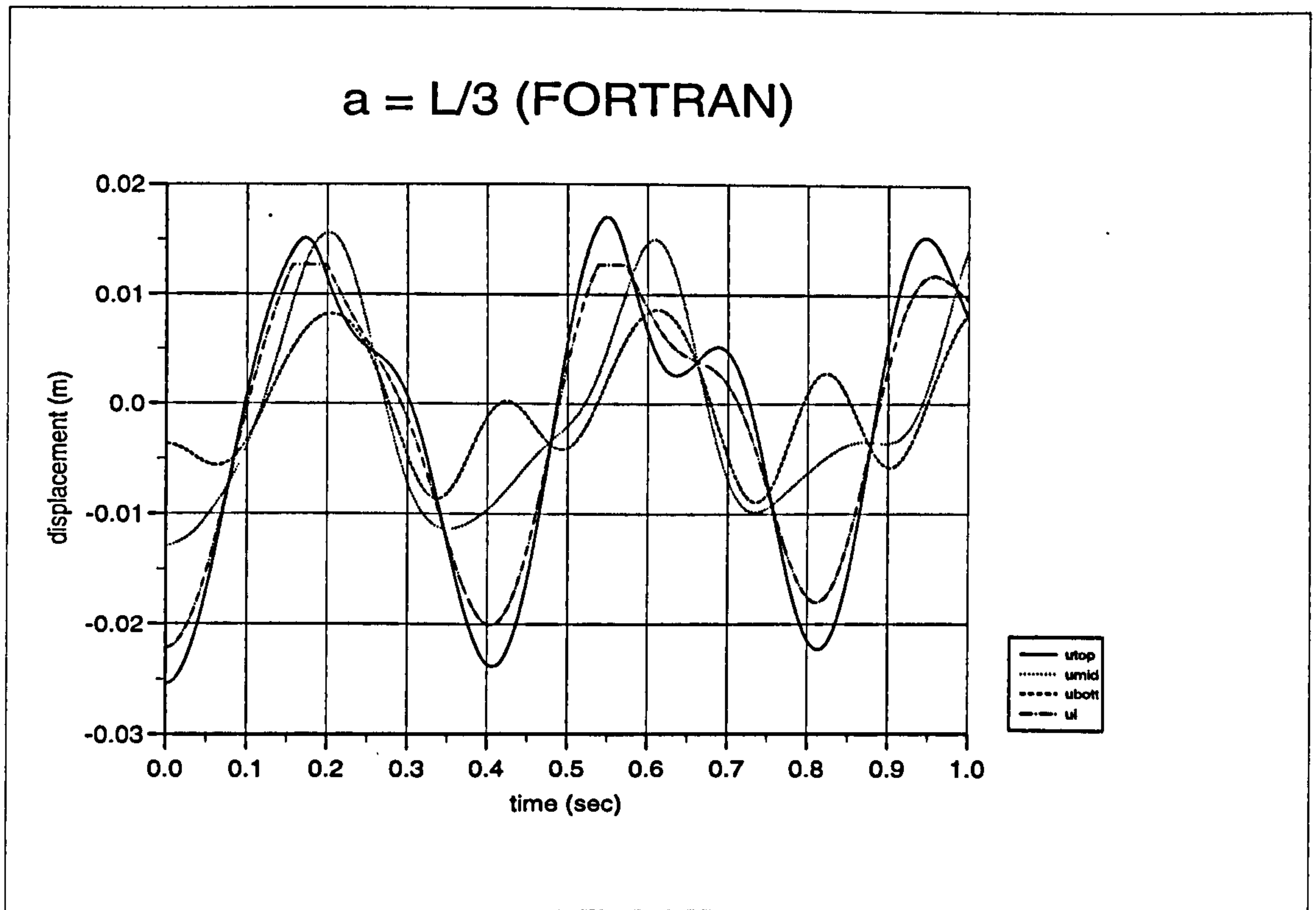


Figure 3.22a) : Displacement curves for pounding case at  $a = L/3$  (FORTRAN)

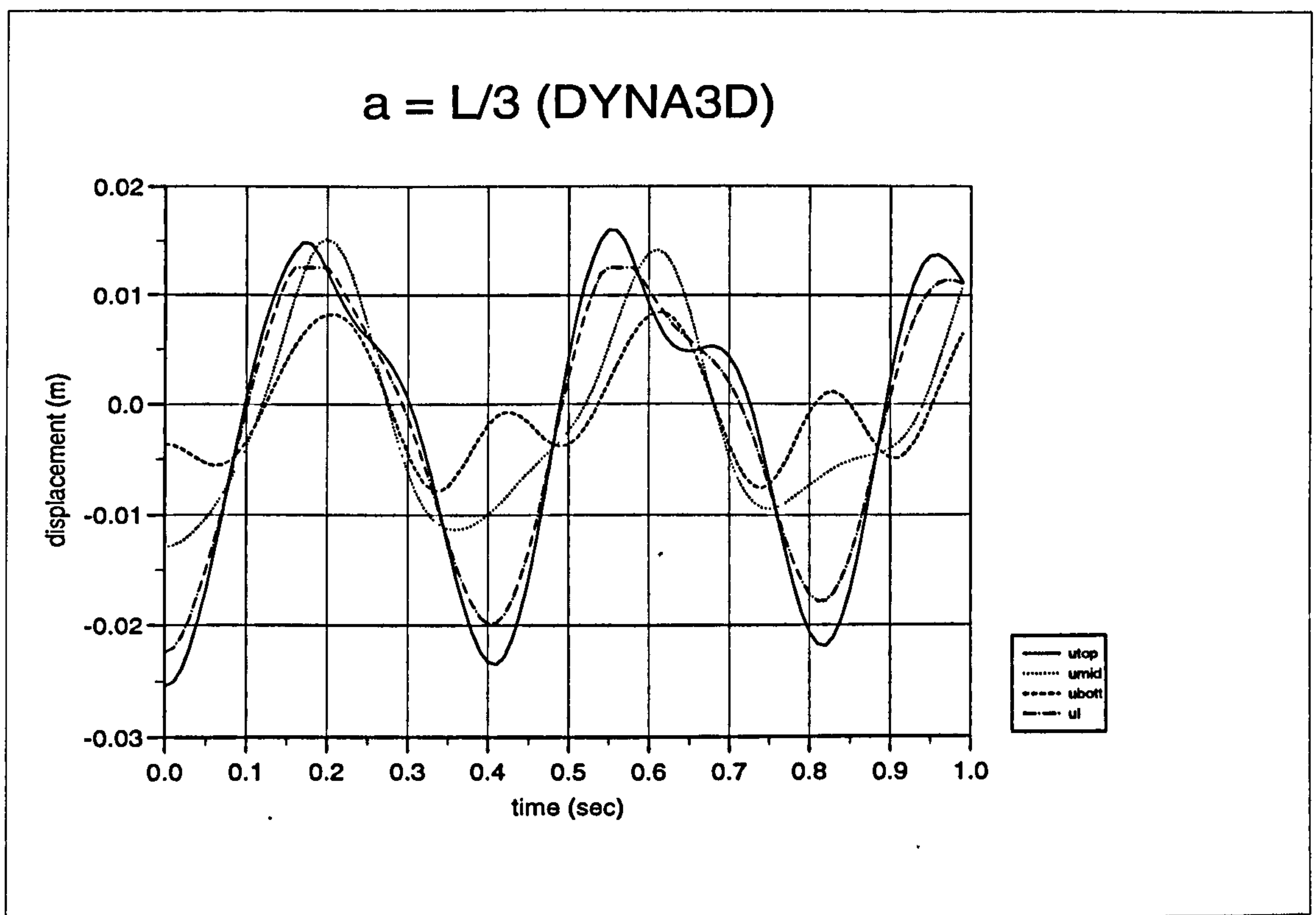


Figure 3.22b) : Displacement curves for pounding case at  $a = L/3$  (DYNA3D)

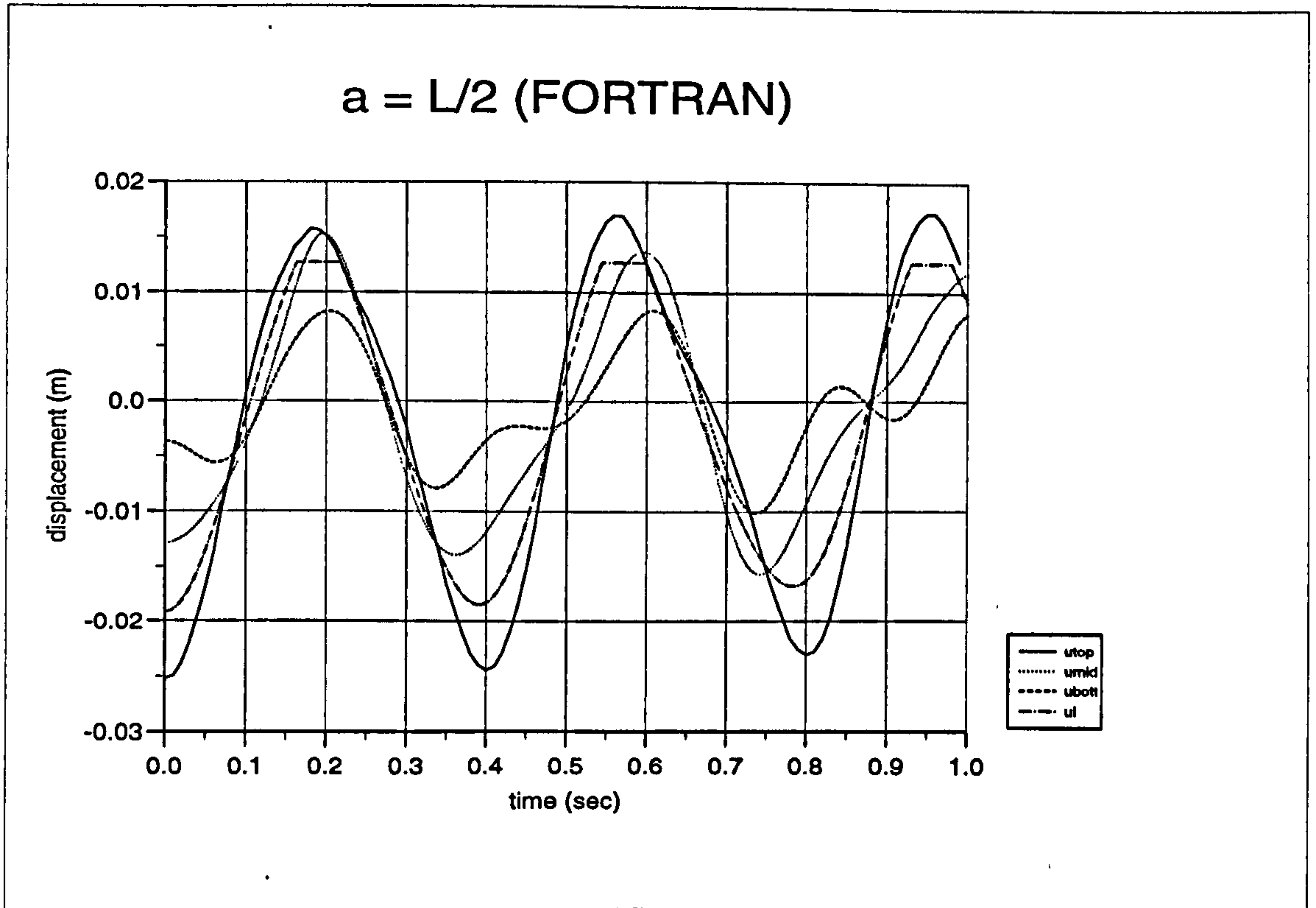


Figure 3.23a) : Displacement curves for pounding case at  $a = L/2$  (FORTRAN)

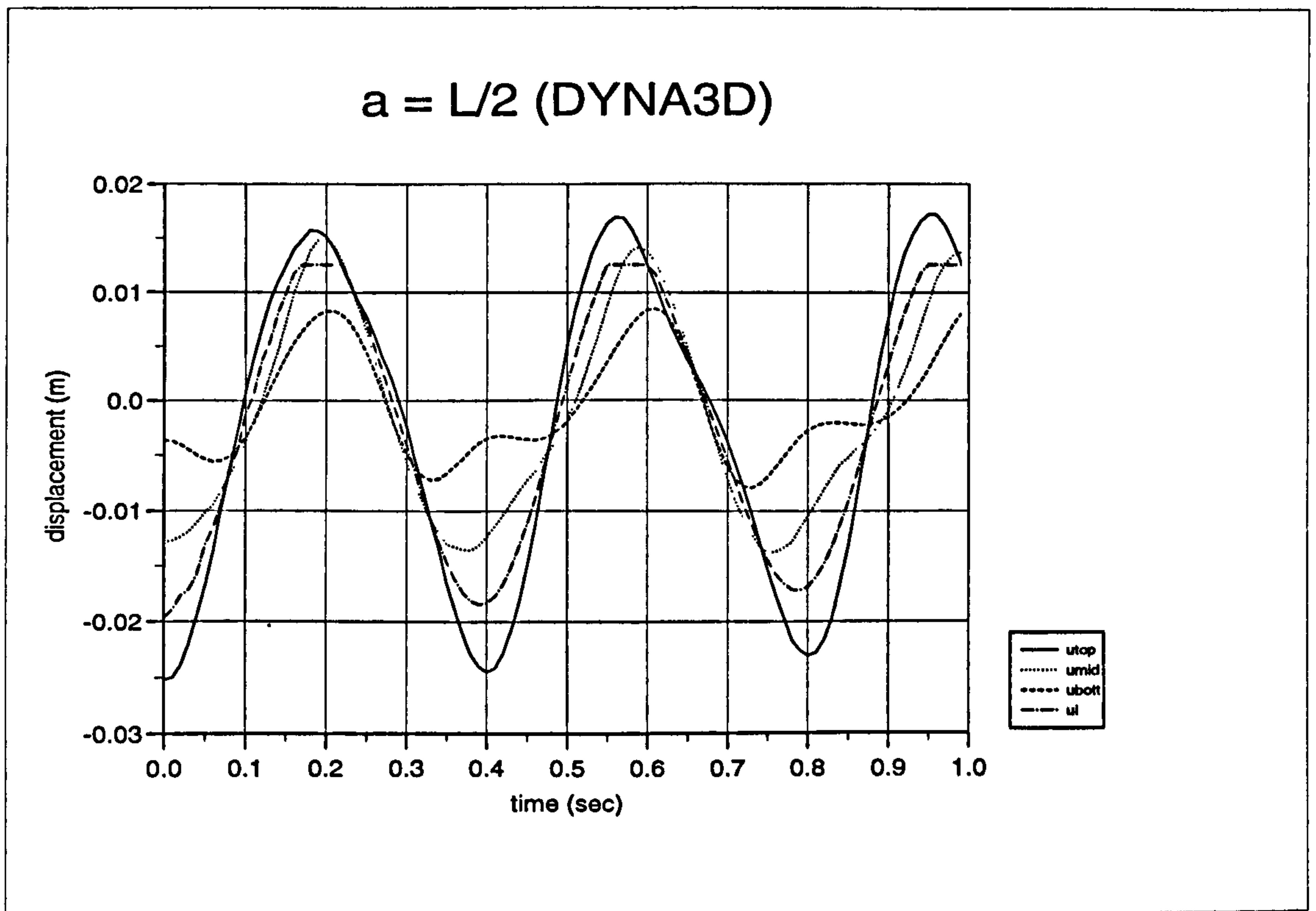


Figure 3.23b) : Displacement curves for pounding case at  $a = L/2$  (DYNA3D)

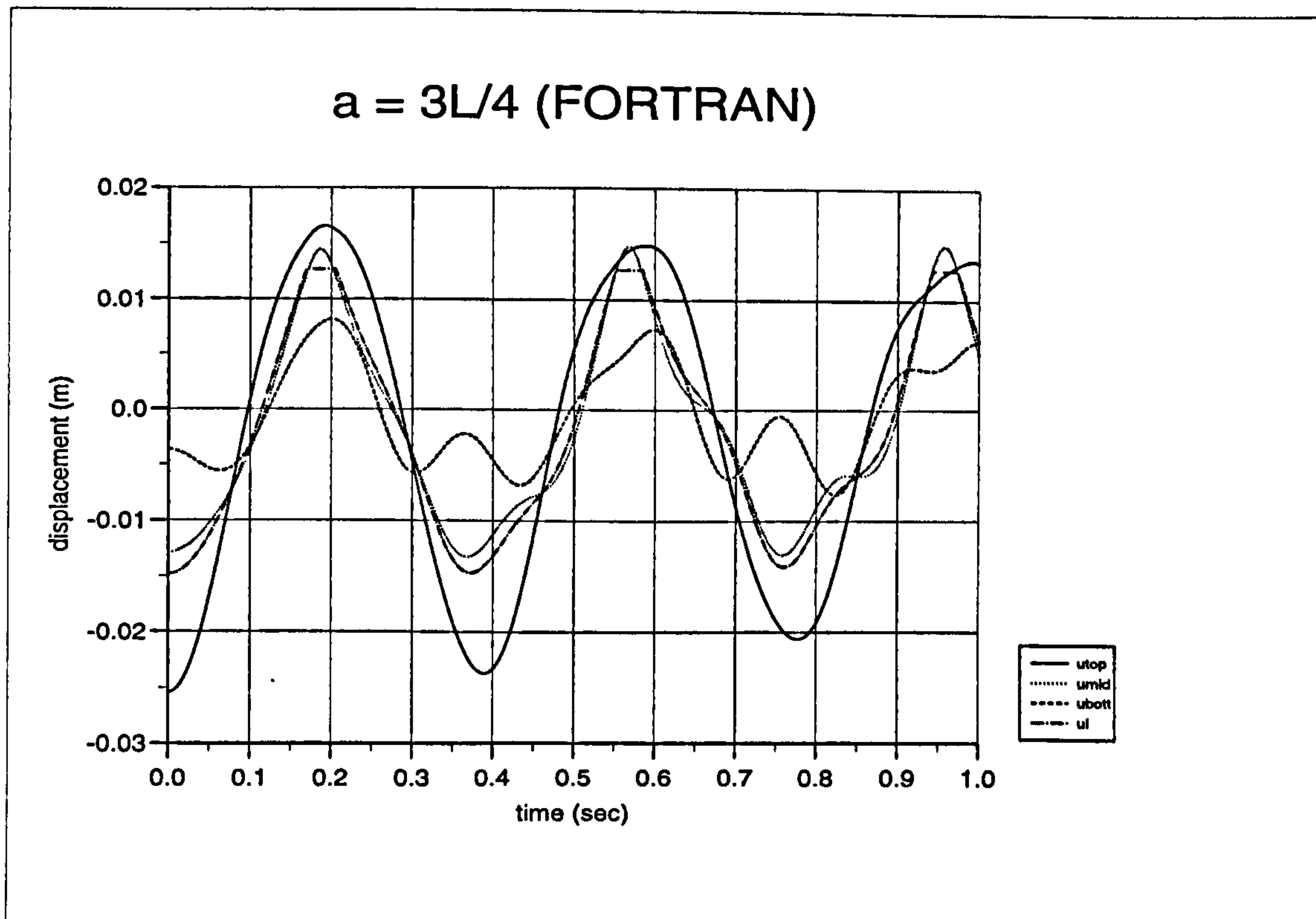


Figure 3.24a) : Displacement curves for pounding case at  $a=3L/4$  (FORTRAN)

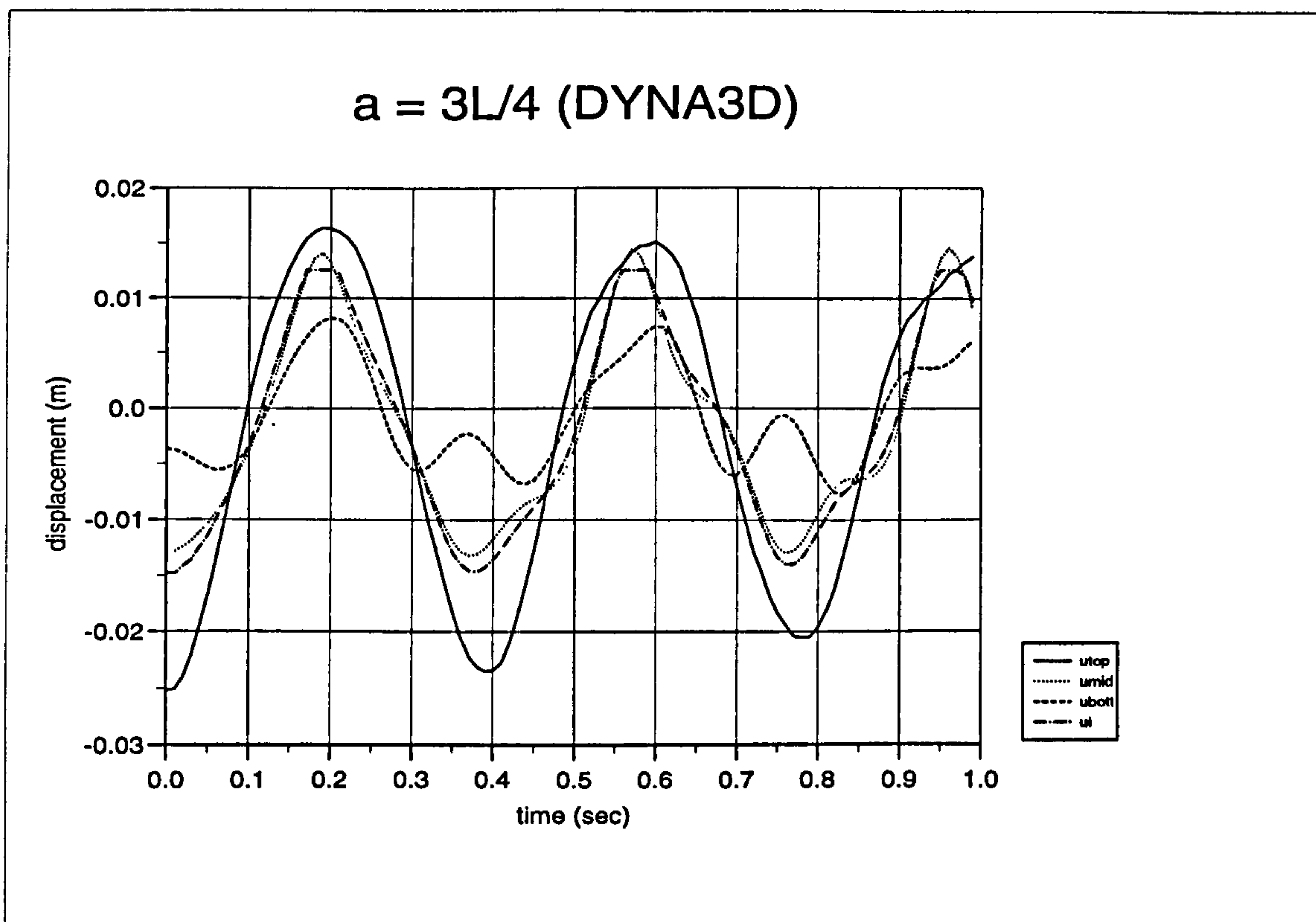


Figure 3.24b) : Displacement curves for pounding case at  $a=3L/4$  (DYNA3D)

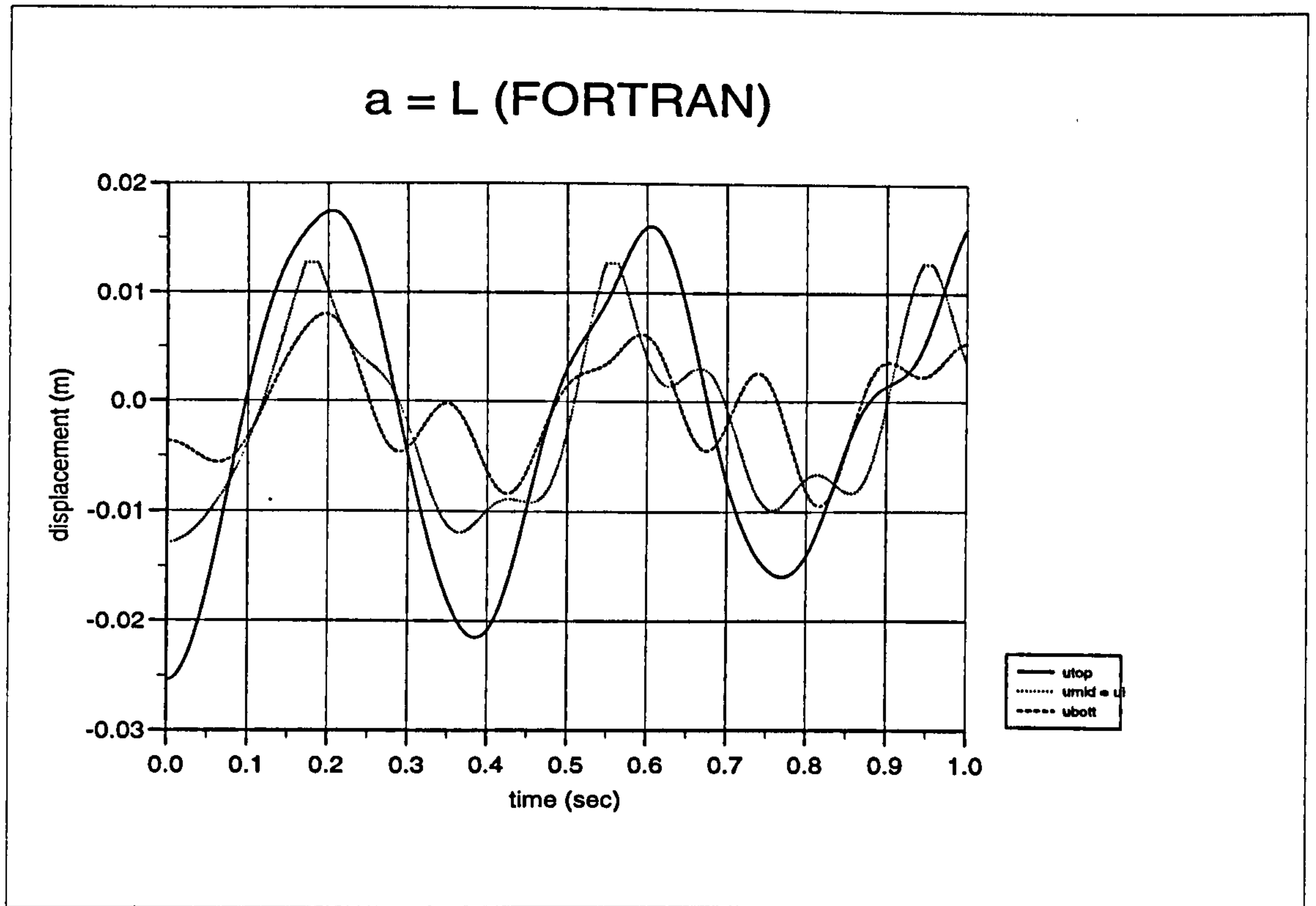


Figure 3.25a) : Displacement curves for pounding case at  $a = L$  (FORTRAN)

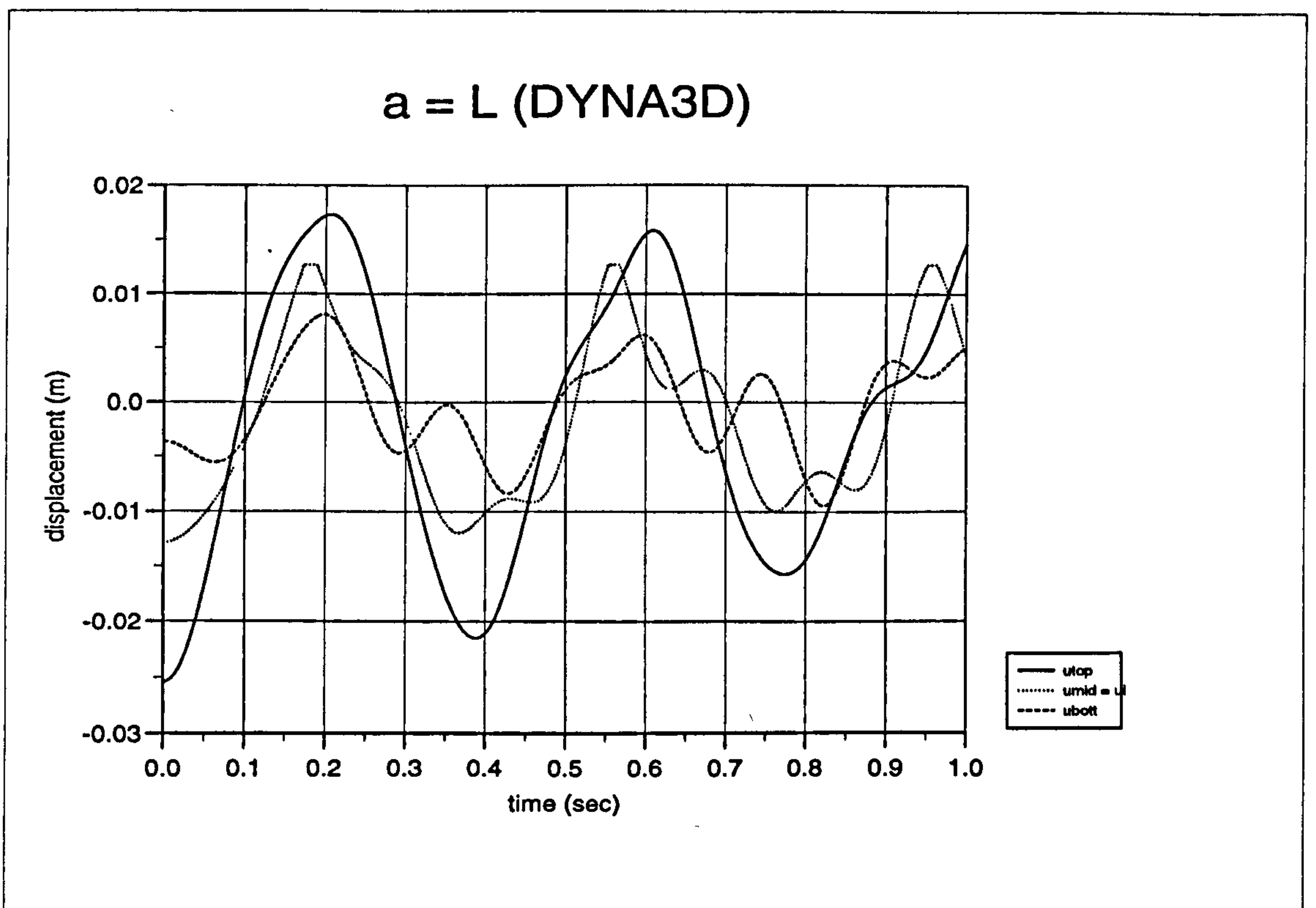


Figure 3.25b) : Displacement curves for pounding case at  $a = L$  (DYNA3D)

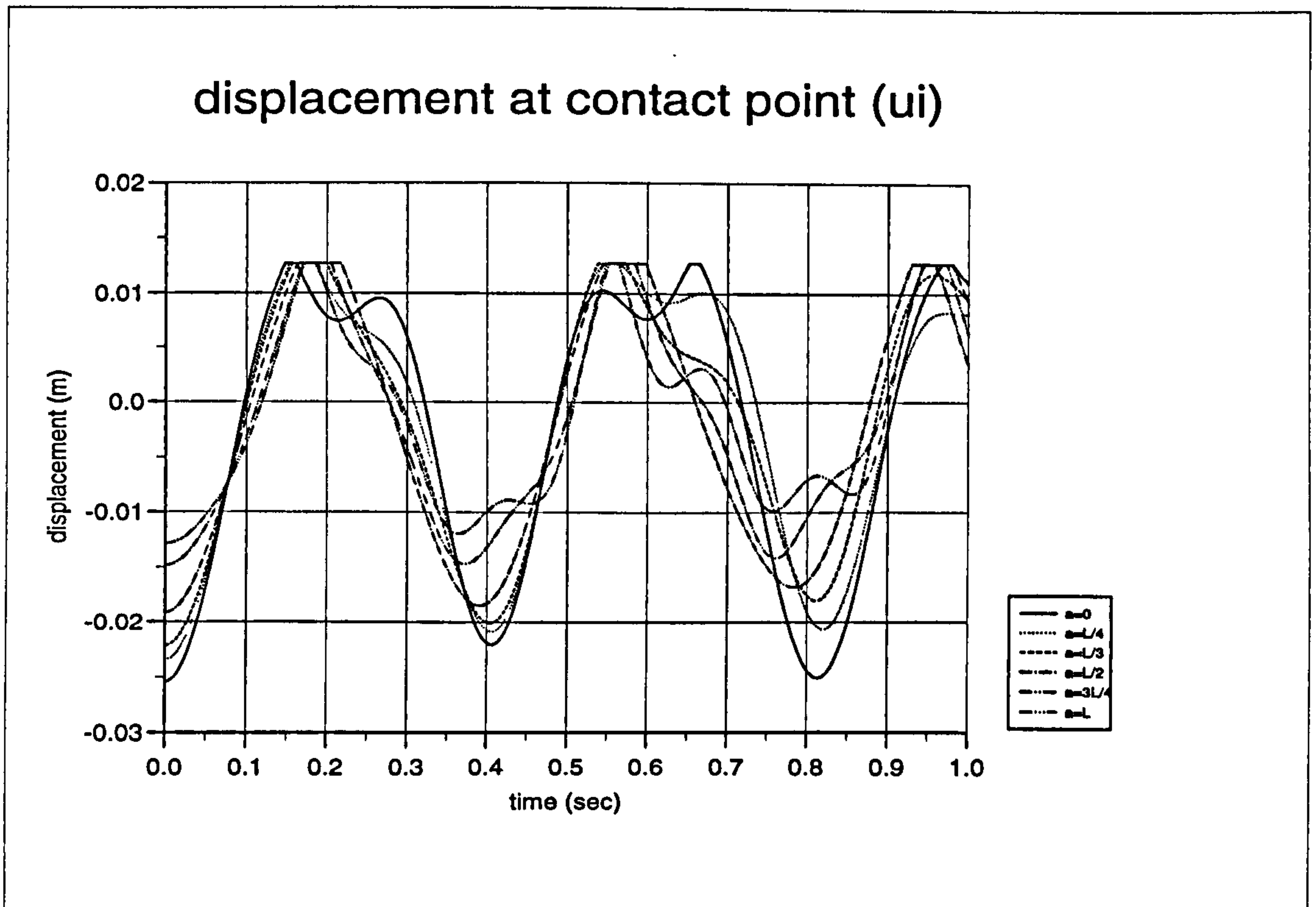


Figure 3.26a) : Displacement curves of  $u_i$  for different "a" values (FORTRAN)

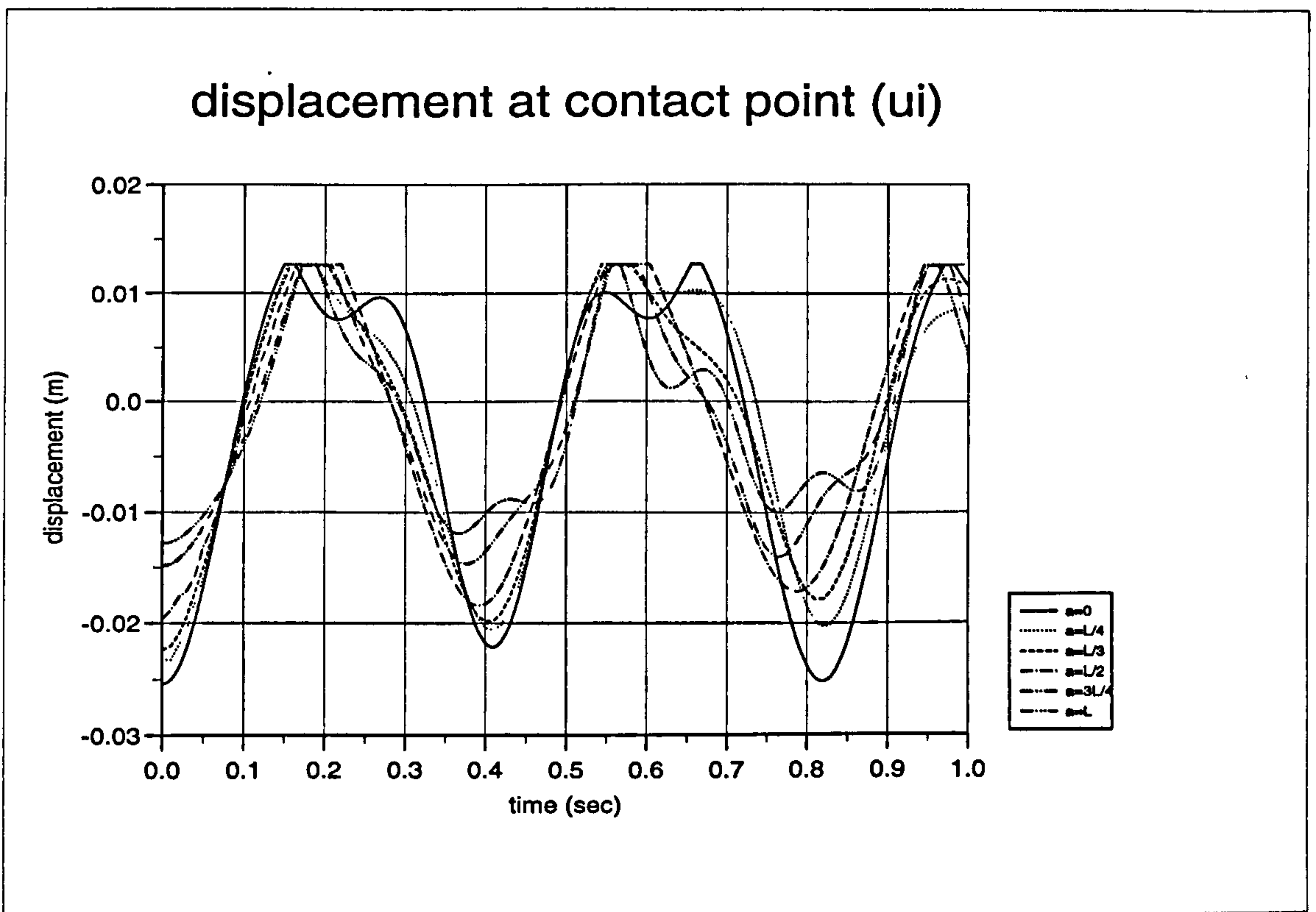


Figure 3.26b) : Displacement curves of  $u_i$  for different "a" values (DYNA3D)

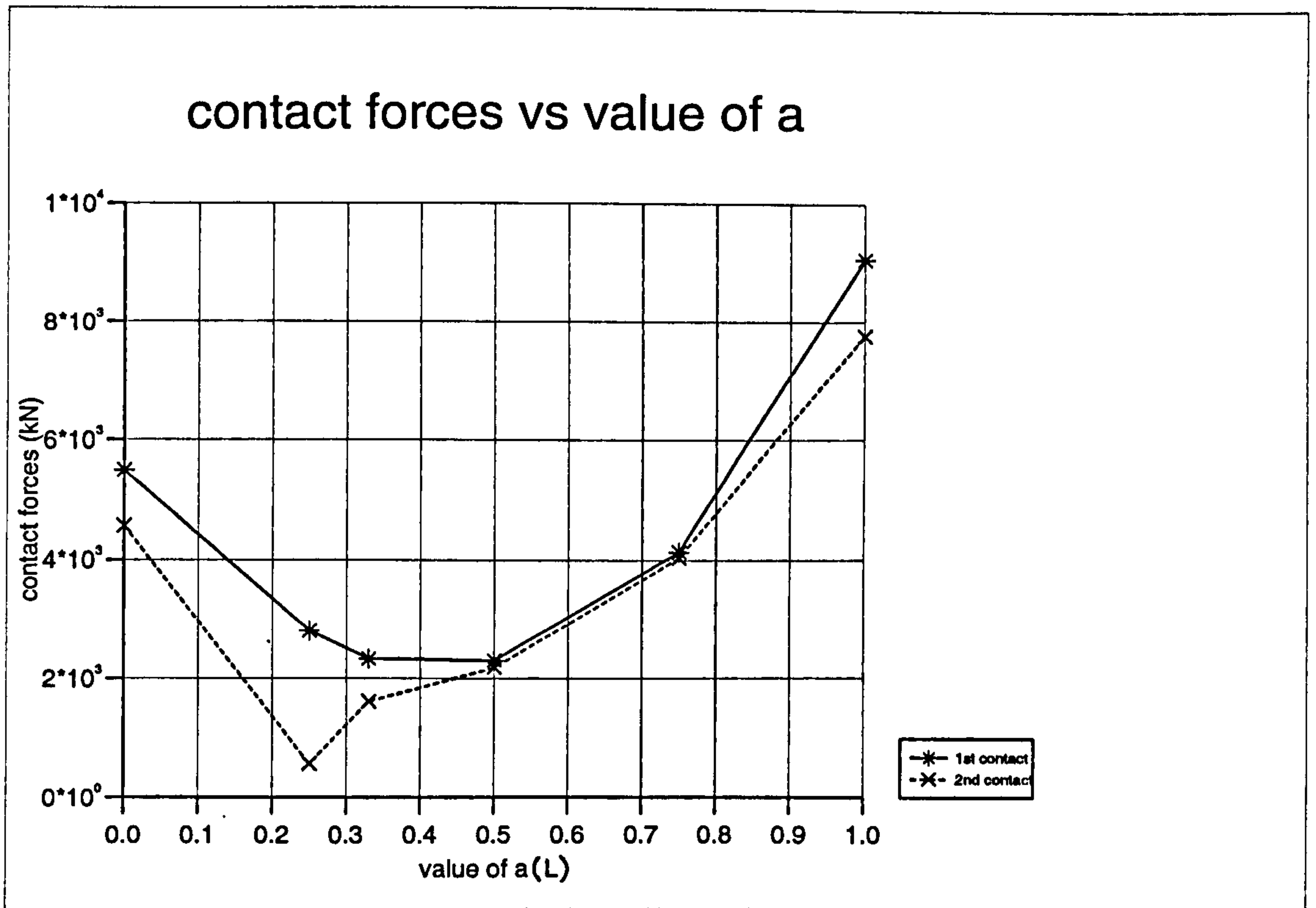


Figure 3.27a) : Maximum contact forces against variation of "a" (FORTRAN)

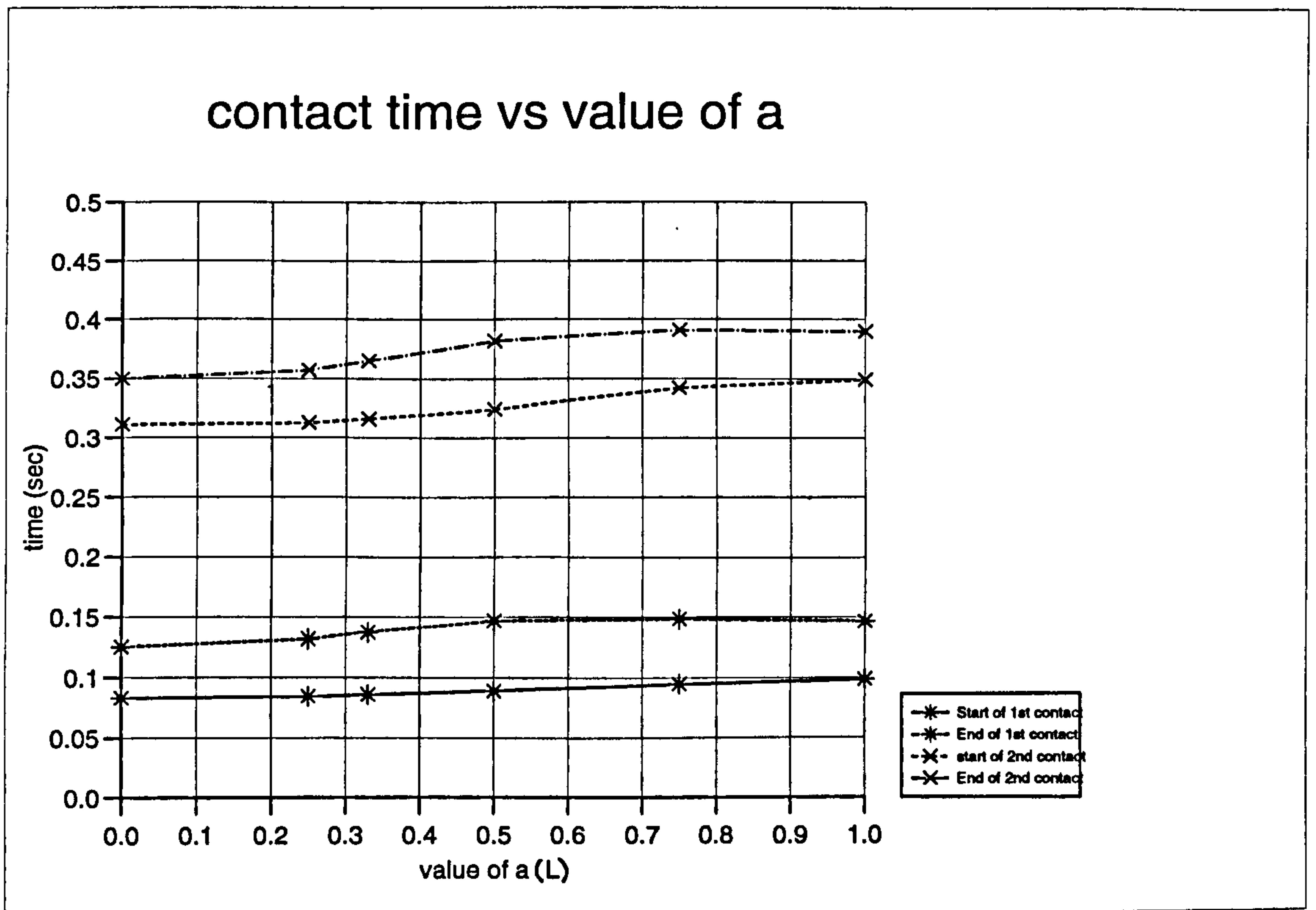


Figure 3.27b) : Contact stage time against variation of "a" (FORTRAN)



### 3.4.2 Damped Free Vibration

The effect of damping always exists in real dynamic problems. In actuality, all systems have some damping that dissipates energy and causes the motion to die out eventually. These are called damping mechanisms. The most common analytical model of damping employed in structural dynamics analyses is the linear viscous dashpot model. The damping constant,  $c$  is expressed as force per displacement rate, and its units are Newtons second per metre (N-s/m).

Normally a damping coefficient/factor/ratio,  $\xi$  can be used to distinguish three cases : *underdamped* ( $0 < \xi < 1$ ), *critically damped* ( $\xi = 1$ ), and *overdamped* ( $\xi > 1$ ). Figure 3.28 illustrates these three cases. For the underdamped case, the motion is oscillatory with a decaying amplitude. For the overdamped case, there is no oscillation, and the amplitude slowly decays. For the critically damped system, there is no oscillation, and the amplitude decays more rapidly than in either the undamped or overdamped cases.

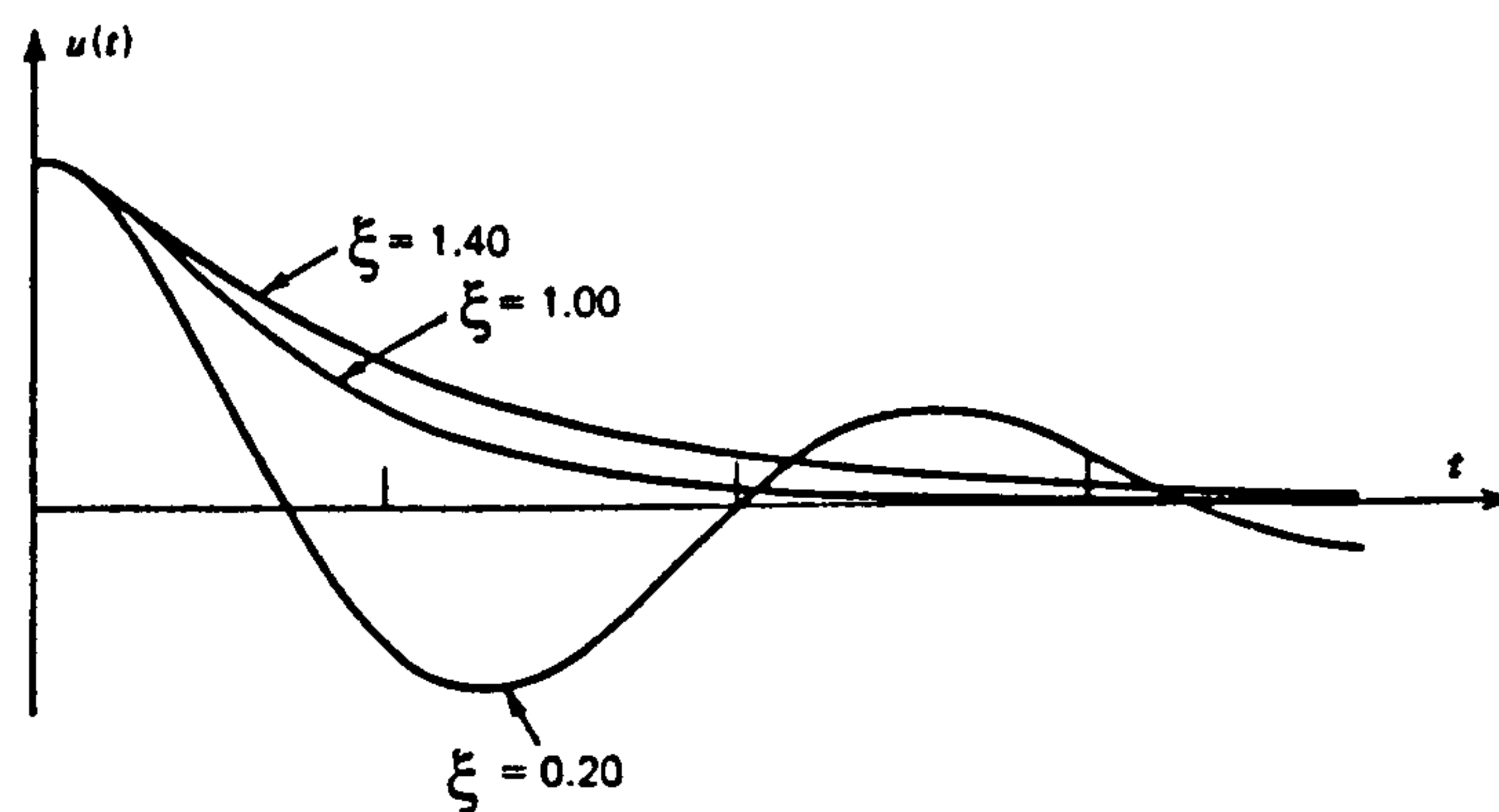


Figure 3.28 : Response of viscous-damped SDOF system with various damping levels ( *After Roy et al. , 1981* )

Since the underdamped case is the most important case for structural dynamics applications, it is the only case treated in this study. For the steel frame structures, the value of  $\xi$  usually falls in the range of 1 % to 2 %.

The pounding analysis for damped free vibration is presented in this section. The equations of motion for the pounding of undamped free vibration using the displacement compatibility method as described in previous section, is extend to this section with the presence of damping coefficient,  $\xi$ .

The numerical examples are based on value of  $\xi = 0.005$  and  $0.02$  and then the results are compared when  $\xi = 0$  (undamped case). Two examples are selected from one SDOF system (aligned floor) and one MDOF system (unaligned floor at  $a=L/2$ ).

### ***3.4.2.1 Free Vibration of Linear - Viscous Damped SDOF system***

Consider a SDOF system as shown in Figure 3.3 of section (3.3.2.1). The equation of motion for the free vibration system ( $\ddot{u}_g=0$ ) is defined by Equation (3.42) produces :

$$m \ddot{u} + c \dot{u} + k u = 0 \quad (3.94)$$

The above equation can be rearranged as

$$\ddot{u} + 2 \xi \omega_d \dot{u} + \omega_d^2 u = 0 \quad (3.95)$$

The damped natural frequency,  $\omega_d$  is given by Equation (3.45) where:

$$\omega_d = \omega \sqrt{(1 - \xi^2)}$$

Where  $\omega$  is the undamped natural frequency. The damped natural period,  $T_d$  is given by:

$$T_d = \frac{2\pi}{\omega_d} \quad (3.96)$$

The damping coefficient,  $\xi$  is expressed as:

$$\xi = \frac{c}{2m\omega} \quad (3.97)$$

Where the  $c$  is a damping constant. In most practical structures in which  $\xi$  is very small, the damped and undamped frequencies are considered to be identical.

### 3.4.2.1.1 No contact Stage

Using the direct integration method over a small time step, the solution for the displacement and velocity of Equation (3.95) at the end of time step is equal to Equation (3.65) and (3.66) respectively. Since  $\ddot{u}_g=0$ , the equations are simplified to become

$$u_2 = e^{-\xi \omega dt} \frac{1}{\omega_d} [\dot{u}_1 + \xi \omega u_1] \sin \omega_d dt + e^{-\xi \omega dt} [u_1] \cos \omega_d dt \quad (3.98)$$

And

$$\dot{u}_2 = e^{-\xi \omega dt} [\dot{u}_1] \cos \omega_d dt - e^{-\xi \omega dt} \frac{1}{\omega_d} [\omega^2 u_1 + \xi \omega \dot{u}_1] \sin \omega_d dt \quad (3.99)$$

The damping coefficient,  $\xi$  as expressed in Equation (3.97) where the subscript  $_{nc}$  represent the no contact stage. Usually  $\xi$  is very small, therefore  $\omega_d = \omega$  becomes  $\omega_{nc}$

$$c = \xi_{nc} 2m \omega_{nc} \quad (3.100)$$

### 3.4.2.1.2 Contact Stage

In order to keep the value of  $c$  constant through out the analysis (no contact stage and contact stage) the value of  $\xi$  must be changed accordingly in the contact stage. The relationship are derived from above equation where the subscript  $_c$  represents the contact stage.

$$\xi_c = \frac{c}{2m \omega_c} = \xi_{nc} \frac{\omega_{nc}}{\omega_c} \quad (3.101)$$

The general equation for the contact stage is expressed as

$$\ddot{u} + 2 \xi \omega_d \dot{u} + \omega_d^2 u = \frac{b_s}{m} \quad (3.102)$$

where  $b_s$  is the static preload forces and is equal to  $(ks Us)$ .

The solution of the displacement is expressed as

$$\begin{aligned}
 u_2 = & e^{-\xi \omega_c dt} \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega_c \left( u_1 - \frac{b_s}{\omega_c^2 m} \right) \right] \sin \omega_d dt \\
 & + e^{-\xi \omega_c dt} \left[ u_1 - \frac{b_s}{\omega_c^2 m} \right] \cos \omega_d dt + \frac{b_s}{\omega_c^2 m}
 \end{aligned} \tag{3.103}$$

And the velocity at the end of the time step is given by:

$$\begin{aligned}
 \dot{u}_2 = & e^{-\xi \omega_c dt} [\dot{u}_1] \cos \omega_d dt \\
 & - e^{\xi \omega_c dt} \omega_d \left[ \left( u_1 - \frac{b_s}{\omega_c^2 m} \right) + \frac{\xi}{\omega_c} \dot{u}_1 \right] \sin \omega_d dt
 \end{aligned} \tag{3.104}$$

Usually,  $\xi$  is very small and  $\omega_d \approx \omega_c$

#### 3.4.2.1.3 Numerical example

A numerical example described in Table 3.1 (section 3.4.1.5.1) is used except that the building separation  $U_s = 0.01\text{m}$  and  $\xi = 0.02$  and  $0.005$ .

#### 3.4.2.1.4 Results and discussions

The displacement and the velocity graphs are plotted in Figure 3.29a and 3.29b respectively, for three values of  $\xi$  (0, 0.005 and 0.02). The results are shown in Table 3.5. It is clearly shown that the presence of the damping as small as 0.005 in the pounding analysis will reduce the contact force. The general trend is that the contact forces reduce as the damping value increases. The starting time for the contact stage is increased as the damping value increases.

$\xi$	1st CONTACT		2nd CONTACT		3rd CONTACT		4th CONTACT		5th CONTACT	
	FORTAN	DYNA3D	FORTAN	DYNA3D	FORTAN	DYNA3D	FORTAN	DYNA3D	FORTAN	DYNA3D
0.02	0.38300 - 0.46439	0.38601 - 0.46700	1.23839 - 1.31861	1.24600 - 1.32601	2.12063 - 2.19995	2.13803 - 2.21702	3.03689 - 3.11471	3.06497 - 3.14196	-	-
	20,472.5	20,340.1	17,253.8	17,115.6	14,118.2	13,977.8	10,997.8	10,857.5	-	-
.005	0.37700 - 0.45859	0.38000 - 0.46100	1.21359 - 1.29520	1.22101 - 1.30201	2.05623 - 2.13735	2.07204 - 2.15303	2.90429 - 2.98511	2.92897 - 3.00897	3.75806 - 3.83877	3.79191 - 3.87191
	21,731.3	21,597.7	20,901.9	20,765.2	20,083.7	19,939.0	19,275.1	19,125.4	18,471.5	18,318.2
$\xi=0$	0.37500 - 0.45679	0.37701 - 0.45801	1.20679 - 1.28810	1.21300 - 1.29501	2.03813 - 2.11935	2.05204 - 2.13504	2.86930 - 2.95051	2.89197 - 2.97397	3.70046 - 3.78158	3.73192 - 3.81391
	22,150.5	22,016.5	22,159.2	22,010.2	22,167.7	22,010.2	22,176.8	22,005.0	22,188.1	22,005.0

Table 3.5 : Overall results for the contact stages of damped SDOF system, pounding at  $a = 0$  (  $\xi=0$  , 0.005 , 0.02 )

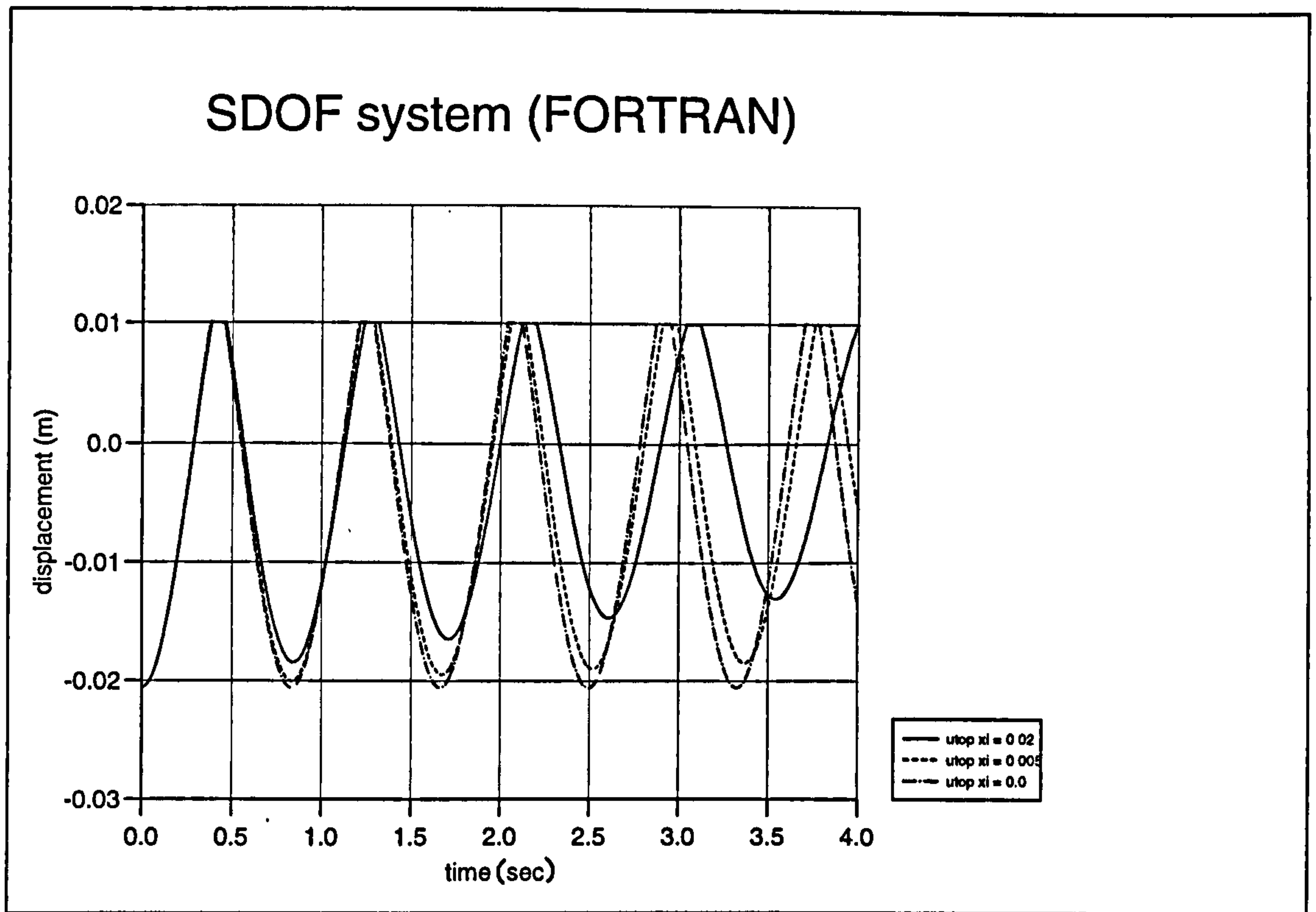


Figure 3.29a : Displacement curves for pounding case (FORTRAN)

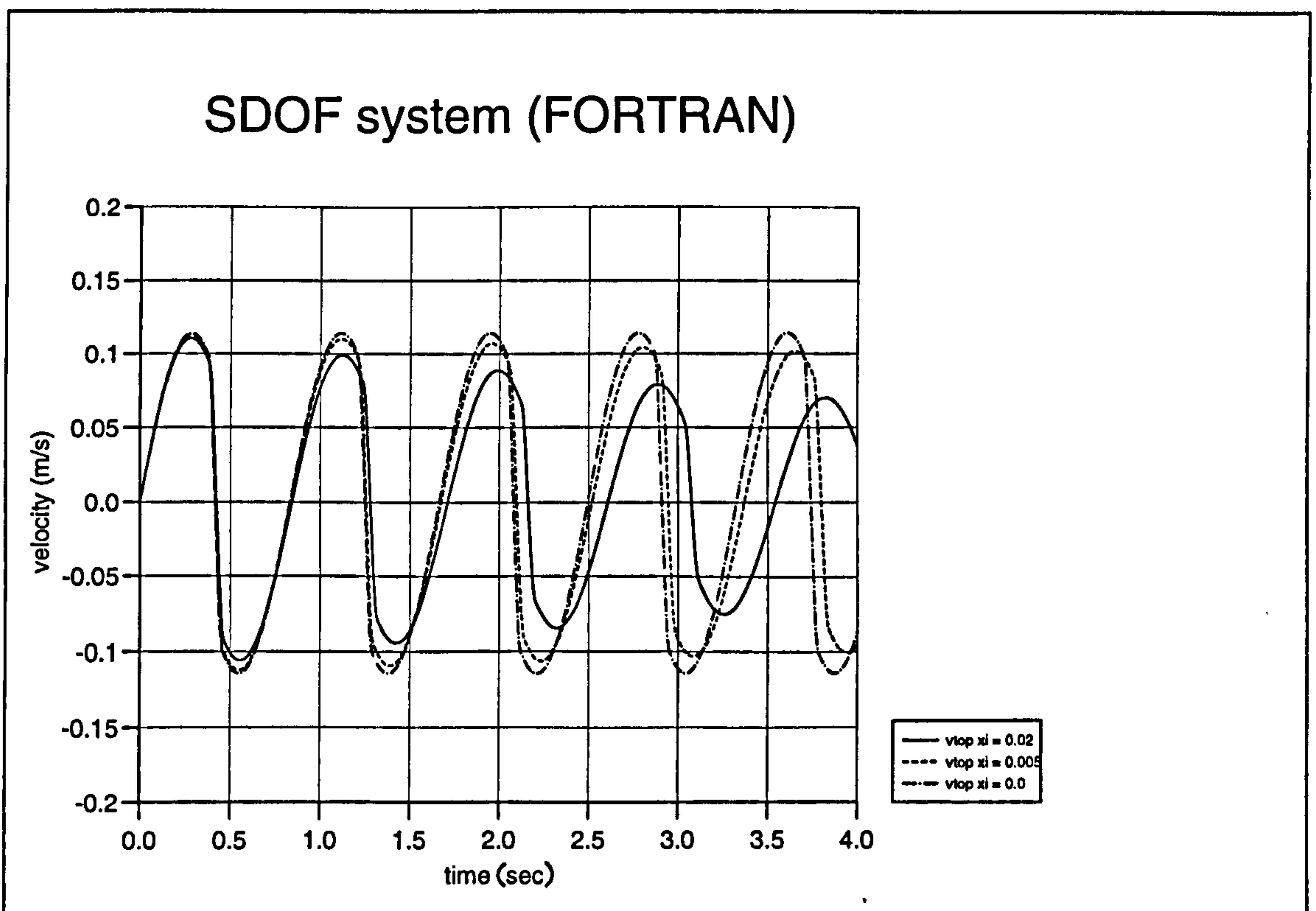


Figure 3.29b : Velocity curves for pounding case (FORTRAN)

### 3.4.2.2 Free Vibration of Linear - Viscous Damped MDOF systems

The same approach is adopted as in damped SDOF system where the physical properties of Table 3.2 are used. Again the MDOF systems in Figure 3.18 is chosen specifically refer to Example 2 (Case  $a = L/2$ ) with the presence of damping.

#### 3.4.2.2.1 No Contact Stage

The equation of motion in the no contact stage is given by Equation (3.54) where

$$m \ddot{u}_i + c \dot{u}_i + k u_i = 0 \quad (3.105)$$

Since damping matrix,  $c$  is proportional to  $k$  and  $m$ , these equations can be transformed into uncoupled equations using the orthogonality conditions (if  $i=j$ ,  $Q_j^T m Q_i = 1$ ) produces the displacement solution of

$$y_{i2} = e^{-\xi_i \omega_i dt} \frac{1}{\omega_{id}} [ \dot{y}_{i1} + \xi_i \omega_i y_{i1} ] \sin \omega_{id} dt + e^{-\xi_i \omega_i dt} [y_{i1}] \cos \omega_{id} dt \quad (3.106)$$

And the velocity solution at the end of the time step as

$$\dot{y}_{i2} = e^{-\xi_i \omega_i dt} [\dot{y}_{i1}] \cos \omega_{id} dt - e^{\xi_i \omega_i dt} \frac{1}{\omega_{id}} [ \omega_{id}^2 y_{i1} + \xi_i \omega_i \dot{y}_{i1} ] \sin \omega_{id} dt \quad (3.107)$$

where  $\omega_{id} = \omega_i$  for small value of  $\xi_i$  and the total displacement and velocity responses are being the sum of the modal contributions of  $i$ th modes as defined in Equation (3.11).

### 3.4.2.2.2 Contact Stage

The contact stage starts when the boundary condition as in section (3.4.1.3) is satisfied. The equations of motion for this stage are expressed as

$$m \ddot{u}_i + \bar{c} \dot{u}_i + \bar{k} u_i = \bar{b} \quad (3.108)$$

where  $\bar{c}$  is the damping matrix;  $\bar{k}$ , is the stiffness matrix and  $\bar{b}$  is a column vector of static preload forces (section 3.4.1.2). Hence, the displacement solution for these equations are

$$y_{i2} = e^{-\xi_i \omega_{ic} dt} \frac{1}{\omega_{id}} [ \dot{y}_{i1} + \xi_i \omega_{ic} (y_{i1} - y_{sii}) ] \sin \omega_{id} dt \quad (3.109)$$

$$+ e^{-\xi_i \omega_{ic} dt} [ u_{i1} - y_{sii} ] \cos \omega_{id} dt + y_{sii}$$

And the velocity at the end of the time step is given by:

$$\dot{u}_{i2} = e^{-\xi_i \omega_{ic} dt} [ \dot{y}_{i1} ] \cos \omega_{id} dt \quad (3.110)$$

$$- e^{-\xi_i \omega_{ic} dt} \omega_{id} [ y_{i1} - y_{sii} + \frac{\xi_i}{\omega_{ic}} \dot{y}_{i1} ] \sin \omega_{id} dt$$

where  $\omega_{id} \approx \omega_{ic}$  for small value of  $\xi_i$  and

$$y_{sii} = \frac{Q_j^T \bar{b}}{\omega_{ic}^2}$$

as in Equation (3.91). The total displacement and velocity responses are given by Equation (3.11).

### 3.4.2.2.3 Numerical Example

A numerical examples as defined in Table 3.2 is used and values of  $\xi_i$  is equal to 0.005 and 0.02. The FORTRAN programs namely DAM1.FOR and DAM2.FOR are written to assist the iteration process for a damped SDOF and MDOF systems respectively (Appendix D).



#### **3.4.2.2.4 Results and discussions**

The displacements of the MDOF systems, are plotted in Figure 3.30a - 3.30c, for various value of  $\xi$  (0, 0.005 and 0.02) respectively. The results are shown in Table 3.6. Again, similar to the SDOF system, the contact forces reduce as the damping value is increased.

$\xi$	1st CONTACT		2nd CONTACT		3rd CONTACT	
	FORTAN	DYNA3D	FORTAN	DYNA3D	FORTAN	DYNA3D
0.02	Contact time (sec)	0.16730-0.22006 (0.05276)	0.16701-0.22000 (0.05299)	0.56953-0.61414 (0.04461)	0.56600-0.61400 (0.04800)	-
	Contact forces (kN)	1,946.858	2,104.040	1,002.001	1,484.860	-
.005	Contact time (sec)	0.16350-0.21759 (0.05409)	0.16500-0.21901 (0.054009)	0.54866-0.59995 (0.05129)	0.55400-0.60599 (0.05199)	0.95499-0.99699 (0.04200)
	Contact forces (kN)	2,238.841	2,280.170	1,910.326	1,972.510	1,429.940
$\xi=0$	Contact time (sec)	0.16230-0.21676 (0.05446)	0.16390-0.21981 (0.05591)	0.54294-0.59564 (0.05270)	0.55008-0.60369 (0.05361)	0.94505-0.99506 (0.05001)
	Contact forces (kN)	2,337.394	2,364.380	2,217.216	2,079.050	1,532.160

Table 3.6 : Overall results for the contact stages of damped MDOF systems, pounding at  $a = L/2$  (  $\xi=0$  , 0.005 , 0.02 )

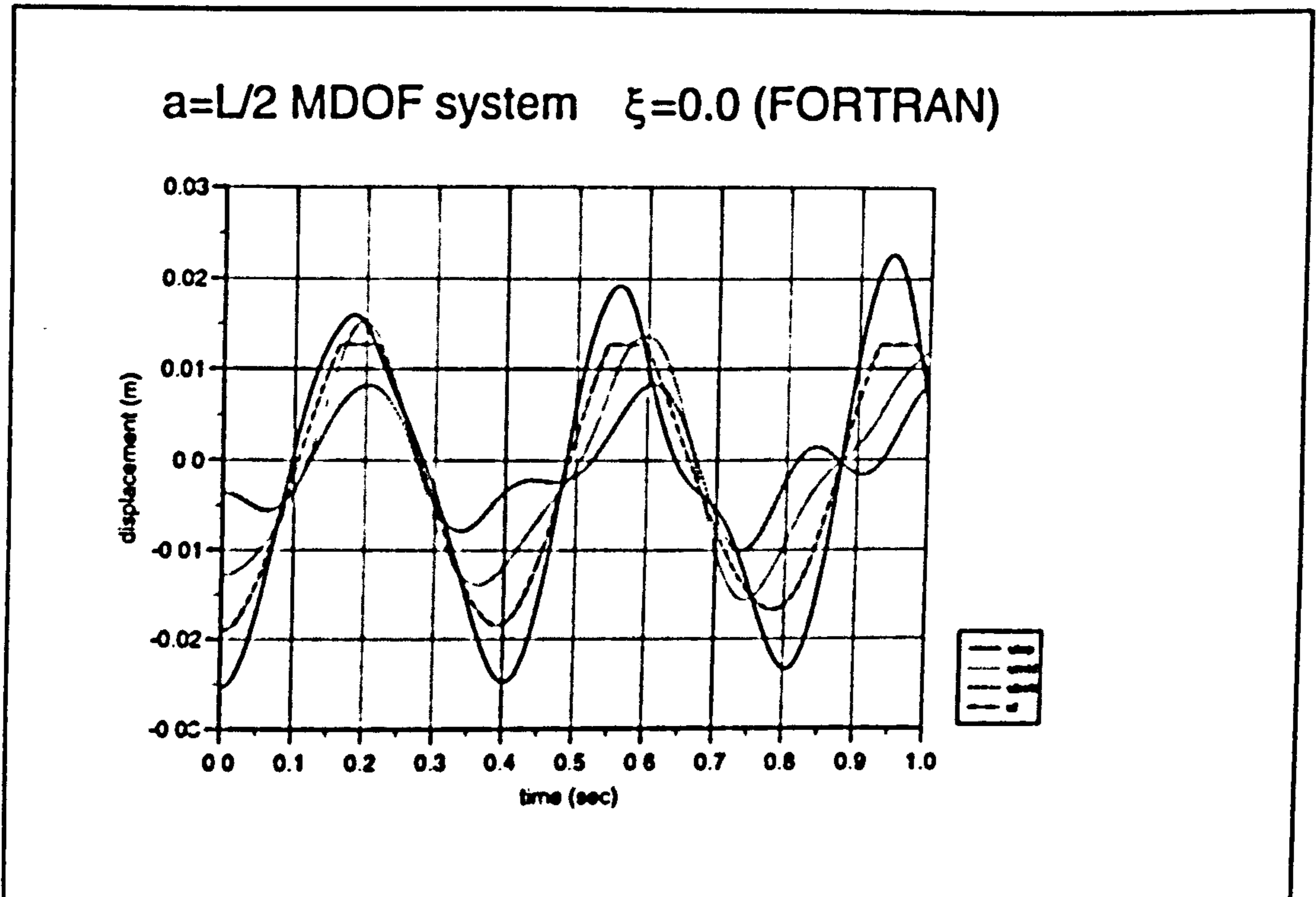


Figure 3.30a : Displacement curves for pounding case  $\xi=0$  (FORTRAN).

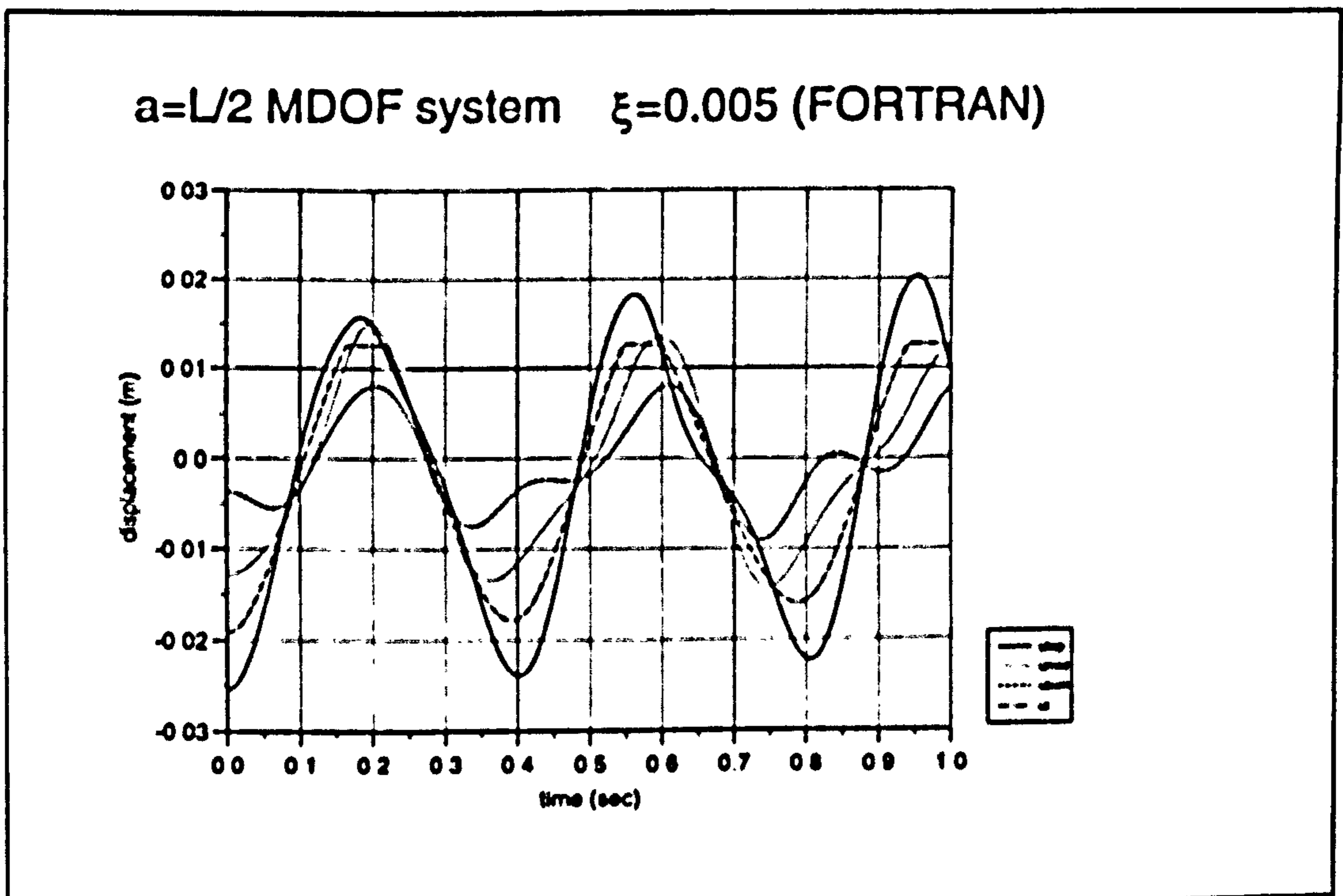


Figure 3.30b : Displacement curves for pounding case  $\xi=0.005$  (FORTRAN).

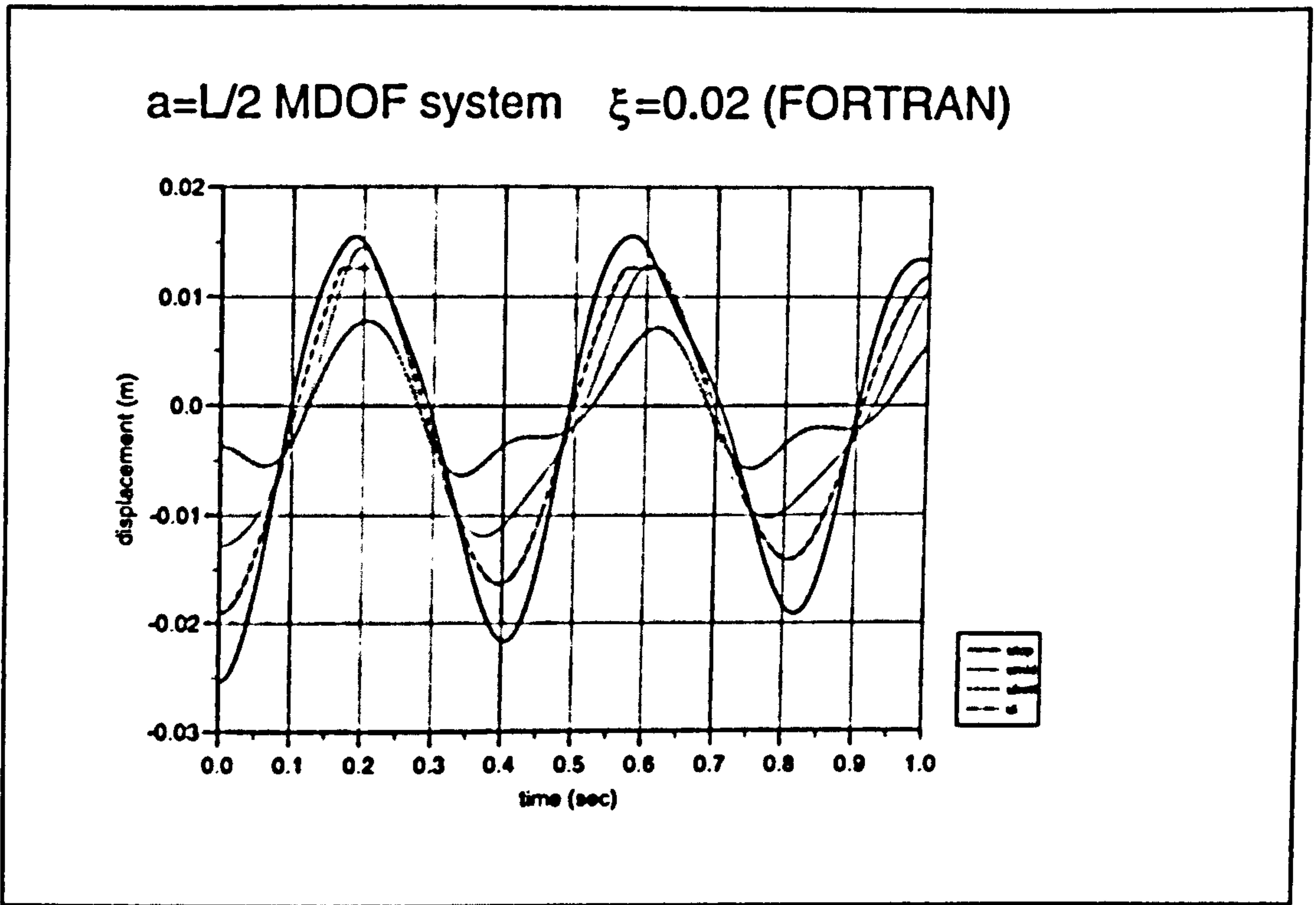


Figure 3.30c : Displacement curves for pounding case  $\xi=0.02$  (FORTRAN).

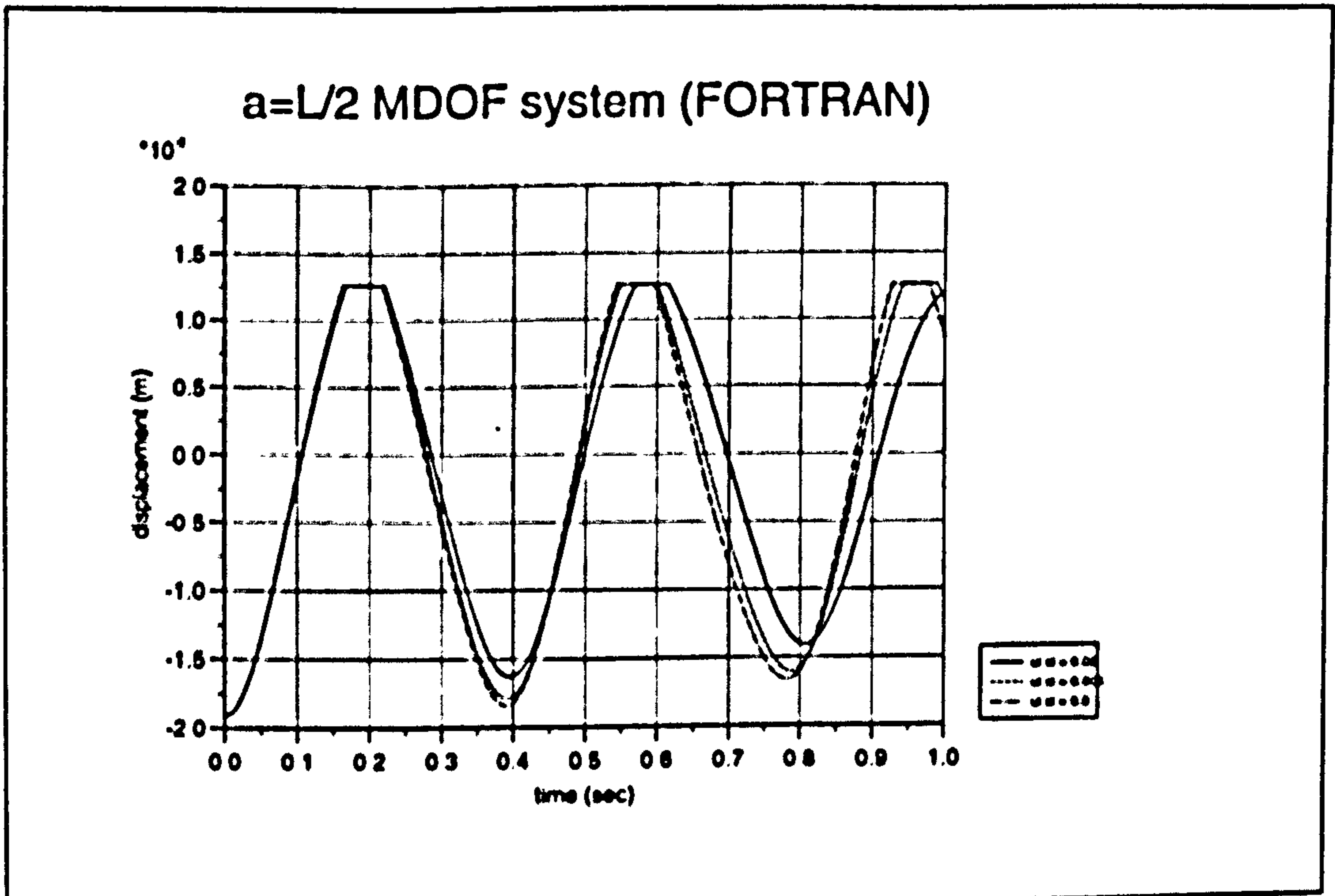


Figure 3.30d : Displacement curves of  $u_i$  for various  $\xi$  (FORTRAN).

### 3.4.3 Undamped Forced Vibration

So far the analysis of pounding are initiated by the snap back analysis where the value of  $\dot{u}_g = 0$  is assumed. Dynamic response can be induced in a structure not only by a time-varying applied load or initiated by a snap-back analysis, but also by motions of its support. In this section, an example of such excitation is studied on the earthquake-induced pounding analysis using the horizontal ground acceleration recorded for the Mexico earthquake, 1985, N90-W component (Figure 3.31).

Two examples of MDOF systems are selected from one aligned floor case ( $a=0$ ) and one from unaligned floor case ( $a=L/2$ ). Consider a shear frame in Figure (3.6) having the physical properties as in Table 3.7 below:

$m_3, m_2, m_1$ = building masses of 1.0, 1.5, 2.0 kips-sec <sup>2</sup> /in. = 175197, 262795 , and 350394 kg (N-sec <sup>2</sup> /m)
$2k_3, 2k_2, 2k_1$ = building stiffness of 150, 300, 450 kips/in. = $26.279 \cdot 10^3$ , $51.559 \cdot 10^3$ , and $78.839 \cdot 10^3$ (kN/m)
$k_s$ = slab stiffness of 50,000 kips/in. ( $8.76 \cdot 10^6$ kN/m)
Building separation, $U_s = 0.02$ m
The acceleration of gravity, $g = 9.81$ m/s <sup>2</sup>
Undamped vibration, $c = 0$
Height between floors, $L = 72$ in. (1.8288 m)
Young Modulus of steel, $E = 30 \cdot 10^3$ kip/in <sup>2</sup> ( $205 \cdot 10^6$ kN/m <sup>2</sup> )

Table 3.7 : Physical properties of the building. (\*\* 1 kip = 4450N, 1 in. = 0.0254 m)

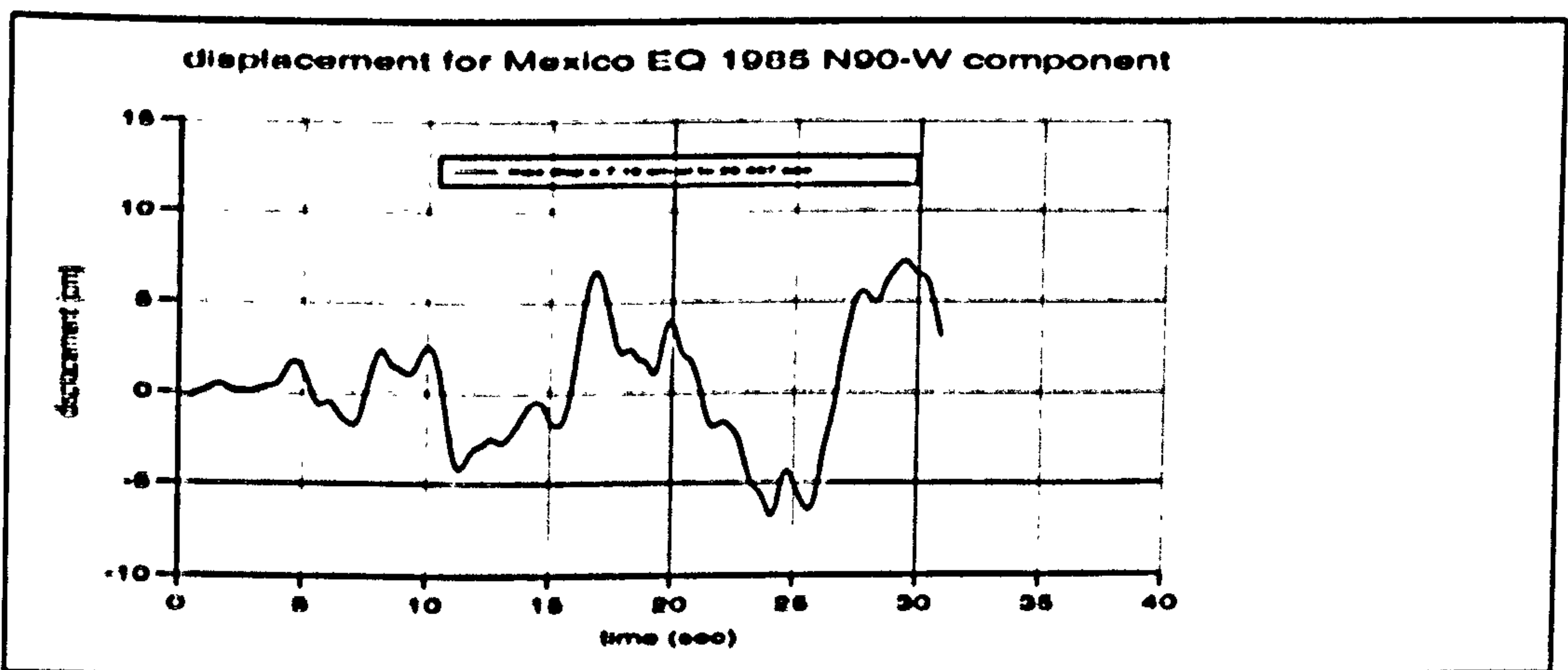
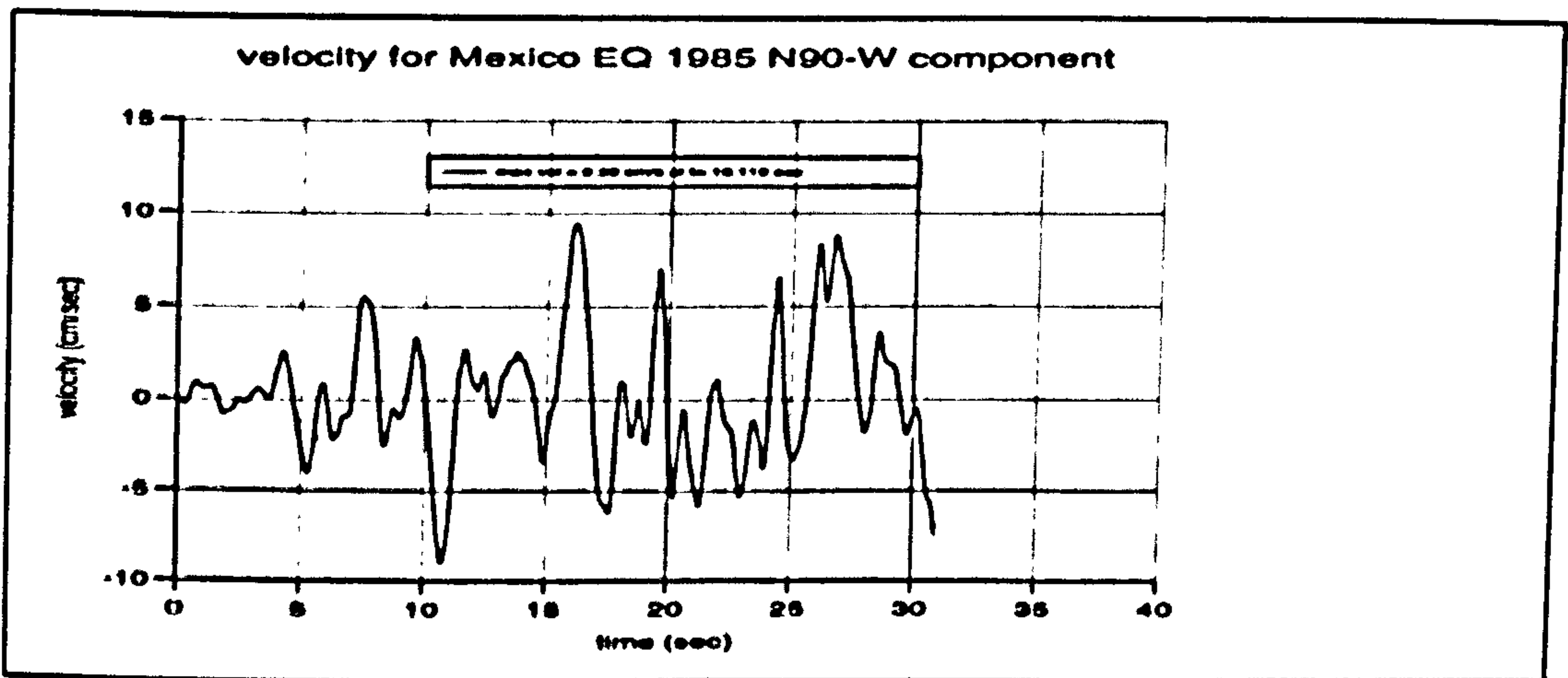
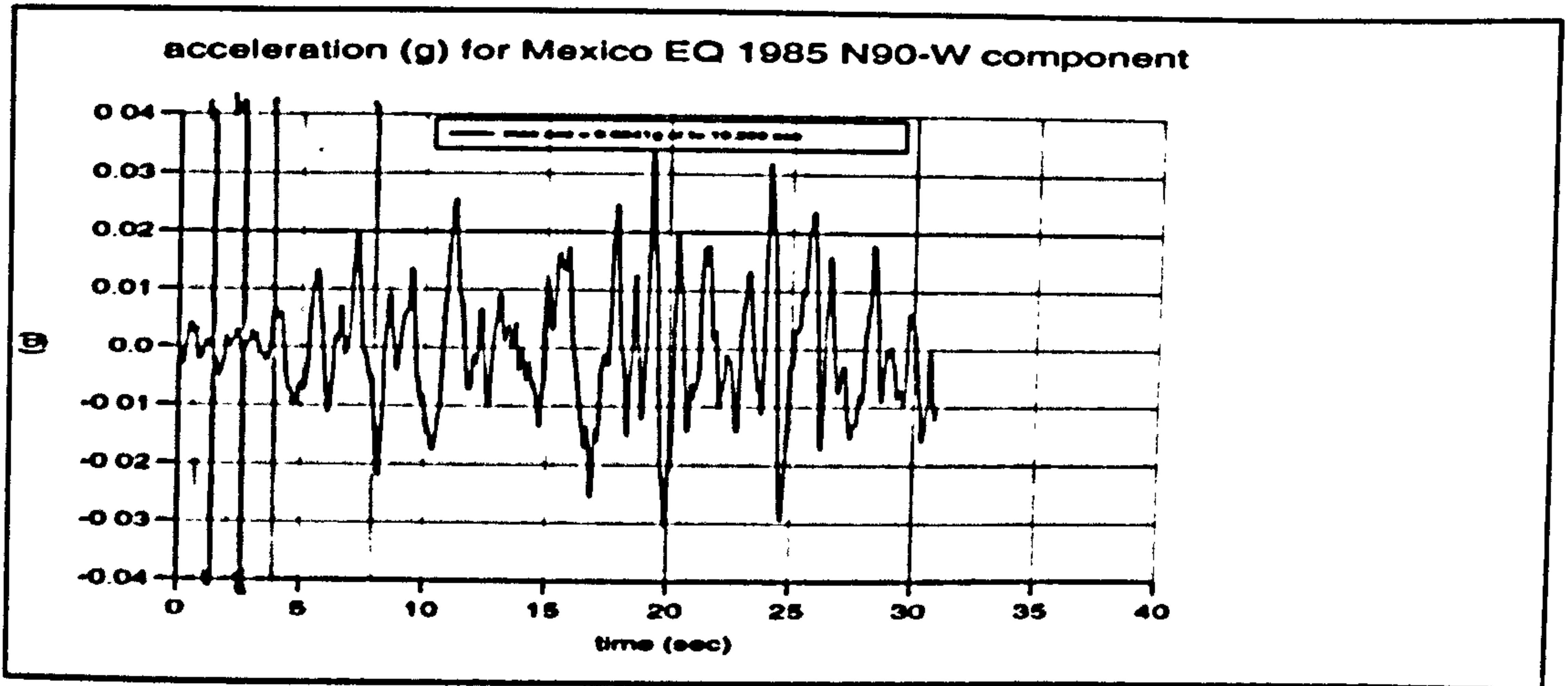


Figure 3.31 : Ground acceleration, velocity and displacement curves for the Mexico earthquake, 1985, N90 - W component.

### 3.4.3.1 MDOF systems, contact at floor level (aligned floor)

In this section, the earthquake-induced pounding analysis of MDOF systems is presented. One case of  $\alpha=0$  for the aligned floor is taken as a numerical example.

#### 3.4.3.1.1 No Contact Stage

The equations of motion for undamped force vibration in the no contact stage are:

$$m \ddot{u}_i + k u_i = - m r \ddot{u}_g \quad (3.111)$$

where  $r$  is Pseudo static vector as described in section (3.3.1.2). These coupled equations are then transformed into a set of  $i$ th uncoupled equations which can then be treated in the same way of SDOF system. Assumed the variation of the ground motion is linear over a small time step, as shown in Figure 3.8 (section 3.3.2.1) produces

$$\ddot{y}_i + \omega_i^2 y_i = \beta_i \ddot{u}_g \quad (3.112)$$

where,  $\beta_i$  is the participation factor as defined in Equation (3.56). The solution for displacement and velocity at the end of time step are given by substituting value  $\xi_i = 0$  ( $\therefore \omega_{id} = \omega_i$ ) into Equation (3.67) and (3.68) produces

$$\begin{aligned} y_{i2} &= \frac{1}{\omega_i} \left[ \dot{y}_{i1} + \frac{\beta_i R}{\omega_i^2} \right] \sin \omega_i dt + \left[ y_{i1} + \beta_i \frac{\ddot{u}_{g1}}{\omega_i^2} \right] \cos \omega_i dt \\ &- \beta_i \frac{R dt}{\omega_i^2} - \beta \frac{\ddot{u}_g}{\omega_i^2} \end{aligned} \quad (3.113)$$

and

$$\begin{aligned} \dot{y}_{i2} &= \left[ \dot{y}_{i1} + \beta_i \frac{R}{\omega_i^2} \right] \cos \omega_i dt + \frac{1}{\omega_i} \left[ -\omega_i^2 y_{i1} - \beta_i \ddot{u}_{g1} \right] \sin \omega_i dt \\ &- \beta_i \frac{R}{\omega_i^2} \end{aligned} \quad (3.114)$$

respectively. Total relative displacement and velocity responses are given by Equation (3.11).

### 3.4.3.1.2 Contact Stage

The contact stage starts when the boundary conditions as in section (3.4.1.3) are satisfied. The equations of motion for this stage are expressed as

$$m \ddot{u}_i + \bar{k} u_i = -m r \ddot{u}_g + \bar{b} \quad (3.115)$$

where  $\bar{k}$  is the stiffness matrix as in Example 1 b) of section (3.4.1.5.2) and  $\bar{b}$  is the column vector of static preload force ,

$$\bar{b} = \begin{vmatrix} ks \\ 0 \\ 0 \end{vmatrix} U_s \quad (3.116)$$

These equations are transform using mode superposition technique gives

$$y_i + \omega_i^2 y_i = - \frac{Q_j^T m r}{Q_j^T m Q_i} \ddot{u}_g + \frac{Q_j^T \bar{b}}{Q_j^T m Q_i} \quad (3.117)$$

Applying the orthogonolity properties (if  $i=j$ ,  $Q_j^T m Q_i=1$ ) produces

$$\ddot{y}_i + \omega_i^2 y_i = - \beta_i \ddot{u}_g + Q_j^T \bar{b} \quad (3.118)$$

The generalized solution for displacement and velocity at the end of time step are

$$\begin{aligned} y_{i2} = & \frac{1}{\omega_{ic}} \left[ \dot{y}_{i1} + \frac{\beta_i R}{\omega_{ic}^2} \right] \sin \omega_{ic} dt + \left[ y_{i1} + \beta_i \frac{\ddot{u}_{g1}}{\omega_{ic}^2} - y_{sii} \right] \cos \omega_{ic} dt \\ & - \beta_i \frac{R dt}{\omega_{ic}^2} - \beta \frac{\ddot{u}_{g1}}{\omega_{ic}^2} + y_{sii} \end{aligned} \quad (3.119)$$

and

$$\begin{aligned} \dot{y}_{i1} = & \left[ \dot{y}_{i1} + \beta_i \frac{R}{\omega_{ic}^2} \right] \cos \omega_{ic} dt + \frac{1}{\omega_{ic}} \left[ -\omega_{ic}^2 y_{i1} - \beta_i \ddot{u}_{g1} + y_{sii} \omega_{ic}^2 \right] \sin \omega_{ic} dt \\ & - \beta_i \frac{R}{\omega_{ic}^2} \end{aligned} \quad (3.120)$$

respectively. Where  $\omega_{ic}$  are the natural frequencies for  $i$ th modes during contact stage and



$$y_{sii} = \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \quad (3.121)$$

Total relative displacement and velocity responses are given by Equation (3.11).

### 3.4.3.1.3 Numerical Example

#### *a) No Contact Stage*

The equations of motion for this stage is defined by Equation (3.111). Substituting values from Table 3.7 into Equation (3.111) gives the natural frequencies

$$\begin{aligned} \omega_1 &= 7.2608 \text{ rad/sec} \\ \omega_2 &= 15.524 \text{ rad/sec} \\ \omega_3 &= 23.050 \text{ rad/sec} \end{aligned}$$

The solutions for displacement and velocity are defined by Equation (3.113) and (3.114) respectively. Total relative displacement and velocity responses are given by Equation (3.11).

#### *a) Contact Stage*

The equations of motion for this stage is defined by Equation (3.115). Substituting values from Table 3.7 into Equation (3.111) gives the natural frequencies during the contact stage

$$\begin{aligned} \omega_{1c} &= 12.653 \text{ rad/sec} \\ \omega_{2c} &= 22.685 \text{ rad/sec} \\ \omega_{3c} &= 223.94 \text{ rad/sec} \end{aligned}$$

The solutions for displacement and velocity are defined by Equation (3.119) and (3.120) respectively. Total relative displacement and velocity responses are given by Equation (3.11).

The contact force is given by

$$\begin{aligned} P &= ks ( u_3 - U_s ) \\ &= 50,000 ( u_3 - U_s ) \text{ kips} \end{aligned}$$

### ***3.4.3.2 MDOF systems, contact between floor levels (unaligned floor)***

In this section, the contact point occurs at the mid-height of the top column ( $a=L/2$ ) as in Figure (3.18). The displacement at the point of contact,  $u_i$  is given by Equation (3.74) in such

$$\begin{aligned} ( 1 - k_{AB} ) &= k_{AB} = \frac{1}{2} \\ \therefore u_i &= \frac{1}{2} u_3 + \frac{1}{2} u_2 \end{aligned} \tag{3.122}$$

#### **3.4.3.2.1 No Contact Stage**

The equations of motion are the same as in the no contact stage for the aligned floor case as described in section (3.4.3.1.1).

#### **3.4.3.2.2 Contact Stage**

The boundary condition for this stage is given by Equation (3.93a) that is when the displacement at the point of contact,  $u_i = U_s$  and the velocity,  $\dot{u}_i \geq 0$ . The equations of motion are

$$\underline{m} u_i + \bar{k} u_i = - m r \ddot{u}_g + \bar{b} \tag{3.123}$$

where  $\bar{k}$  is the stiffness matrix and  $\bar{b}$  is the column vector of static preload forces. Both matrices are derived as in Equation (3.85) of section (3.4.1.2) using the displacement compatibility method. The solutions are similar to Equation (3.119) and (3.120). The contact forces are defined in Equation (3.79).

### 3.4.3.2.3 Numerical Example

#### *a) No Contact*

The equations of motion for this stage is defined by Equation (3.111) as in previous example of  $\mathbf{a}=\mathbf{0}$ . Substituting values from Table 3.7 into Equation (3.111) gives the same natural frequencies as for the aligned floor case (section 3.4.3.1.3). Thus, the solutions for displacement and velocity are also the same as defined in Equation (3.113) and (3.114) respectively. Total relative displacement and velocity responses are given by Equation (3.11).

#### *a) Contact Stage*

The equations of motion for this stage is defined by Equation (3.123). The lateral stiffness of the top column is given by

$$\frac{12 EI}{L^3} = 75$$
$$\therefore EI = 2332800$$

Substituting values of  $k_{AB}$  and  $EI$  into Equation (3.84) produces

$$k1^* = k2^* = k3^* = k4^* = \frac{48 EI}{L^3} = 300 \text{ kips/in.}$$

Substituting values from Table 3.7 into Equation (3.123) gives the natural frequencies during the contact stage of

$$\omega_{1c} = 12.540 \text{ rad/sec}$$
$$\omega_{2c} = 19.365 \text{ rad/sec}$$
$$\omega_{3c} = 25.841 \text{ rad/sec}$$

The solutions for displacement and velocity are defined by Equation (3.119) and (3.120) respectively. The  $y_{sij}$  term is

$$\begin{aligned}
y_{sii} &= \frac{Q_j^T \bar{b}}{\omega_{ic}^2} \\
&= \frac{\alpha_i^T}{\omega_{ic}^2} \begin{vmatrix} 225 & 375 & 0 \\ 375 & 225 & 0 \\ 0 & 0 & 0 \end{vmatrix} \\
&= \frac{\alpha_{1i}^T (225 u_{s3} + 375 u_{s2}) + \alpha_{2i}^T (375 u_{s3} + 225 u_{s2})}{\omega_{ic}^2}
\end{aligned}$$

Total relative displacement and velocity responses are given by Equation (3.11). The contact force is given by Equation (3.79) where

$$\begin{aligned}
P &= 1200 \left[ \frac{1}{2} ud3 + \frac{1}{2} ud2 \right] \text{ kips} \\
\text{where } ud3 &= u_3 - us3 \quad \text{and} \quad ud2 = u_2 - us3
\end{aligned}$$

The FORTRAN programs EQA1.FOR and EQA2.FOR are written in Appendix E for case of  $a=0$  and  $a=L/2$  respectively.

### 3.4.3.3 Results and discussions

Two cases of  $a=0$  and  $a=L/2$  are selected as the numerical examples for the earthquake-induced pounding analysis. The Mexico earthquake, 1985, N90-W component is used. The analysis is run for 30 seconds. The response quantities of interest are the displacement and the contact forces time histories. These results are tabulated in Table 3.8 and compared to the no pounding case.

Figure 3.32a) and 3.32b) show the total displacement ( $u_i + u_g$ ) and the relative displacement ( $u_i$ ) time history, respectively, for no pounding case and these results are then compared to the pounding case as follow:

Figure 3.33a and 3.34a show the total displacements time histories when pounding occurs at the roof level ( $a=0$ ) and ( $a= L/2$ ) respectively. Figure 3.33b and 3.34b represent the contact force time history for  $a=0$  and  $a=L/2$  respectively. When

contact occurs between the floor levels at  $a=L/2$ , the number of contacts is 14 with the maximum value of 2398 kN at  $t = 27.60$  seconds. When contact occurs at the top floor, the number of contacts reduces to only 9 with the maximum value of 10,350 kN at  $t = 19.87$  seconds (Table 3.8). These forces are related directly to the impact spring stiffness or slab stiffness,  $k_s=50,000$  kips/in. for  $a = 0$  and both  $k_s$  and column rigidity,  $k_p$  of 1200 kips/in for  $a = L/2$ . As the fundamental modes dominate the overall response before they are in contact, the number of contact forces at  $a=0$  is expected to be less than at  $a=L/2$  (i.e  $9 < 14$  ).

In general pounding reduces the maximum relative displacement compared to the no pounding case (Figure 3.35a) but adverse response at  $a=L/2$  (Figure 3.35b). Figure 3.36 shows the relative displacement curves for the period of  $t=22.5$ sec until  $t=25$  sec. It shows that the pounding occurs at the top floor level,  $a=0$  produce short contact compared to the pounding at mid height column ( $a=L/2$ ). Again, these behaviour are related to the very rigid slab,  $k_s$  and low column rigidity for the case of  $a=L/2$ .

Figure 3.36 also shows that the displacement at the point of contact,  $u_i$  for  $a=L/2$  and  $u_i=u_{top}$  for  $a=0$  during pounding, do not exceed 0.02 m , which is the building separation. These trends are consistent with those obtained from the snap back analysis as in previous section (3.4.1) which indicates that the snap-back analysis is capable enough of capturing the important response behaviour trends resulting from earthquake-induced pounding. Therefore, in the following chapters, only the snap back analysis is performed to study the dynamic behaviour of a building during pounding.

Value of a	a = 0	a = L/2
Number of contact forces	9	14
Maximum contact forces (kN)	10,349.647	2397.718
Contact forces, P	$P = ks (u_3 - U_s)$ $= 50,000 (u_3 - U_s)$	$P = 1200 (ud3 + ud2)/2$

Table 3.8 : Overall results for earthquake induced pounding analysis.

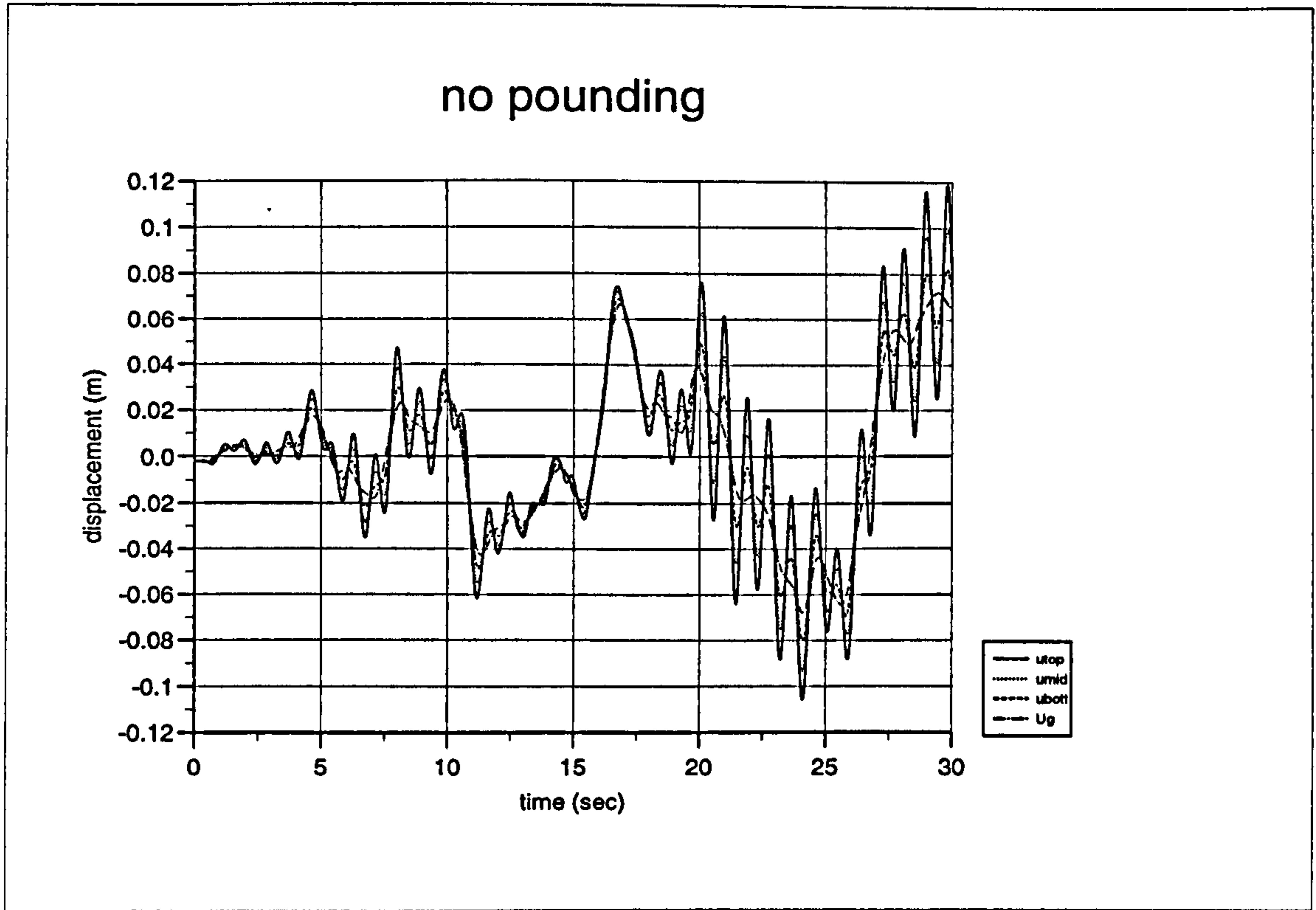


Figure 3.32a) : Total displacement curves for earthquake induced-pounding.

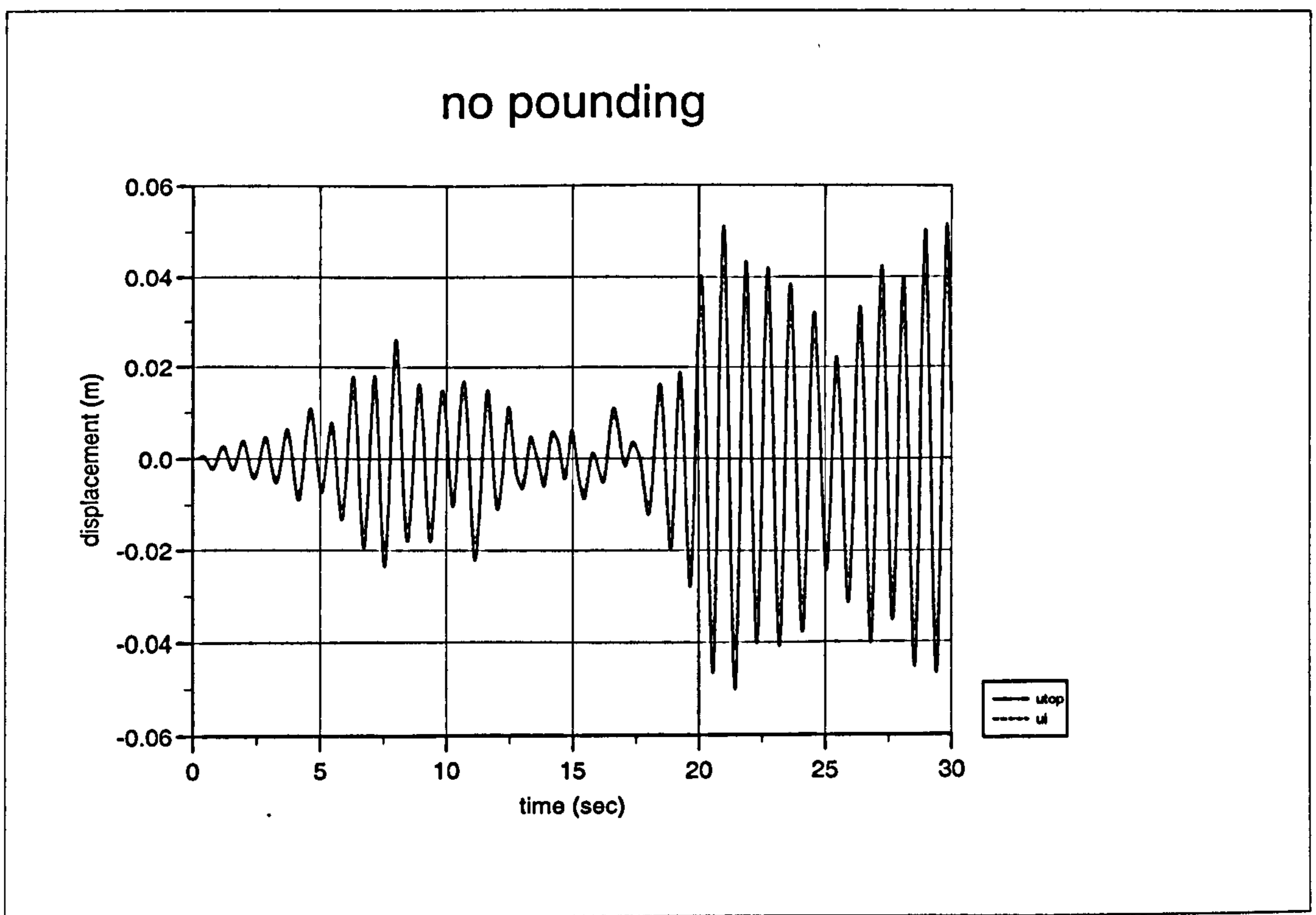


Figure 3.32b) : Relative displacement curves for earthquake induced-pounding.

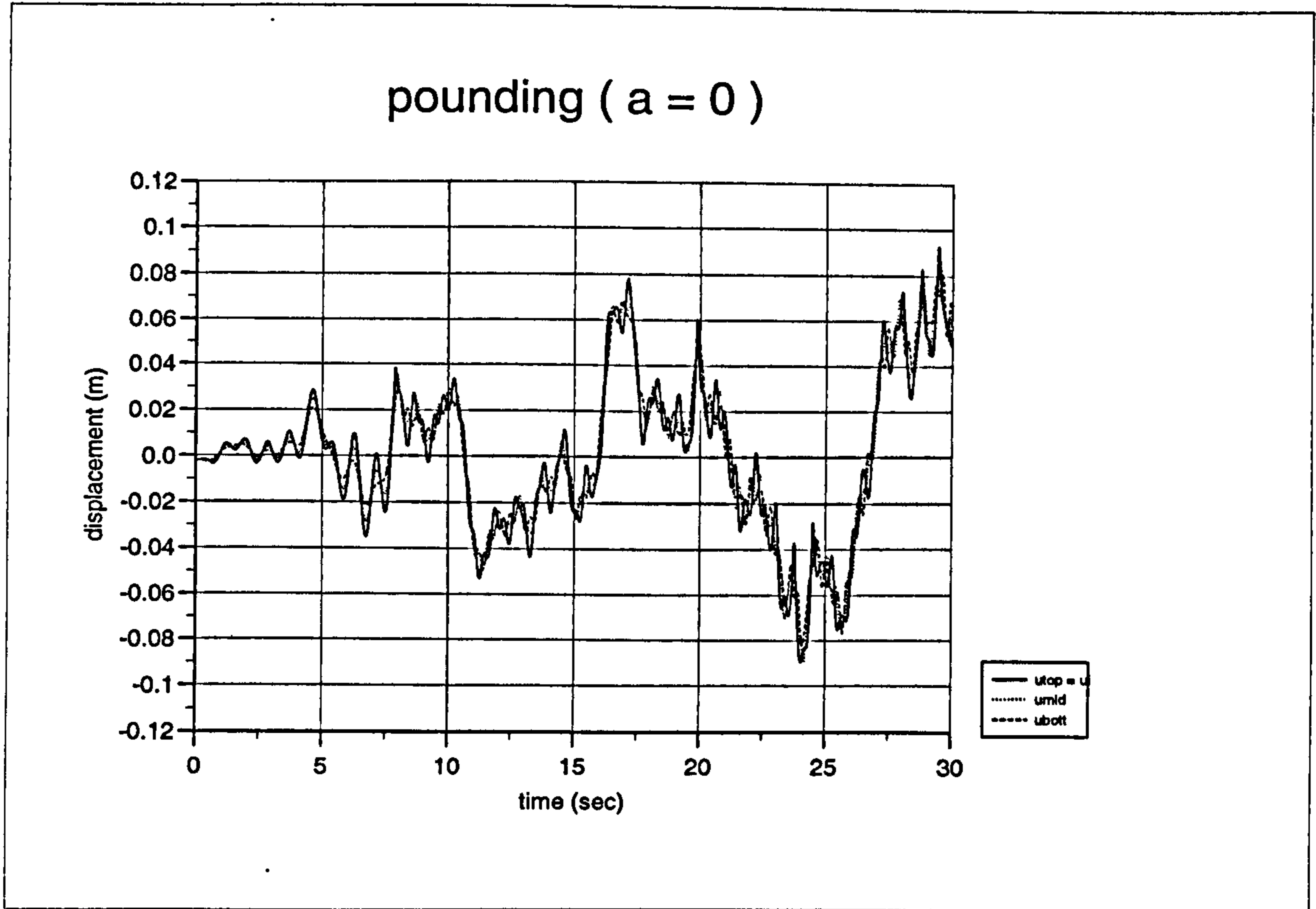


Figure 3.33a) : Total displacement curves for pounding case at  $a = 0$  (FORTRAN)

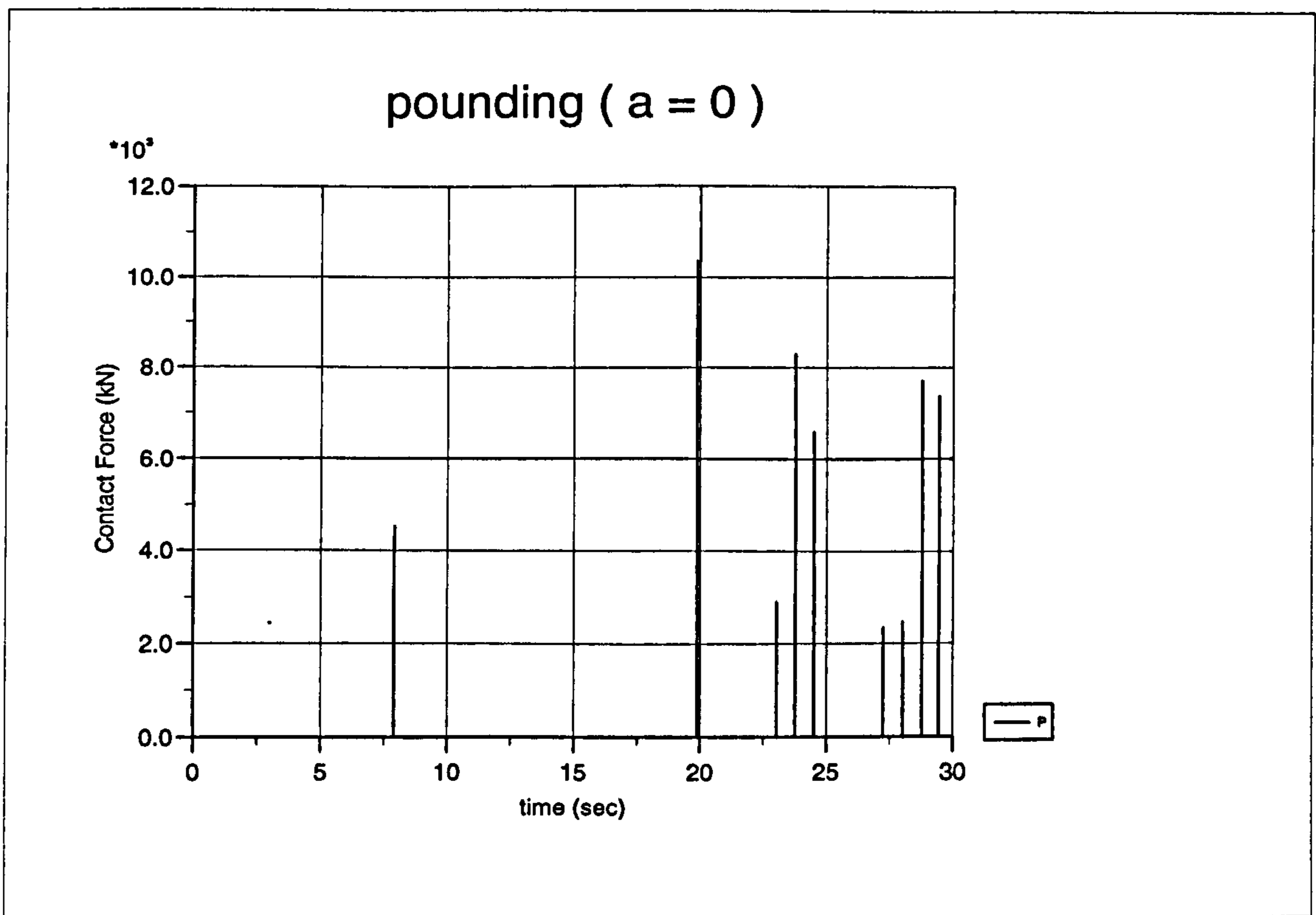


Figure 3.33b) : Contact forces time history for pounding case at  $a = 0$  (FORTRAN)



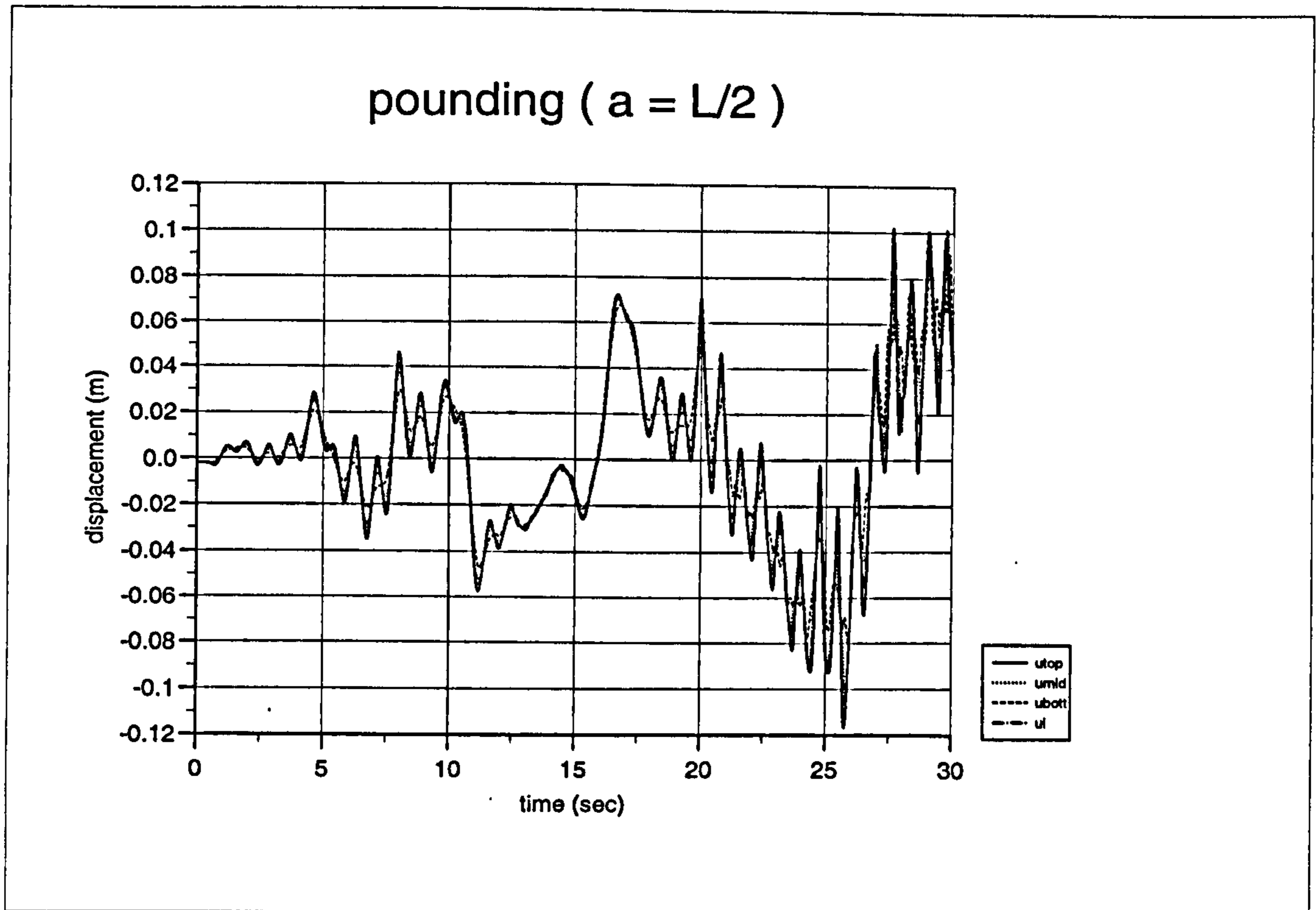


Figure 3.34a) : Total displacement curves for pounding case at  $a = L/2$  (FORTRAN)

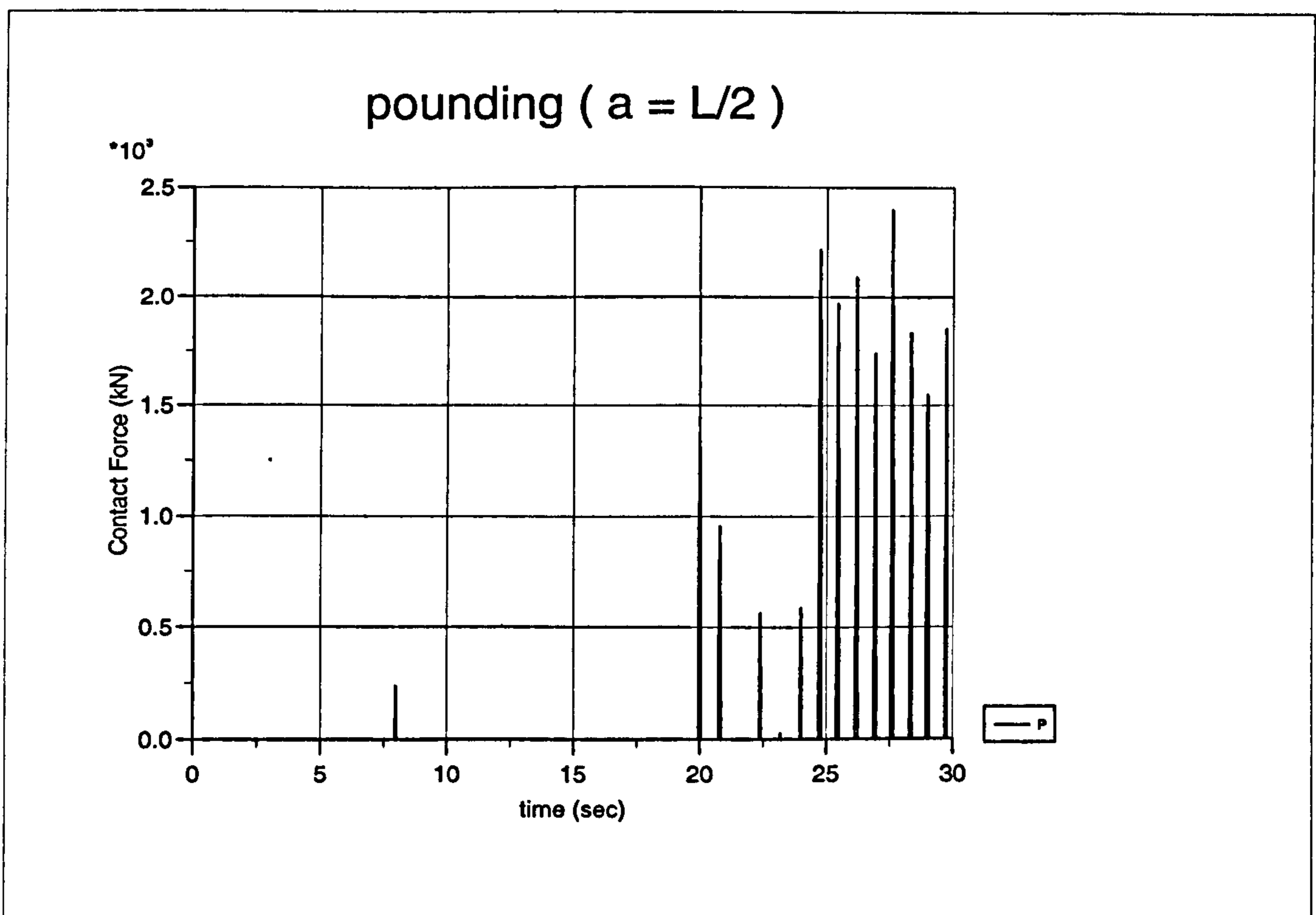


Figure 3.34b) : Contact forces time history for pounding case at  $a = L/2$  (FORTRAN)

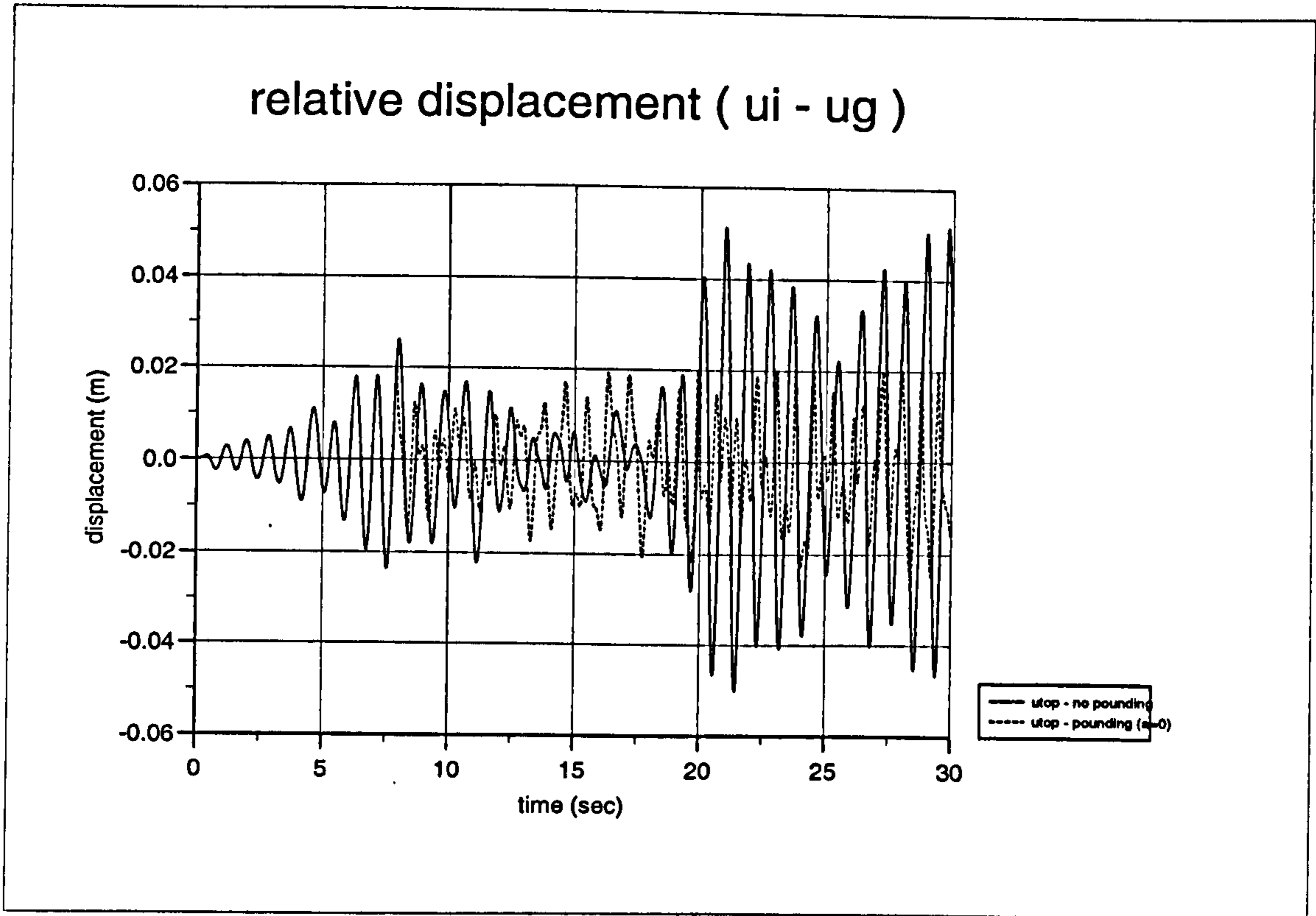


Figure 3.35a) : Relative displacement curves for  $a = 0$  (FORTRAN)

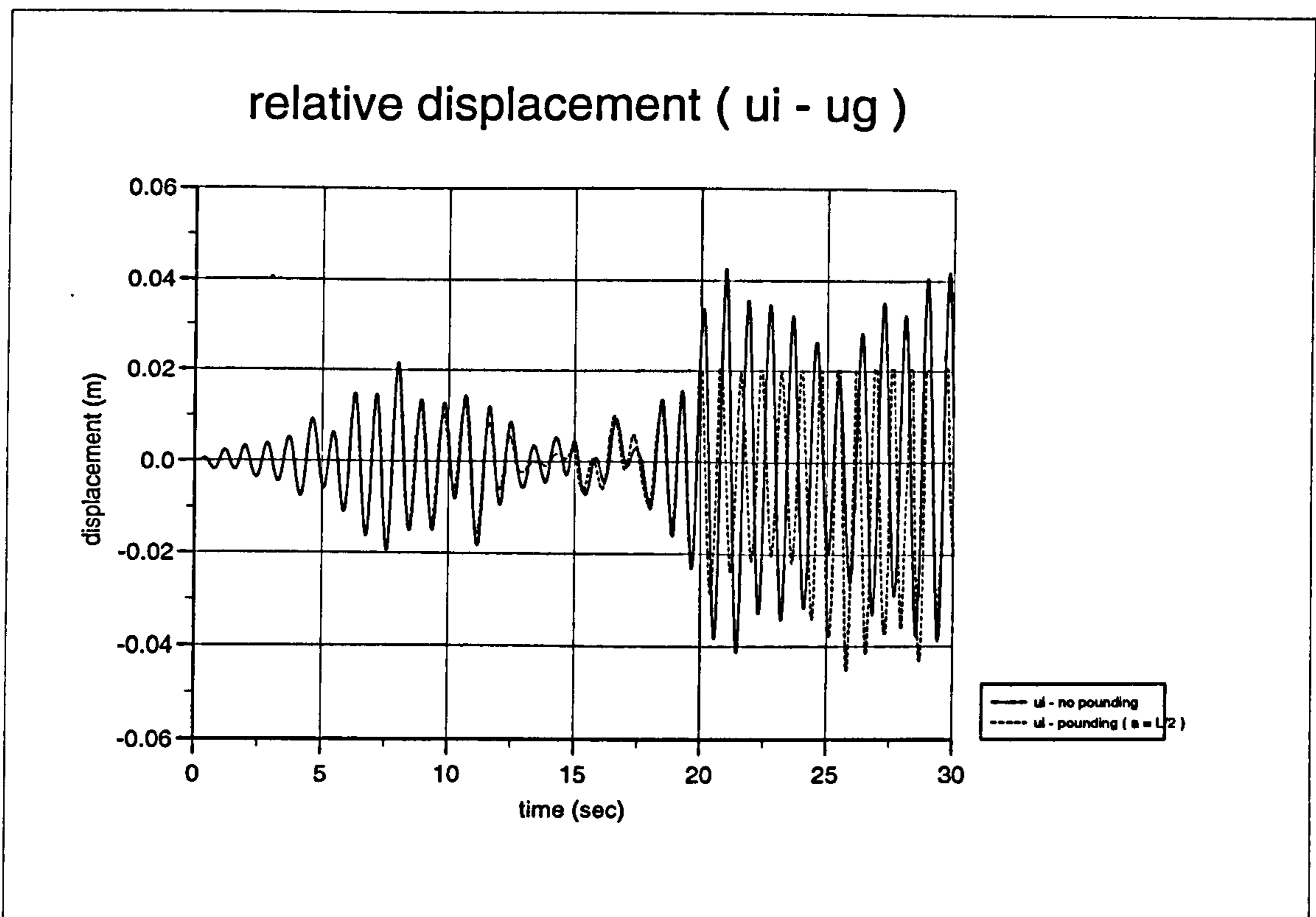


Figure 3.35b) : Relative displacement curves for  $a = L/2$  (FORTRAN)

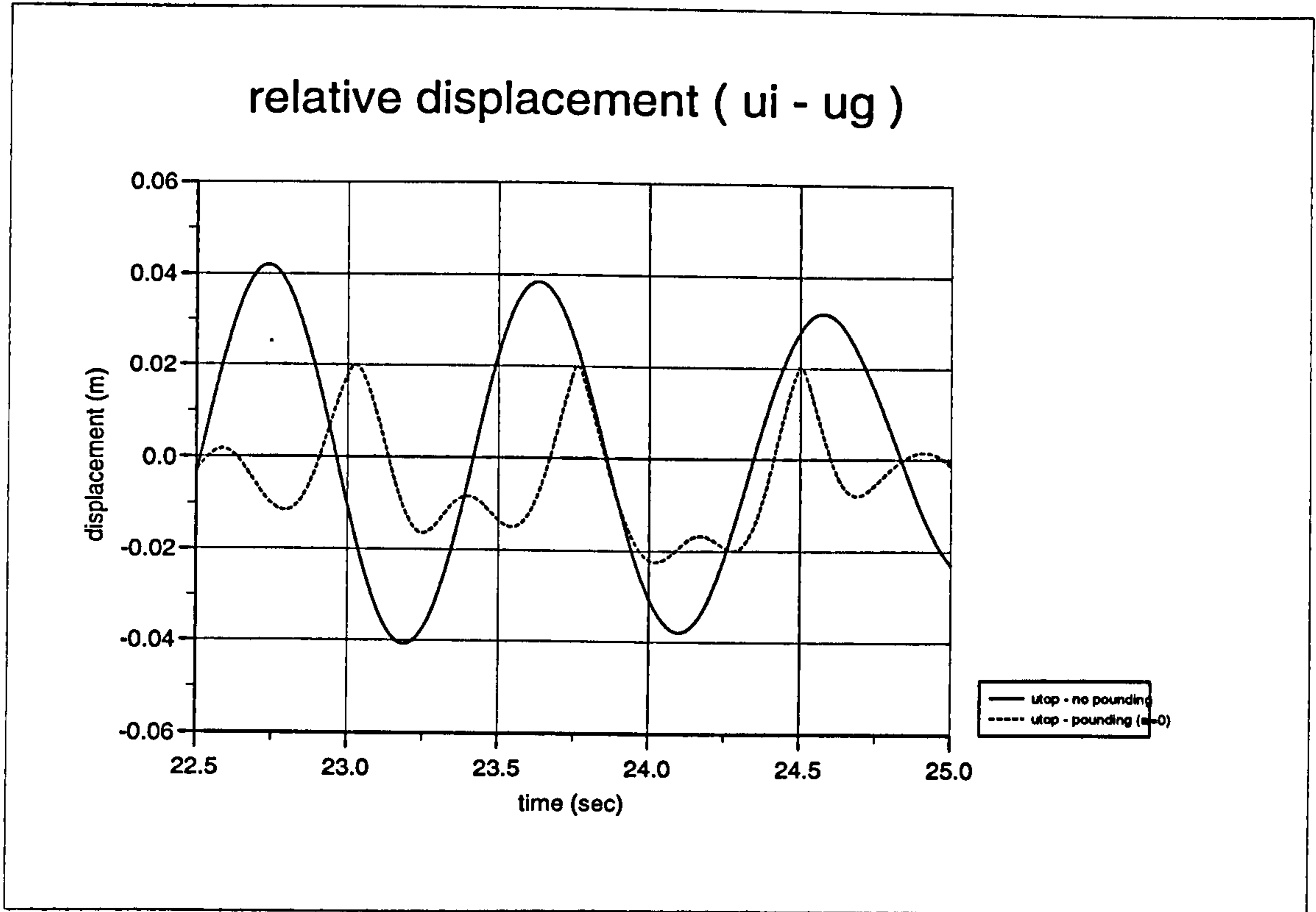


Figure 3.36a) : Relative displacement curves for  $a = 0$  (FORTRAN)

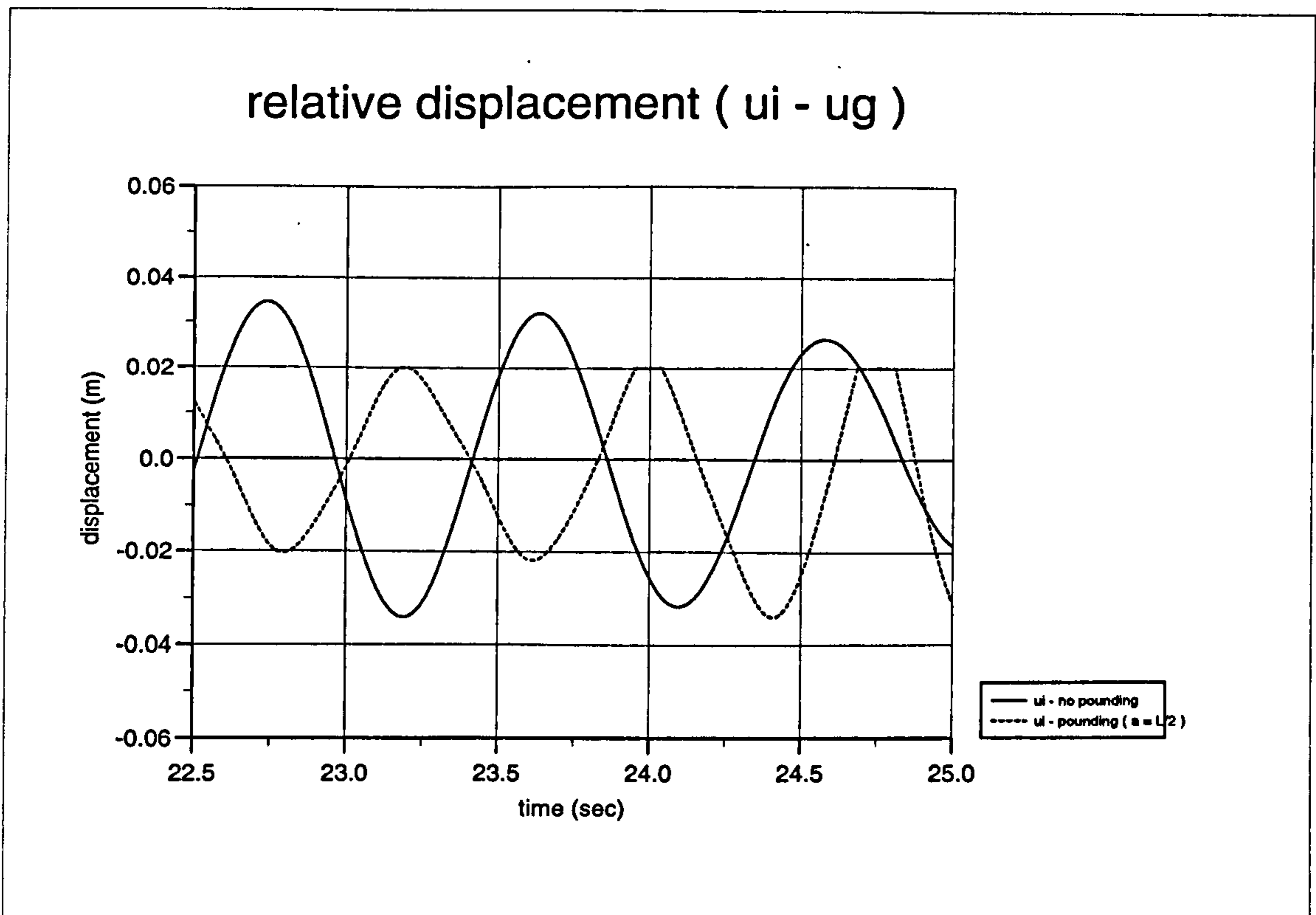


Figure 3.36b) : Relative displacement curves for  $a = L/2$  (FORTRAN)

## **CHAPTER 4**

# **POUNDING OF ADJACENT RIGID BUILDING WITH FLEXIBLE SLAB**

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### **4.1 INTRODUCTION**

The analysis of pounding usually assumes that the slab is rigid and this in turn simplifies the analysis procedure. But this assumption during the seismic analysis of building structures may not be valid in many cases. The validity of this assumption has been discussed recently [34].

In this study, this axial deformations of a flexible (non rigid) slab, are considered and the analysis of pounding under dynamic loading which were covered in previous chapter are thus extended to this section. To illustrate this condition, both for SDOF and MDOF systems are employed in determining its solution.

Like previous analysis of pounding of rigid slab (section 3.4), the analysis of pounding of flexible slab is also considered to be an impact (contact) problem. Two states, i.e. building not in contact and building in contact are considered.

### **4.2 SDOF SYSTEM**

The importance of considering the effect of the flexible slab is illustrated by using the SDOF system model (Figure 4.1) for the basic derivation of the equation of motion. The rigid stopper is to represent the adjacent rigid building and the slab stiffness is termed  $k_s$ . It is convenient to use  $u_1$  and  $u_{f1}$ , to represent the displacement of the left hand side of the slab and the right hand side of the slab respectively. The system is assumed undamped with a free vibration analysis.

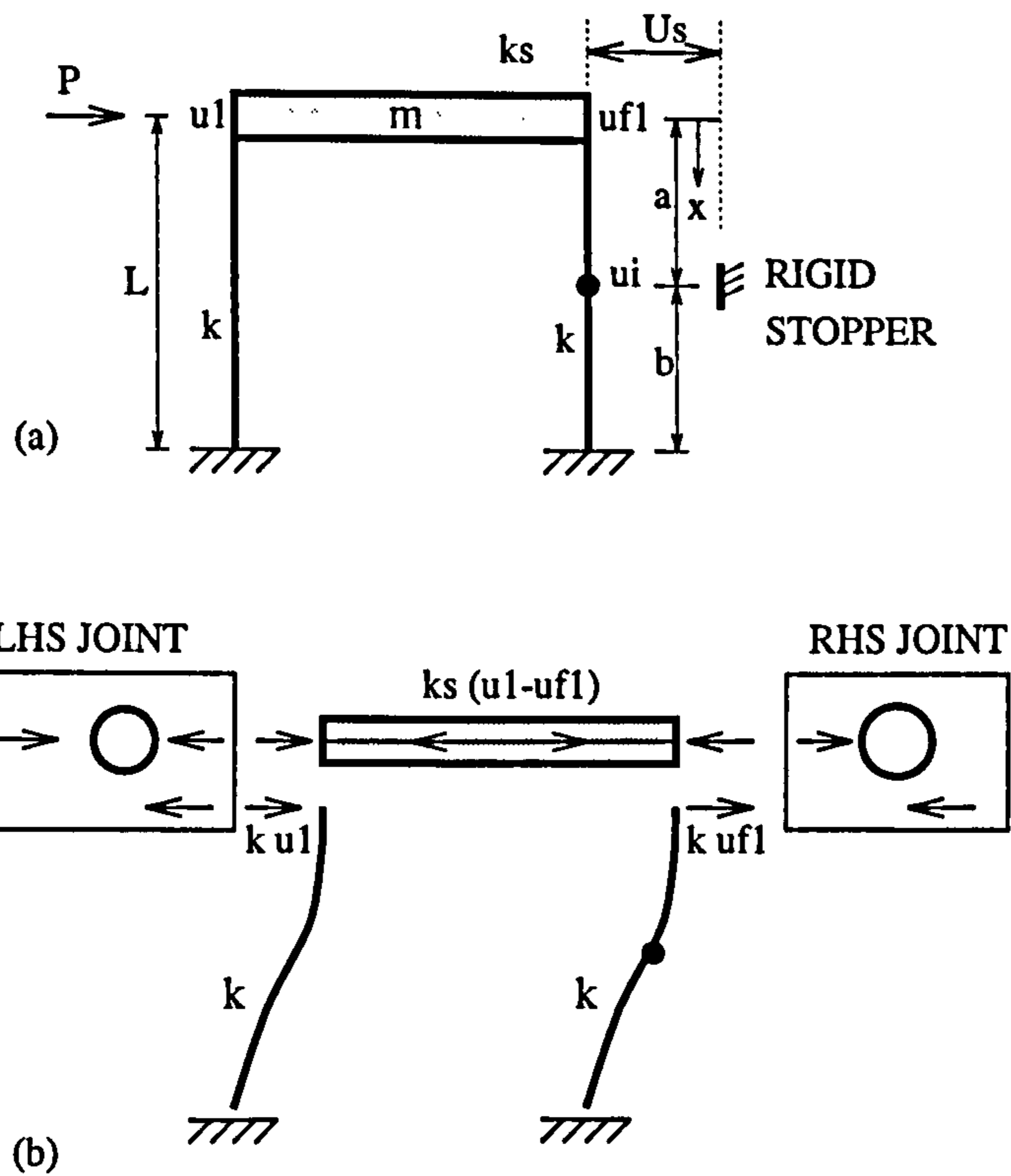


Figure 4.1 : (a) SDOF system and (b) free body diagram

#### 4.2.1 No Contact Stage

The relationship of the displacement for the rigid slab in previous chapter during no contact stage is simply  $u_{f1} = u_1$ . But in the case of flexible slab, a new relationship is derived: Referring to Figure 4.1(b), the free body forces of the SDOF system:

For the top left hand joint (LHS JOINT) :

$$P = k u_1 + k_s (u_1 - u_{f1}) \quad (4.1)$$

From overall equilibrium,

$$P = k u_1 + k u_{f1} \quad (4.2)$$

Combining Equation (4.1) with Equation (4.2)

$$\begin{aligned} ks (u_1 - u_{f1}) &= k u_{f1} \\ u_{f1} &= \frac{ks}{(k+ks)} u_1 \end{aligned} \quad (4.3)$$

To check this Equation (4.3), consider the equilibrium of forces on the top right hand side joint (RHS JOINT):

$$\begin{aligned} ks (u_1 - u_{f1}) - k u_{f1} &= 0 \\ ks u_1 &= (k + ks) u_{f1} \\ \therefore u_{f1} &= \frac{ks}{(k + ks)} u_1 \end{aligned} \quad (4.4)$$

This relationship is always true for both the no contact and the contact stage of the pounding of a flexible slab. The shear forces produced at this stage are shown in Figure 4.2 below:

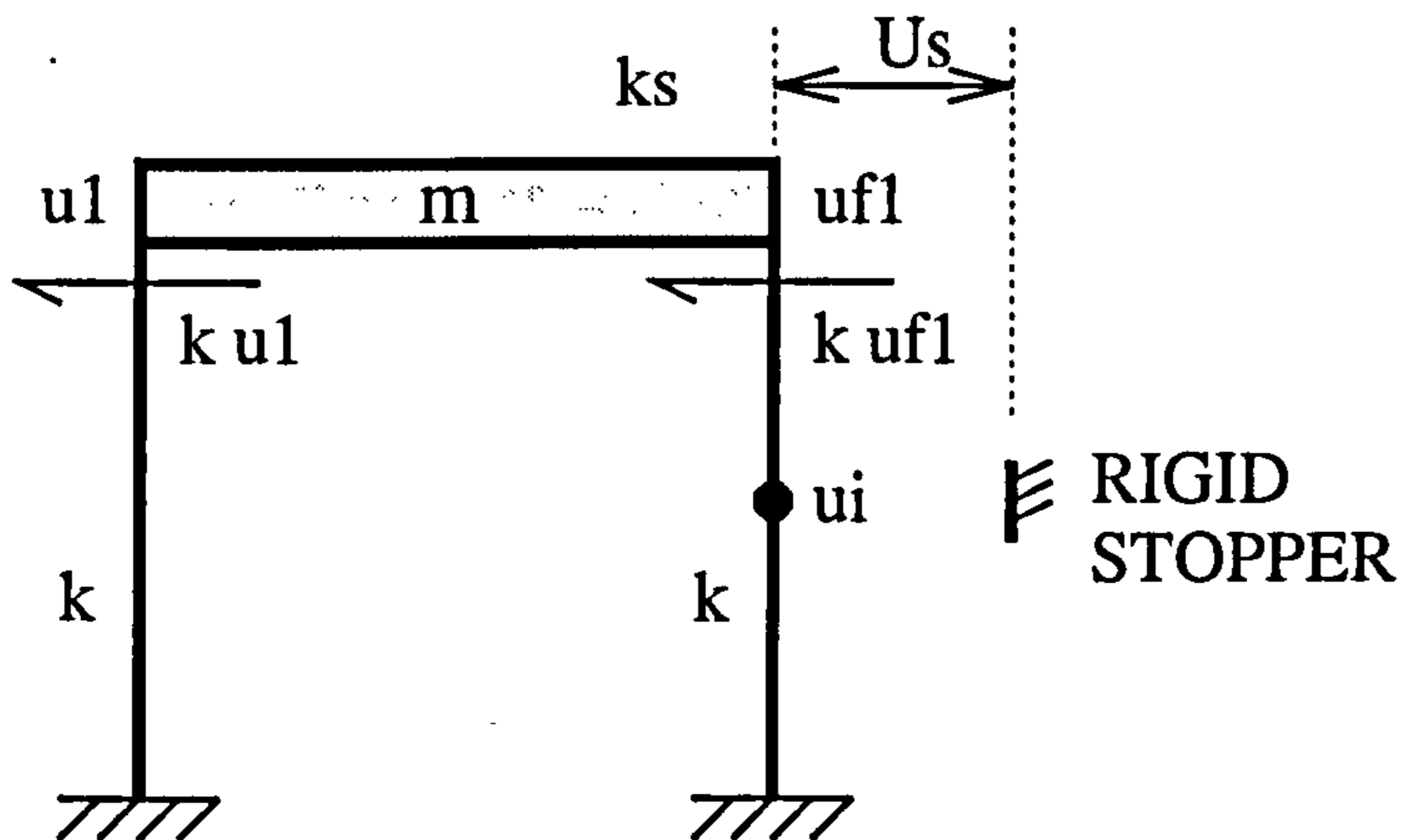


Figure 4.2 : Shear forces diagram for SDOF system (no contact stage)

The equation of motion for the no contact stage is simply:

$$m \ddot{u}_1 = -k u_1 - k u_{f1} \quad (4.5)$$

Rearranged and substituting Equation (4.4) :

$$\begin{aligned}
 m \ddot{u}_1 + k u_1 + k \frac{ks}{(k + ks)} u_1 &= 0 \\
 m \ddot{u}_1 + \left[ k + k \frac{ks}{(k + ks)} \right] u_1 &= 0
 \end{aligned}
 \tag{4.6}$$

In general it is in the form of

$$\begin{aligned}
 m \ddot{u}_1 + k^* u_1 &= 0 \\
 \text{where } k^* &= \left[ k + k \frac{ks}{(k + ks)} \right] \\
 \text{and } \omega &= \sqrt{\frac{k^*}{m}}
 \end{aligned}
 \tag{4.7}$$

The displacement and velocity solutions for above equation are:

$$\begin{aligned}
 u &= A \sin \omega t + B \cos \omega t \\
 \dot{u} &= A \omega \cos \omega t - B \omega \sin \omega t
 \end{aligned}
 \tag{4.8}$$

These equations can also be derived from Equation (3.65) and (3.66) of chapter 3 for the undamped system ( $\xi = 0$ ) and free vibration by a snap back analysis ( $\ddot{u}_g=0$ ). The constant A and B can be calculated if the initial boundary condition are known. The displacement at the point of contact is given by:

$$\begin{aligned}
 u_{f1} &= \frac{ks}{(k + ks)} u_1 \\
 u_i &= (1 - k_{AB}) u_{f1}
 \end{aligned}
 \tag{4.9}$$

## 4.2.2 Contact Stage

Before deriving the equation of motion for the contact stage, the intermediate displacement at the point of contact,  $u_i$  must satisfy the condition:

$$u_i \geq U_s \quad (4.10)$$

In the rigid slab case previously discussed in chapter 3, the only terms that vary throughout the contact stage are the  $u_{di}$  terms (flexible displacements). For the flexible slab, as well as the  $u_{di}$  terms, there are also,  $u_{ki}$  terms which are the axial deformations of the slab. (Figure 4.3). Consider the behaviour of the right hand side column, which hits the rigid stopper. There are three main stages occurred during the contact stage. (Figure 4.3 (c)). First, the column displacement without the rigid stopper, termed  $u_{di}$  and secondly, the column displacement,  $u_p$  due to point (contact) load  $P$  and the associated end reaction of  $R_p$ , and finally, the axial deformation of slab,  $u_{ki}$  due to force  $R_k$ .

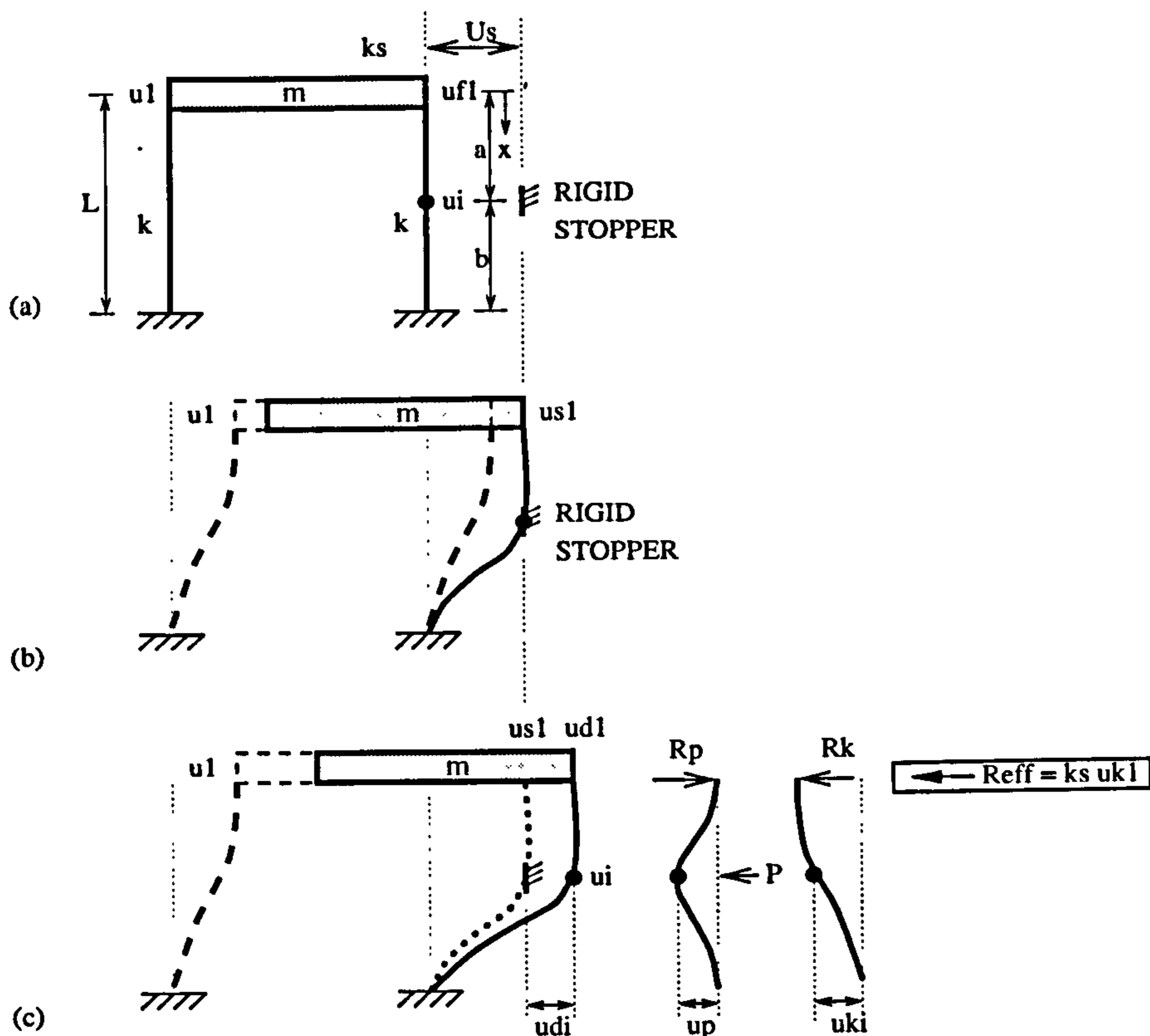


Figure 4.3 : Displacement Compatibility Stages (a) no contact (b) just contact (c) into contact stage of the right hand side column behaviour.



All these terms can be expressed as :

$$\begin{aligned} u_{di} &= u_{d1} (1 - k_{AB}) \\ u_p &= \frac{P b^3 a^3}{3 L^3 E I} \\ u_{ki} &= u_{k1} (1 - k_{AB}) \end{aligned} \quad (4.11)$$

where

$$\begin{aligned} k_{AB} &= 3 \left(\frac{a}{L}\right)^2 - 2 \left(\frac{a}{L}\right)^3 \\ \text{when } a &= \frac{L}{2} \quad \text{thus } k_{AB} = \frac{1}{2} \end{aligned} \quad (4.12)$$

and like rigid slab,

$$u_{f1} = u_{s1} + u_{d1} \quad (4.13)$$

The derivation of these terms ( $u_{di}$ ,  $u_p$  and  $k_{AB}$ ) are same as in Appendix B.

The contact forces,  $P$  acting at the contact point is derived from the displacement compatibility where:

$$u_{di} = u_p + u_{ki} \quad (4.14)$$

Substituting Equation (4.11) into the above relationship:

$$u_{d1} (1 - k_{AB}) = \frac{P a^3 b^3}{3 L^3 E I} + u_{k1} (1 - k_{AB}) \quad (4.15)$$

Because the  $u_{k1}$  term is unknown, thus using the slab reactions

$$R_{eff} = (R_p - R_k) = k_s u_{k1} \quad (4.16)$$

where  $R_{eff}$  is the effective forces of the slab reactions.

$$R_p = P (1 - k_{AB}) \quad R_k = k (u_{k1} - 0) = k u_{k1} \quad (4.17)$$

The derivation of these forces can be referred to Equation (3.80) in Chapter 3.

Substituting into Equation (4.16) gives:

$$\begin{aligned}
 P(1 - k_{AB}) - k u k l &= k s u k l \\
 \therefore u k l &= \frac{(1 - k_{AB})}{(k + k s)} P
 \end{aligned}
 \tag{4.18}$$

For the value of the effective force,  $R_{eff}$ , the Equation (4.16) can be used to explain the relationship between the rigid and flexible case of a SDOF system. Substituting the expression of  $u k l$  from Equation (4.18) into Equation (4.16) gives

$$\begin{aligned}
 R_{eff} &= k s u k l \\
 &= k s \frac{(1 - k_{AB})}{(k + k s)} P \\
 &= \frac{(1 - k_{AB})}{\left(\frac{k}{k s} + 1\right)} P
 \end{aligned}
 \tag{4.19}$$

If  $k s$  is very high as in rigid case,  $k/k s \approx 0$  and therefore,  $R_{eff} = (1 - k_{AB}) P$ . This is equivalent to the force  $F_3$  of Equation (3.80) of Chapter 3 for a rigid slab case.

Substituting Equation (4.18) into Equation (4.15):

$$\begin{aligned}
 u d l (1 - k_{AB}) &= P \left[ \frac{\alpha^3 b^3}{3L^3 EI} + \frac{(1 - k_{AB})^2}{(k + k s)} \right] \\
 P &= \frac{(1 - k_{AB})}{\left[ \frac{\alpha^3 b^3}{3L^3 EI} + \frac{(1 - k_{AB})^2}{(k + k s)} \right]} u d l
 \end{aligned}
 \tag{4.20}$$

Again this equation can represent the rigid case when  $k s$  is very high, the term

$$\frac{(1 - k_{AB})^2}{(k + k s)} \approx 0$$

For simplicity, let the coefficient  $k p$  equals to :

$$k p = \frac{(1 - k_{AB})}{\left[ \frac{\alpha^3 b^3}{3L^3 EI} + \frac{(1 - k_{AB})^2}{(k + k s)} \right]}
 \tag{4.21}$$

so that Equation (4.20) becomes:

$$P = k p u d l
 \tag{4.22}$$

Now the slab reactions,  $R_p$  and  $R_k$  are calculated in terms of  $u d l$ :

$$\begin{aligned}
R_p &= P (1 - k_{AB}) \\
&= k_p (1 - k_{AB}) u_{d1} \\
&= k_{f1}^* u_{d1} \\
R_k &= k u_{k1} \\
&= k k_p \frac{(1 - k_{AB})}{(k + k_s)} u_{d1} \\
&= k_{f2}^* u_{d1}
\end{aligned} \tag{4.23}$$

The shear forces produced, are illustrated in Figure 4.4 below:

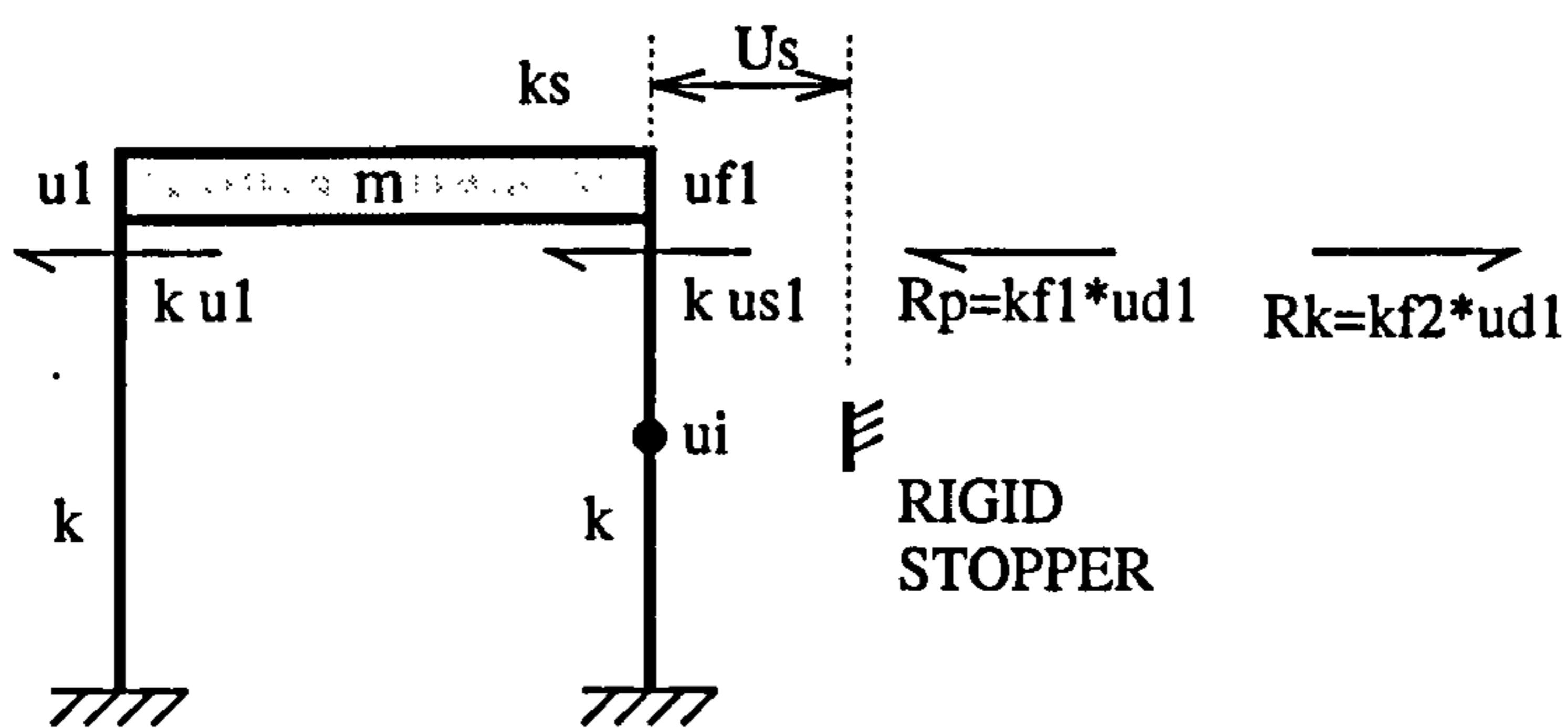


Figure 4.4 : Shear forces diagram of SDOF system (contact stage).

The equation of motion for the contact stage can be written as:

$$m \ddot{u}_1 = -k u_1 - k u_{s1} - k_{f1}^* u_{d1} + k_{f2}^* u_{d1} \tag{4.24}$$

But from Equation (4.13) and (4.4), substituting those values:

$$\begin{aligned}
m \ddot{u}_1 + k u_1 + (k_{f1}^* - k_{f2}^*) u_{d1} &= -k u_{s1} \\
m \ddot{u}_1 + k u_1 + (k_{f1}^* - k_{f2}^*) (u_{f1} - u_{s1}) &= -k u_{s1} \\
m \ddot{u}_1 + k u_1 + (k_{f1}^* - k_{f2}^*) (u_{f1}) &= (-k + k_{f1}^* - k_{f2}^*) u_{s1} \\
m \ddot{u}_1 + k u_1 + \left[ \frac{(k_{f1}^* - k_{f2}^*) k_s}{(k + k_s)} \right] u_1 &= (-k + k_{f1}^* - k_{f2}^*) u_{s1} \\
m \ddot{u}_1 + \left[ k + \frac{(k_{f1}^* - k_{f2}^*) k_s}{(k + k_s)} \right] u_1 &= (-k + k_{f1}^* - k_{f2}^*) u_{s1}
\end{aligned} \tag{4.25}$$

Or in general form of :

$$\begin{aligned}
m \ddot{u}_1 + k_c^* u_1 &= b_s \\
\ddot{u}_1 + \frac{k_c^*}{m} u_1 &= \frac{b_s}{m} \\
\ddot{u}_1 + \omega_c^2 u_1 &= \frac{b_s}{m}
\end{aligned} \tag{4.26}$$

And

$$\begin{aligned}
k_c^* &= \left[ k + \frac{(kf1^* - kf2^*)ks}{(k + ks)} \right] \\
\omega_c &= \sqrt{\frac{k_c^*}{m}} \\
b_s &= [-k + (kf1^* - kf2^*)] us_1
\end{aligned} \tag{4.27}$$

where  $\omega_c^2$  is the contact stage vibration frequency and  $b_s$  is the static preload.

The solution for displacement and velocity are given by:

$$\begin{aligned}
u_1 &= \frac{\dot{u}_t}{\omega_c} \sin \omega_c dt + (u_t - us_i) \cos \omega_c dt + us_i \\
\dot{u}_1 &= \dot{u}_t \cos \omega_c dt - \omega_c (u_t - us_i) \sin \omega_c dt
\end{aligned} \tag{4.28}$$

where:

$$us_i = \frac{b_s}{\omega_c^2 m} \tag{4.29}$$

The displacement at the contact point is

$$\begin{aligned}
u_i &= (1 - k_{AB}) u_{f1} \\
new\ u_{f1} &= u_{f1} - uk_1
\end{aligned} \tag{4.30}$$

where  $k_{AB}$ ,  $u_{f1}$  and  $uk_1$  are expressed in Equation (4.12), (4.4) and (4.18) respectively.

The contact forces,  $P$  is calculated using the Equation (4.22) where,

$$\begin{aligned}
P &= k_p u_{d1} \\
&= k_p (u_{f1} - us_1)
\end{aligned} \tag{4.31}$$

### 4.2.3 Change of state rules

The contact stage starts when the displacement and velocity are such

$$u_i = U_s \quad \text{but} \quad \dot{u}_i \geq 0 \quad (4.32a)$$

And the contact stage is finished when the displacement and velocity are such

$$u_i = U_s \quad \text{but} \quad \dot{u}_i \leq 0 \quad (4.32b)$$

### 4.2.4 Numerical Examples of SDOF System

Consider the SDOF system as described in Chapter 3 and shown in Figure 4.1 which has the building mass,  $m$  of 39 kips-sec<sup>2</sup>/in. ( $6.83 \cdot 10^3$  kN-sec<sup>2</sup>/m), a building stiffness,  $k$  of 600 kips/in. ( $105.12 \cdot 10^3$  kN/m) and the slab stiffness,  $k_s$  of 10,000 kips/in. ( $1.752 \cdot 10^6$  kN/m). The snap back analysis is performed with the initial displacement of -0.81 in. (-0.020574 m). The rigid stopper is at a distance of  $a=L/2$  from the top floor and 0.008m away from the building ( $U_s$ ). The system is assumed to be undamped.

#### 4.2.4.1 No Contact stage

Using Equation (4.12), the value of  $k_{AB}$  is equal to 1/2 and therefore, the intermediate displacement for the point of contact is given by:

$$\begin{aligned} u_i &= (1 - k_{AB}) u_{f1} \\ \therefore u_i &= \frac{u_{f1}}{2} \end{aligned}$$

The final equation of motion as described in Equation (4.7) is :

$$\begin{aligned} m \ddot{u}_1 + k^* u_1 &= 0 \\ \text{where } k^* &= [600 + 600(0.943397186)] \\ &= 1166.038312 \text{ kips/in.} \\ \text{and } \omega &= 5.46794 \text{ rad/sec} \end{aligned}$$

The solution for Equation (4.8) is:

$$u1 = -0.81 \cos \omega dt$$

$$\dot{u}1 = 0.81 \omega \sin \omega dt$$

The right hand side displacement are given by Equation (4.4) where:

$$uf1 = \frac{k_s}{(k + k_s)} u1$$

$$= 0.943397186 u1$$

#### 4.2.4.2 Contact stage

When the state of change rule is satisfied, (section 4.2.3), the response is in the contact stage. The term  $k_p$  from Equation (4.21) is constant throughout this contact stage:

$$k_p = 3913.858409$$

$$\text{where } EI = 18662400 \text{ kips.in}^2$$

Also the axial deformation of the slab,  $u_{ki}$  in Equation (4.18) becomes :

$$u_{k1} = \frac{(1 - k_{AB})}{(10600)} k_p u_{d1}$$

$$= 0.184612831 u_{d1}$$

The  $k_{f1}^*$  and  $k_{f2}^*$  values are given by Equation (4.23) where:

$$k_{f1}^* = k_p(1 - k_{AB}) = 1956.929204$$

$$k_{f2}^* = k k_p \frac{(1 - k_{AB})}{(k + k_s)} = 110.7676989$$

so that  $R_p = k_{f1}^* u_{d1} = 1956.929204 u_{d1}$

$R_k = k_{f2}^* u_{d1} = 110.7676989 u_{d1}$

Thus the equation of motion (Equation 4.26) becomes:

$$m \ddot{u}_1 + k_c^* u_1 = b_s$$

$$\begin{aligned} \text{where } k_c^* &= [600 + (kf1^* - kf2^*) 0.943397186] \\ &= 2341.66357 \text{ kips/in.} \\ b_s &= [-600 + (kf1^* - kf2^*)] us1 \\ &= 1246.161505 \text{ us1} \end{aligned}$$

The natural frequency,  $\omega_c$  during contact stage is

$$\begin{aligned} \omega_c &= \sqrt{\frac{k_c^*}{39}} \\ &= 7.748719612 \text{ rad/sec.} \end{aligned}$$

The solution is described in Equation (4.28) and the right hand side displacement of the column is

$$\begin{aligned} uf1 &= 0.43397186 u_1 \\ \therefore \text{ new } uf1 &= uf1 - uk1 \end{aligned}$$

The maximum contact force is calculated using Equation (4.31) where

$$P_{\max} = kp (uf1_{\max} - us1)$$

Other examples for unaligned floor cases of  $a=L/3$  and  $a=L/4$  and are summarised in Table 4.1 as well as the aligned floor case where  $a=0$ . FORTRAN programs called SFLEX1.FOR, SFLEX2.FOR, SFLEX3.FOR and SFLEX4.FOR are written in Appendix F, for case  $a=0$ ,  $L/4$ ,  $L/3$  and  $L/2$  respectively.

These results are then compared to those obtained with DYNA3D.

#### 4.2.5 Results and discussions

A free vibration snap-back analysis was performed with the initial displacement at the top floor is equal to -0.81 inch (0.020574m) for duration of 1 sec. The displacement time history curves are shown in Figure 4.6 to 4.9 for cases  $a=0$ ,  $L/4$ ,  $L/3$  and  $L/2$ , respectively. The corresponding contact time and forces are tabulated in Table 4.2.

Figure 4.5a shows the no pounding case for the value  $a=L/2$ , so that a comparison can be made between flexible and rigid slabs. It clearly shown that, the floor flexibility produces a longer vibration period ( $T_{flex}$ ) than a rigid case ( $T_{rigid}$ ).

To clarify this, only the first contact is shown in Figure 4.5b, for the pounding case of  $a= L/2$ . It shows that the top displacement on the right hand side column ( $u_{f1}$ ) is always factored by  $(ks / (k+ks))$  of the left hand side column as derived in Equation (4.4) throughout both the no contact and the contact stages. The same factor is applied to other case of  $a$  ( $a=0$ ,  $L/4$  and  $L/3$ ) as shown in Figure 4.6 to 4.8.

It also shows that the contact time starts later in the flexible case than in the rigid case (Table 4.3). As expected, the contact forces for the flexible case are lower than the rigid case due to the different in slab stiffness.

All the displacement curves at the contact point,  $u_i$  (Figure 4.6 - 4.10) are as expected not exceeding 0.008m which is equivalent to the building separation. Figure 4.10a shows a displacement curves at the contact point ( $u_i$ ) for a various value of  $a$  along the height of the column. It is found that, the value of the maximum contact forces,  $P$  decreases as the value of  $a$  increases from 0 to  $L/2$ . (Table 4.2). The starting time of the contact stage also increases as the value of  $a$  increases from 0 to  $L/2$  according to the dominant first mode shapes.

All results are verified using DYNA3D (finite element package) and are shown in Figures (4.6b - 4.10b). It shows that a good agreement is achieved by this study to those results obtained by DYNA3D.



PARAMETERS (UNITS)	ALIGNED FLOOR	UNALIGNED FLOOR		
	CASE a=0	CASE a=L/4	CASE a=L/3	CASE a=L/2
$k_{AB}$ , $(1 - k_{AB})$ (dimensionless)	0 , 1	5/32 , 27/32	7/27 , 20/27	1/2 , 1/2
$k_p$ (kips/in)	10600.0	7594.107706	5929.599682	3913.858409
$uk_1$ (in)	1.0 ud1	0.604473557 ud1	0.414360525 ud1	0.184612831 ud1
$k_{f1}^*$ (kips/in)	10600.0	6407.528376	4392.296061	1956.929204
$k_{f2}^*$ (kips/in)	600.0	362.6841342	248.6163151	110.7676989
$\omega_c$ (rad/sec)	16.04012714	12.71249037	10.7526214	7.748719612
$b_s$ static preload (kips)	9400.0 us1	5444.844242 us1	3543.679746 us1	1246.161505 us1

Table 4.1 : Summary of parameters during contact stage of SDOF system for case a= 0, L/4, L/3, and L/2.

DISTANCE "a"		1st CONTACT	
		FORTRAN	DYNA3D
0	Contact time (sec)	0.36500-0.54000 (0.17500)	0.36700-0.53900 (0.17200)
	Contact forces (kN)	9,599.598	9,345.770
L/4	Contact time (sec)	0.38100-0.59120 (0.21020)	0.38400-0.58500 (0.20100)
	Contact forces (kN)	7,616.785	7,376.370
L/3	Contact time (sec)	0.39600-0.62741 (0.23141)	0.39600-0.61897 (0.22297)
	Contact forces (kN)	6,061.672	5,807.470
L/2	Contact time (sec)	0.46500-0.66400 (0.19900)	0.46600-0.65797 (0.19197)
	Contact forces (kN)	2,142.123	2,070.510

Table 4.2 : Overall results for the contact stages of flexible slab of SDOF system

DISTANCE "a"		FLEXIBLE (1st contact)		RIGID (1st contact)	
		FORTRAN	DYNA3D	FORTRAN	DYNA3D
L/2	Contact time (sec)	0.46500-0.66400 (0.19900)	0.46600-0.65797 (0.19197)	0.44400-0.65081 (0.20681)	0.44958-0.64860 (0.19902)
	Contact forces (kN)	2,142.123	2,070.510	3,343.157	2,932.260

Table 4.3 : Results for the contact stage of flexible and rigid slab of SDOF system

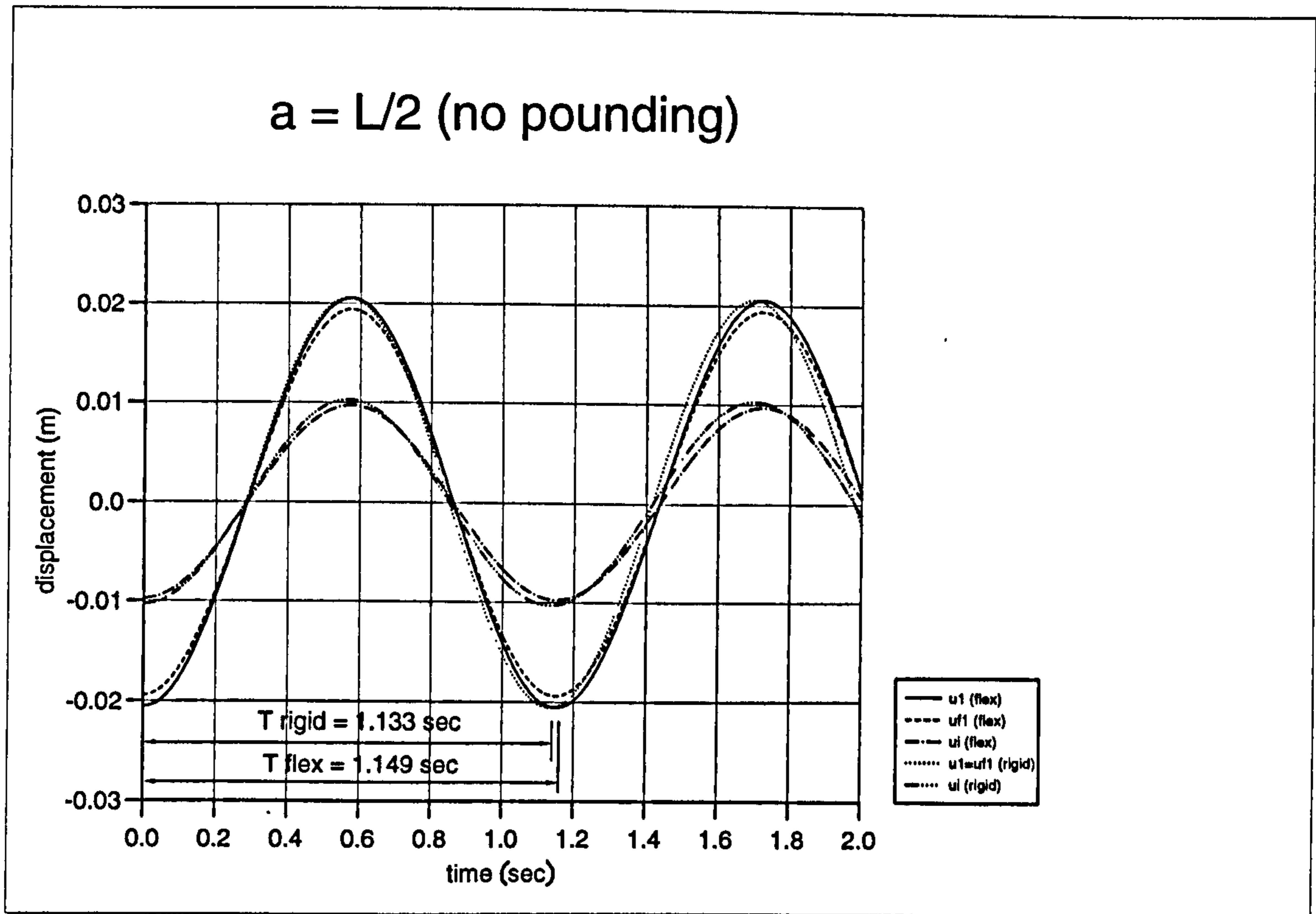


Figure 4.5a) : Displacement curves for free vibration ( no pounding) (FORTRAN)

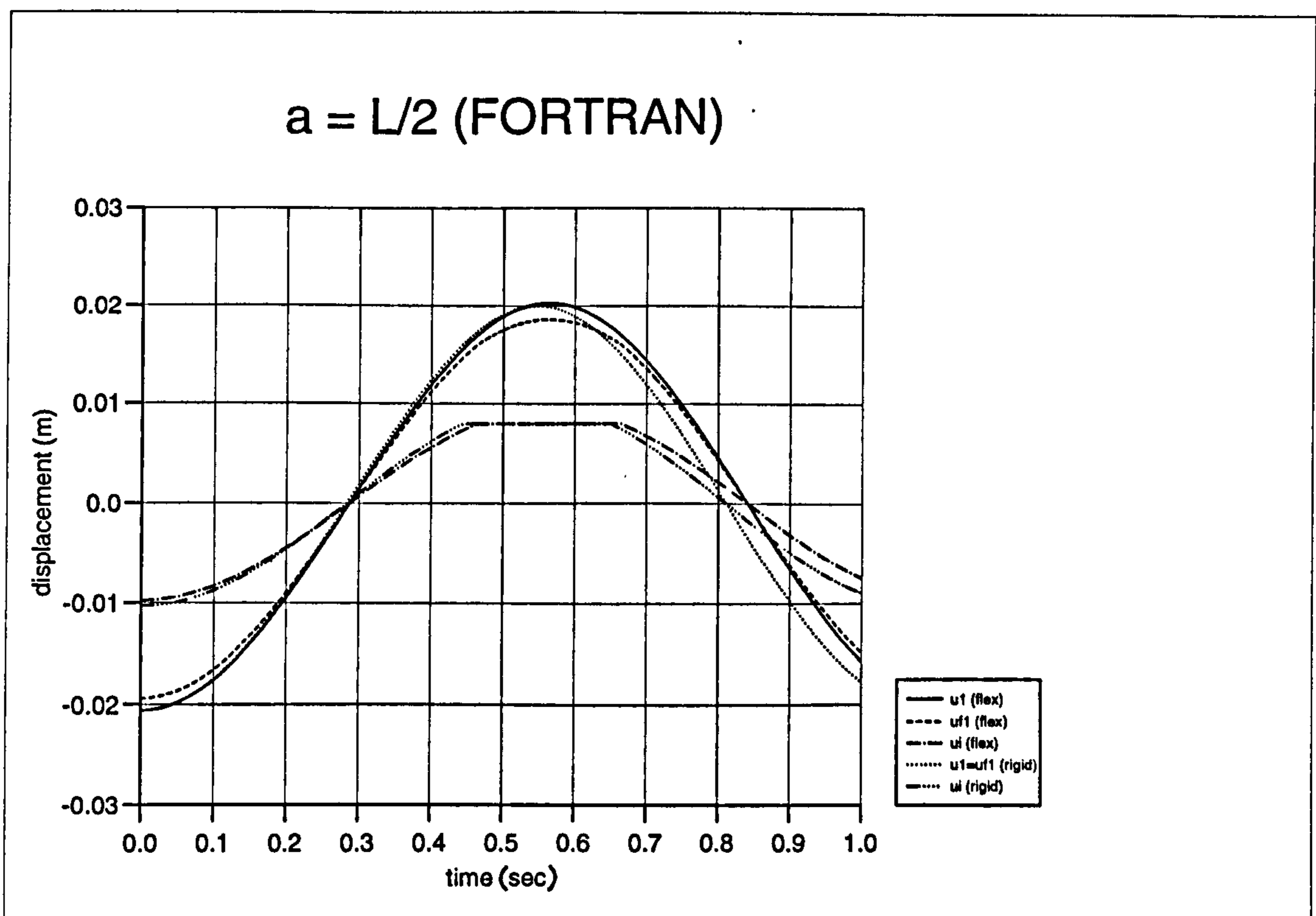


Figure 4.5b) : Displacement curves for pounding case of flexible and rigid slab.

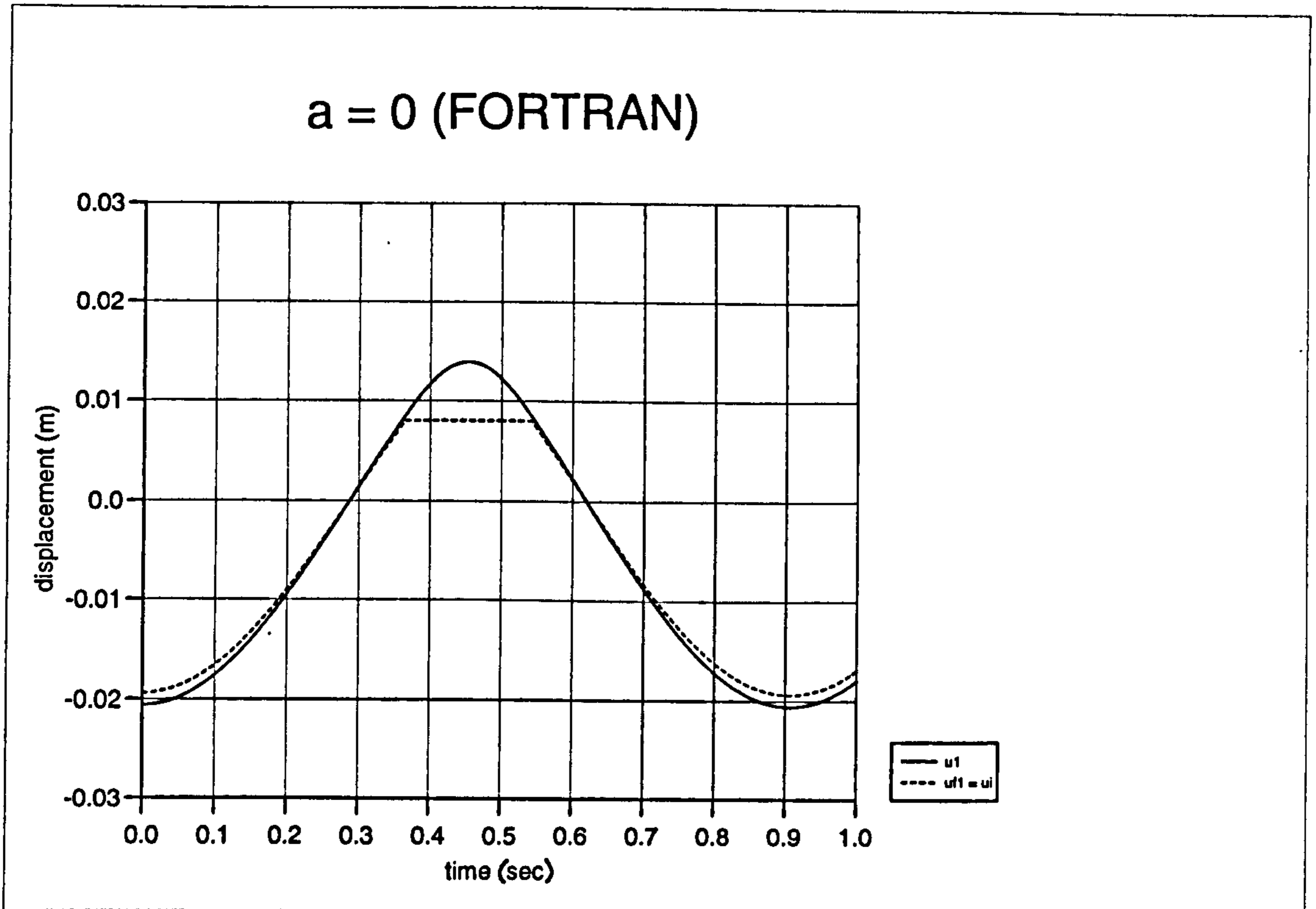


Figure 4.6a) : Displacement curves for pounding case at a = 0 (FORTRAN)

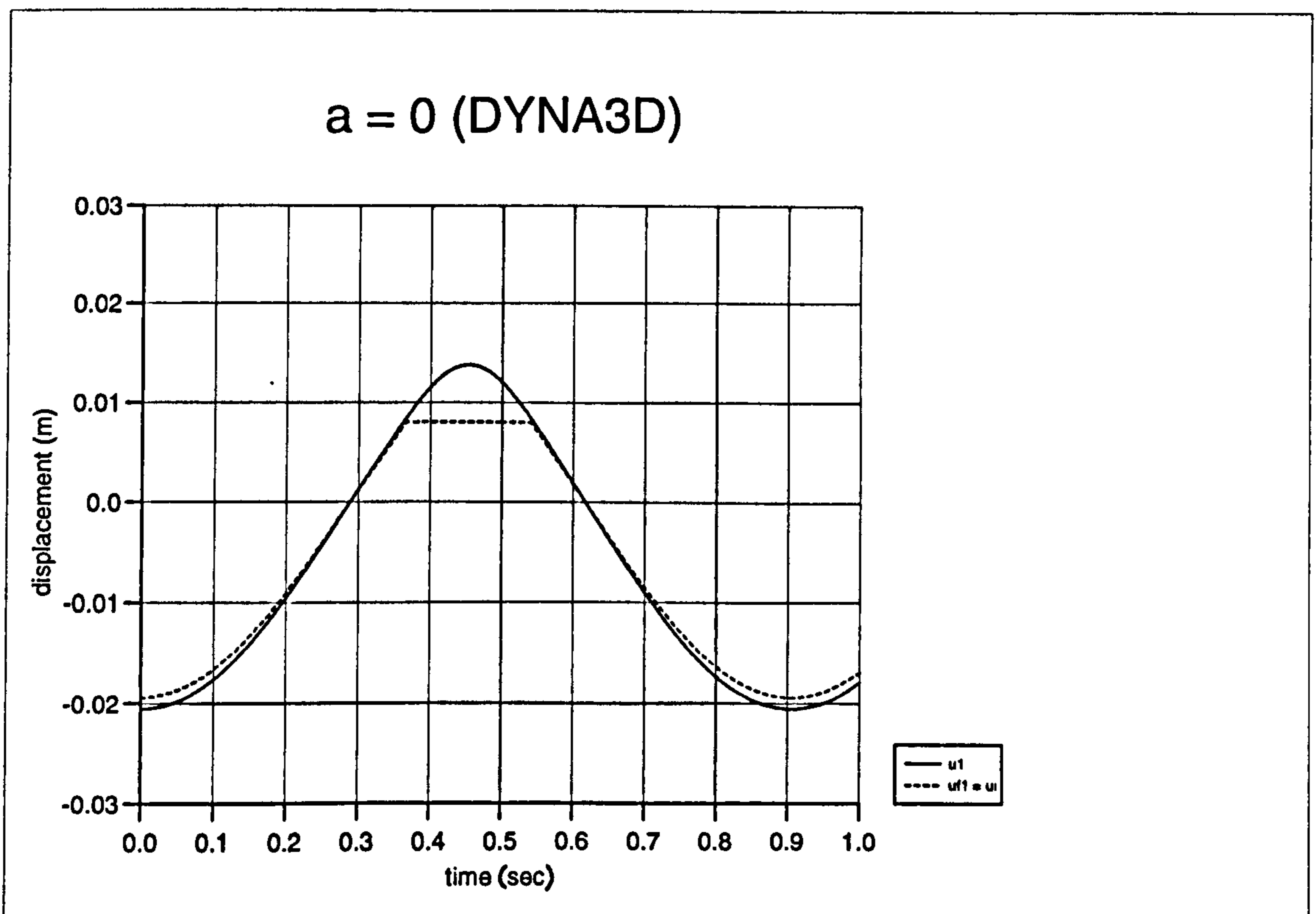


Figure 4.6b) : Displacement curves for pounding case at a = 0 (DYNA3D)

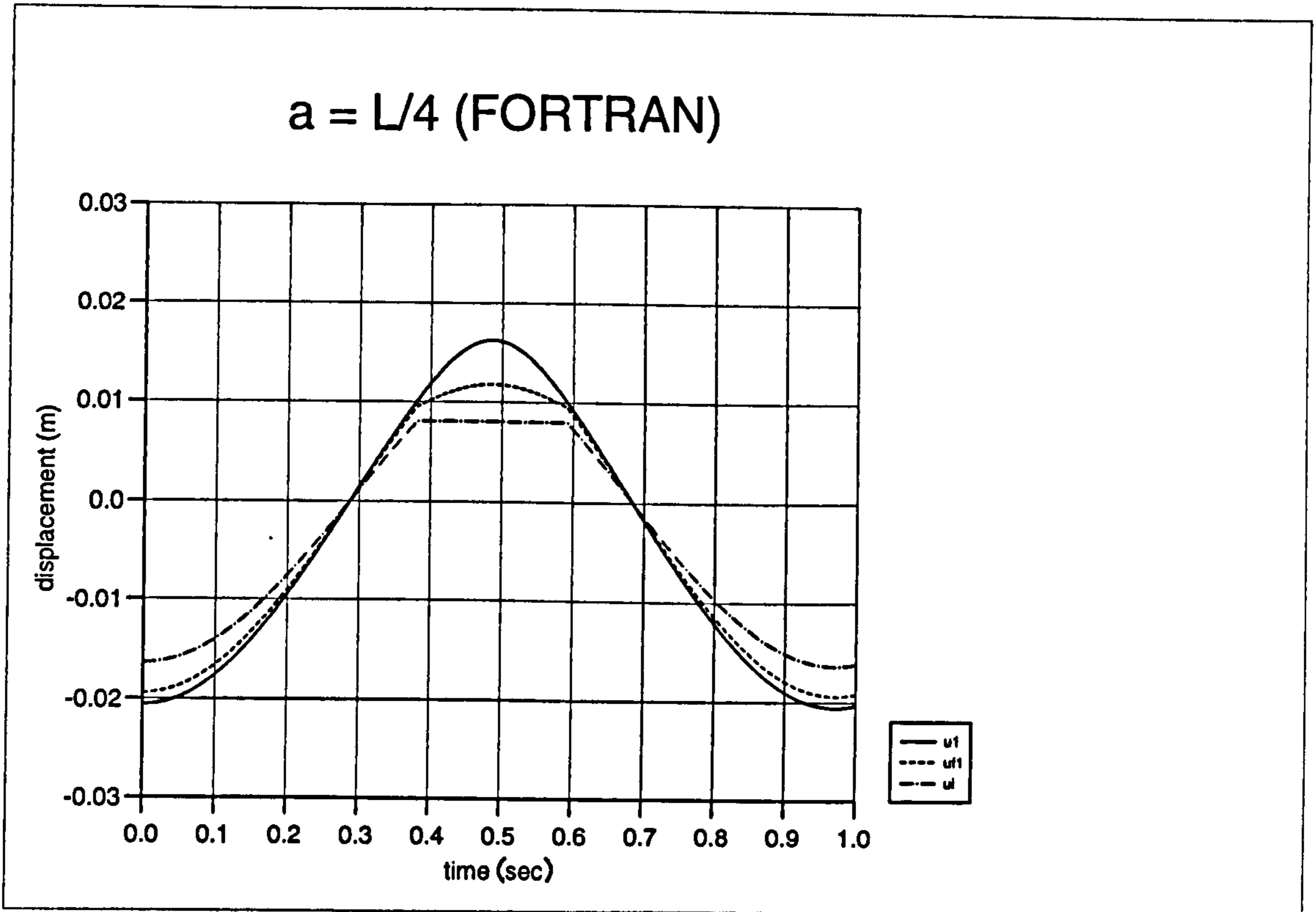


Figure 4.7a) : Displacement curves for pounding case at  $a = L/4$  (FORTRAN)

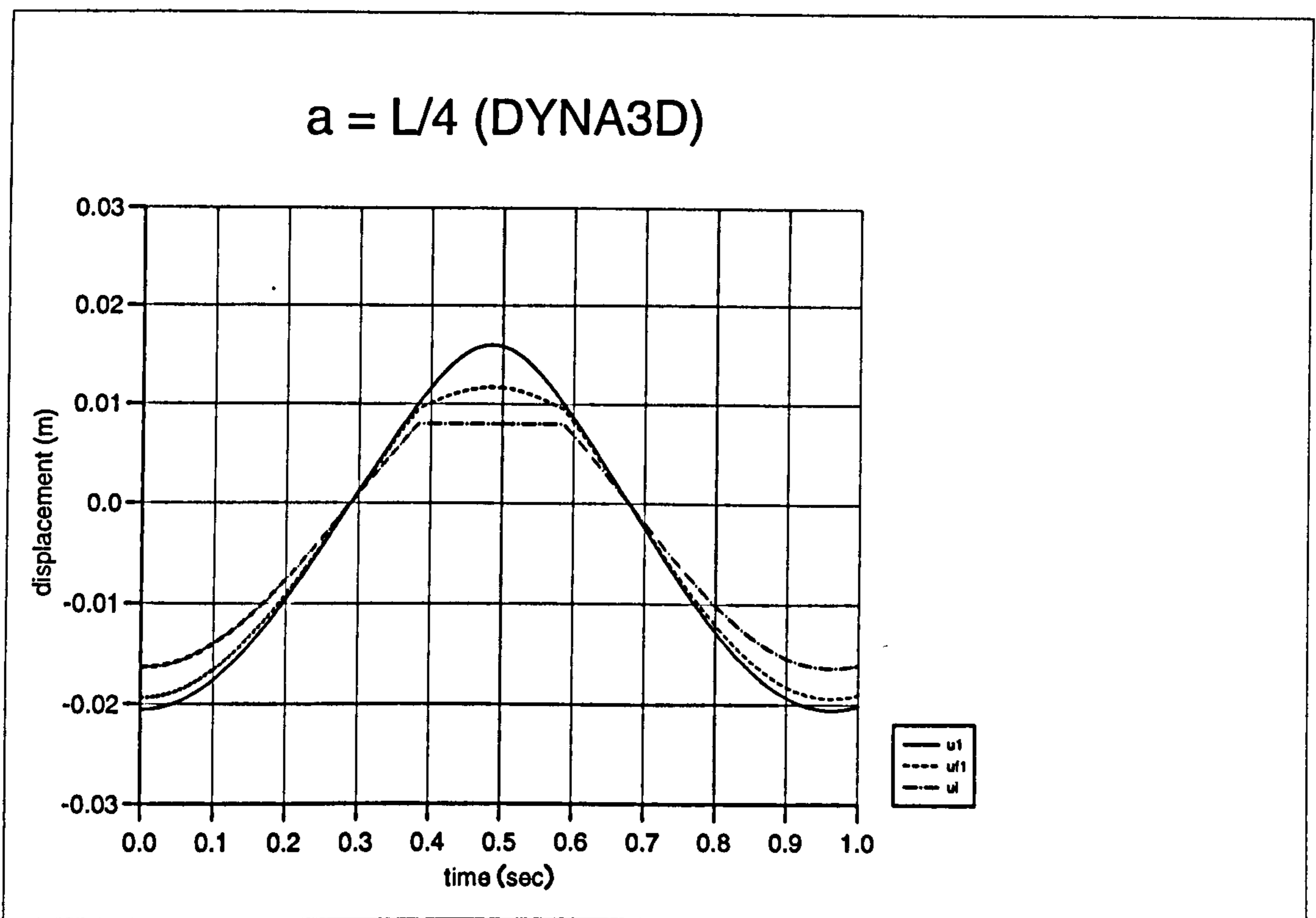


Figure 4.7b) : Displacement curves for pounding case at  $a = L/4$  (DYNA3D)

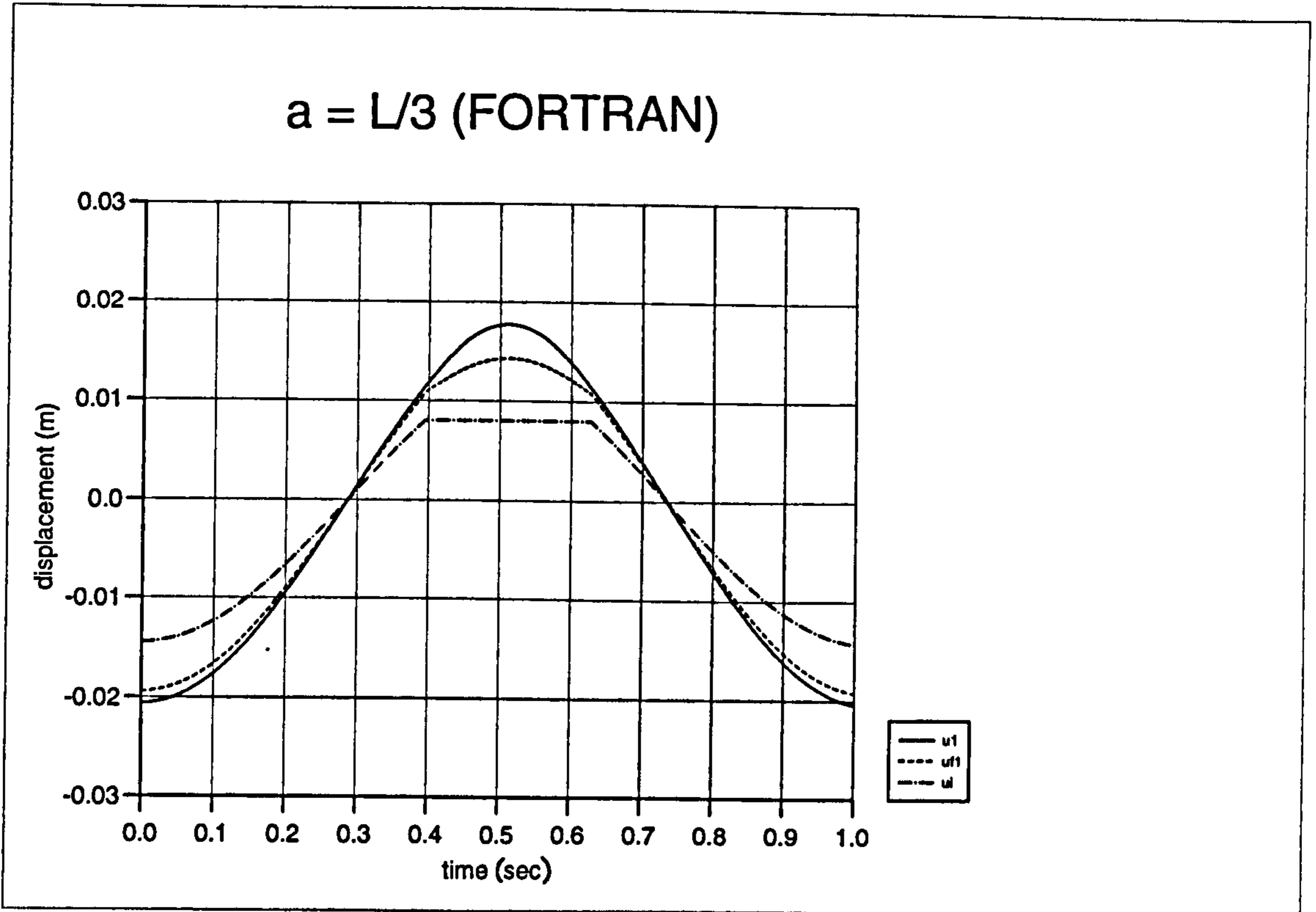


Figure 4.8a) : Displacement curves for pounding case at  $a= L/3$  (FORTRAN)

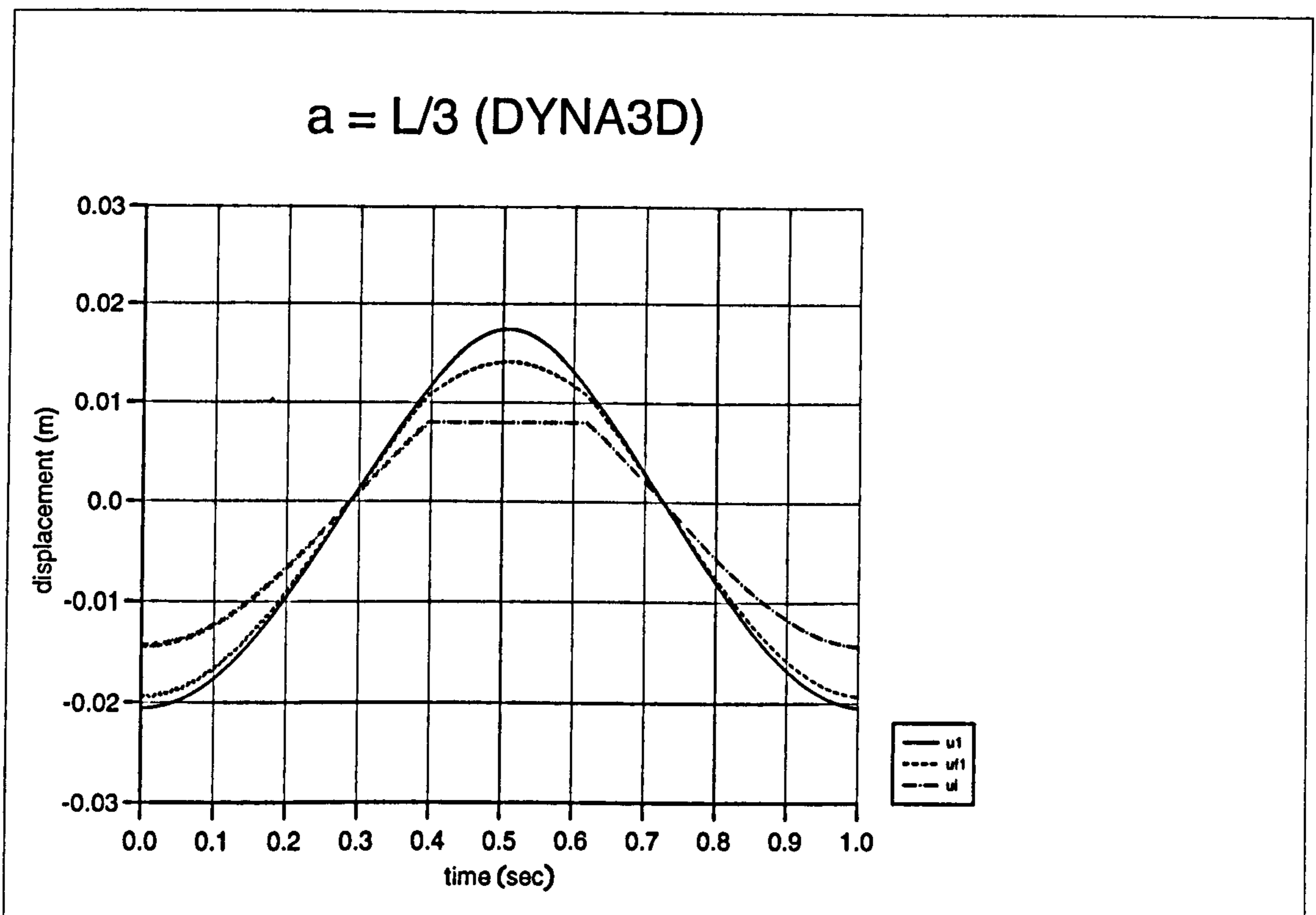


Figure 4.8b) : Displacement curves for pounding case at  $a= L/3$  (DYNA3D)

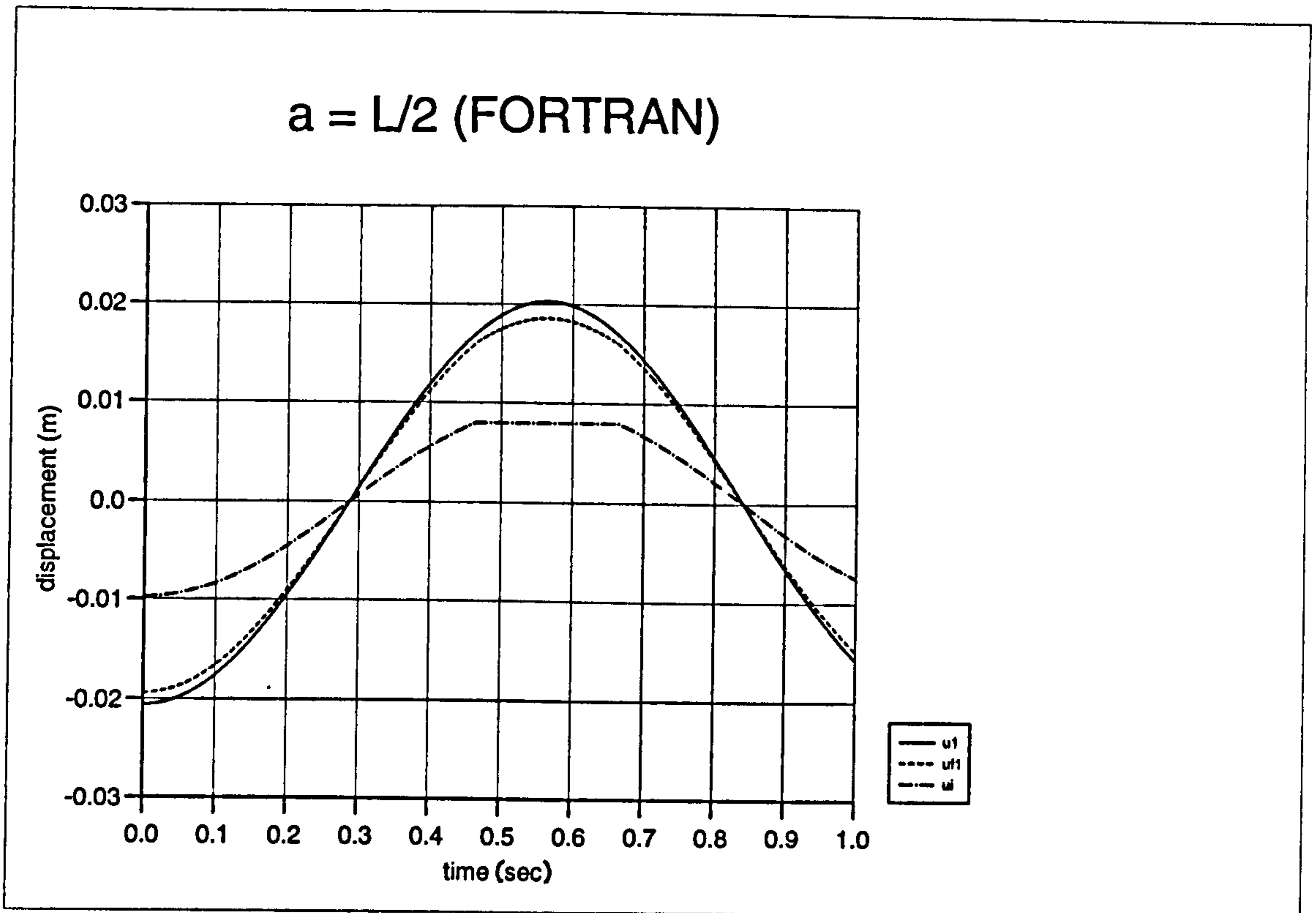


Figure 4.9a) : Displacement curves for pounding case at a= L/2 (FORTRAN)

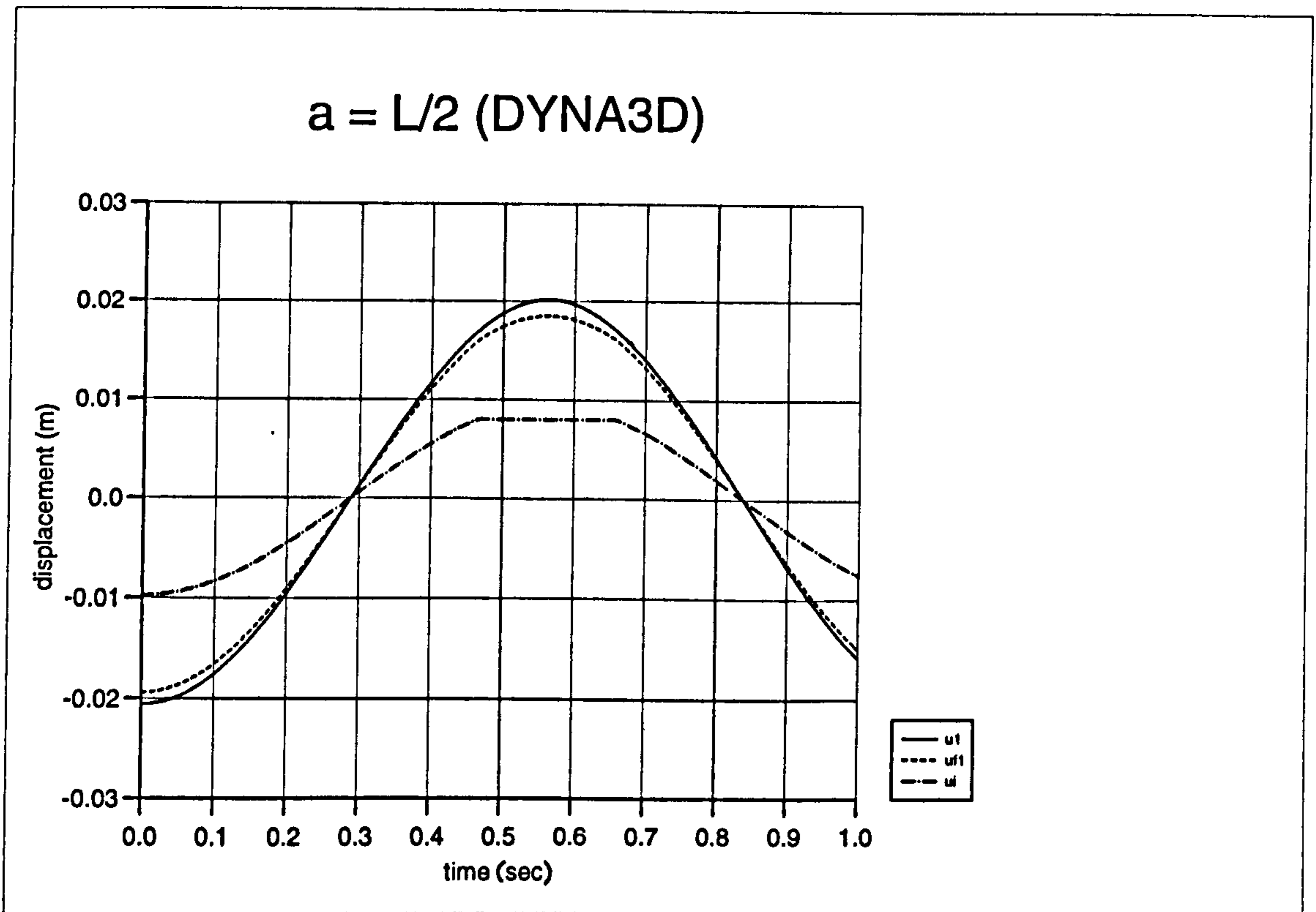


Figure 4.9b) : Displacement curves for pounding case at a= L/2 (DYNA3D)

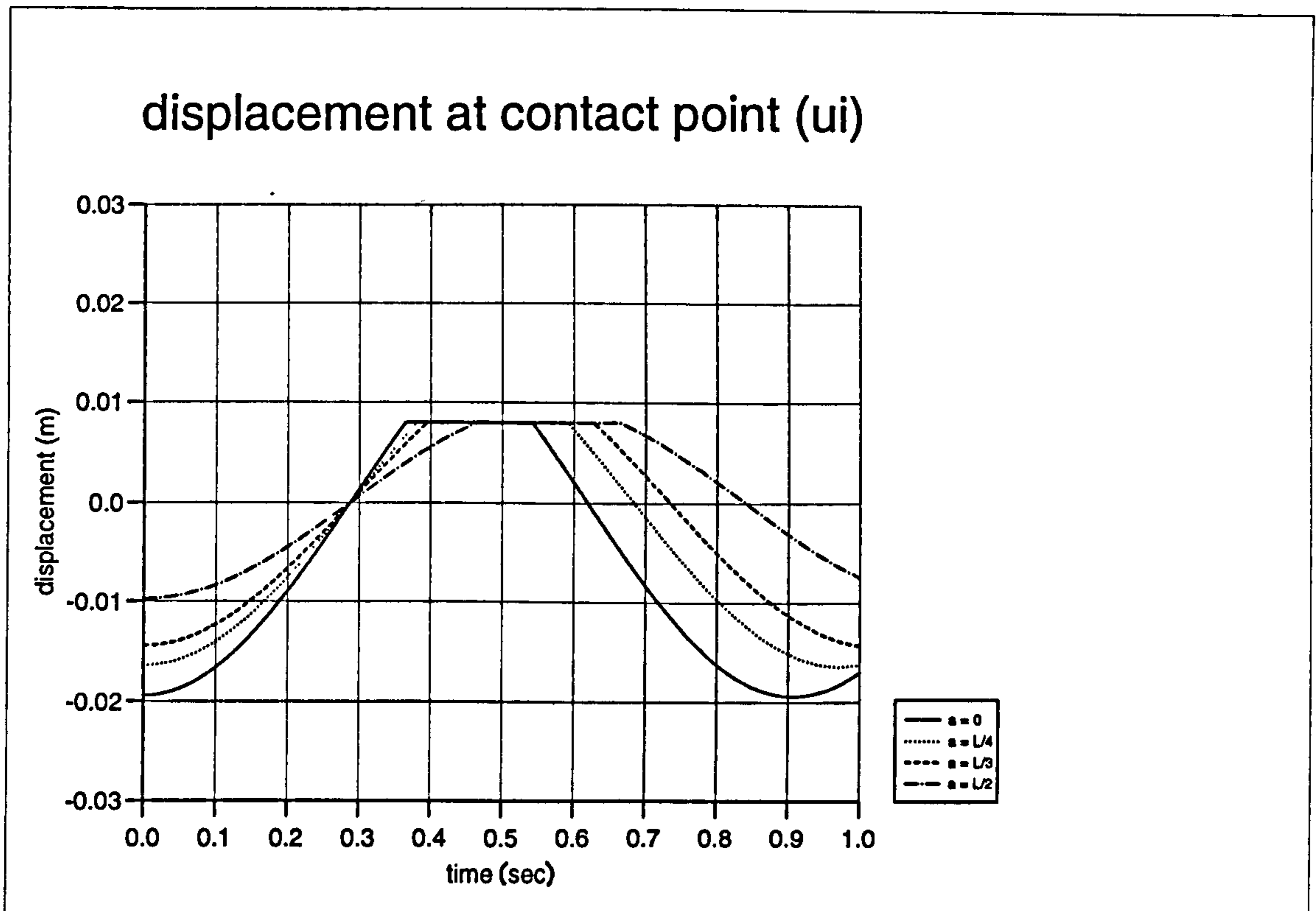


Figure 4.10a) : Displacement curves of  $u_i$  for different "a" values (FORTRAN)

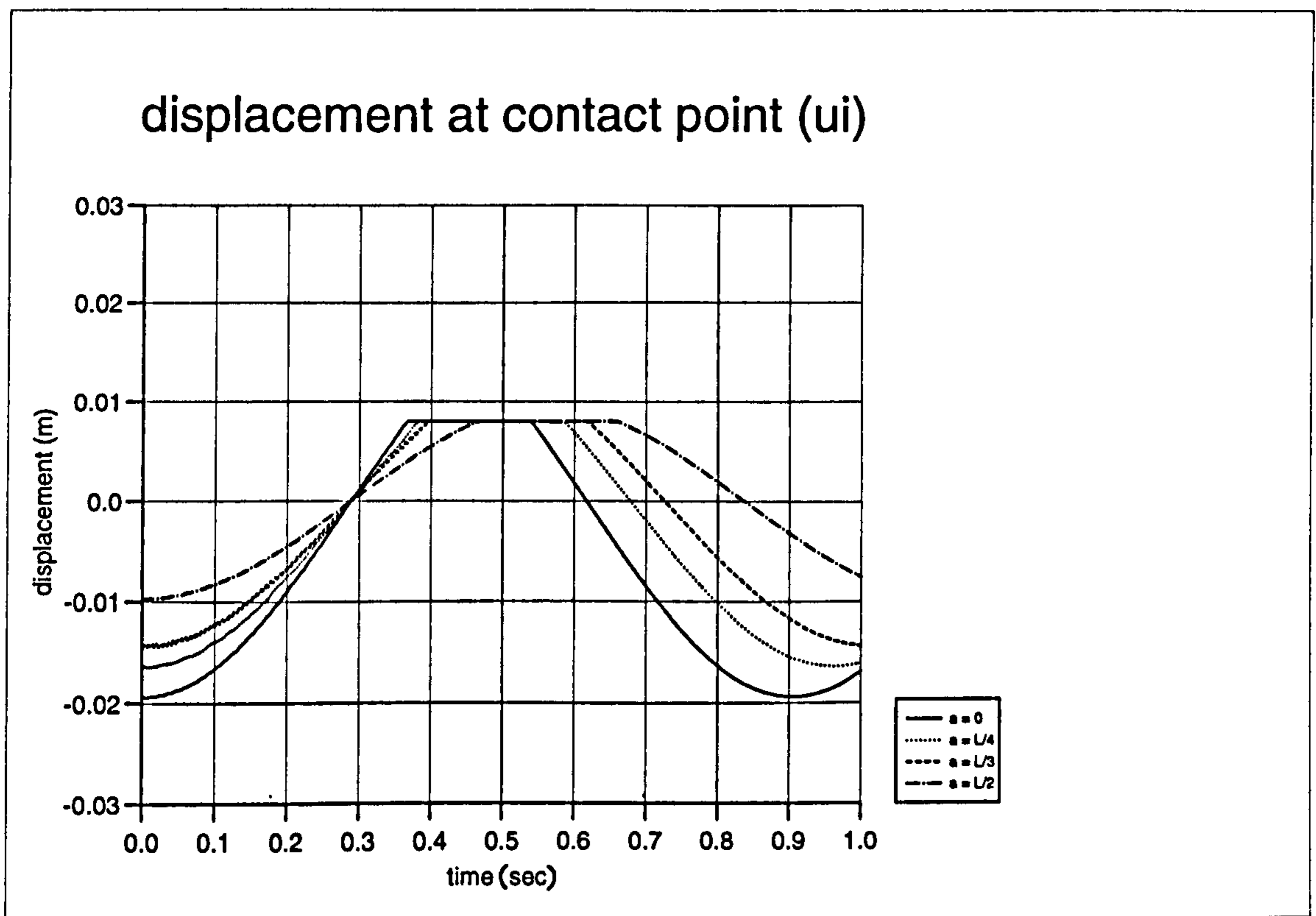


Figure 4.10b) : Displacement curves of  $u_i$  for different "a" values (DYNA3D)



### 4.3 MDOF SYSTEMS

To find the effect of the slab flexibility,  $k_s$  during pounding on MDOF systems, consider a two degree of freedom system as shown in Figure 4.11 below. Again, the rigid stopper represents the adjacent rigid building.

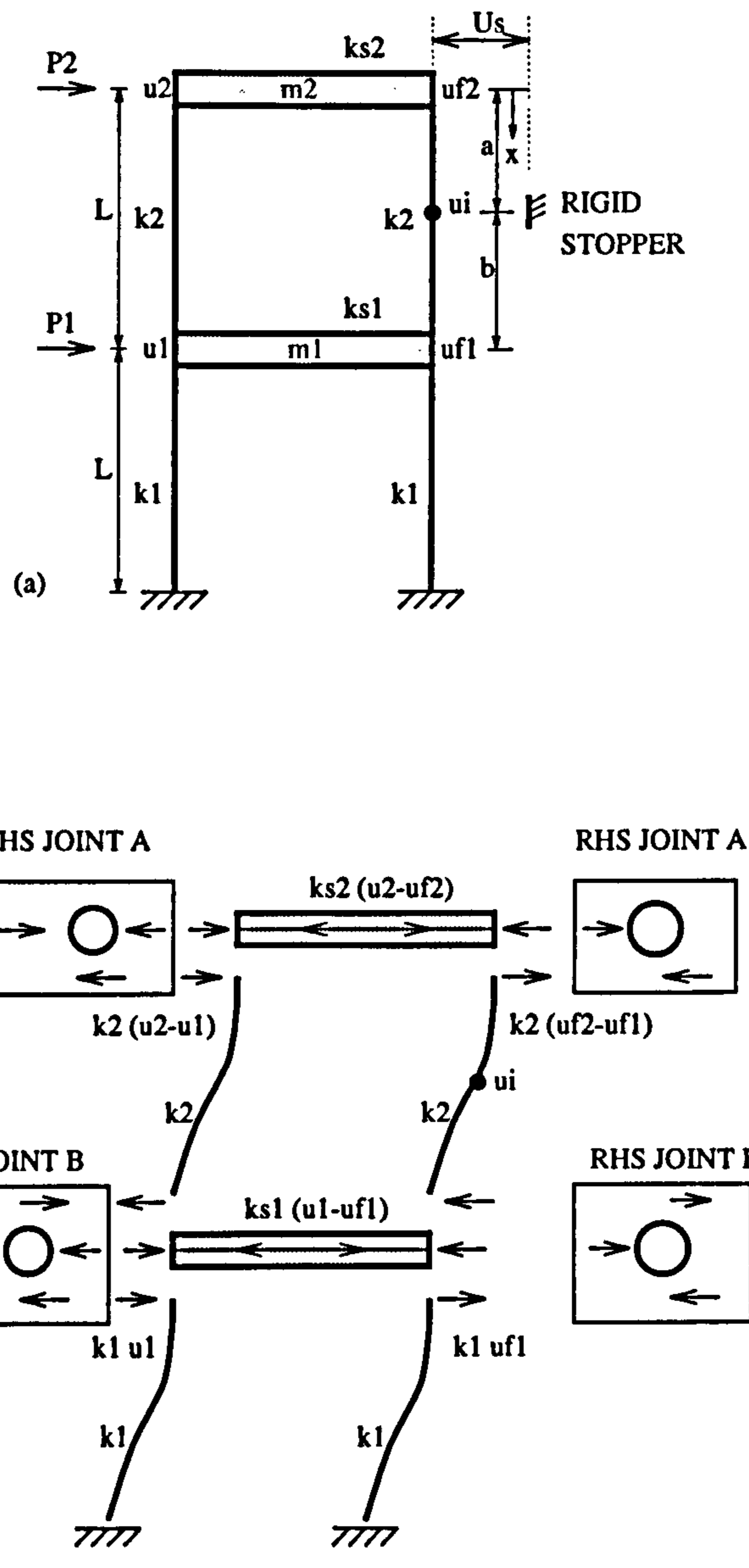


Figure 4.11 : (a) MDOF system and (b) free body diagram

### 4.3.1 No contact stage

The relationships for the displacements of a rigid slab during the no contact stage is  $uf1 = u1$  and  $uf2 = u2$ . For the case of a flexible slab, new relationships are derived. Referring to Figure 4.11( b), the free body forces of the MDOF system:

For the top right hand joint (RHS JOINT A) :

$$ks2 (u2 - uf2) - k2 (uf2 - uf1) = 0 \quad (4.33)$$

From the bottom right hand joint B (RHS JOINT B),

$$ks1(u1 - uf1) + k2(uf2-uf1) - k1 uf1 = 0 \quad (4.34)$$

Rearranging the above equations into a matrix form;

$$\begin{vmatrix} ks2+k2 & -k2 \\ -k2 & k2+ks1+k1 \end{vmatrix} \begin{Bmatrix} uf2 \\ uf1 \end{Bmatrix} = \begin{vmatrix} ks2 & 0 \\ 0 & ks1 \end{vmatrix} \begin{Bmatrix} u2 \\ u1 \end{Bmatrix} \quad (4.35)$$

Thus, the  $uf1$  and  $uf2$  can be found by rearranging the above matrices:

$$\begin{aligned} \begin{Bmatrix} uf2 \\ uf1 \end{Bmatrix} &= \frac{1}{k_D} \begin{vmatrix} (k2+ks1+k1) & k2 \\ k2 & (ks2+k2) \end{vmatrix} \begin{vmatrix} ks2 & 0 \\ 0 & ks1 \end{vmatrix} \begin{Bmatrix} u2 \\ u1 \end{Bmatrix} \\ \therefore \begin{Bmatrix} uf2 \\ uf1 \end{Bmatrix} &= \frac{1}{k_D} \begin{vmatrix} (k2+ks1+k1)ks2 & k2ks1 \\ k2ks2 & (ks2+k2)ks1 \end{vmatrix} \begin{Bmatrix} u2 \\ u1 \end{Bmatrix} \end{aligned} \quad (4.36)$$

where

$$k_D = (ks2 + k2) (k2 + ks1 + k1) - k2^2 \quad (4.37)$$

This relationship is true throughout both the no contact and the contact stages. The shear forces diagram at this stage is illustrated in Figure 4.12 below:

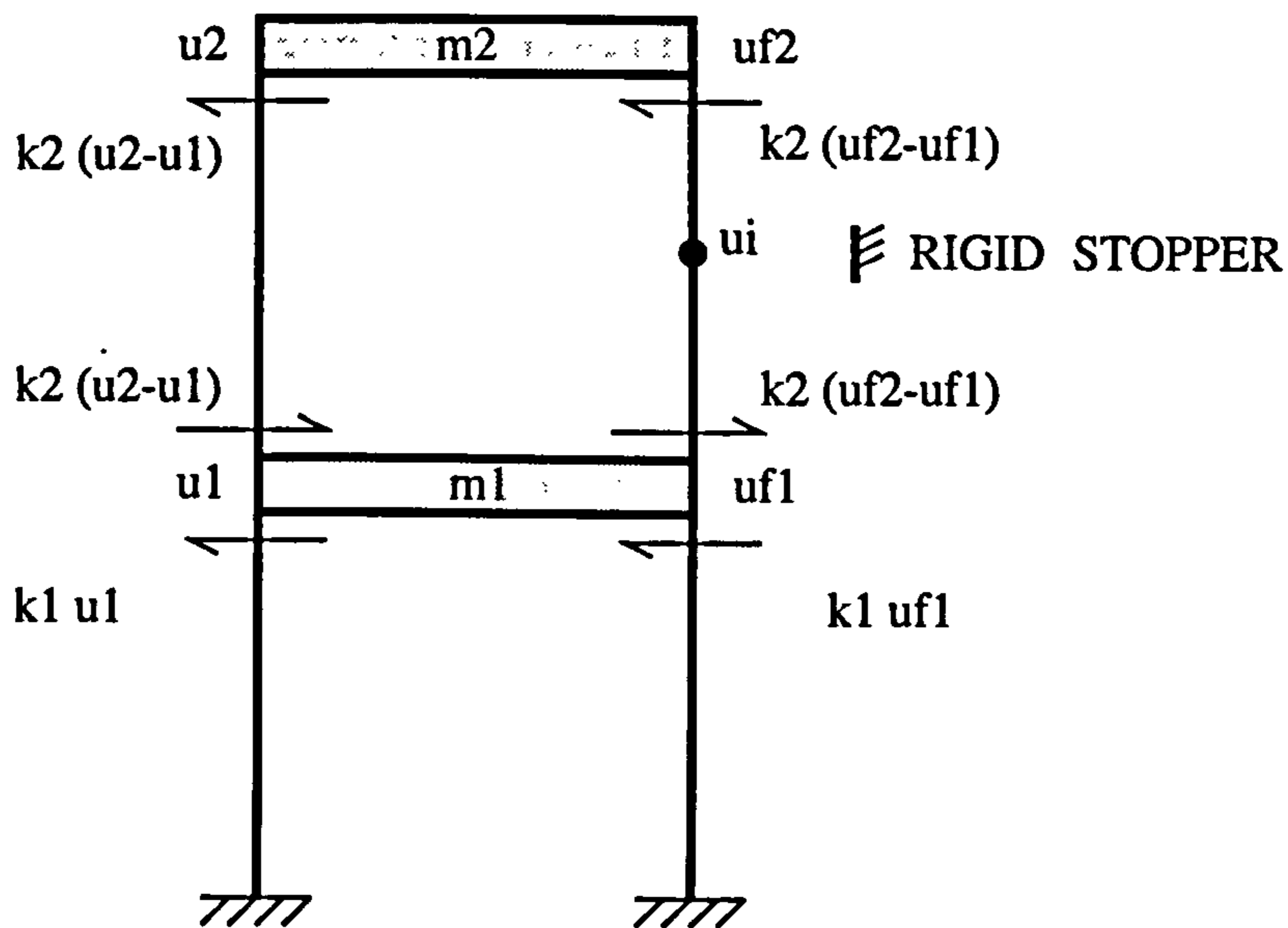


Figure 4.12 : Shear forces diagram of MDOF system (no contact stage)

The equations of motion for the no contact stage are :

$$\begin{aligned} m_2 \ddot{u}_2 &= -k_2 (u_2 - u_1) - k_2 (u_{f2} - u_{f1}) \\ m_1 \ddot{u}_1 &= k_2 (u_2 - u_1) + k_2 (u_{f2} - u_{f1}) - k_1 u_1 - k_1 u_{f1} \end{aligned} \quad (4.38)$$

Rearranged into a matrix form:

$$\begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 + k_1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} + \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 + k_1 \end{bmatrix} \begin{Bmatrix} u_{f2} \\ u_{f1} \end{Bmatrix} = 0 \quad (4.39)$$

Substituting Equation (4.36) into the third term of the above equation becomes:

$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 + k_1 \end{bmatrix} \begin{Bmatrix} u_{f2} \\ u_{f1} \end{Bmatrix} = \frac{1}{k_D} \begin{bmatrix} k_2(k_{s1} + k_1)k_{s2} & -k_2 k_{s2} k_{s1} \\ -k_2 k_{s2} k_{s1} & k_{s1} [(k_2 + k_1)k_{s2} + k_1 k_2] \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} \quad (4.40)$$

Back substitution into Equation (4.39)

$$\begin{vmatrix} m_2 & 0 \\ 0 & m_1 \end{vmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} k_2 + \frac{k_2(k_{s1} + k_1)k_{s2}}{k_D} & -k_2 - \frac{k_2k_{s2}k_{s1}}{k_D} \\ -k_2 - \frac{k_2k_{s2}k_{s1}}{k_D} & (k_2 + k_1) + \frac{k_{s1}[(k_2 + k_1)k_{s2} + k_1k_2]}{k_D} \end{vmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} = 0 \quad (4.41)$$

These matrices are always symmetrical at any contact position ( $a=0$  to  $L$ ) along the height of the top column.

In general these can be represented by :

$$\underline{m} \ddot{u}_i + \overline{k}^* u_i = 0 \quad (4.42)$$

where  $\underline{m}$  is the mass matrix and  $\overline{k}^*$  is the stiffness matrix. Since these matrices ( $\underline{m}$  and  $\overline{k}^*$ ) are always symmetrical at any contact position ( $a=0$  to  $L$ ) along the height of the column, the orthogonality properties (if  $i = j$   $Q_j^T \underline{m} Q_i = 1$ ) described in the Mode Superposition techniques can be employed (section 3.2 of chapter 3).

Using the modal analysis, uncoupling of the equations produces:

$$\ddot{y}_i + \omega_i^2 y_i = 0 \quad (4.43)$$

where displacement and velocity solutions are given by:

$$\begin{aligned} y_i &= \frac{\dot{y}_{it}}{\omega_i} \sin \omega_i dt + y_{it} \cos \omega_i dt \\ \dot{y}_i &= \dot{y}_{it} \cos \omega_i dt - \omega_i y_{it} \sin \omega_i dt \end{aligned} \quad (4.44)$$

where  $\omega_i$  is the no contact stage vibration frequencies. The total displacement,  $u_i$  and velocity,  $\dot{u}_i$  during the no contact stage and the displacement at the contact point,  $u_i$  are :

$$\begin{aligned} u_i &= \sum_{i=1}^n \alpha_i y_i \quad \text{and} \quad \dot{u}_i = \sum_{i=1}^n \alpha_i \dot{y}_i \\ u_i &= (1 - k_{AB}) u_{f2} + k_{AB} u_{f1} \\ \text{where} \quad k_{AB} &= 3\left(\frac{a}{L}\right)^2 - 2\left(\frac{a}{L}\right)^3 \end{aligned} \quad (4.45)$$

where  $n$ th is the number of mode shapes.

### 4.3.2 Contact Stage

At the beginning of the contact stage, the condition of  $u_i \geq U_s$  must be satisfied, where  $u_i$  is the point of contact. Similar to the SDOF system (section 4.2.2), an expression for variable  $u_{ki}$  (the axial deformation of the slab) must be found first. The same concepts are adopted for the MDOF shear frame as described in section (4.2.2), except that the effective forces on each floor change, these are shown in Figure 4.13 below.

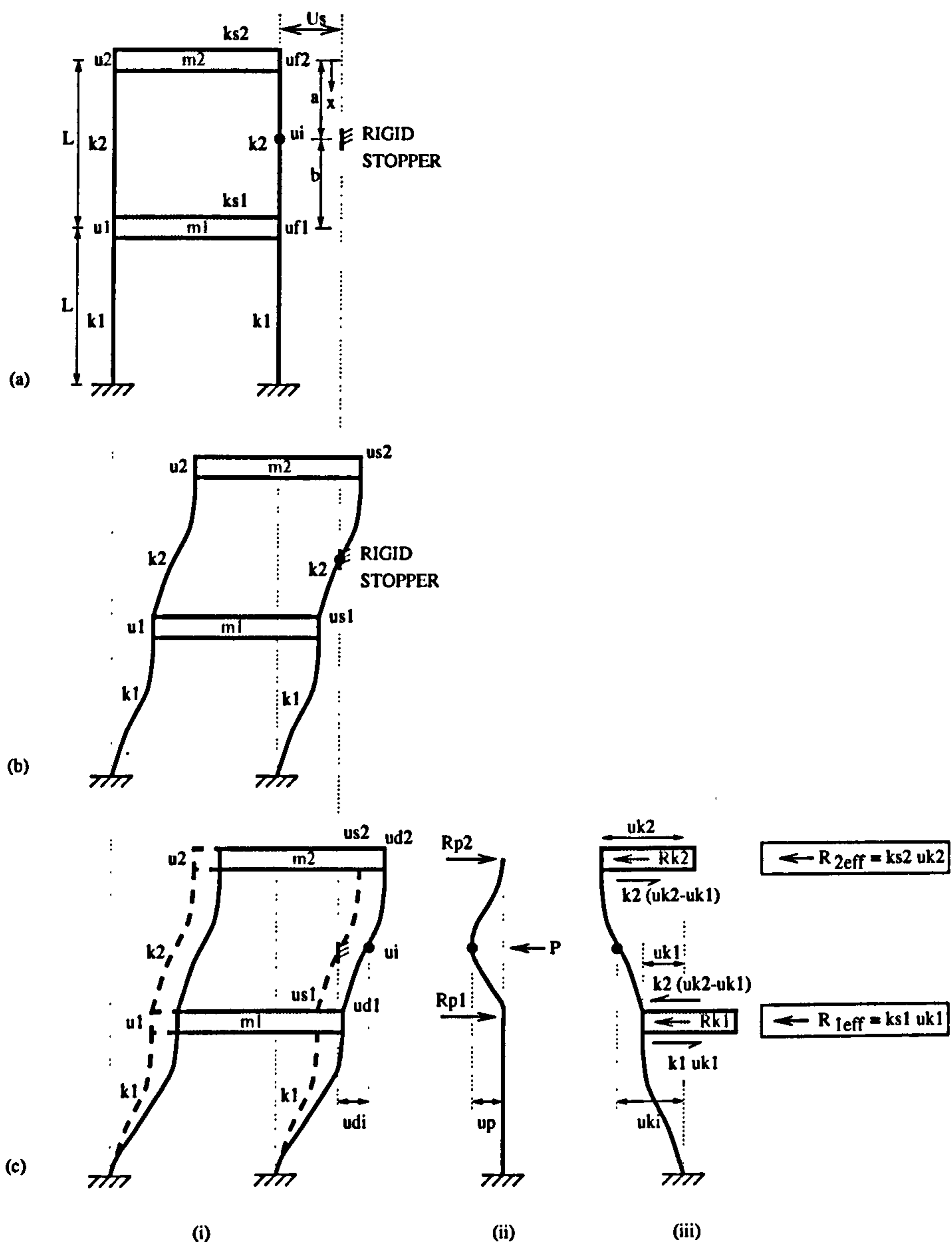


Figure 4.13 : Displacement Compatibility Stages (a) no contact (b) initial contact (c) into contact stage of the right hand side column behaviour.

Before the contact stage (i.e during the no contact stage), the masses are moved horizontally until the point of contact,  $u_i$ , reaches the rigid stopper (Figure 4.13b). At this stage, the displacements of the top and bottom slab on the right hand side of the top column are equal to  $u_{s2}$  and  $u_{s1}$  where:

$$\begin{aligned} u_{f2} &= u_{s2} + u_{d2} \\ u_{f1} &= u_{s1} + u_{d1} \end{aligned} \quad (4.46)$$

Without the presence of the rigid stopper, the masses actually move further distances  $u_{d2}$  and  $u_{d1}$  at the top and bottom slab respectively (Figure 4.13c (i)). The presence of the rigid stopper, produces the displacement  $u_p$  creating a force  $P$  as shown in Figure 4.13c (ii). Thus it produces a slab reactions  $R_{p2}$  and  $R_{p1}$ . Finally, the axial deformation of the slab,  $u_{ki}$  at the point of contact is added which in turn produces the slab reactions of  $R_{k2}$  and  $R_{k1}$  (Figure 4.13c (iii)).

All these terms are expressed below:

$$\begin{aligned} u_{di} &= (1 - k_{AB}) u_{d2} + k_{AB} u_{d1} \\ u_p &= \frac{P b^3 a^3}{3 L^3 E I} \\ u_{ki} &= (1 - k_{AB}) u_{k2} + k_{AB} u_{k1} \\ u_i &= (1 - k_{AB}) u_{f2} + k_{AB} u_{f1} \\ R_{k2} &= (1 - k_{AB}) P \quad \text{and} \quad R_{k1} = k_{AB} P \\ k_{AB} &= 3 \left(\frac{a}{L}\right)^2 - 2 \left(\frac{a}{L}\right)^3 \end{aligned} \quad (4.47)$$

There are three unknowns in the above equations, i.e  $u_{k2}$ ,  $u_{k1}$  and  $P$ . To derive the variable  $u_{k2}$  and  $u_{k1}$ , consider the equilibrium of slab forces in Figure 4.13c (iii) :

$$\begin{aligned} R_{k2} - k_2(u_{k2} - u_{k1}) &= k_{s2}u_{k2} \\ R_{k1} + k_2(u_{k2} - u_{k1}) - k_1u_{k1} &= k_{s1}u_{k1} \end{aligned} \quad (4.48)$$

Or in matrix form :

$$\begin{Bmatrix} R_{k2} \\ R_{k1} \end{Bmatrix} = \begin{vmatrix} (k_{s2} + k_2) & -k_2 \\ -k_2 & (k_{s1} + k_1 + k_2) \end{vmatrix} \begin{Bmatrix} u_{k2} \\ u_{k1} \end{Bmatrix} \quad (4.49)$$

Rearranging Equation (4.49) by inverting the above stiffness matrix gives :

$$\begin{Bmatrix} uk_2 \\ uk_1 \end{Bmatrix} = \frac{1}{k_D} \begin{vmatrix} (ks_1 + k_1 + k_2) & k_2 \\ k_2 & (ks_2 + k_2) \end{vmatrix} \begin{Bmatrix} Rk_2 \\ Rk_1 \end{Bmatrix} \quad (4.50)$$

where

$$k_D = (ks_2 + k_2) (ks_1 + k_1 + k_2) - k_2^2 \quad (4.51)$$

At the point of contact, the axial deformation of the slab,  $uki$  from Equation (4.47) becomes

$$\begin{aligned} uki &= (1 - k_{AB})uk_2 + k_{AB}uk_1 \\ uki &= \frac{(1 - k_{AB})^2(ks_1 + k_1 + k_2)}{k_D} P + \frac{(1 - k_{AB})k_{AB}k_2}{k_D} P \\ &+ \frac{k_{AB}(1 - k_{AB})k_2}{k_D} P + \frac{k_{AB}^2(ks_2 + k_2)}{k_D} P \end{aligned} \quad (4.52)$$

Rearranging the above equation produces the following :

$$uki = \frac{P}{k_D} [ks_1(1 - k_{AB})^2 + k_1(1 - k_{AB})^2 + k_2 + k_{AB}^2ks_2] \quad (4.53)$$

Using the displacement compatibility at point of contact  $ui$ , (Figure 4.13c) the contact force  $P$  can be found, that is :

$$udi = up + uki \quad (4.54)$$

Substituting terms from Equations (4.47) and (4.53) into above equation gives,

$$\begin{aligned} P &= kp udi \\ &= kp (1 - k_{AB})ud_2 + kp k_{AB}ud_1 \end{aligned} \quad (4.55)$$

where

$$kp = \frac{1}{\frac{a^3 b^3}{3L^3 EI} + \frac{1}{k_D} [(1 - k_{AB})^2(ks_1 + k_1) + k_2 + k_{AB}^2ks_2]} \quad (4.56)$$

Substituting the above equation for  $P$  into Equation (4.50) produces the following expressions for  $uk_2$  and  $uk_1$ .

$$\begin{aligned}
uk_2 &= \frac{k_D}{k_D} (1 - k_{AB}) [(ks_1 + k_1 + k_2)(1 - k_{AB}) + k_2 k_{AB}] ud_2 \\
&\quad + \frac{k_D}{k_D} (k_{AB}) [(ks_1 + k_1 + k_2)(1 - k_{AB}) + k_2 k_{AB}] ud_1 \\
uk_1 &= \frac{k_D}{k_D} (1 - k_{AB}) [k_2(1 - k_{AB}) + (ks_2 + k_2)k_{AB}] ud_2 \\
&\quad + \frac{k_D}{k_D} k_{AB} [k_2(1 - k_{AB}) + (ks_2 + k_2)k_{AB}] ud_1
\end{aligned} \tag{4.57}$$

where  $ud_2$  and  $ud_1$  are defined in Equation (4.46). The expressions for the effective forces in the slabs shown in Figure 4.13c are:

$$\begin{aligned}
R_{2eff} &= ks_2 uk_2 \\
&= ka^* ud_2 + kb^* ud_1 \\
R_{1eff} &= ks_1 uk_1 \\
&= kc^* ud_2 + kd^* ud_1
\end{aligned} \tag{4.58}$$

where

$$\begin{aligned}
ka^* &= \frac{k_D}{k_D} (1 - k_{AB}) ks_2 [(ks_1 + k_1 + k_2)(1 - k_{AB}) + k_2 k_{AB}] \\
kb^* &= \frac{k_D}{k_D} k_{AB} ks_2 [(ks_1 + k_1 + k_2)(1 - k_{AB}) + k_2 k_{AB}] \\
kc^* &= \frac{k_D}{k_D} (1 - k_{AB}) ks_1 [k_2(1 - k_{AB}) + (ks_2 + k_2)k_{AB}] \\
kd^* &= \frac{k_D}{k_D} k_{AB} ks_1 [k_2(1 - k_{AB}) + (ks_2 + k_2)k_{AB}]
\end{aligned} \tag{4.59}$$

These forces are shown in the shear force diagram (Figure 4.14) below in order to assist the derivation of the equation of motion in this contact stage.

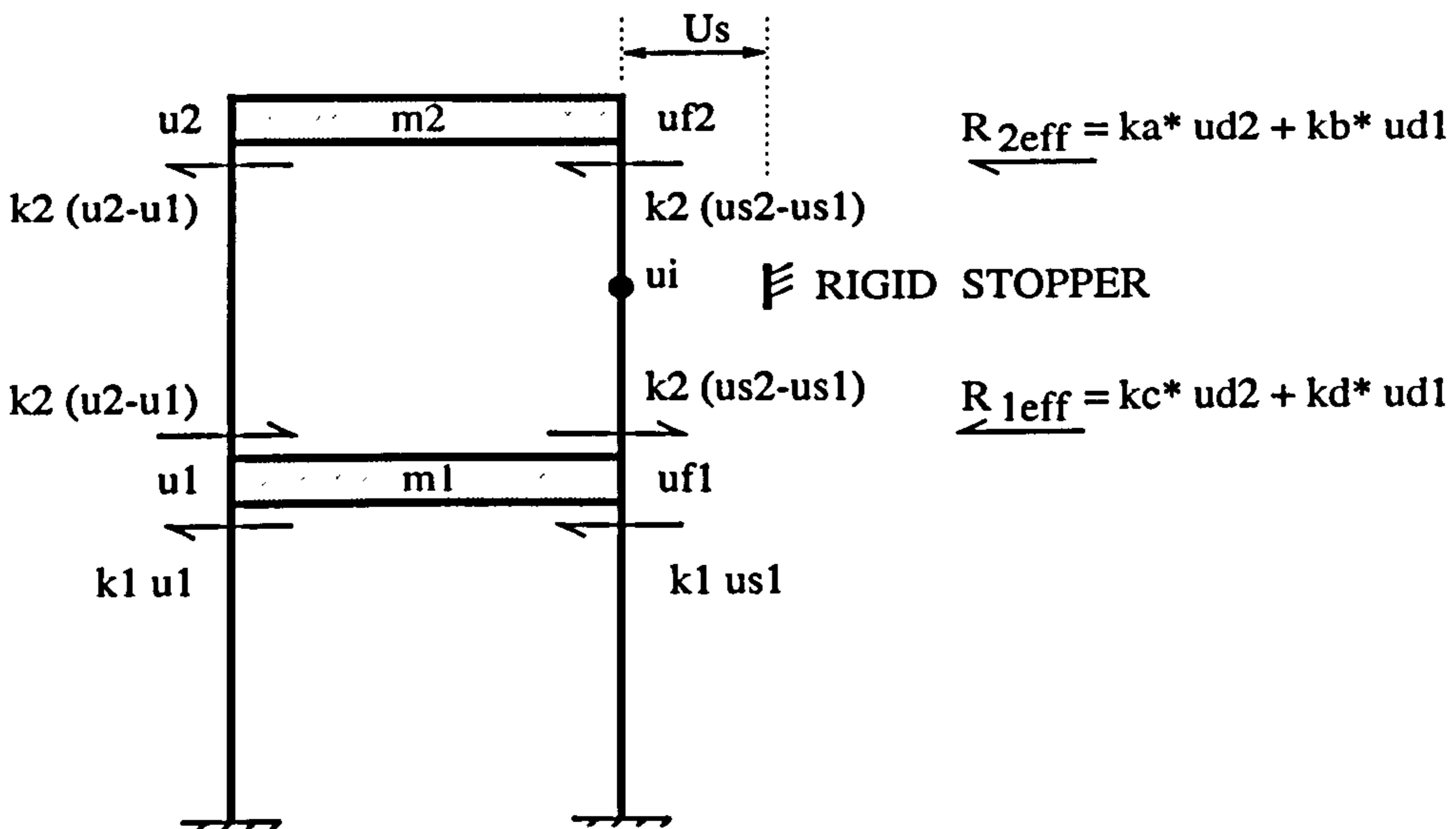


Figure 4.14 : Shear force diagram of MDOF system (contact stage).



The equation of motions for the contact stage can be written as:

$$\begin{aligned} m_2 \ddot{u}_2 &= -k_2(u_2 - u_1) - k_2(us_2 - us_1) - k_a^* ud_2 + k_b^* ud_1 \\ m_1 \ddot{u}_1 &= k_2(u_2 - u_1) + k_2(us_2 - us_1) - k_c^* ud_2 - k_d^* ud_1 \\ &\quad - k_1 u_1 - k_1 us_1 \end{aligned} \quad (4.60)$$

Substituting the value of  $ud_2$  and  $ud_1$  from Equation (4.46) and rearranging into a matrix form;

$$\begin{aligned} \begin{vmatrix} m_2 & 0 \\ 0 & m_1 \end{vmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} k_2 & -k_2 \\ -k_2 & k_2 + k_1 \end{vmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} + \begin{vmatrix} k_a^* & k_b^* \\ k_c^* & k_d^* \end{vmatrix} \begin{Bmatrix} uf_2 \\ uf_1 \end{Bmatrix} \\ = \begin{vmatrix} -k_2 + k_a^* & k_2 + k_b^* \\ k_2 + k_c^* & -k_2 + k_d^* - k_1 \end{vmatrix} \begin{Bmatrix} us_2 \\ us_1 \end{Bmatrix} \end{aligned} \quad (4.61)$$

Substituting Equation (4.36) into the third term of the above equation

$$\begin{aligned} &\begin{vmatrix} k_a^* & k_b^* \\ k_c^* & k_d^* \end{vmatrix} \begin{Bmatrix} uf_2 \\ uf_1 \end{Bmatrix} \\ &= \frac{1}{k_D} \begin{vmatrix} k_a^*(k_2 + ks_1 + k_1)ks_2 + k_b^*(k_2 ks_2) & k_a^* k_2 ks_1 + k_b^*(ks_2 + k_2)ks_1 \\ k_c^*(k_2 + ks_1 + k_1)ks_2 + k_d^* k_2 ks_2 & k_c^* k_2 ks_1 + k_d^*(ks_2 + k_2) \end{vmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} \\ &= \frac{k_p}{k_D^2} \begin{vmatrix} ks_2^2[(1 - k_{AB})(ks_1 + k_1) + k_2]^2 & [(1 - k_{AB})(ks_1 + k_1) + k_2] * \\ - & (ks_2 ks_1)(k_2 + ks_2 k_{AB}) \\ [(1 - k_{AB})(ks_1 + k_1) + k_2] * & \\ (ks_2 ks_1)(k_2 + ks_2 k_{AB}) & ks_1^2(k_2 + k_{AB} ks_2)^2 \end{vmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} \end{aligned} \quad (4.62)$$

Substituting back into Equation (4.61)

$$\begin{aligned}
\begin{vmatrix} m2 & 0 \\ 0 & m1 \end{vmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} \frac{k_D}{k_D^2} k_s^2 [(1-k_{AB})(k_{s1}+k_1)+k_2]^2 & \frac{k_D}{k_D^2} [(1-k_{AB})(k_{s1}+k_1)+k_2] * \\ + k_2 & (k_{s2}k_{s1})(k_2+k_{s2}k_{AB}) - k_2 \\ \frac{k_D}{k_D^2} [(1-k_{AB})(k_{s1}+k_1)+k_2] * & \frac{k_D}{k_D^2} k_{s1}^2 (k_2+k_{AB}k_{s2})^2 \\ (k_{s2}k_{s1})(k_2+k_{s2}k_{AB}) - k_2 & + (k_2+k_1) \end{vmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} \\
= \begin{vmatrix} -k_2+k_{\alpha}^* & k_2+k_{\beta}^* \\ k_2+k_{\gamma}^* & -k_2+k_{\delta}^*-k_1 \end{vmatrix} \begin{Bmatrix} u_{s2} \\ u_{s1} \end{Bmatrix}
\end{aligned}
\tag{4.63}$$

In general, the equation of the motion for the contact stage can be written as:

$$\mathbf{m} \ddot{\mathbf{u}}_i + \mathbf{k}_c^* \mathbf{u}_i = \mathbf{b}_s \tag{4.64}$$

where  $\mathbf{m}$  is the mass matrix,  $\mathbf{k}_c^*$  is the stiffness matrix and  $\mathbf{b}_s$  is a column vector of static preload forces. Since these matrices ( $\mathbf{m}$  and  $\mathbf{k}_c^*$ ) are always symmetrical at any contact position ( $a=0$  to  $L$ ) along the height of the column, the orthogonality properties (if  $i = j$   $Q_j^T \mathbf{m} Q_i = 1$ ) can be employed, whilst the static preload forces,  $\mathbf{b}_s$ , don't have to be symmetrical, where in this case ( $k_{\alpha}^* \neq k_{\beta}^* \neq k_{\gamma}^* \neq k_{\delta}^*$ ).

Hence, the solution for these equations are found by using modal analysis and uncoupling the equations produces

$$\ddot{y}_i + \omega_i^2 y_i = Q_i^T \mathbf{b}_s \tag{4.65}$$

The solution of the above equations are

$$\begin{aligned}
y_i &= \frac{\dot{y}_{it}}{\omega_{ic}} \sin \omega_{ic} dt + (y_{it} - y_{sii}) \cos \omega_{ic} dt + y_{sii} \\
\dot{y}_i &= \dot{y}_{it} \cos \omega_{ic} dt - \omega_{ic} (y_{it} - y_{sii}) \sin \omega_{ic} dt
\end{aligned}
\tag{4.66}$$

Where  $\omega_{ic}$  is the natural frequencies during the contact stage and

$$y_{sii} = \frac{Q_j^T \mathbf{b}_s}{\omega_{ic}^2} = \frac{\alpha_{1i}^T \mathbf{b}_s}{\omega_{ic}^2} \tag{4.67}$$

The total displacements and velocities responses are the sum of the modal contributions defined in Equations (4.45).

The right hand side displacements during the contact stage are also equal to Equation (4.36) of the no contact stage. Considering the effects of slab flexibility, that is the  $uk2$  and  $uk1$  terms in Equation (4.57), the final right hand side displacements are given by

$$\begin{aligned} \text{new } uf2 &= uf2 - uk2 \\ \text{new } uf1 &= uf1 - uk1 \end{aligned} \quad (4.68)$$

The maximum contact force is given by Equation (4.55) and the displacement of point of contact,  $ui$  is given by Equation (4.45)

### 4.3.3 Change of state rules

The contact stage starts when the displacement and velocity are such:

$$ui = Us \quad \text{but} \quad \dot{ui} \geq 0 \quad (4.69a)$$

And the contact stage is finishing when the displacement and velocity are such:

$$ui = Us \quad \text{but} \quad \dot{ui} \leq 0 \quad (4.69b)$$

### 4.3.4 Numerical Examples of MDOF Systems

The MDOF undamped free vibration pounding response induced by a snap back analysis is presented. As for the rigid slab (section 3.4.1.5.2 and 3.4.1.5.3 of chapter 3), two examples for the aligned floor ( $a = 0$  and  $a = L$ ) and four examples for the unaligned floor ( $a = L/4, L/3, L/2, 3L/4$ ) are considered. In this section, a two storey shear frame as shown in Figure 4.11(a) and having the physical properties as in Table 4.4 below is studied.

$m_2, m_1$	= building masses of 1.0, 1.5 kips-sec <sup>2</sup> /in. = 175197, 262795 kg (N-sec <sup>2</sup> /m)
$k_2, k_1$	= building stiffness of 600, 900 kips/in. = $105.12 \cdot 10^3, 157.68 \cdot 10^3$ (kN/m)
$ks_2 = ks_1$	= top and middle slab stiffness of 5,000 kips/in. ( $8.76 \cdot 10^5$ kN/m), respectively
Initial $u_2, u_1$ displacements	= -1.0 in. (-0.0254 m) and -0.5 in (-0.0127m)
Building separation, $U_s$	= 0.008 m
Ground acceleration, ( $\ddot{u}_{gt} = 0$ )	
Undamped free vibration, $c$	= 0
Height between floors, $L$	= 72 in. (1.8288 m)
Young Modulus of steel, $E$	= $30 \cdot 10^3$ kip/in <sup>2</sup> ( $205 \cdot 10^6$ kN/m <sup>2</sup> ) ∴ $EI = 18662400$ kips . in <sup>2</sup>

Table 4.4 : Physical properties of the building. ( \*\* 1 kip = 4450N )

#### 4.3.4.1 No Contact stage

As this is the flexible slab cases, even during the no contact stage, the relationship between the right and left hand side of the column displacement is always given by Equation (4.36) for all cases ( $0 \leq a \leq L$ ). Substituting the physical properties given in Table 4.4 into Equation (4.36) gives :

$$\begin{Bmatrix} uf2 \\ uf1 \end{Bmatrix} = \frac{5000}{k_D} \begin{vmatrix} (6500) & 600 \\ 600 & (5600) \end{vmatrix} \begin{Bmatrix} u2 \\ u1 \end{Bmatrix}$$

where  $k_D = 36040000$  kips/in.

Thus the final equation of motion, as in Equation (4.41), becomes:

$$\begin{vmatrix} 1.0 & 0 \\ 0 & 1.5 \end{vmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} 1091.120977 & -1016.204218 \\ -1016.204218 & 2615.427303 \end{vmatrix} \begin{Bmatrix} u2 \\ u1 \end{Bmatrix} = 0$$

These matrices are always symmetrical at any contact position ( $a=0$  to  $L$ ) along the height of the top column. Solving these equations give the vibration frequencies of

$$\omega_1 = 22.930 \text{ rad/sec}$$

$$\omega_2 = 48.051 \text{ rad/sec}$$

with the associate normalized mode shapes of :

<i>1st mode</i>	<i>2nd mode</i>
0.82642	-0.56306
0.45974	0.67477

In general these can be represented by Equation (4.42). The displacements and velocities response are the sum of the modal contributions defined in Equations (4.45).

The final displacement at the contact point  $u_i$ , is given by Equation (4.45) where for example,  $a=L/2$  ,

$$k_{AB} = (1 - k_{AB}) = \frac{1}{2}$$

$$\therefore u_i = \frac{1}{2} uf2 + \frac{1}{2} uf1$$

#### 4.3.4.2 Contact Stage

When the condition ( $u_i = U_s$  and  $\dot{u}_i > 0$ ) is satisfied, then the change from the no contact stage to the contact stage takes place. In this section, a typical unaligned pounding case of  $a = L/2$  is taken as a numerical example. Therefore, Equation (4.47) becomes:

$$\begin{aligned}k_{AB} &= (1 - k_{AB}) = \frac{1}{2} \\ u_{di} &= \left(\frac{1}{2}\right) u_{d2} + \left(\frac{1}{2}\right) u_{d1} \\ u_{ki} &= \left(\frac{1}{2}\right) u_{k2} + \left(\frac{1}{2}\right) u_{k1} \\ u_i &= \left(\frac{1}{2}\right) u_{f2} + \left(\frac{1}{2}\right) u_{f1} \\ Rk_2 &= Rk_1 = \left(\frac{1}{2}\right) P\end{aligned}$$

At the beginning of the contact, the right hand side displacements are equal to

$$u_{s2} = u_{f2} \quad , \quad u_{s1} = u_{f1}$$

The following constants from Equations (4.51) , (4.56) and (4.59)

$$\begin{aligned}k_D &= 336040000 \text{ kips/in.} \\ k_p &= 5090.994703 \text{ kips/in.} \\ k_{a^*} &= 1253.678634 \text{ kips/in.} \\ k_{b^*} &= 1253.678634 \text{ kips/in.} \\ k_{c^*} &= 1094.761624 \text{ kips/in.} \\ k_{d^*} &= 1094.761624 \text{ kips/in.}\end{aligned}$$

are substituted into the equation of motion (Equation 4.63) giving

$$\begin{vmatrix} 1.0 & 0 \\ 0 & 1.5 \end{vmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{vmatrix} 1834.894326 & 478.358426 \\ 478.358426 & 2441.665104 \end{vmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} = \begin{vmatrix} 653.678634 & 1853.678634 \\ 1694.761624 & -405.238376 \end{vmatrix} \begin{Bmatrix} u_{s2} \\ u_{s1} \end{Bmatrix}$$

The general equation are in the form of Equation (4.64). The natural frequencies for the contact stage ( $a = L/2$ ) are calculated as

$$\begin{aligned}\omega_{1c} &= 34.572 \text{ rad/sec} \\ \omega_{2c} &= 56.226 \text{ rad/sec}\end{aligned}$$

with the associate normalized mode shapes of :

<i>1st mode</i>	<i>2nd mode</i>
-0.15126	0.98849
0.80710	0.12351

The displacement and velocity solutions are defined by Equation (4.66). The total displacement and velocity responses are the sum of the modal contributions defined in Equations (4.45) respectively.

Considering the effects of slab flexibility, that is  $uk2$  and  $uk1$  terms as in Equation (4.57), where for  $a = L/2$  case:

$$uk2 = 0.250735726(uf2 - us2) + 0.250735726(uf1 - us1)$$

$$uk1 = 0.218952324(uf2 - us2) + 0.218952324(uf1 - us1)$$

The final right hand side displacements are given by

$$new\ uf2 = uf2 - uk2$$

$$new\ uf1 = uf1 - uk1$$

The maximum contact forces are given in Equation (4.55)

$$P = 5090.994703\left(\frac{1}{2}\right)[(uf2 - us2) + (uf1 - us1)]$$

and the displacement of point of contact,  $ui$  is given by Equation (4.47).

This process of changing from one stage to another continues until the required time of analysis is reached. The summary of the parameters calculated for other cases of  $a$  (0,  $L/3$ ,  $L/4$ ,  $3L/4$ , and  $L$ ) are listed in Table 4.5.

To assist the iteration process, a FORTRAN program called MFLEX1.FOR and MFLEX2.FOR, MFLEX3.FOR, MLFLEX4.FOR, MFLEX5.FOR and MFLEX6.FOR are written in Appendix G for case ( $a=0$ ,  $L/3$ ,  $L/4$ ,  $L/2$ ,  $3L/4$ , and  $L$ ) respectively.

Again, all these results are verified using DYNA3D.

### 4.3.5 Results and discussions

Analyses for various values of  $a$  (0,  $L/4$ ,  $L/3$ ,  $L/2$ ,  $3L/4$  and  $L$ ) are performed and the results are shown in Figure 4.15 - 4.21. The full results are tabulated in Table 4.6. Results for a comparison study of rigid and flexible slab are tabulated in Table 4.7. Figure 4.15 shows a typical case of pounding of flexible slab at  $a=L/2$  against the rigid slab case.

In the contact stage, the top ( $u_2$ ) and middle ( $u_1$ ) displacements are moved further than the rigid slab case. This caused a longer contact time (0.0577 sec and 0.05500 sec) than a rigid case (0.04250 sec and 0.04350 sec) for the first and second contact respectively as shown in Table 4.7.

Figure 4.15b shows the displacements on the right hand side of the shear frame, termed  $uf_2$  and  $uf_1$  for the top and middle displacements respectively. It is found that these displacements of a flexible slab case are always less by the factor described in Equation (4.36) than a rigid slab case. This is true for both the no contact and the contact stages. It also shows that, during the contact stage, due to the floor flexibility, the  $uf_2$  and  $uf_1$  are further reduced by the axial deformation of slab namely,  $uk_2$  and  $uk_1$  as in Equation (4.57). As a result, the maximum contact force for the flexible slab is lower than the rigid case (Table 4.7). This is also applied to other value of  $a$  (0,  $L/4$ ,  $L/3$ ,  $3L/4$ ,  $L$ ) as shown in Figures 4.16 to 4.21.

As expected, all the displacements curves at the contact point,  $u_i$ , are not exceed 0.008 m which is the building separation of a rigid adjacent building as shown in Figure 4.22.

Figure 4.23a shows the value of the maximum contact force for various value of  $a$  from Table 4.6. In general, the value of the maximum contact force  $P$  decreases as the value of  $a$  increases. Also the value of the starting time of the contact stage also increases as the value of  $a$  increases (Figure 4.15b).

All the results obtained are verified by results obtained with DYNA3D as shown in Figure (4.16b - 4.22b)



PARAMETERS (UNITS)	UNALIGNED FLOOR				ALIGNED FLOOR	
	CASE a=0	CASE a=L/4	CASE a=L/3	CASE a=L/2		CASE a=3L/4
$k_{AB}$ , (1 - $k_{AB}$ ) (dimensionless)	0 , 1	5/32 , 27/32	7/27 , 20/27	1/2 , 1/2	27/32 , 5/32	1 , 0
$k_p$ (kips/in)	5544.615	5539.372	5292.171	5090.995	6121.545	6435.714
$u_{k2}$ (in)	1.0 ud2 + 0.0 ud1	0.723398826 ud2 + 0.133962745 ud1	0.540634903 ud2 + 0.189222216 ud1	0.250735726 ud2 + 0.250735726 ud1	0.040390133 ud2 + 0.218106722 ud1	0.0 ud2 + 0.107142857 ud1
$u_{k1}$ (in)	0.092307692 ud2 + 0.0 ud1	0.179127328 ud2 + 0.033171727 ud1	0.206263092 ud2 + 0.072192082 ud1	0.218952324 ud2 + 0.218952324 ud1	0.127888267 ud2 + 0.690596646 ud1	0.0 ud2 + 1.0 ud1
$ka^*$ (kips/in)	5000.0	3616.994	2703.175	1253.679	201.951	0.0
$kb^*$ (kips/in)	0.0	669.814	946.111	1253.679	1090.534	535.714
$kc^*$ (kips/in)	461.539	895.637	1031.315	1094.762	639.441	0.0
$kd^*$ (kips/in)	0.0	165.857	360.960	1094.762	3452.983	5000.0
$\bar{k}_c^*$ stiffness matrix (kips/in)	5108.879 -183.796 -183.796 1538.419	3917.474 221.470 221.470 1703.412	3116.412 360.062 360.062 1860.283	1834.894 478.358 478.358 2441.665	872.891 264.062 264.062 4235.901	644.593 -183.796 -183.796 5384.573
$\bar{b}_s$ static preload matrix (kips)	4400.0 600.0 1061.538 -1500.0	3016.994 1269.814 1495.637 -1334.141	2103.175 1546.111 1631.315 -1139.040	653.678 1853.679 1694.762 -405.238	-398.049 1690.534 1239.441 1952.983	-600.0 1135.714 600.0 3500.0
$\omega_{1c}$ , $\omega_{2c}$ (rad/sec)	31.939 , 71.515	33.525 , 62.683	34.572 , 56.226	36.432 , 46.210	36.432 , 46.210	5.238 , 59.978

Table 4.5 : Summary of parameters during contact stage of MDOF systems for case a= 0, L/4, L/3, L/2, 3L/4 and L.

DISTANCE "a"		1st CONTACT		2nd CONTACT	
		FORTRAN	DYNA3D	FORTRAN	DYNA3D
0	Contact time (sec)	0.08300-0.12550 (0.04250)	0.08300-0.12501 (0.04201)	0.31100-0.34959 (0.03859)	0.30901-0.34600 (0.03699)
	Contact forces (kN)	5,869.711	5,767.690	6,878.545	6,265.340
L/4	Contact time (sec)	0.08450-0.13170 (0.04720)	0.08400-0.13100 (0.04700)	0.31270-0.35699 (0.04429)	0.31100-0.35399 (0.04299)
	Contact forces (kN)	6,028.951	5,867.500	6,782.760	6,249.790
L/3	Contact time (sec)	0.08600-0.13740 (0.05140)	0.08600-0.13600 (0.05000)	0.31590-0.36509 (0.04919)	0.31300-0.36000 (0.04700)
	Contact forces (kN)	5,871.608	5,709.840	6,348.909	5,881.250
L/2	Contact time (sec)	0.08900-0.14670 (0.05770)	0.08900-0.14400 (0.05500)	0.32420-0.38209 (0.05789)	0.32200-0.37800 (0.05600)
	Contact forces (kN)	5,550.488	5,316.260	5,539.060	5,325.780
3L/4	Contact time (sec)	0.09500-0.14840 (0.05340)	0.09500-0.14800 (0.05300)	0.34190-0.39089 (0.04899)	0.34200-0.39000 (0.04800)
	Contact forces (kN)	4,787.283	4,545.070	4,875.177	4,517.500
L	Contact time (sec)	0.09850-0.14630 (0.04780)	0.09801-0.14500 (0.04699)	0.34930-0.39009 (0.04079)	0.34801-0.38901 (0.04100)
	Contact forces (kN)	3,928.172	3,809.890	3,643.063	3,437.520

Table 4.6 : Overall results for the contact stages of flexible slab of MDOF systems

DISTANCE "a"		FLEXIBLE SLAB		RIGID SLAB	
		FORTRAN	DYNA3D	FORTRAN	DYNA3D
L/2	Contact time (sec)	0.08900-0.14670 (0.05770)	0.08900-0.14400 (0.05500)	0.08650-0.12900 (0.04250)	0.08650-0.13000 (0.04350)
	1st contact forces (kN)	5,550.488	5,316.260	8,651.723	7,989.020
	Contact time (sec)	0.32420-0.38209 (0.05789)	0.32200-0.37800 (0.05600)	0.30150-0.34379 (0.04229)	0.30400-0.34800 (0.04400)
	2 nd contact forces (kN)	5,539.060	5,325.780	8,652.834	8,109.940

Table 4.7 : Results for the contact stage of flexible and rigid slab of MDOF systems

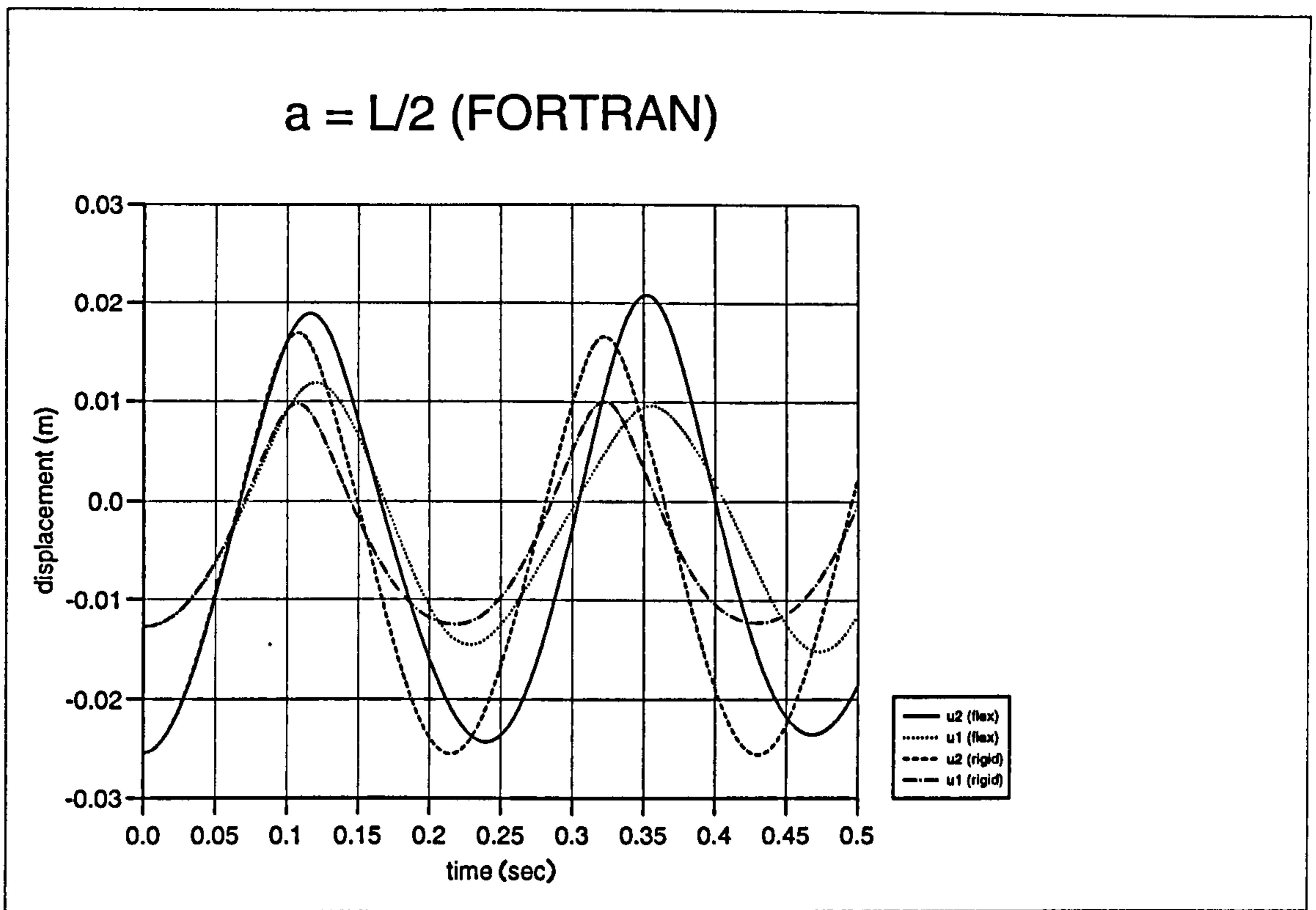


Figure 4.15a) : Displacement curves for pounding case of flexible and rigid slab.

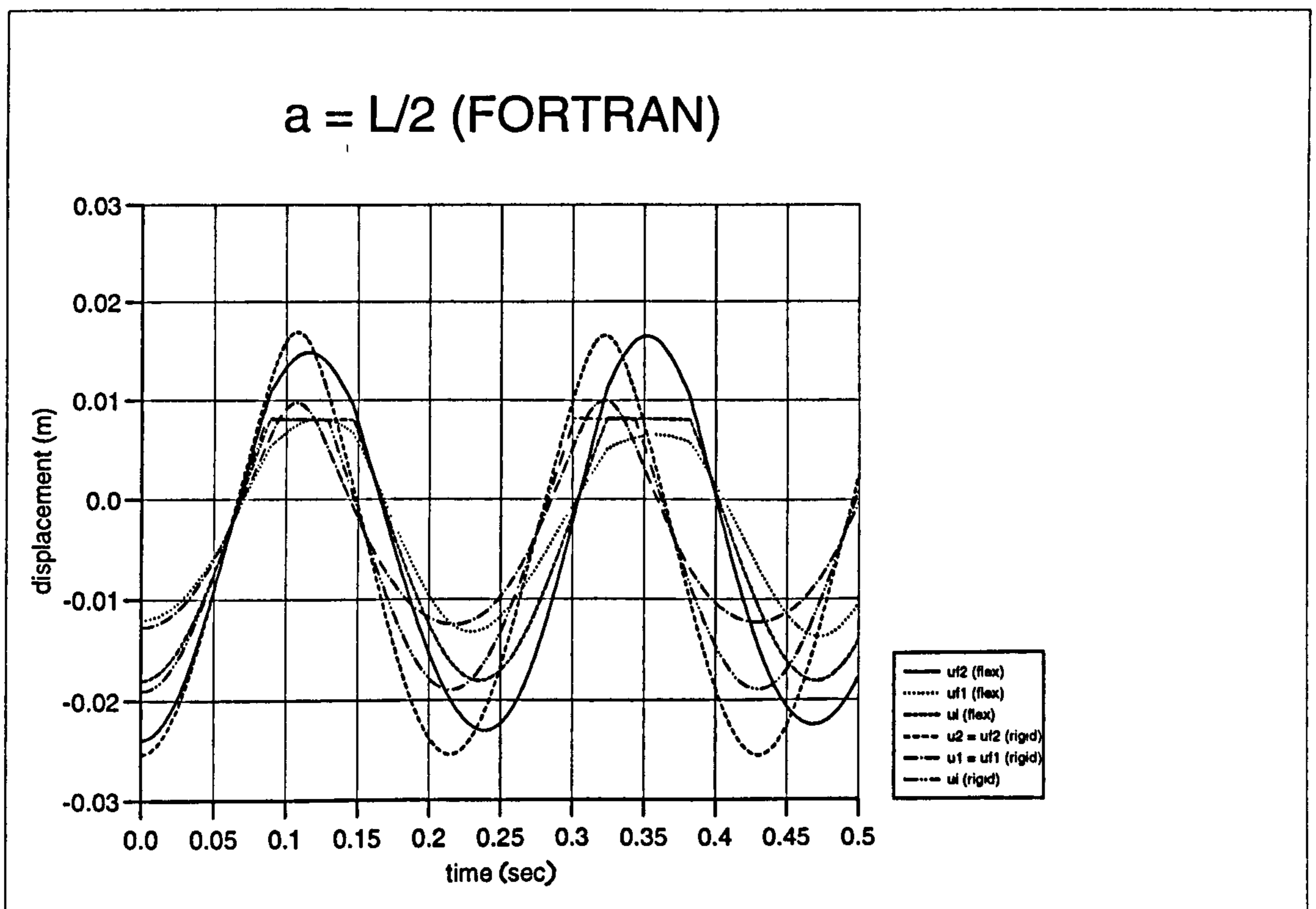


Figure 4.15b) : Displacement curves for pounding case of flexible and rigid slab.

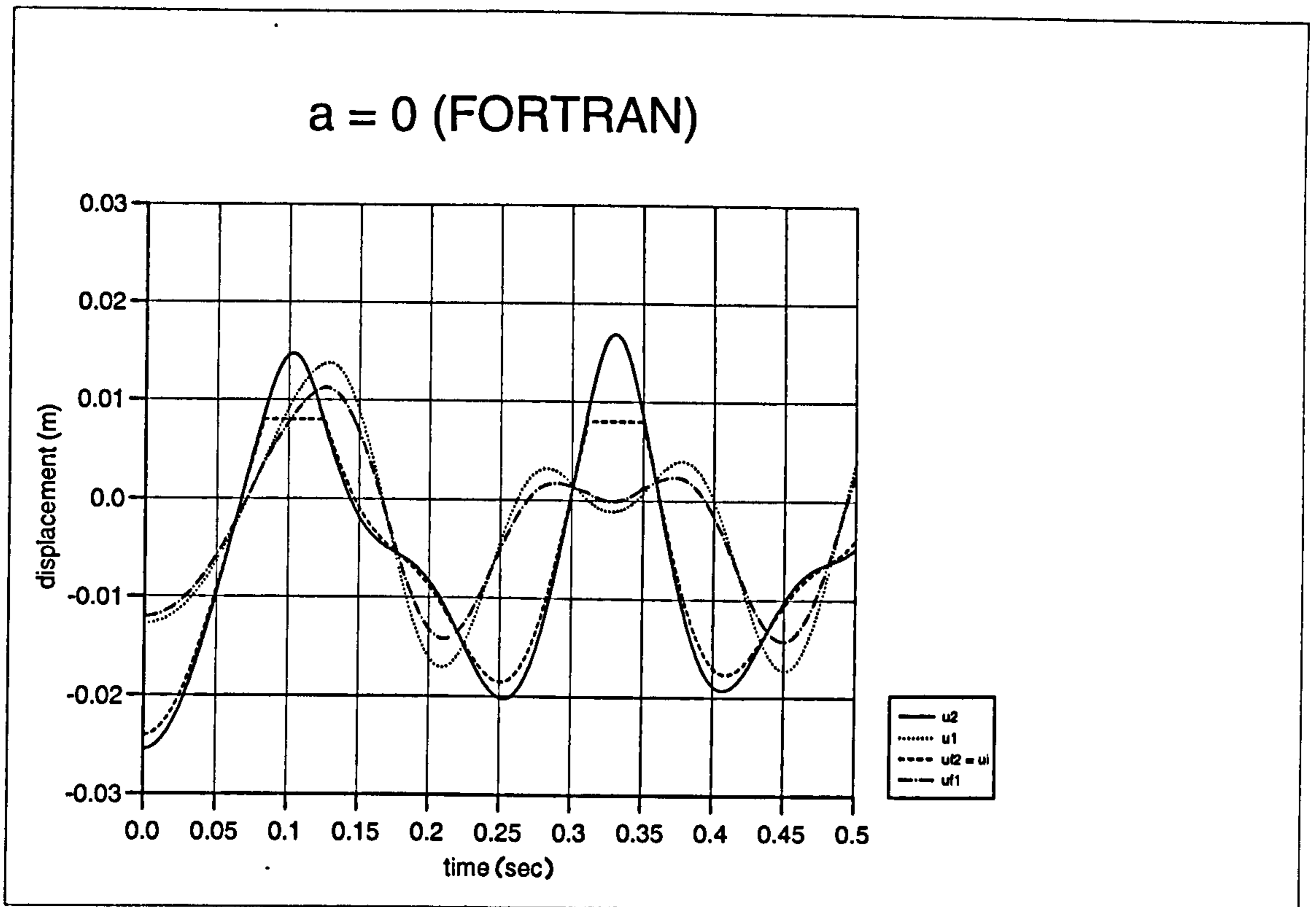


Figure 4.16a) : Displacement curves for pounding case at a = 0 (FORTRAN)

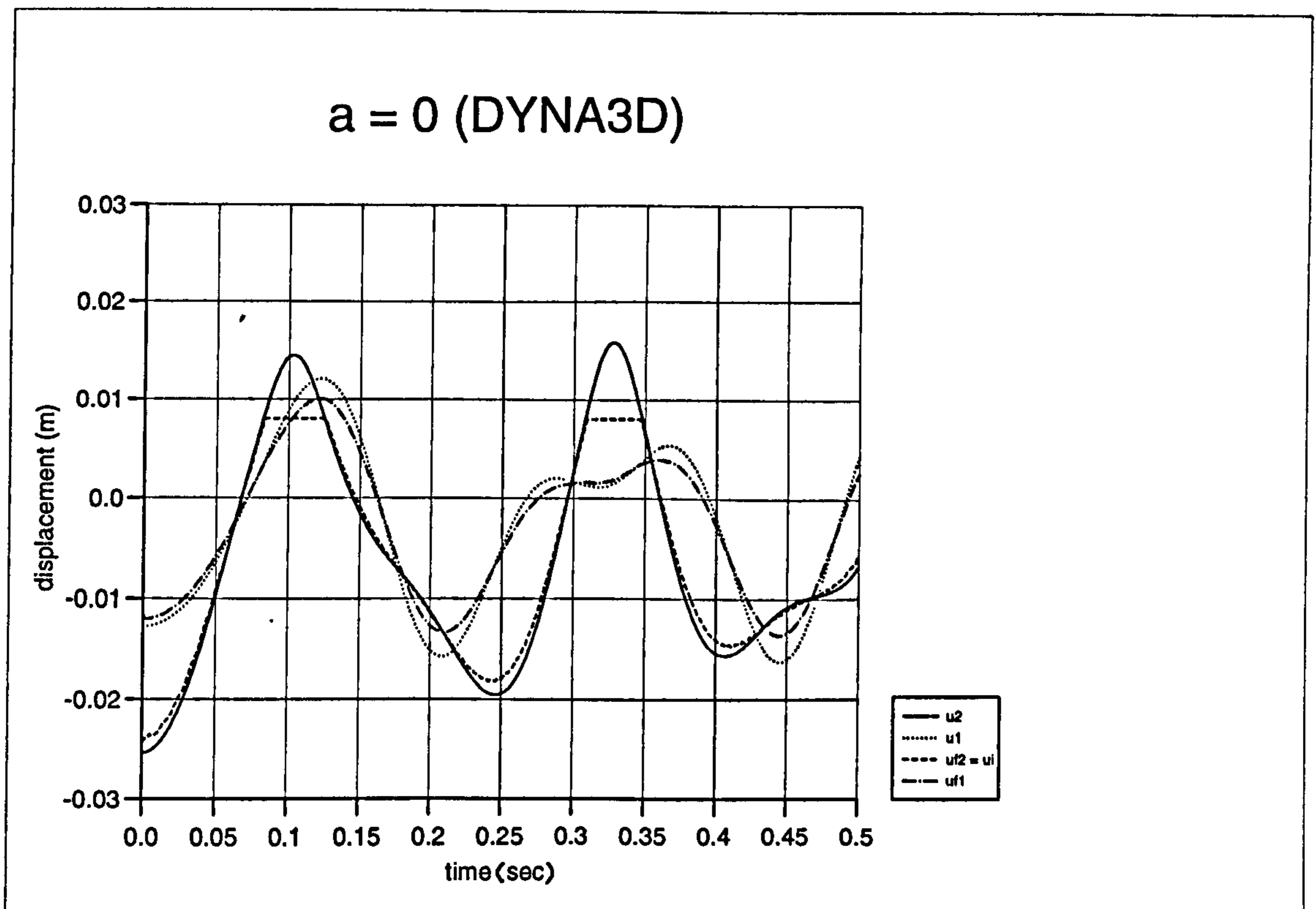


Figure 4.16b) : Displacement curves for pounding case at a = 0 (DYNA3D)

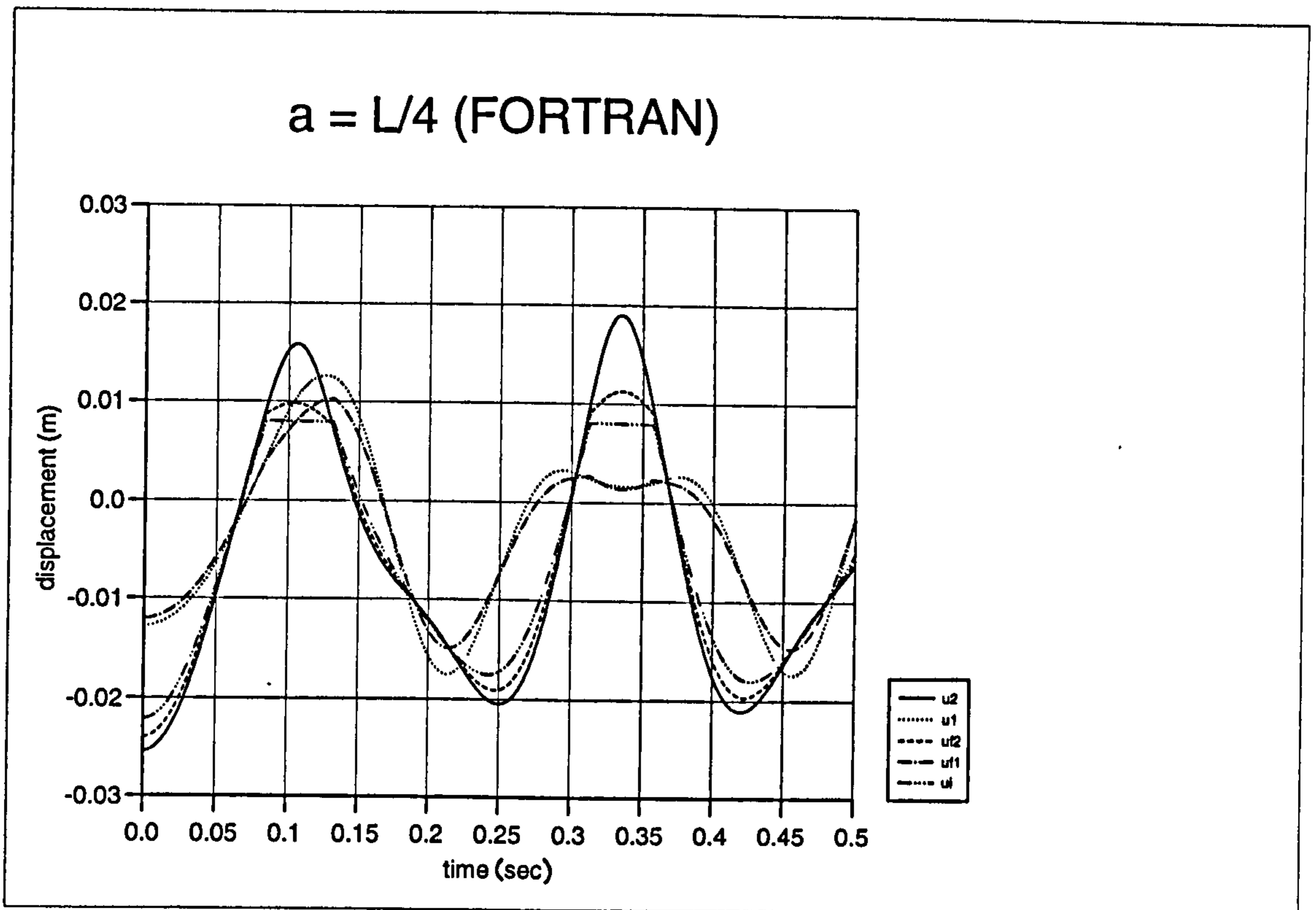


Figure 4.17a) : Displacement curves for pounding case at  $a = L/4$  (FORTRAN)

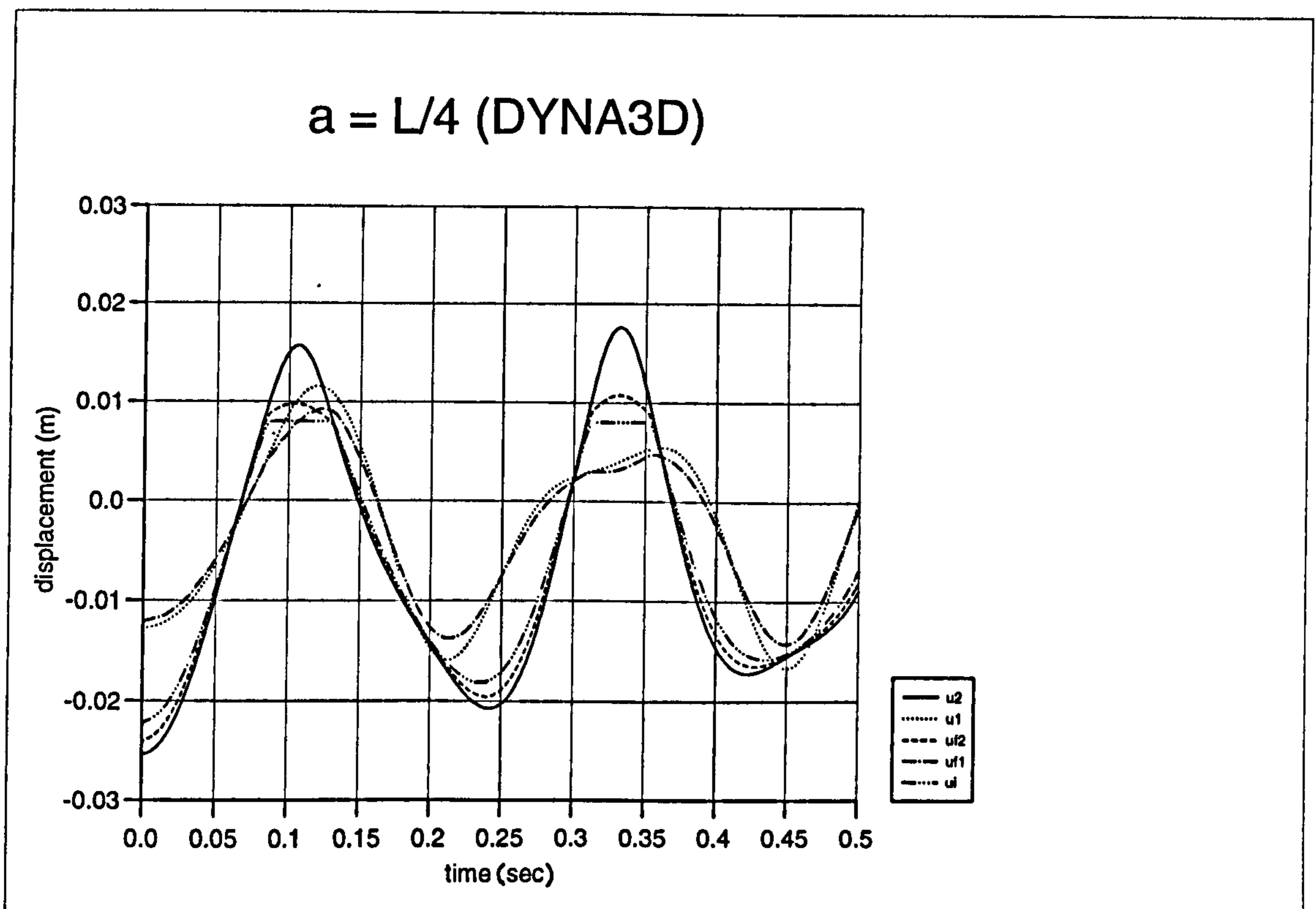


Figure 4.17b) : Displacement curves for pounding case at  $a = L/4$  (DYNA3D)

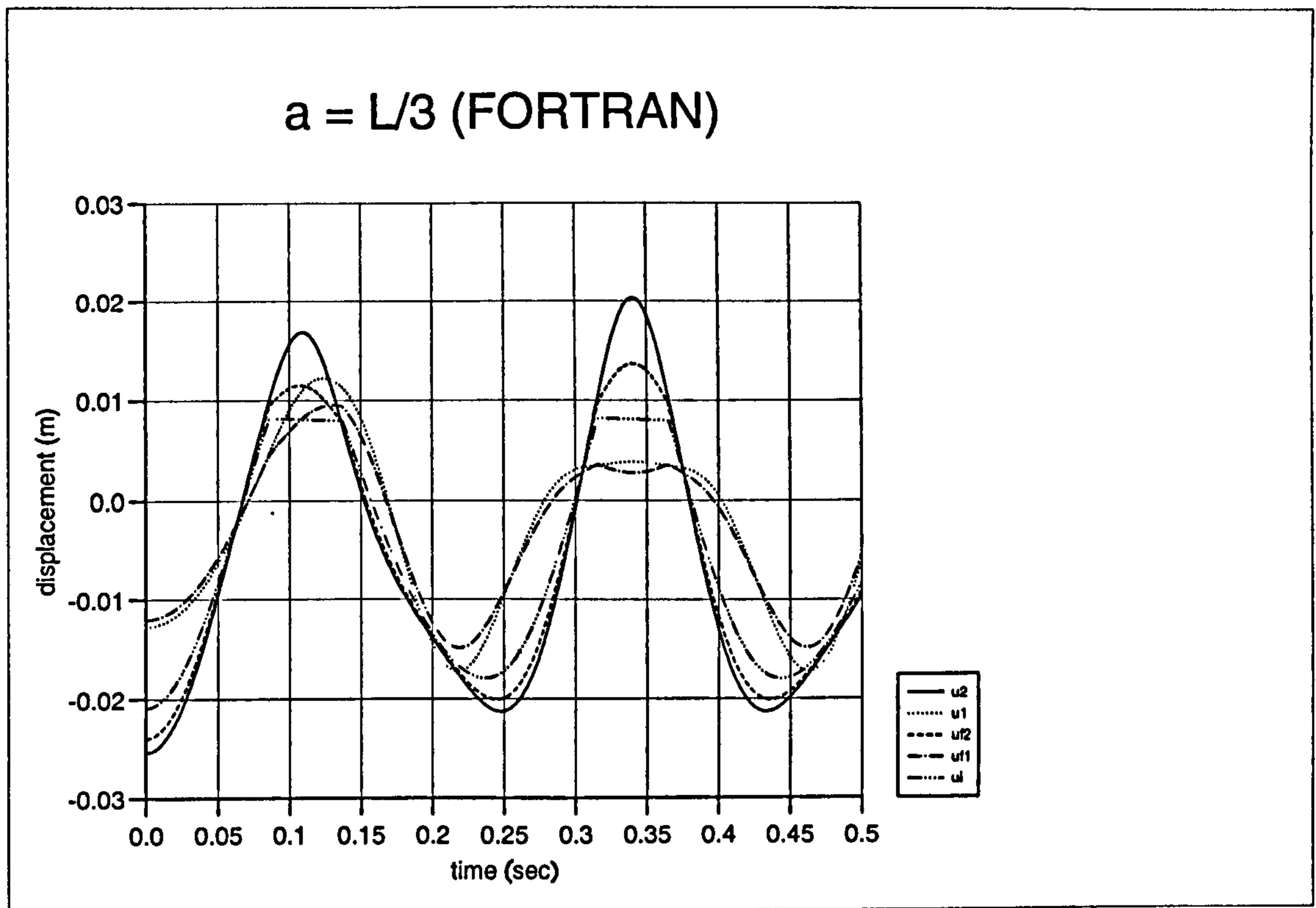


Figure 4.18a) : Displacement curves for pounding case at  $a = L/3$  (FORTRAN)

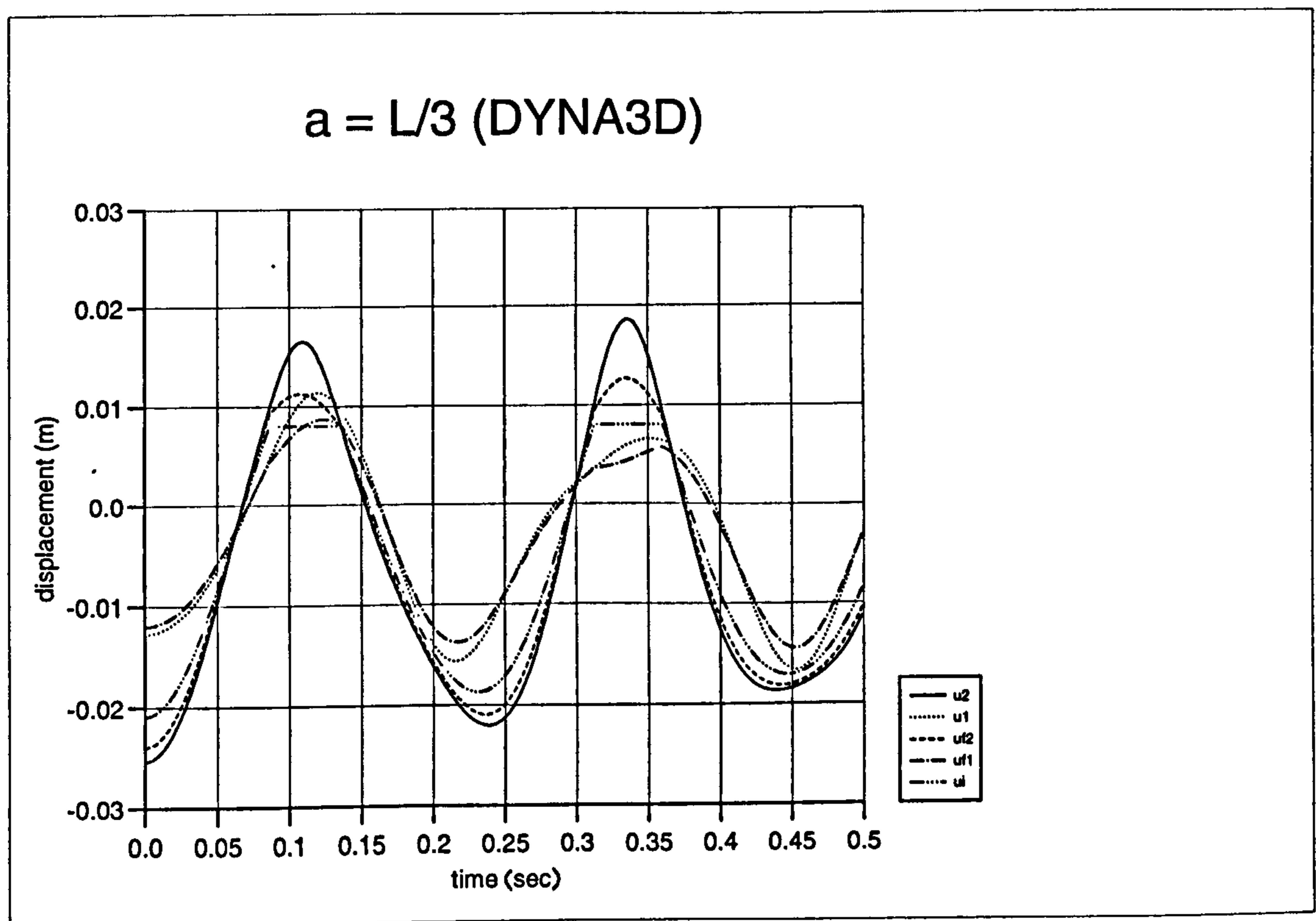


Figure 4.18b) : Displacement curves for pounding case at  $a = L/3$  (DYNA3D)

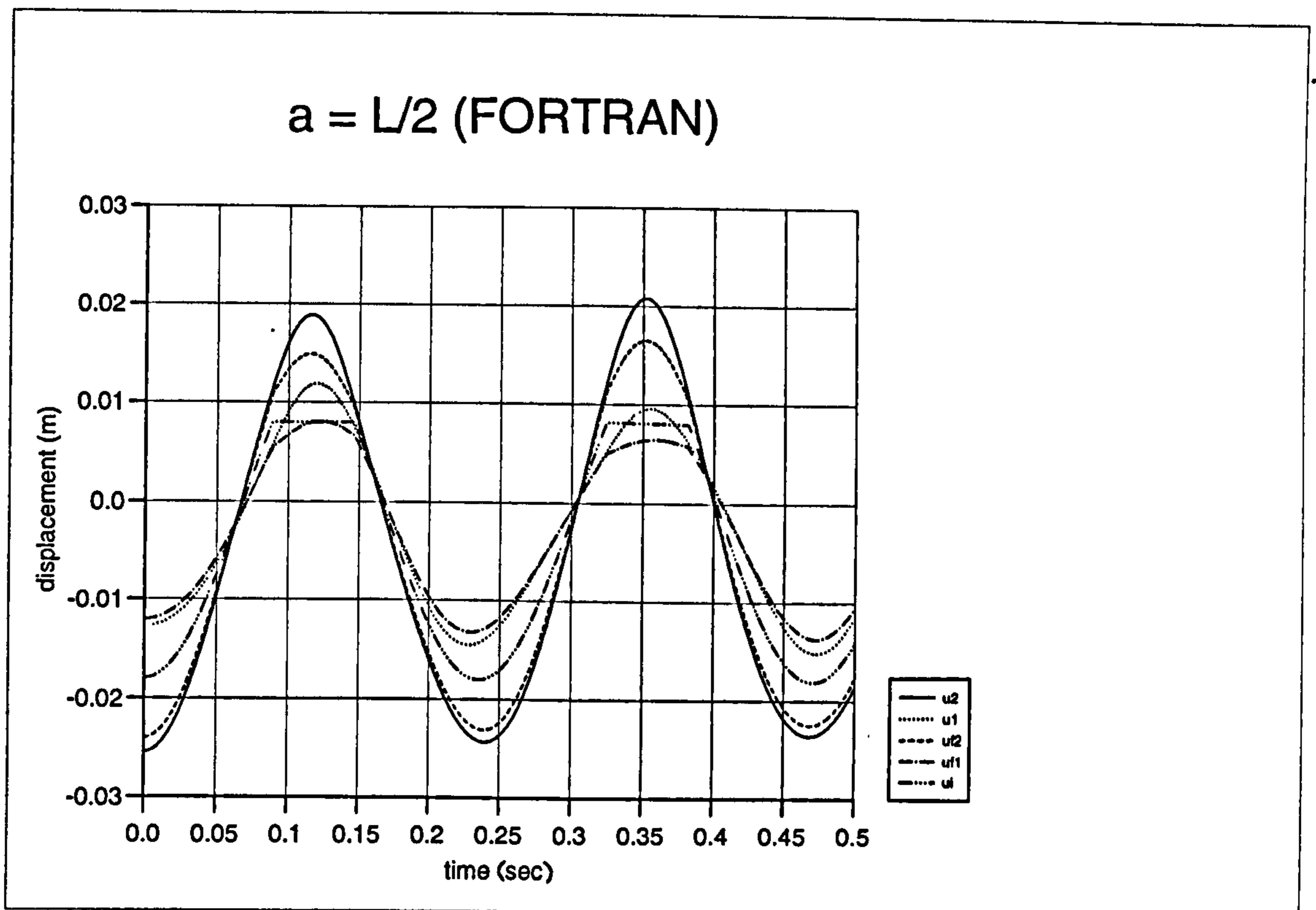


Figure 4.19a) : Displacement curves for pounding case at  $a = L/2$  (FORTRAN)

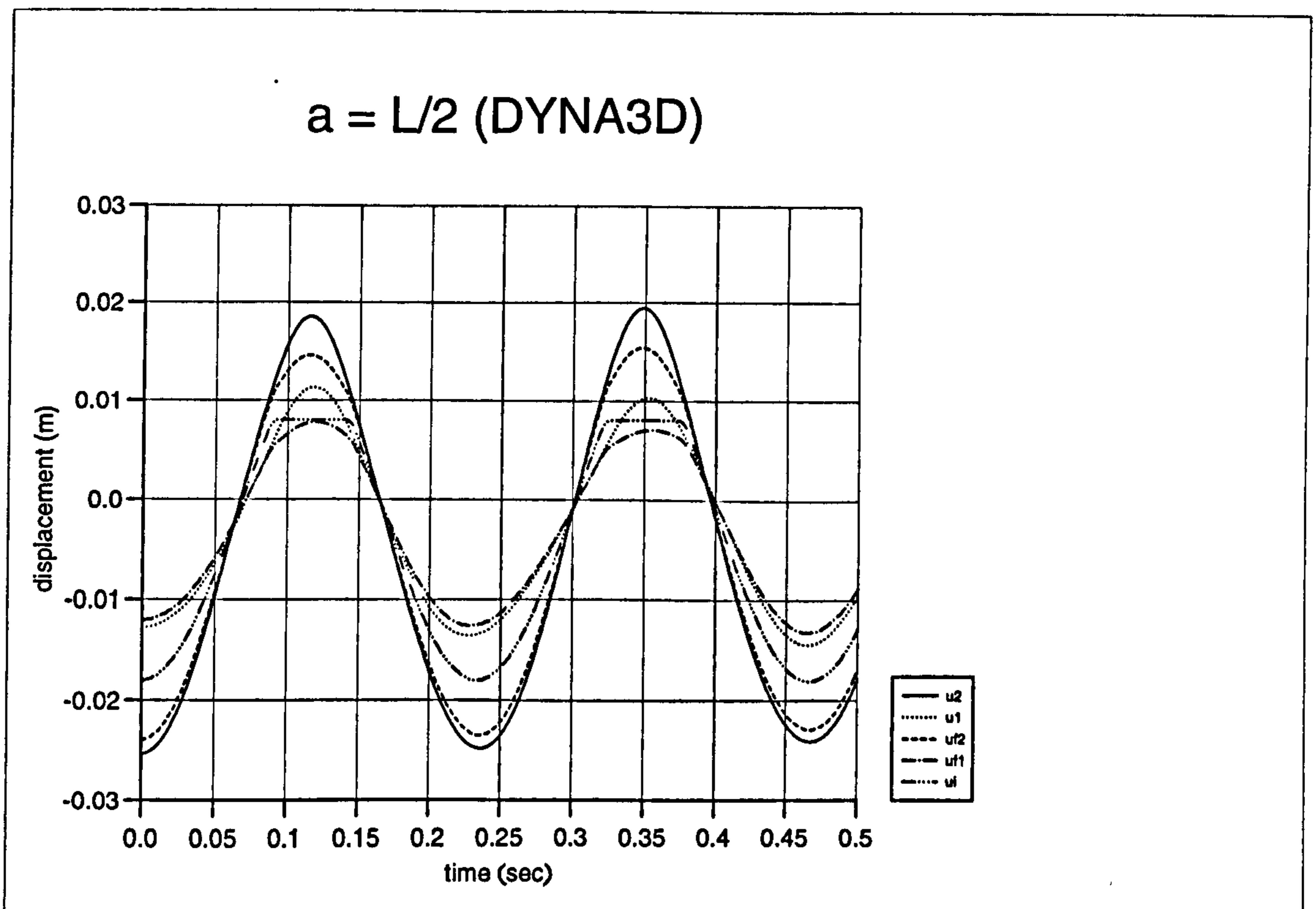


Figure 4.19b) : Displacement curves for pounding case at  $a = L/2$  (DYNA3D)

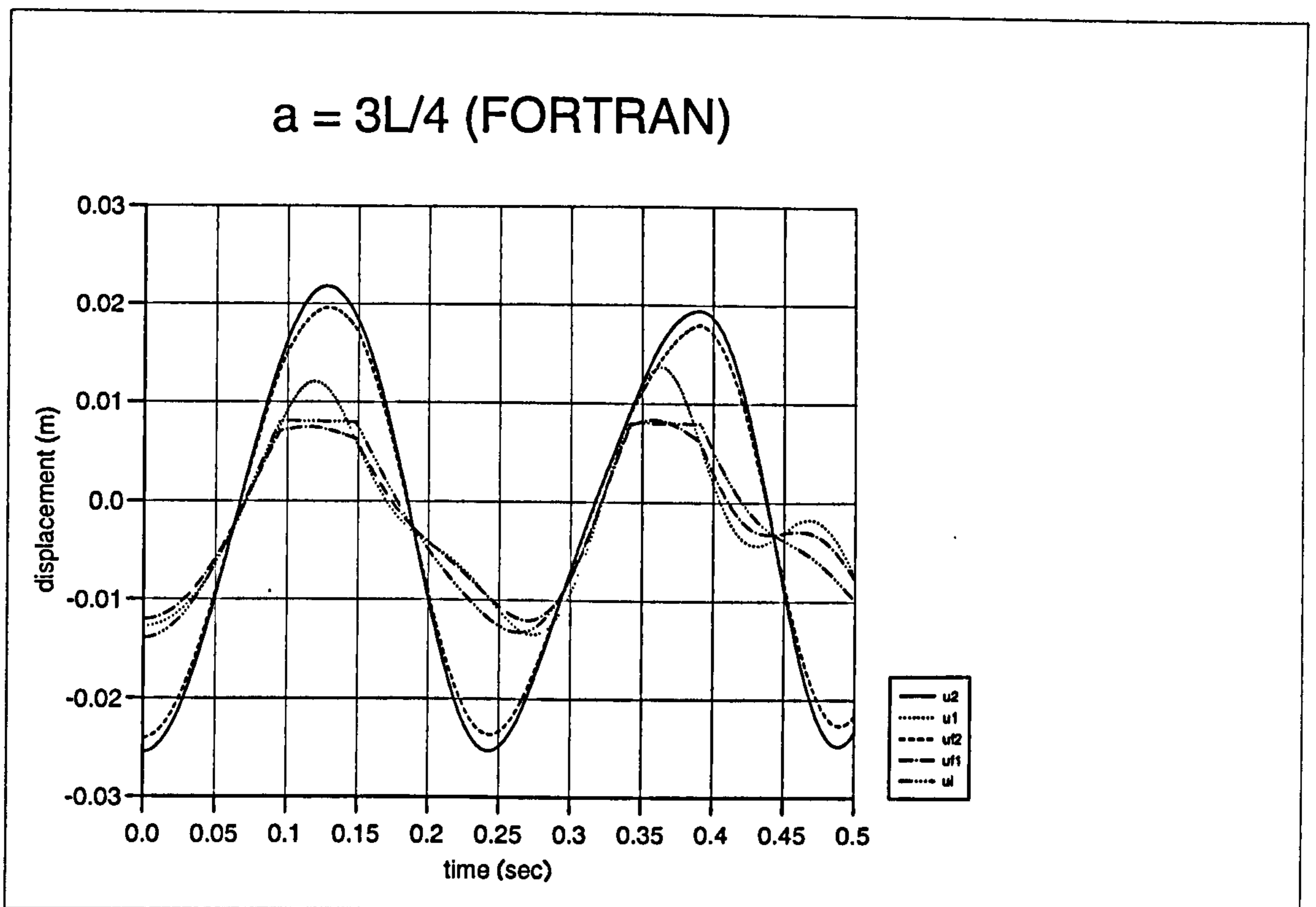


Figure 4.20a) : Displacement curves for pounding case at  $a = 3L/4$  (FORTRAN)

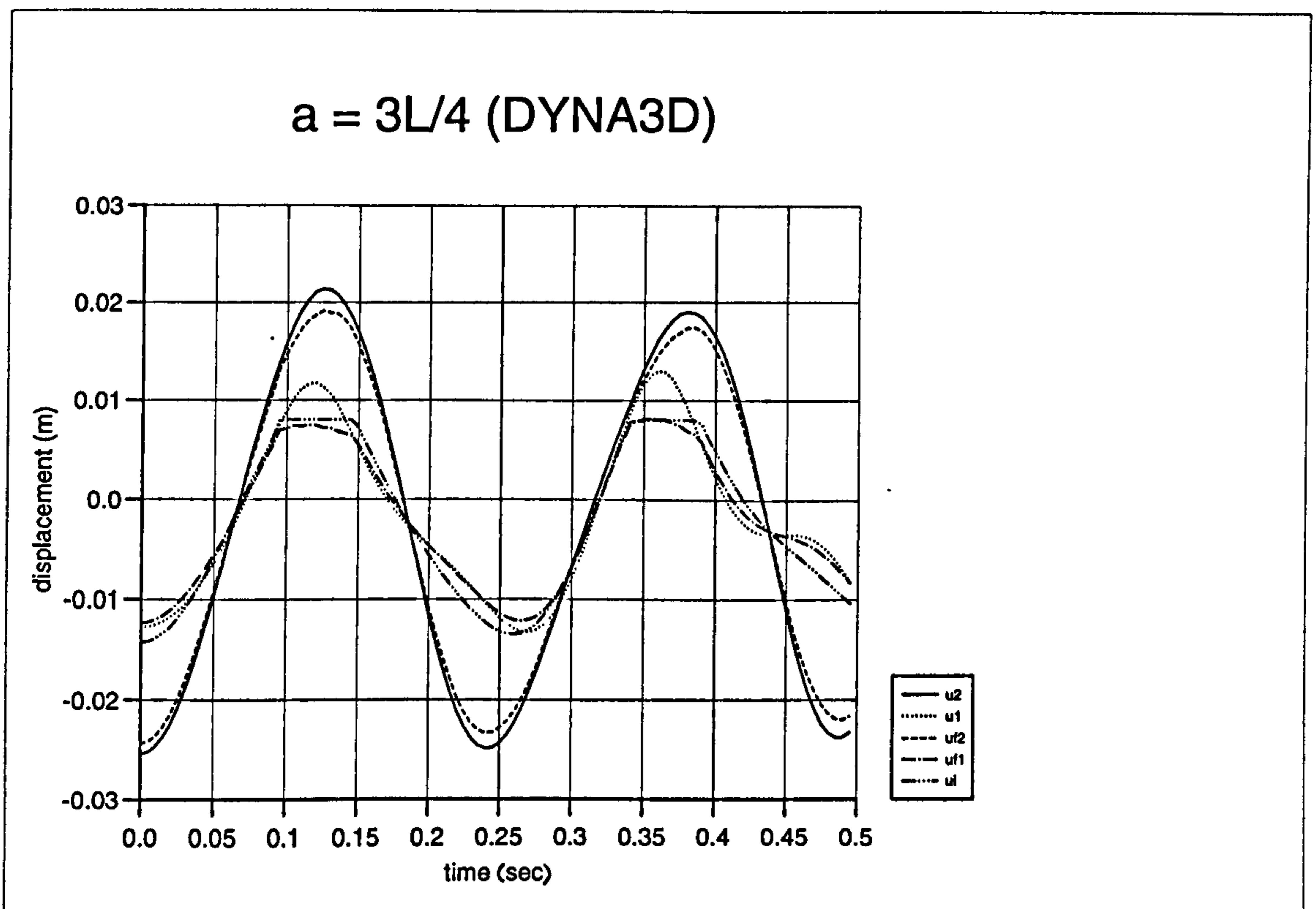


Figure 4.20b) : Displacement curves for pounding case at  $a = 3L/4$  (DYNA3D)



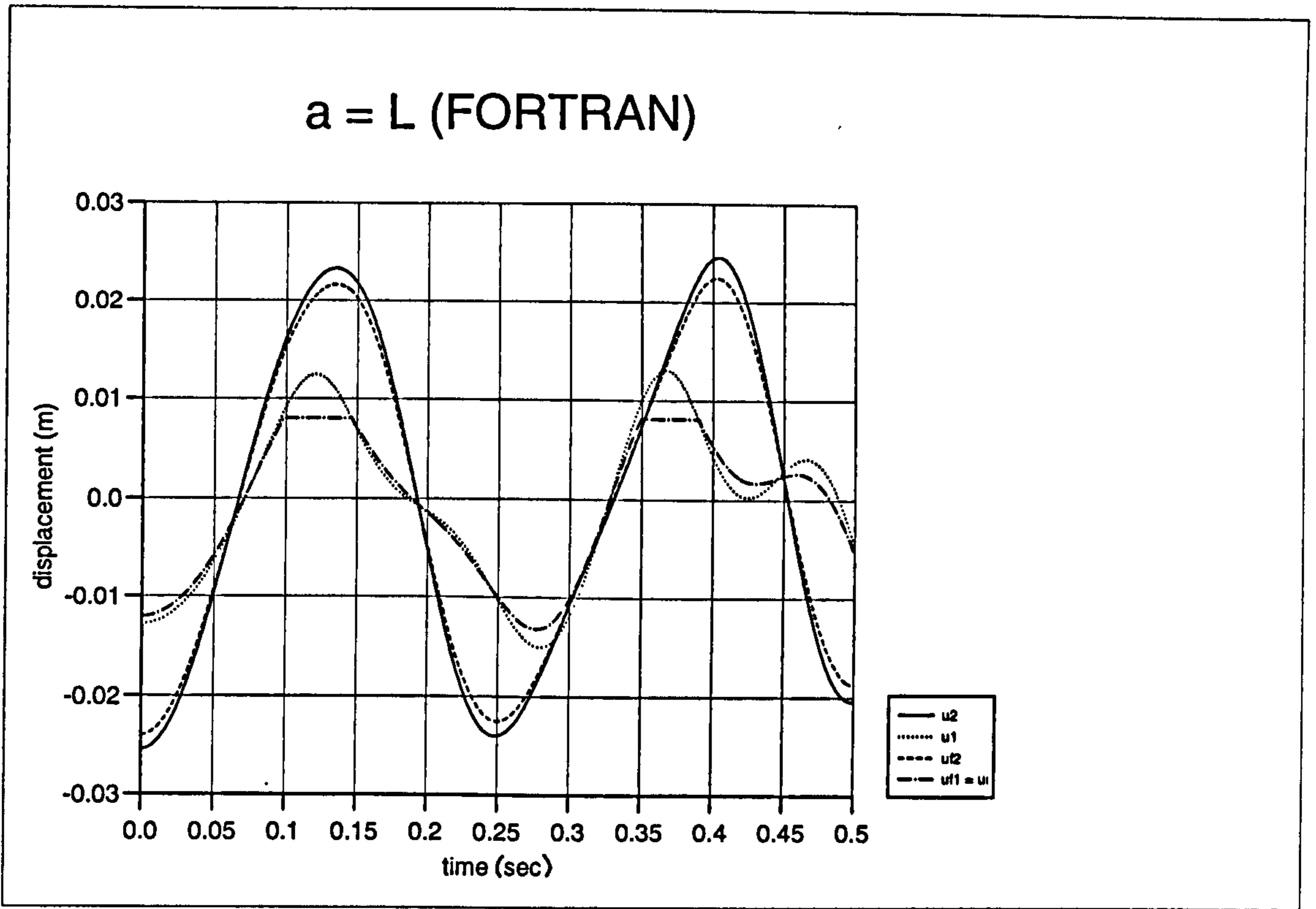


Figure 4.21a) : Displacement curves for pounding case at  $a = L$  (FORTRAN)

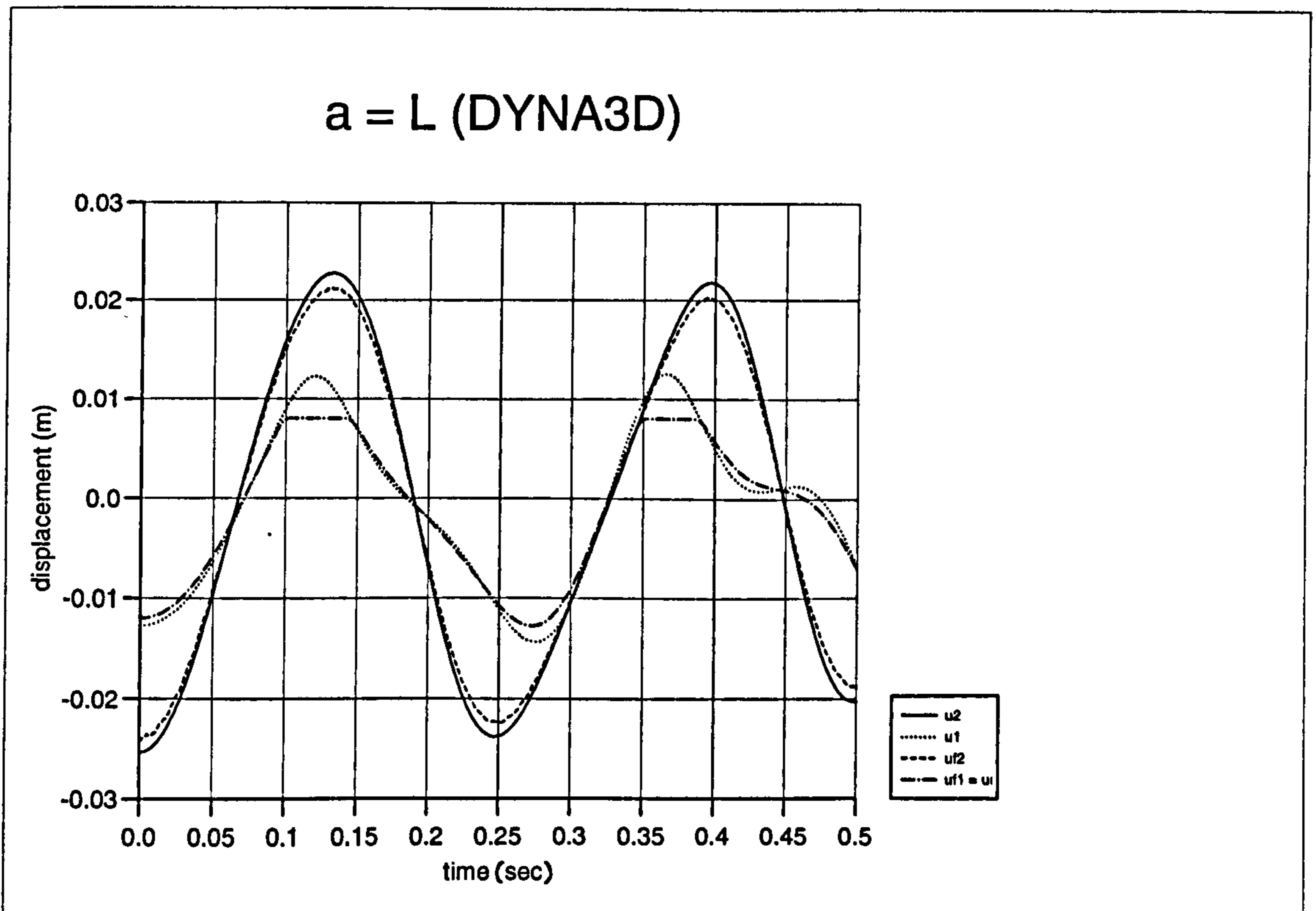


Figure 4.21b) : Displacement curves for pounding case at  $a = L$  (DYNA3D)

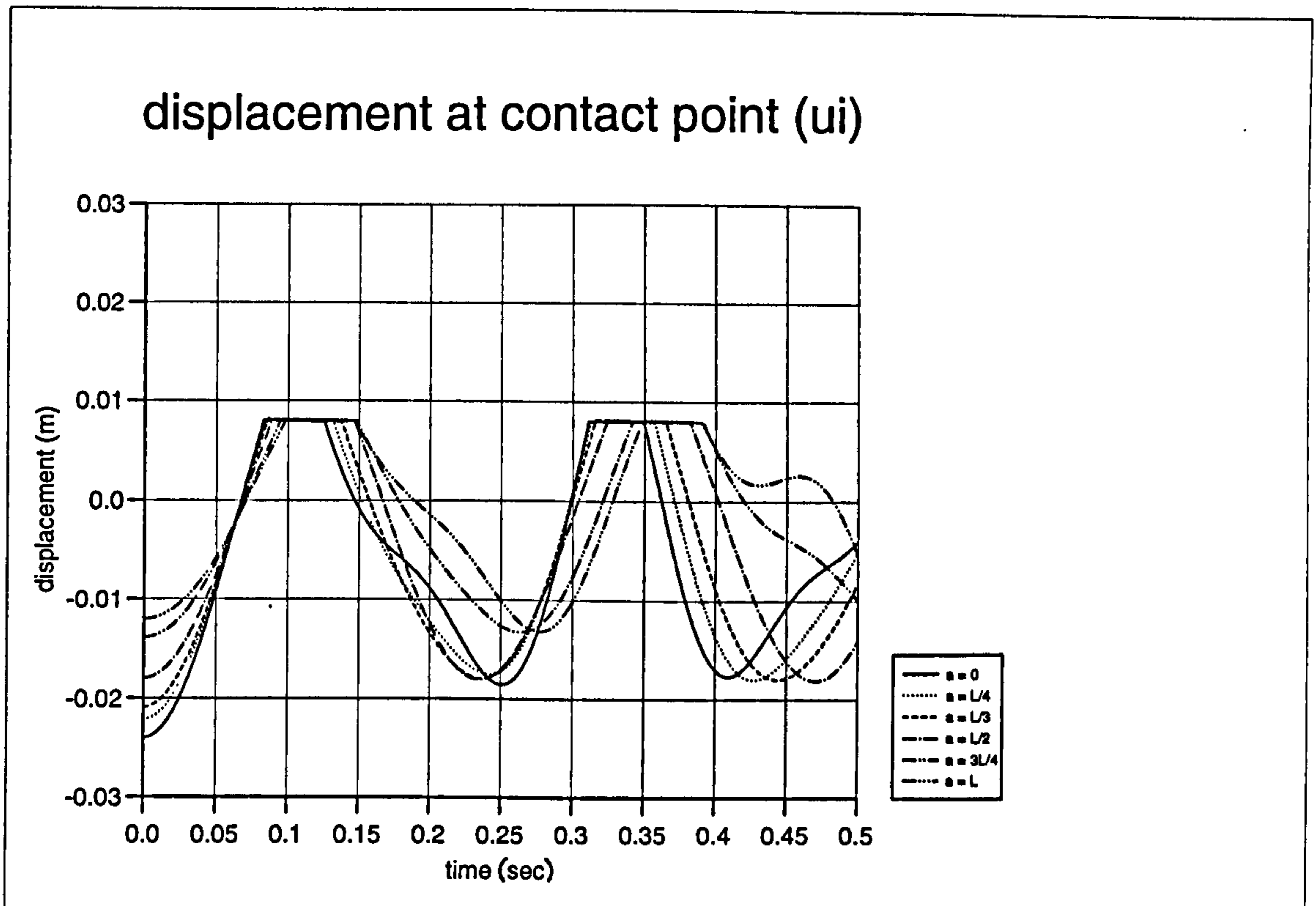


Figure 4.22a) : Displacement curves of  $u_i$  for different "a" values (FORTRAN)

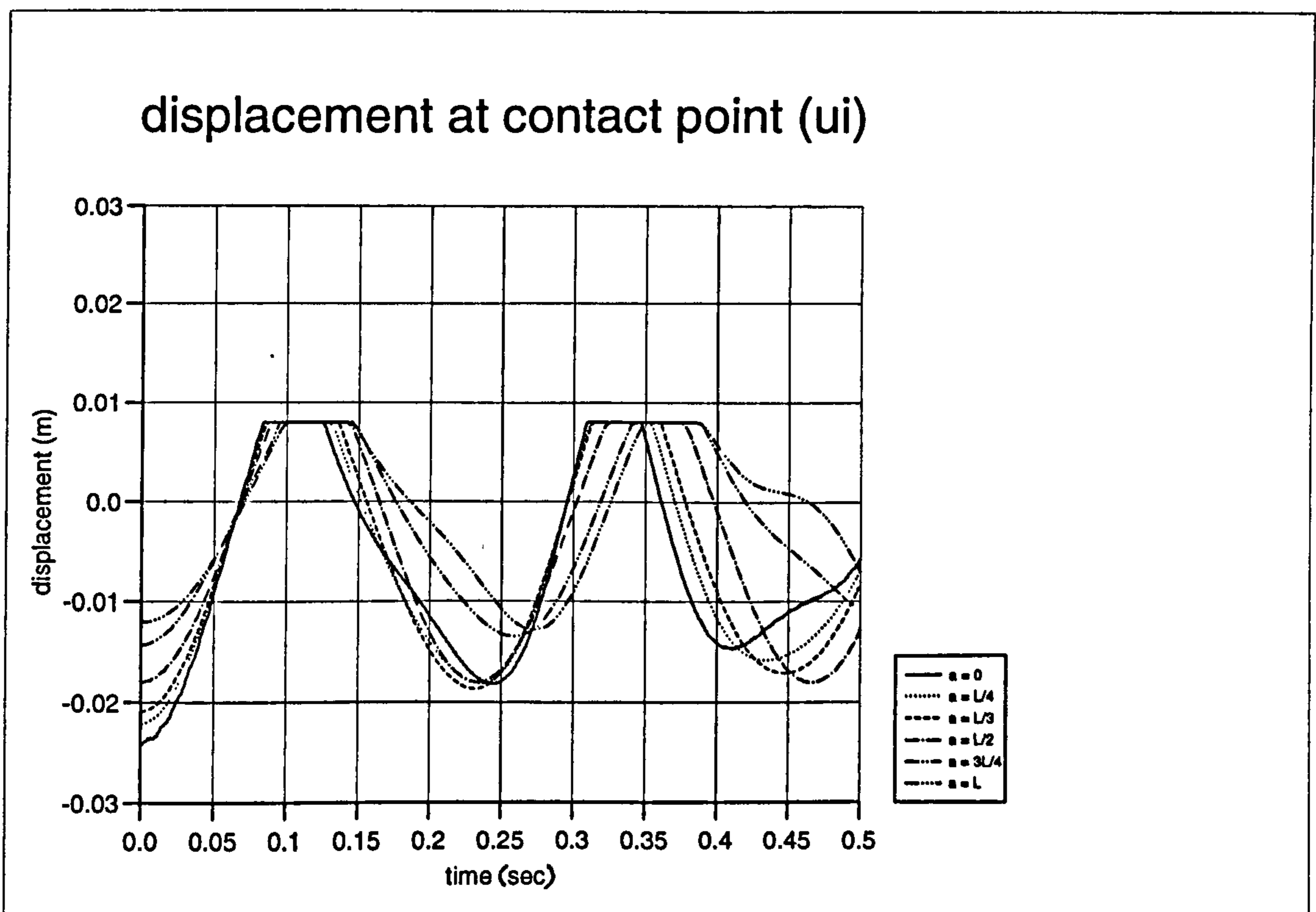


Figure 4.22b) : Displacement curves of  $u_i$  for different "a" values (DYNA3D)

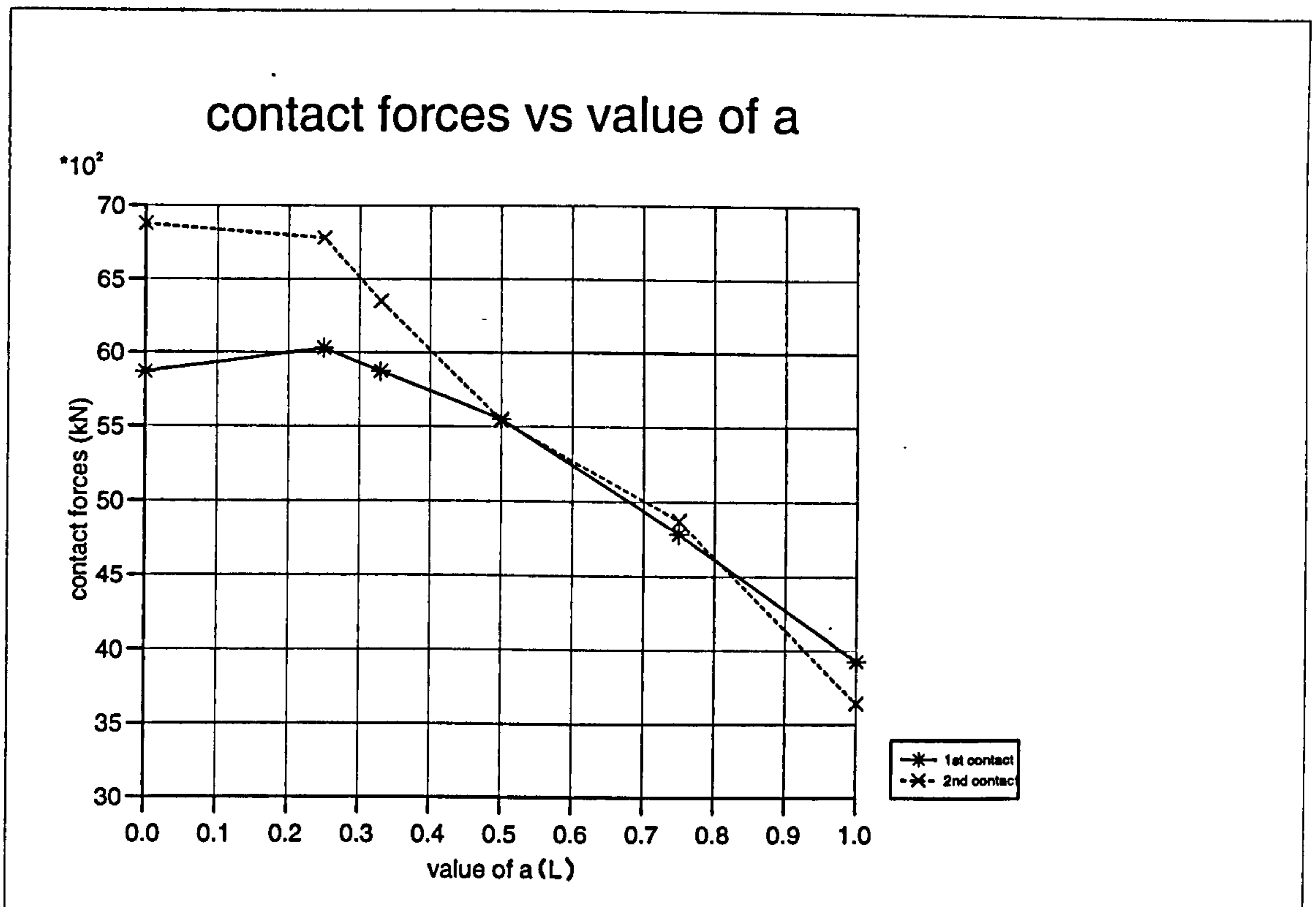


Figure 4.23a) : Maximum contact forces against variation of "a" (FORTRAN)

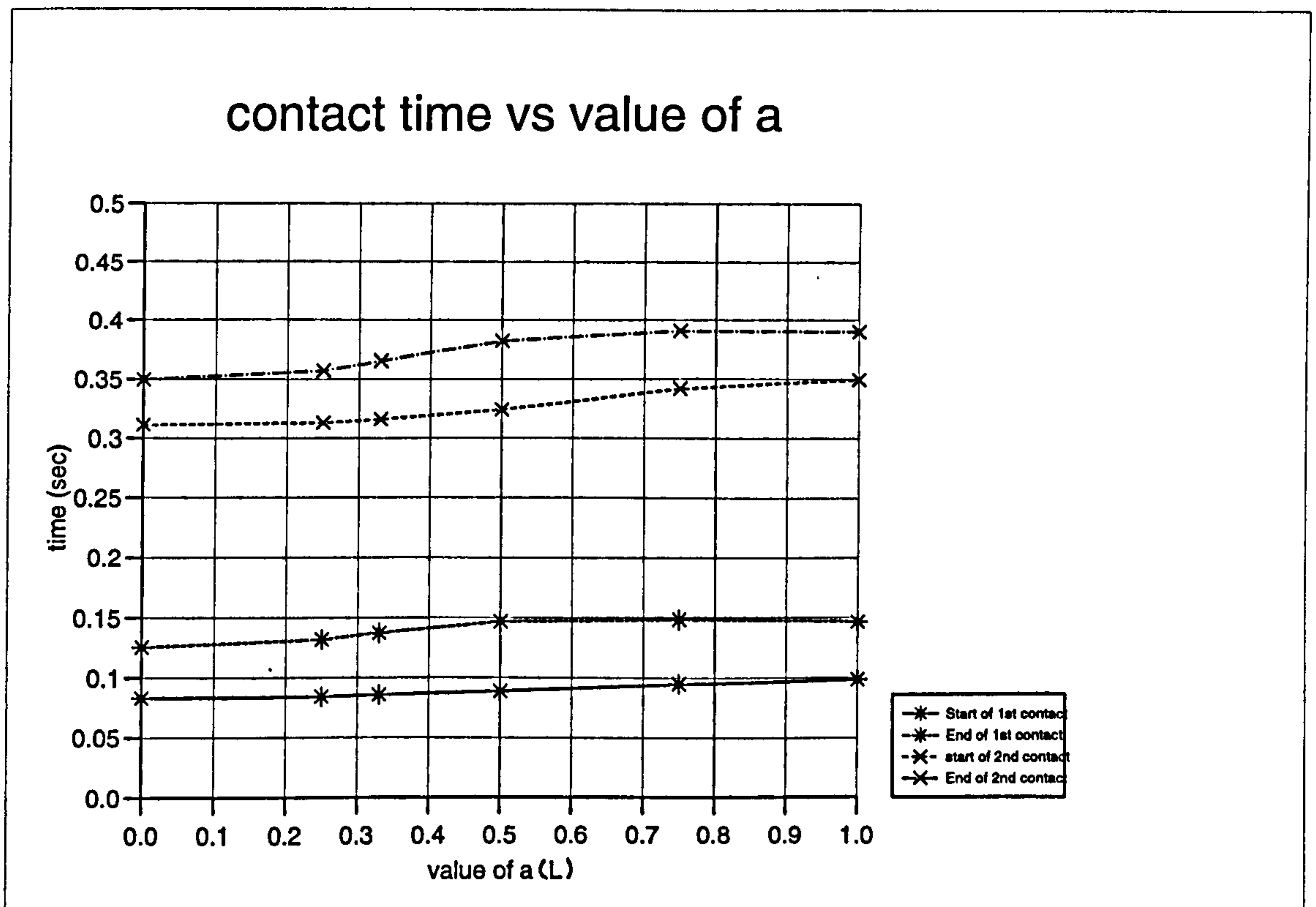


Figure 4.23b) : Contact stage time against variation of "a" (FORTRAN)

## CHAPTER 5

### POUNDING OF TWO FLEXIBLE BUILDINGS WITH RIGID SLABS

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#### 5.1 INTRODUCTION

In previous chapters, pounding is considered between a flexible building and a rigid adjacent building. In this chapter, further applications of the displacement compatibility method for the more realistic case of pounding, in an urban area, is presented between two flexible buildings due to a small gap, between them. Two such buildings usually do not have the same physical or dynamic properties.

Two cases of pounding of two flexible buildings, both aligned and unaligned floor cases of a single degree of freedom (SDOF) system and multi degree of freedom (MDOF) systems are considered.

#### 5.2 SDOF SYSTEM

The basic derivation of the equation of motions for pounding between two flexible buildings is best illustrated as a SDOF system where both aligned and unaligned floor cases are considered. The solution for the no contact stage will be the same for both cases.

##### 5.2.1 No Contact Stage

Two SDOF system buildings considered in this study are shown in Figure 5.1 below. The buildings on the left hand side and right hand side of Figure 5.1 are labelled Building A and B respectively. Building A and B have masses and column stiffness of  $m_1$ ,  $m_2$ ,  $k_1$  and  $k_2$  respectively. The slab stiffness are  $ks_1$  and  $ks_2$  for building A and B. The buildings are separated at a distance  $Us$ .

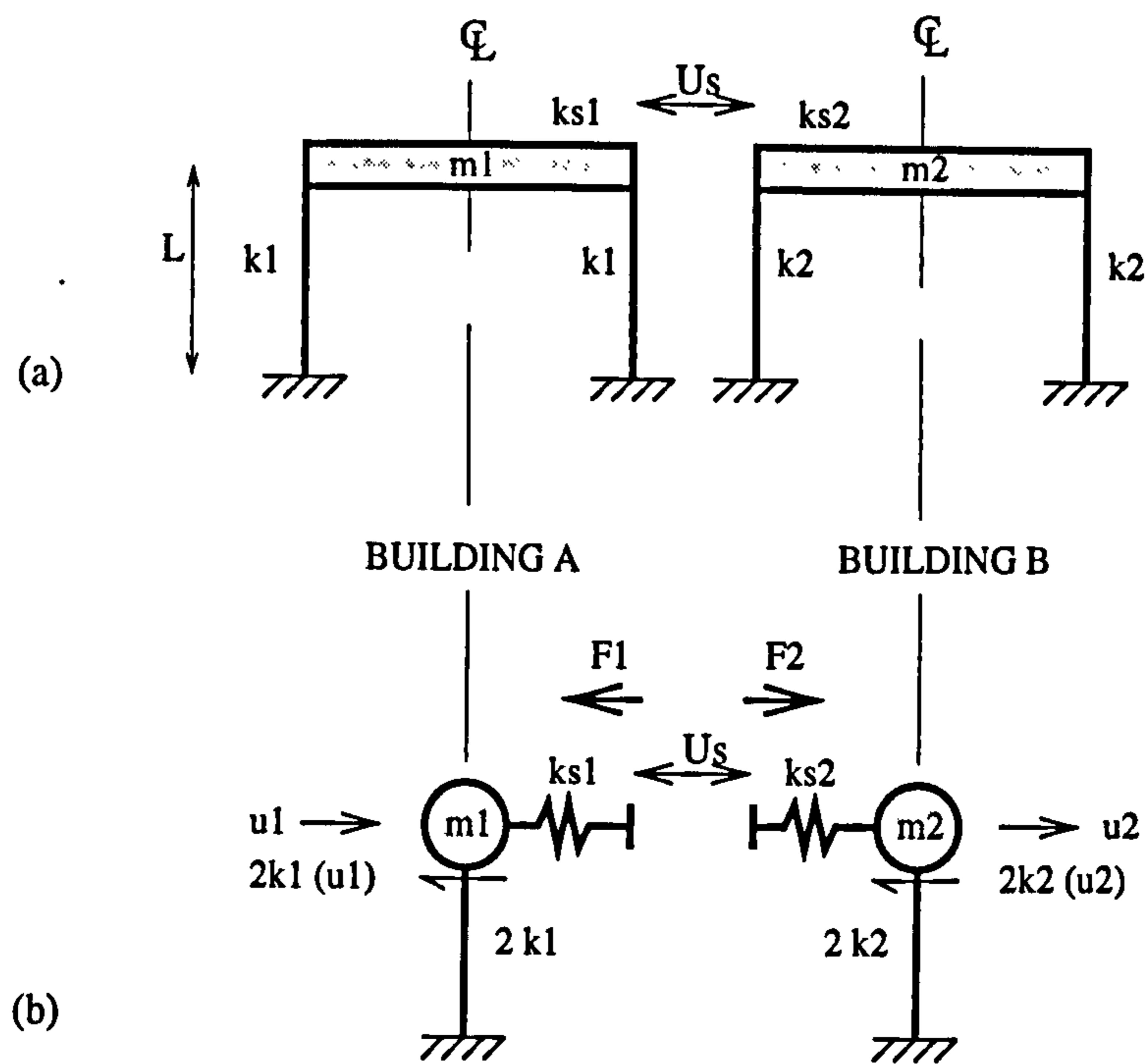


Figure 5.1 : (a) Pounding of two SDOF systems and (b) The shear forces diagram (Aligned floor case).

The derivation of the equation of motions are

$$\begin{aligned} \text{BUILDING A} \quad m_1 \ddot{u}_1 &= -2k_1 u_1 \\ \text{BUILDING B} \quad m_2 \ddot{u}_2 &= -2k_2 u_2 \end{aligned} \quad (5.1)$$

Rearrange in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 2k_1 & 0 \\ 0 & 2k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = 0 \quad (5.2)$$

In general these can be written in the form of :

$$m \ddot{u}_i + k u_i = 0 \quad (5.3)$$

These equations are transformed into a set of uncoupled equations through modes shapes generalized displacement, orthogonality and normalising with respect to mass,  $\underline{m}$  i.e.(if  $j=i$ ,  $Q_j^T \underline{m} Q_i = 1$ ) and become:

$$\ddot{y}_i + \omega_i^2 y_i = 0 \quad (5.4)$$

The solutions of these equation are same as in previous chapter where:

$$\begin{aligned} y_i &= \frac{\dot{y}_t}{\omega_i} \sin \omega_i dt + y_t \cos \omega_i dt \\ \dot{y}_i &= \dot{y}_t \cos \omega_i dt - \omega_i y_t \sin \omega_i dt \end{aligned} \quad (5.5)$$

The total displacements and velocities response are being the sum of the modal contributions of the  $n$ th mode shapes:

$$u_i = \sum_{i=1}^n \alpha_i y_i \quad \text{and} \quad \dot{u}_i = \sum_{i=1}^n \alpha_i \dot{y}_i \quad (5.6)$$

These equations are applied to both aligned and unaligned cases of the SDOF system in the no contact stage.

## 5.2.2 Contact stage

The derivation of the equation of the motion, during the contact stage for the aligned and unaligned cases are treated separately. This is because for the aligned case, it is simply a direct calculation of the forces on the floor and for the unaligned case, the displacement compatibility method derived in Chapter 3 is adopted.

### 5.2.2.1 Contact Stage (Aligned floor case)

To find the contact forces, first the forces  $F_1$  and  $F_2$  at the floor level for building A and B, are calculated using the Newton's law of equilibrium i.e. the application of external force or action, will results in an equal and opposite reaction. The schematic diagram of the two buildings during the contact stage are shown in Figure 5.2 below:

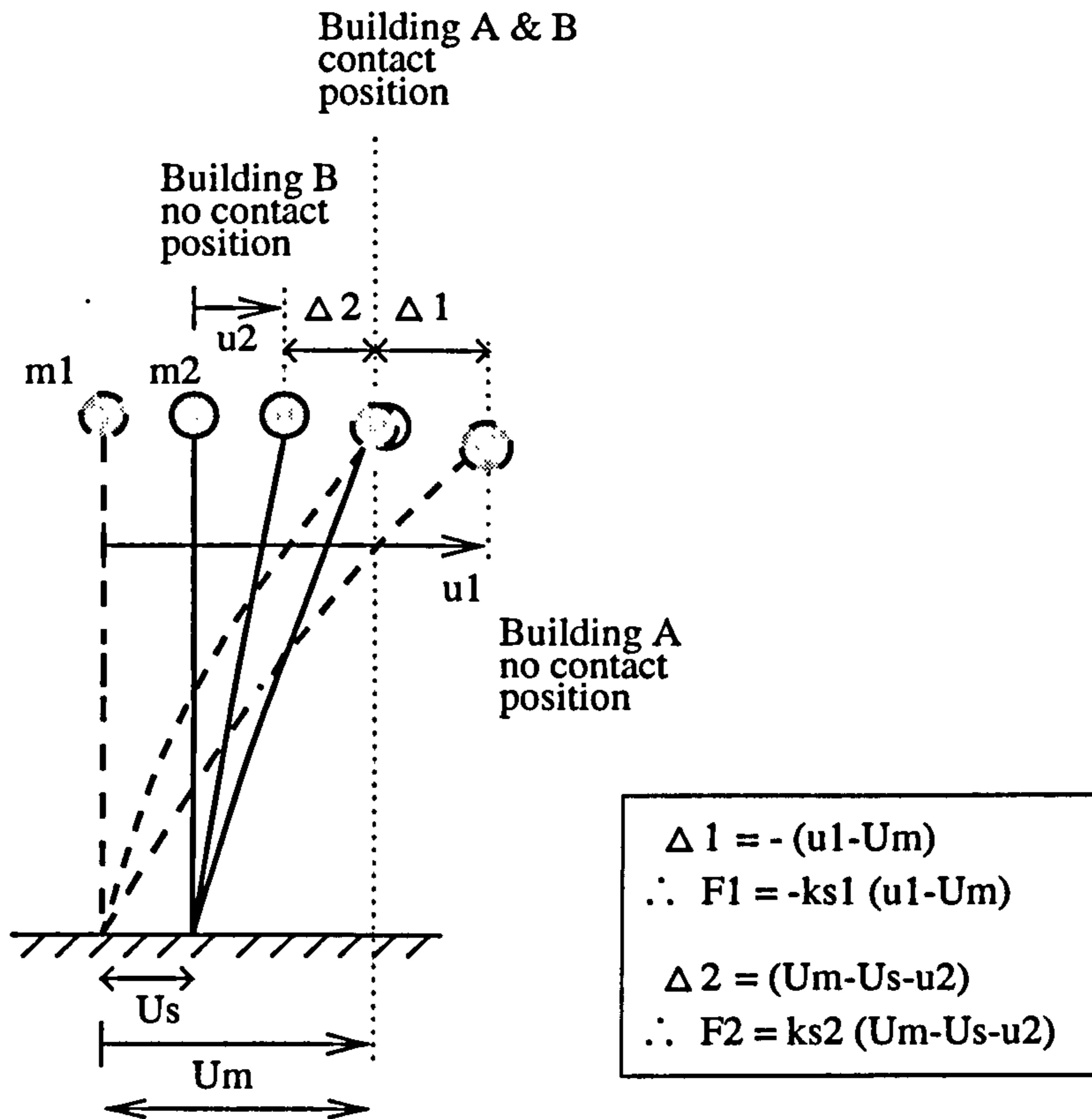


Figure 5.2 : Schematic diagram of Building A and B during contact stage.

During the contact, let  $U_m$  equal the deflection at the floor level for both building A and B. For equilibrium at contact:

$$F_1 + F_2 = 0 \quad (5.7)$$

Therefore,

$$k_{s2} (U_m - U_s - u_2) = k_{s1} (u_1 - U_m)$$

$$\therefore U_m = \frac{k_{s1} u_1 + k_{s2} U_s + k_{s2} u_2}{(k_{s2} + k_{s1})} \quad (5.8)$$

Substituting Equation (5.8) into  $F_1$  ;

$$F_1 = -k_{s1} (u_1 - U_m)$$

$$= -\frac{k_{s1} k_{s2}}{(k_{s1} + k_{s2})} (u_1 - u_2 - U_s) \quad (5.9)$$

Similarly, for  $F_2$  :

$$\begin{aligned}
F_2 &= k_{s2} (U_m - U_s - u_2) \\
&= \frac{k_{s1} k_{s2}}{(k_{s1} + k_{s2})} (u_1 - u_2 - U_s) \\
&= k (u_1 - u_2 - U_s)
\end{aligned} \tag{5.10}$$

Where,

$$k = \frac{k_{s1} k_{s2}}{(k_{s1} + k_{s2})} \tag{5.11}$$

Thus the equations of motion for the contact stage can be written as below, (refer to shear force diagram in Figure 5.1b)

$$\begin{aligned}
\text{BUILDING A} \quad m_1 \ddot{u}_1 &= -2k_1 u_1 - k(u_1 - u_2 - U_s) \\
\text{BUILDING B} \quad m_2 \ddot{u}_2 &= -2k_2 u_2 + k(u_1 - u_2 - U_s)
\end{aligned} \tag{5.12}$$

In matrix form these can be written as:

$$\begin{vmatrix} m_1 & 0 \\ 0 & m_2 \end{vmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{vmatrix} 2k_1 + k & -k \\ -k & 2k_2 + k \end{vmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{vmatrix} k \\ -k \end{vmatrix} U_s \tag{5.13}$$

In general this can be written in the form of :

$$\underline{m} \ddot{u}_i + \underline{\bar{k}} u_i = \underline{\bar{b}} \tag{5.14}$$

where  $\underline{m}$  is the mass matrix,  $\underline{\bar{k}}$  is the stiffness matrix and  $\underline{\bar{b}}$  is a column vector of static preload forces. Since these matrices ( $\underline{m}$  and  $\underline{\bar{k}}$ ) are symmetrical, the orthogonality properties (if  $j=i$ ,  $Q_j^T \underline{m} Q_i = 1$ ) are employed. The solution of these equations are same as in Equation (5.5) except some additional term, that is after taken into account of the static preload,  $\underline{\bar{b}}$  ;

$$\begin{aligned}
y_i &= \frac{\dot{y}_t}{\omega_{ic}} \sin \omega_{ic} dt + (y_{it} - y_{sii}) \cos \omega_{ic} dt + y_{sii} \\
\dot{y}_i &= \dot{y}_{it} \cos \omega_{ic} dt - \omega_{ic} (y_{it} - y_{sii}) \sin \omega_{ic} dt
\end{aligned} \tag{5.15}$$

where

$$\begin{aligned}
y_{sii} &= \frac{Q_i^T \underline{\bar{b}}}{\omega_{ic}^2} \\
&= \frac{\alpha_{i1}^T k U_s - \alpha_{i2}^T k U_s}{\omega_{ic}^2} \quad (i = 1, 2)
\end{aligned} \tag{5.16}$$



The total displacements and velocities response are defined in Equation (5.6) and the maximum contact force,  $P$  is equal to  $F_1$  as in Equation (5.9).

### 5.2.2.2 Contact Stage (Unaligned floor case)

In this section, the derivation of the equation of motion uses the displacement compatibility method which is an extension of the method used in Chapter 3. The same buildings as in section (5.2.1) are used except in order to show the unaligned floor cases, where building B is assumed shorter than building A by a distance of  $a$ . This is illustrated in Figure 5.3a below. Building A remains a shear frame building and building B is simplified to a lumped mass model. (Figure 5.3b).

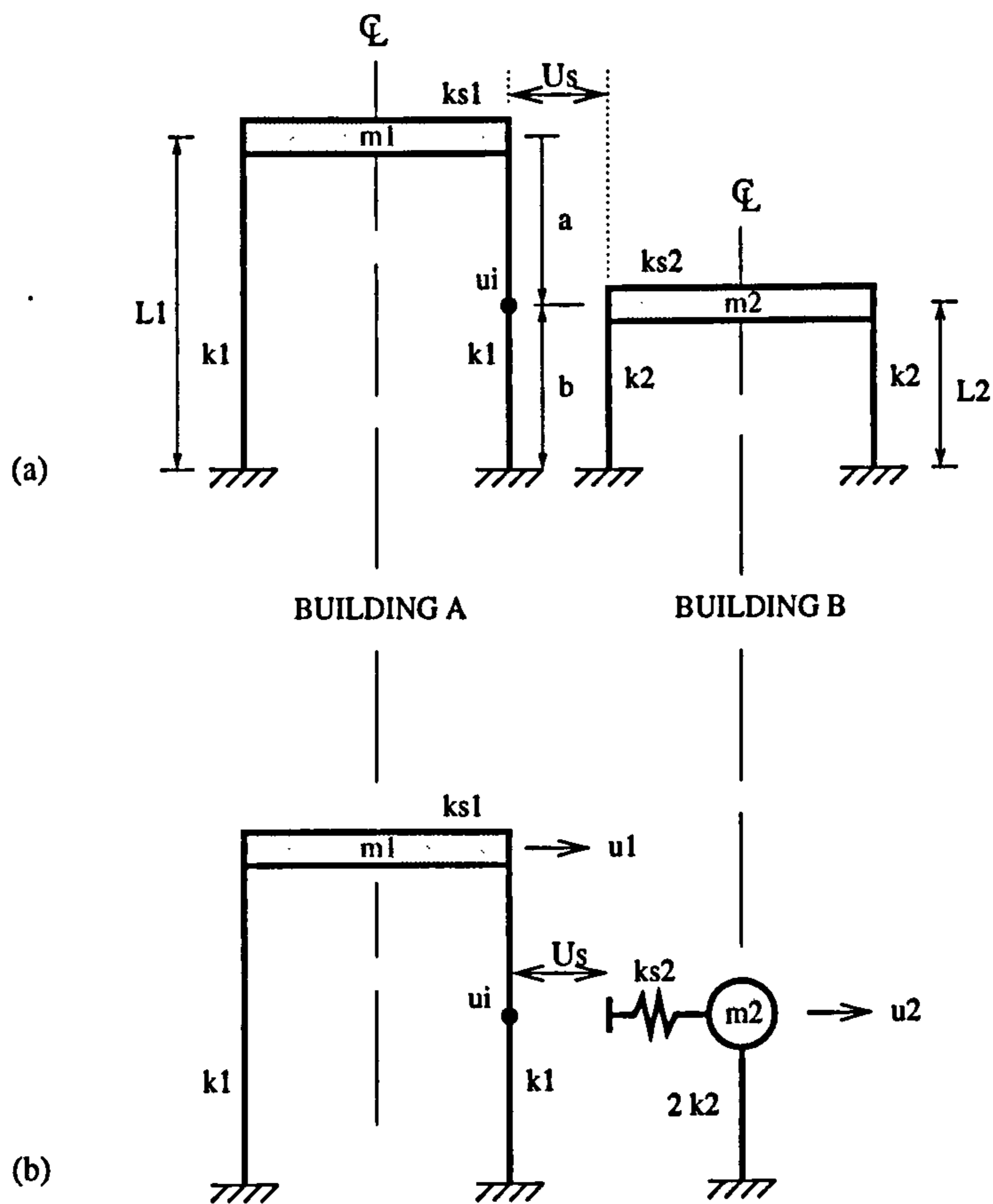


Figure 5.3 : (a) Pounding of two SDOF systems and (b) The model idealization. (Unaligned floor).

Assume the floor is rigid compared to the column, then

$$\begin{aligned} u_1 &= u_{s1} + u_{d1} \\ u_2 &= u_{s2} + u_{d2} \end{aligned} \quad (5.17)$$

The derivation of the equation of motion for this system is illustrated in Figure 5.4 below:

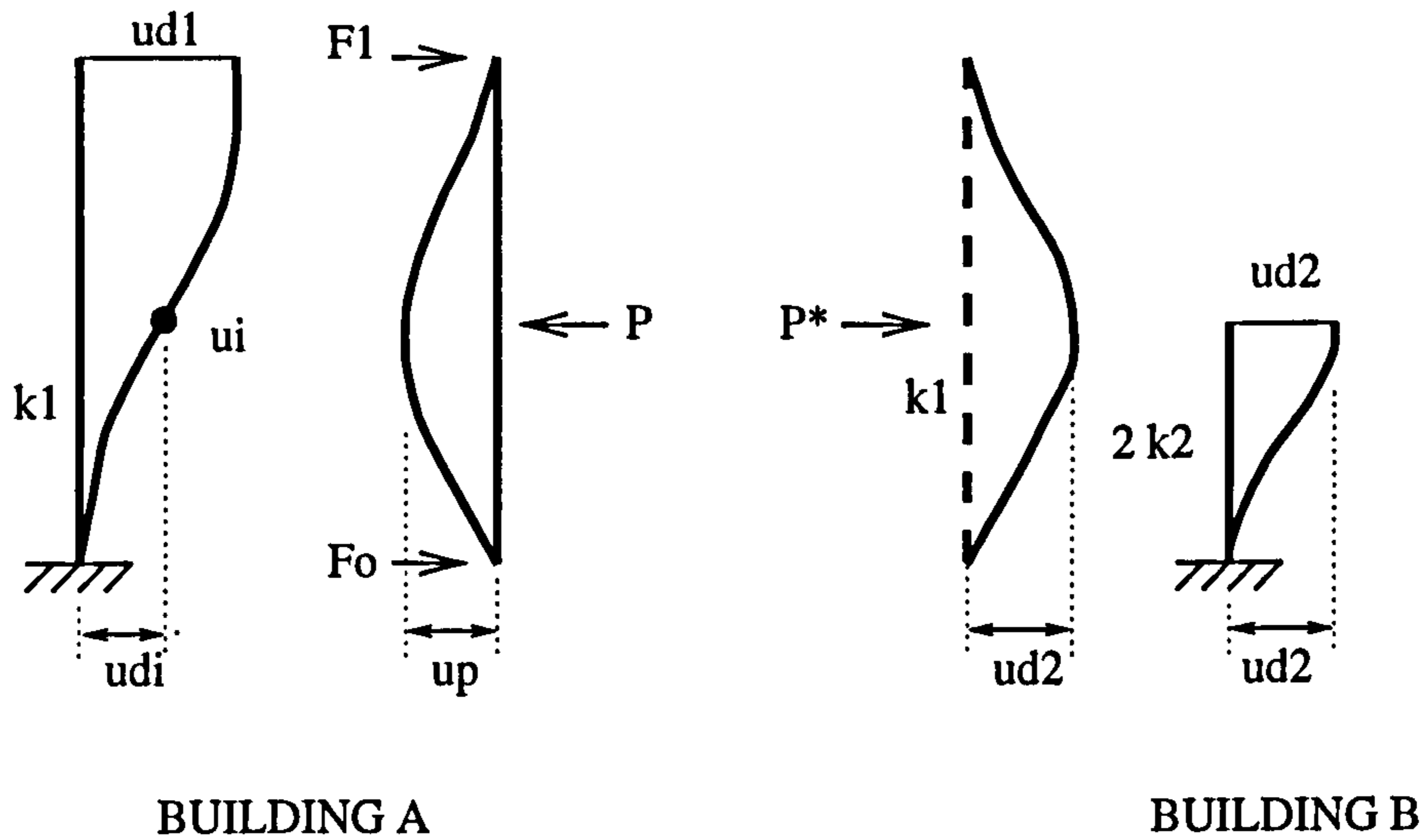


Figure 5.4 : Displacement compatibility stage of building A and B.

First, consider building A where the contact force,  $P$  acting at the contact point,  $ui$  is derived from the displacement compatibility method where ;

$$u_{di} + u_p = 0 \quad (5.18)$$

All these terms can be expressed as:

$$\begin{aligned} u_{di} &= (1 - k_{AB}) u_{d1} \\ u_i &= (1 - k_{AB}) u_1 \\ u_p &= - \frac{P b^3 \alpha^3}{3 L^3 E I} \end{aligned} \quad (5.19)$$

where

$$k_{AB} = 3 \left( \frac{\alpha}{L} \right)^2 - 2 \left( \frac{\alpha}{L} \right)^3 \quad (5.20)$$

The derivation of these terms ( $u_i$ ,  $u_{di}$ ,  $u_p$ ) are same as in Appendix B. Substituting Equation (5.19) into Equation (5.18) gives:

$$(1 - k_{AB}) ud1 = \frac{Pa^3b^3}{3L^3EI} \quad (5.21)$$

$$\therefore P = \frac{(1 - k_{AB}) ud1 3L^3EI}{a^3b^3}$$

Now consider Building B where there must be equal an opposite force acting at the floor level, namely  $P^*$ . This force caused a displacement of  $ud2$  and can be expressed as:

$$ud2 = \frac{P^* a^3 b^3}{3 L^3 E I} \quad (5.22)$$

$$\therefore P^* = \frac{ud2 3 L^3 E I}{a^3 b^3}$$

The net forces, at point of contact is given by:

$$P - P^* = \frac{(1 - k_{AB}) ud1 - ud2}{\left(\frac{a^3 b^3}{3L^3 E I}\right)} \quad (5.23)$$

Further simplified into:

$$P - P^* = kp(1 - k_{AB}) ud1 - kpud2 \quad (5.24)$$

where

$$kp = \left(\frac{3L^3EI}{a^3b^3}\right) \quad (5.25)$$

Thus the slab reaction of building A is given by:

$$F1 = (1 - k_{AB}) (P - P^*) \quad F0 = 0 \quad (5.26)$$

Substituting Equation (5.24) into (5.26) gives:

$$F1 = kp(1 - k_{AB})^2 ud1 - kp(1 - k_{AB})ud2 \quad (5.27)$$

The shear forces diagram are illustrated in Figure 5.5 below:

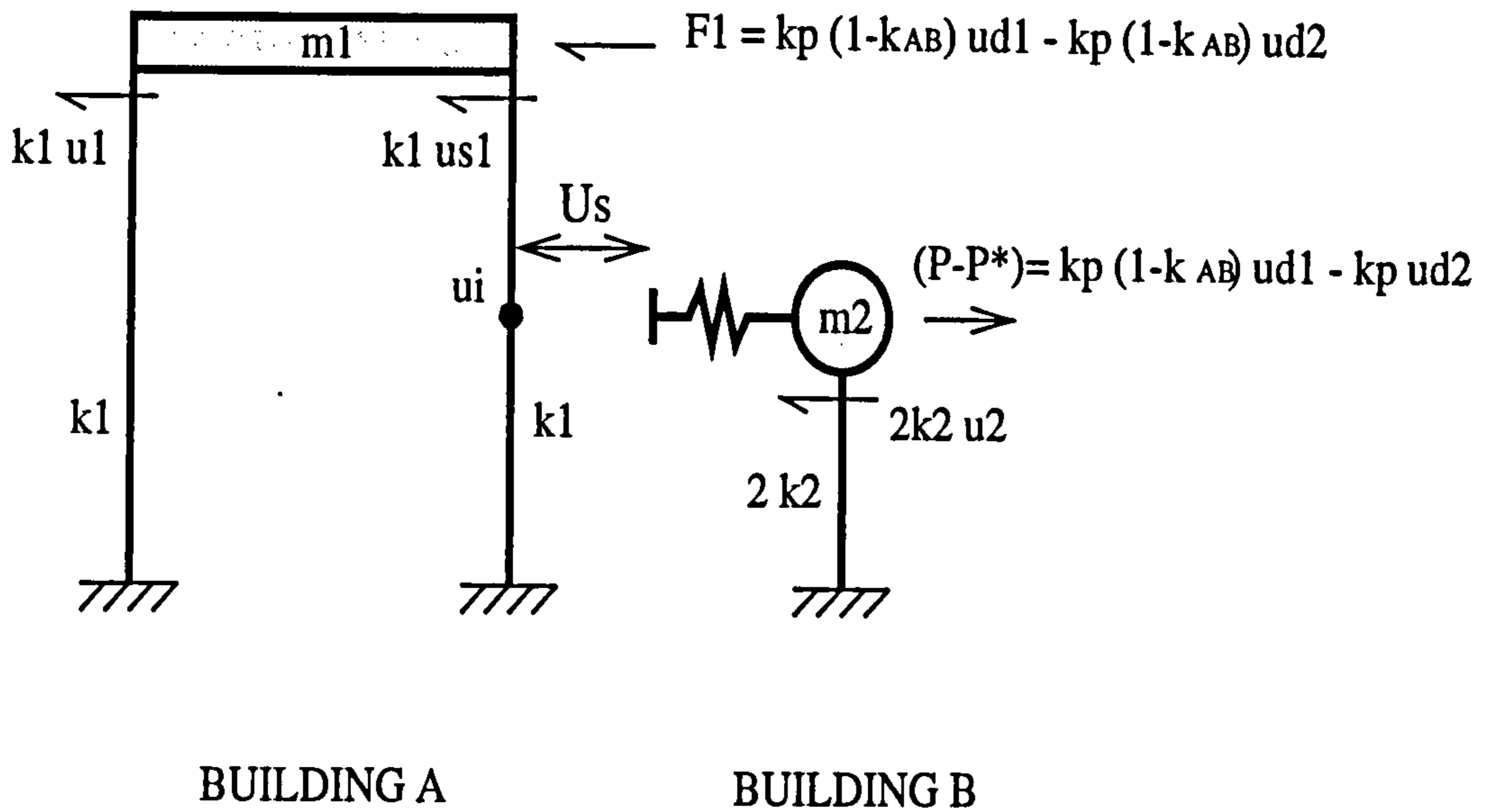


Figure 5.5 : Shear forces diagram of two SDOF systems (contact stage).

Thus the equations of motion for the contact stage can be written as below:

$$\begin{aligned} \text{BUILDING A } m_1 \ddot{u}_1 &= -k_1 u_1 - k_1 u_{s1} - k_p(1-k_{AB})^2 u_{d1} + k_p(1-k_{AB})^2 u_{d2} \\ \text{BUILDING B } m_2 \ddot{u}_2 &= -2k_2 u_2 + k_p(1-k_{AB})^2 u_{d1} - k_p u_{d2} \end{aligned} \quad (5.28)$$

where  $u_{d2} = u_2 - u_{s2}$  and  $u_{d1} = u_1 - u_{s1}$  from Equation (5.17). In matrix form these can be written as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_p(1-k_{AB})^2 & -k_p(1-k_{AB}) \\ -k_p(1-k_{AB}) & 2k_2 + k_p \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} -k_1 + k_p(1-k_{AB})^2 & -k_p(1-k_{AB}) \\ -k_p(1-k_{AB}) & k_p \end{bmatrix} \begin{Bmatrix} u_{s1} \\ u_{s2} \end{Bmatrix} \quad (5.29)$$

In general these can be written in the form of :

$$\underline{m} \ddot{\underline{u}}_i + \overline{k} \underline{u}_i = \overline{b} \quad (5.30)$$

where  $\underline{m}$  is the mass matrix,  $\overline{k}$  is the stiffness matrix and  $\overline{b}$  is a column vector of static preload forces. The solutions of these equations are the same as in Equation (5.15).

The total displacements and velocities response are given by Equation (5.6) and the maximum contact force,  $(P-P^*)$  is equal to Equation (5.24). The displacement at point of contact,  $u_i$  is equal to Equation (5.19).

### 5.2.3 Change of state rules

The general rule or conditions for switching from one state to another state are dependent on the displacement of contact point,  $u_i$  and the corresponding velocity. The contact stage starts when the relative displacements and velocities are such:

$$\begin{aligned} \text{Aligned floor case: } & (u_1 - u_2) = Us \text{ and } (\dot{u}_1 - \dot{u}_2) \geq 0 \\ \text{Unaligned floor case: } & (u_i - u_2) = Us \text{ and } (\dot{u}_i - \dot{u}_2) \geq 0 \end{aligned} \quad (5.31)$$

And the contact stage is finished when the relative displacements and velocities are such:

$$\begin{aligned} \text{Aligned floor case: } & (u_1 - u_2) = Us \text{ and } (\dot{u}_1 - \dot{u}_2) \leq 0 \\ \text{Unaligned floor case: } & (u_i - u_2) = Us \text{ and } (\dot{u}_i - \dot{u}_2) \leq 0 \end{aligned} \quad (5.32)$$

### **5.3 MDOF SYSTEMS**

In this section, two multi storey buildings are studied to represent a case of pounding of two adjacent flexible buildings. Like previous analysis of pounding, it is considered to be an impact (contact) problem, where it can be idealized as having two states, buildings not contact and buildings in contact. Both cases of aligned and unaligned floor are presented.

The analysis of the no contact stage for both aligned and unaligned floor are the same, but as mentioned in section 5.2 (SDOF system), the analysis of the contact stage for aligned and unaligned floor cases are treated separately.

#### **5.3.1 No Contact Stage**

Two MDOF system buildings are shown in Figure 5.6 below. The buildings on the left hand side and right hand side of Figure 5.6 are called building A and B respectively. They have masses and column stiffness of  $m_2, m_1, m_4, m_3$ , and  $k_2, k_1, k_4, k_3$  respectively. The local slab stiffness are  $ks_2, ks_1$ , and  $ks_4, ks_3$  for building A and B respectively. The buildings are separated by a distance  $U_s$ .

It is assumed that the point of contact occurred at the top of the buildings due to the dominant effect of the first modal shapes. The second floors are assumed not in contact throughout the analysis.

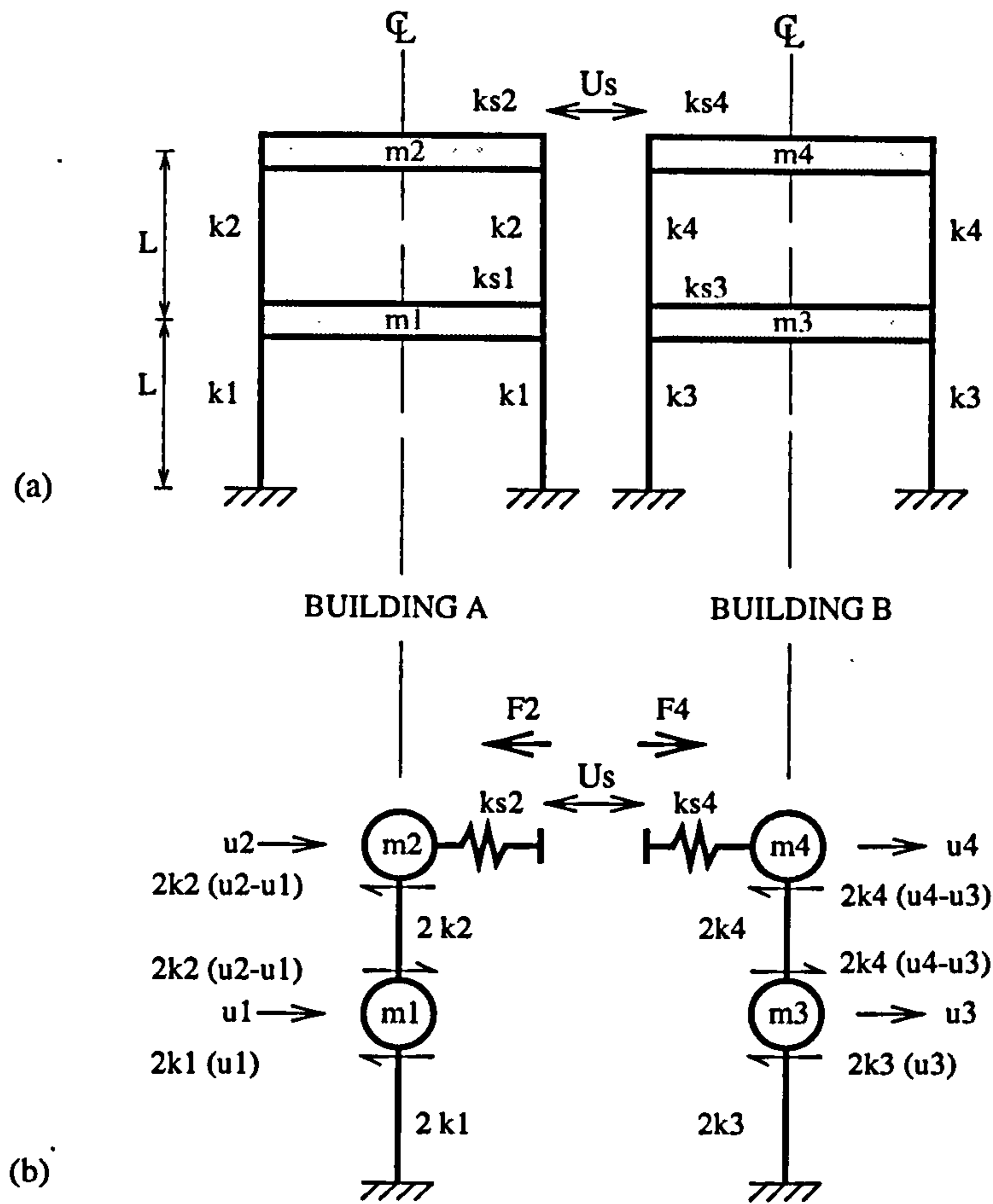


Figure 5.6: (a) Pounding of two MDOF systems and (b) The shear forces diagram (Aligned floor).

The derivation of the equations of motion can be assessed by considering the two models separately, i.e

### BUILDING A

$$\begin{aligned} m_2 \ddot{u}_2 &= -2k_2(u_2 - u_1) \\ m_1 \ddot{u}_1 &= 2k_2(u_2 - u_1) - 2k_1(u_1) \end{aligned} \quad (5.33)$$

### BUILDING B

$$\begin{aligned} m_4 \ddot{u}_4 &= -2k_4(u_4 - u_3) \\ m_3 \ddot{u}_3 &= 2k_3(u_4 - u_3) - 2k_3(u_3) \end{aligned} \quad (5.34)$$

In matrix form these can be combined as:

$$\begin{bmatrix} m2 & 0 & 0 & 0 \\ 0 & m1 & 0 & 0 \\ 0 & 0 & m4 & 0 \\ 0 & 0 & 0 & m3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_4 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 2k2 & -2k2 & 0 & 0 \\ -2k2 & 2k2+2k1 & 0 & 0 \\ 0 & 0 & 2k4 & -2k4 \\ 0 & 0 & -2k4 & 2k4+2k3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \\ u_4 \\ u_3 \end{Bmatrix} = 0 \quad (5.35)$$

In general these can be written in the form of :

$$m \ddot{u}_i + k u_i = 0 \quad (5.36)$$

The solutions of these equations are the same as in Equation (5.5). The total displacements and velocities response are defined in Equation (5.6).

### 5.3.2 Contact stage

To illustrate the derivation of the equation of motion in the contact stage of these model, the aligned and unaligned cases are treated separately. This is because, the contact force for the aligned case is simply the direct application of the Newton's law of equilibrium and for the unaligned case, the displacement compatibility method is applied first in order to find the contact force.

#### 5.3.2.1 Contact Stage (Aligned floor case)

The contact forces  $F_2$  and  $F_4$  at the top floor level of building A and B, are calculated using the Newton's law of equilibrium of forces as in section (5.2.2.1). The schematic diagram of the two buildings in the contact stage is shown in Figure 5.7 below:



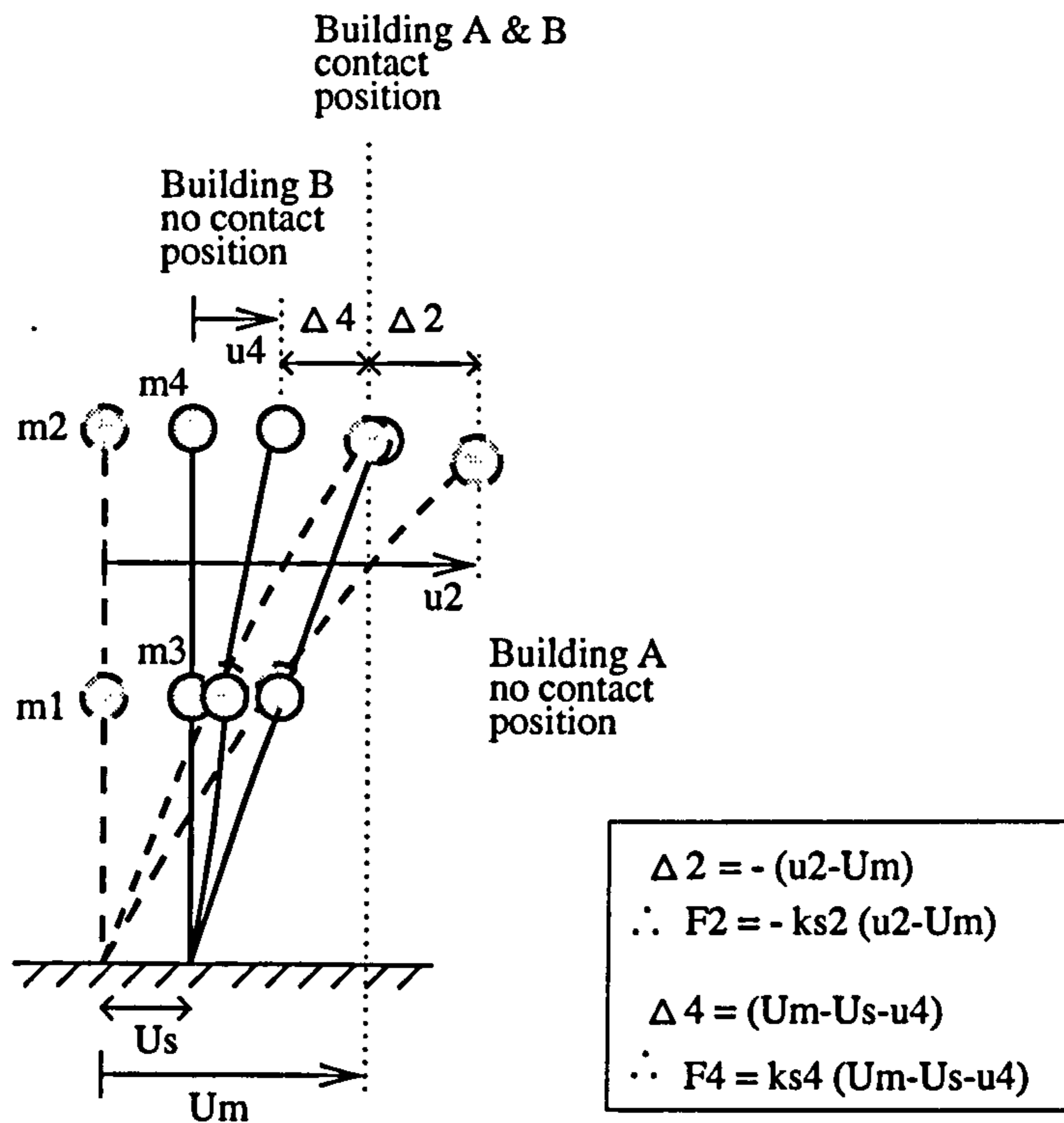


Figure 5.7 : Schematic diagram of building A and B during contact stage.

Again, let  $Um$  equal to the deflection at the top floor of building A and B. The equilibrium of forces at contact are given by

$$F4 + F2 = 0 \quad (5.37)$$

Therefore,

$$\begin{aligned} ks4(Um - Us - u4) &= ks2(u2 - Um) \\ \therefore Um &= \frac{ks2u2 + ks4Us + ks4u4}{(ks4 + ks2)} \end{aligned} \quad (5.38)$$

Substituting Equation (5.38) into equation for  $F2$  ;

$$\begin{aligned} F2 &= -ks2(u2 - Um) \\ &= -k(u2 - Us - u4) \end{aligned} \quad (5.39)$$

Similarly, substituting into equation for  $F5$  :

$$\begin{aligned} F4 &= ks4(Um - Us - u4) \\ &= k(u2 - Us - u4) \end{aligned} \quad (5.40)$$

Where,

$$k = \frac{k_2 k_4}{(k_4 + k_2)} \quad (5.41)$$

So the equations of motion for the contact stage can be written as below:

#### BUILDING A

$$\begin{aligned} m_2 \ddot{u}_2 &= -2k_2(u_2 - u_1) - k(u_2 - u_4 - U_s) \\ m_1 \ddot{u}_1 &= 2k_2(u_2 - u_1) - 2k_1(u_1) \end{aligned} \quad (5.42)$$

#### BUILDING B

$$\begin{aligned} m_4 \ddot{u}_4 &= -2k_4(u_4 - u_3) + k(u_2 - u_4 - U_s) \\ m_3 \ddot{u}_3 &= 2k_3(u_4 - u_3) - 2k_3(u_3) \end{aligned} \quad (5.43)$$

In matrix form these can be combined as:

$$\begin{bmatrix} m_2 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_4 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 2k_2 + k & -2k_2 & -k & 0 \\ -2k_2 & 2k_2 + 2k_1 & 0 & 0 \\ -k & 0 & 2k_4 + k & -2k_4 \\ 0 & 0 & -2k_4 & 2k_4 + 2k_3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \\ u_4 \\ u_3 \end{Bmatrix} = \begin{bmatrix} k \\ 0 \\ -k \\ 0 \end{bmatrix} U_s \quad (5.44)$$

In general these can be written in standard form of the equations of motion as:

$$m \ddot{u}_i + \bar{k} u_i = \bar{b} \quad (5.45)$$

The solution of these equations are same as in Equation (5.15) except the term  $y_{sii}$  is expressed:

$$\begin{aligned} y_{sii} &= \frac{Q_i^T \bar{b}}{\omega_{ic}^2} \\ &= \frac{\alpha_{i1}^T k U_s - \alpha_{i3}^T k U_s}{\omega_{ic}^2} \quad (i = 1, 2, 3, 4) \end{aligned} \quad (5.46)$$

The total displacements and velocities response are being the sum of the modal contributions of  $n$ th mode shapes as in Equation (5.6). The maximum contact force,  $P$  is equal to  $F_2$  as in Equation (5.39).

### 5.3.2.2 Contact Stage (Unaligned floor case)

In this section, building B is assumed to be shorter than building A at a distance  $a$  from the top of building A. The location of contact point during pounding is assumed to occur at the top floor level of the shorter building. This is illustrated in Figure 5.8 below.

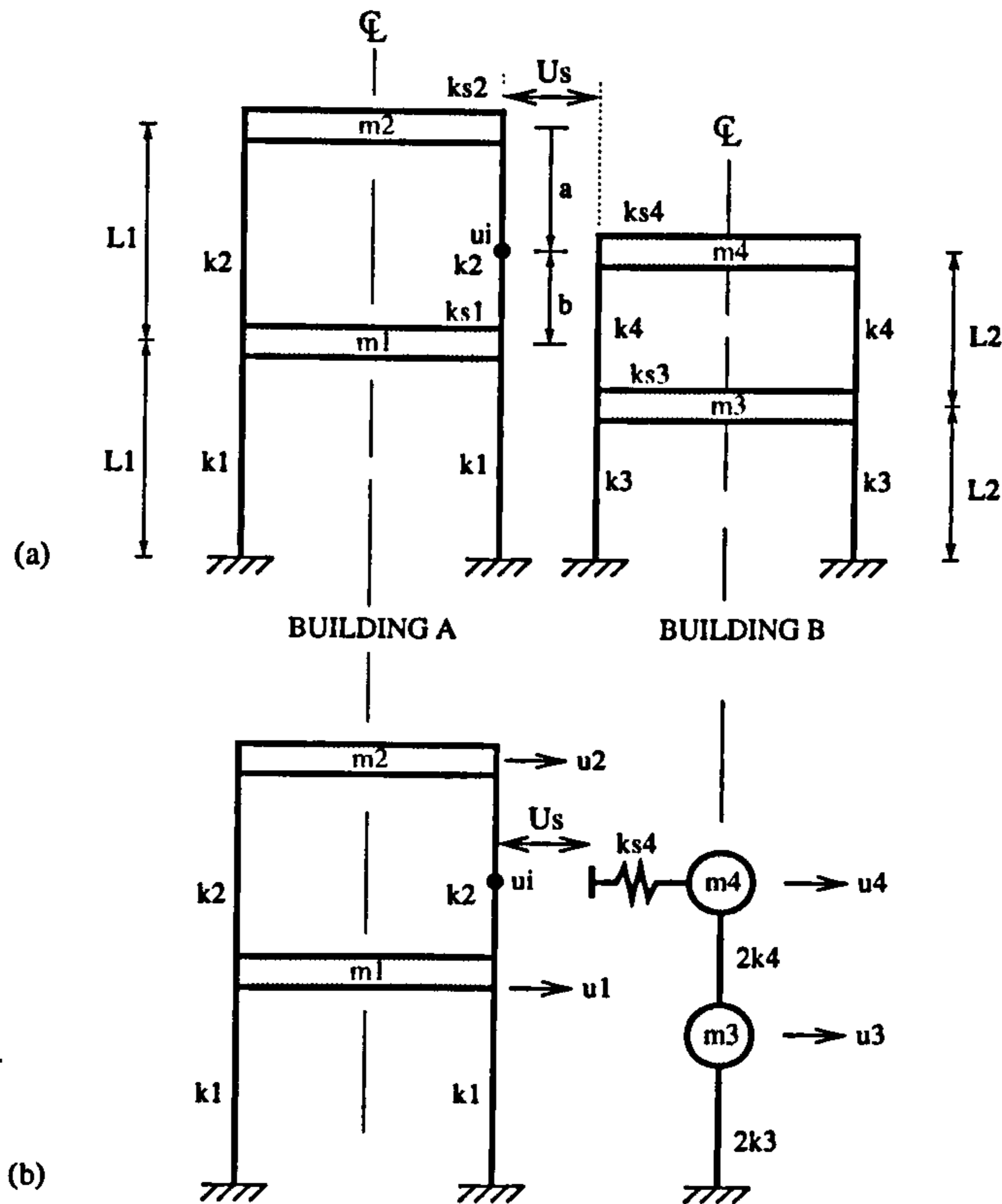


Figure 5.8 : (a) Pounding of two MDOF systems and (b) The model idealization. (Unaligned floor).

Assuming the floor is rigid compared to the column as in Equation (5.17), the derivation of the equation of motion for this system is illustrated in Figure 5.9 below:

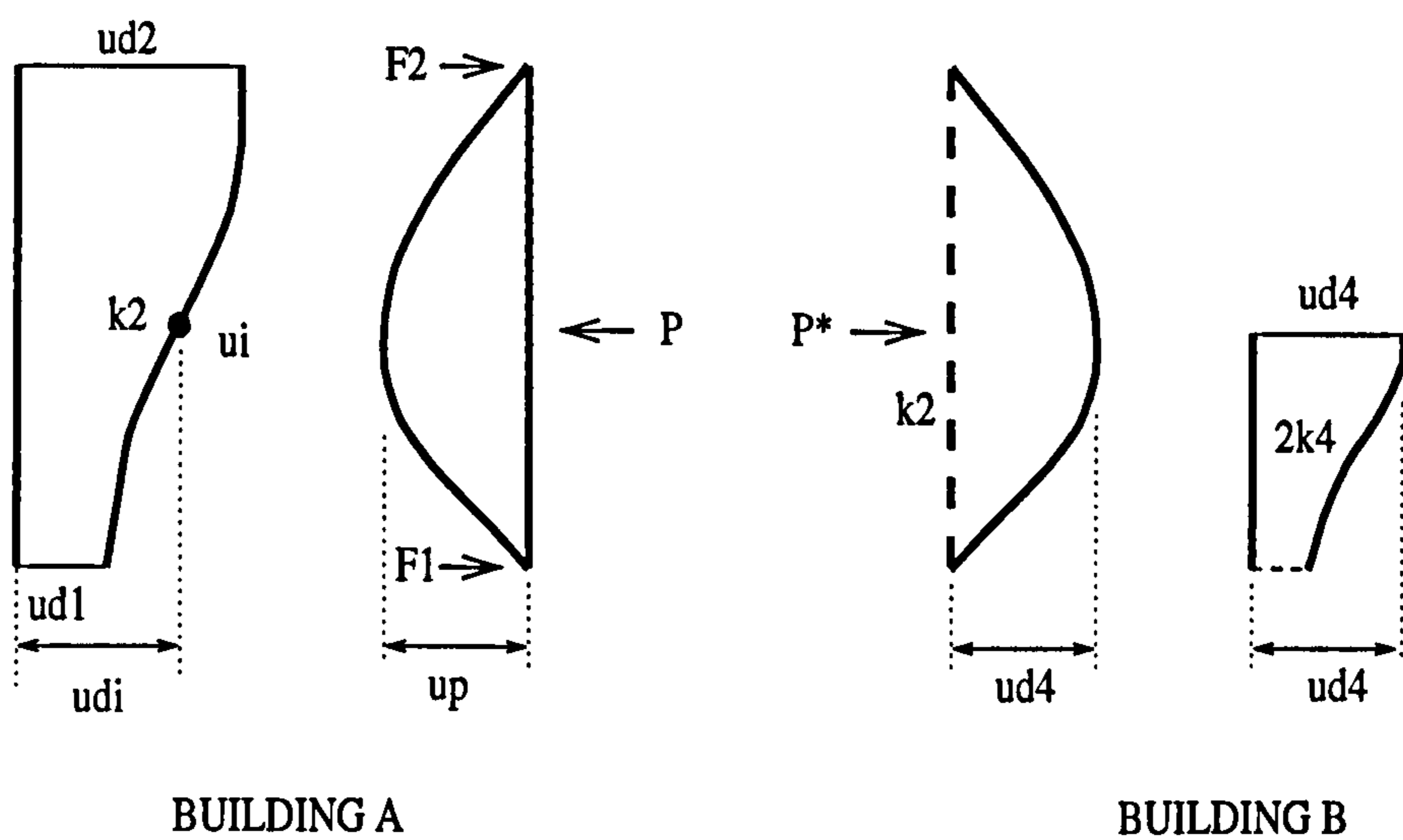


Figure 5.9 : Displacement compatibility stage of building A and B.

Consider building A only, where the contact force,  $P$  acting at the contact point,  $ui$  is derived from the displacement compatibility method where ;

$$u_{di} + u_p = 0 \quad (5.47)$$

All these terms can be expressed as :

$$\begin{aligned} u_{di} &= (1 - k_{AB}) u_{d2} + k_{AB} u_{d1} \\ u_i &= (1 - k_{AB}) u_2 + k_{AB} u_1 \\ u_p &= - \frac{P b^3 a^3}{3 L^3 E I} \end{aligned} \quad (5.48)$$

where  $k_{AB}$  is defined by Equation (5.20). Substituting Equation (5.48) into Equation (5.47) gives:

$$(1 - k_{AB}) u_{d2} + k_{AB} u_{d1} = \frac{P a^3 b^3}{3 L^3 E I} \quad (5.49)$$

$$\therefore P = [(1 - k_{AB}) u_{d2} + k_{AB} u_{d1}] \frac{3 L^3 E I}{a^3 b^3}$$

When in contact with building B, there must be equal and opposite force acting at this point,  $ui$ , namely  $P^*$ . This force caused a displacement of  $ud4$  and can be expressed as:

$$ud4 = \frac{P^* a^3 b^3}{3 L^3 E I} \quad (5.50)$$

$$\therefore P^* = \frac{ud4 \ 3 \ L^3 \ E \ I}{a^3 \ b^3}$$

Thus the net force, at point of contact is given by:

$$P - P^* = \frac{(1 - k_{AB}) ud2 + k_{AB} ud1 - ud4}{\left(\frac{a^3 b^3}{3L^3 E I}\right)} \quad (5.51)$$

Further simplified into:

$$P - P^* = kp(1 - k_{AB}) ud2 + kp k_{AB} ud1 - kpud4 \quad (5.52)$$

where

$$kp = \left(\frac{3L^3 EI}{a^3 b^3}\right) \quad (5.53)$$

Thus the net floor reactions of building A are given by:

$$\begin{aligned} F2 &= (1 - k_{AB}) (P - P^*) \\ F1 &= k_{AB} (P - P^*) \end{aligned} \quad (5.54)$$

Substituting Equation (5.52) into the above equations give:

$$\begin{aligned} F2 &= kp(1 - k_{AB})^2 ud2 + kp k_{AB}(1 - k_{AB})ud1 - kp(1 - k_{AB})ud4 \\ F2 &= kp k_{AB}(1 - k_{AB}) ud2 + kp k_{AB}^2 ud1 - kp k_{AB} ud4 \end{aligned} \quad (5.55)$$

The shear forces diagram are illustrated in Figure 5.10 below:

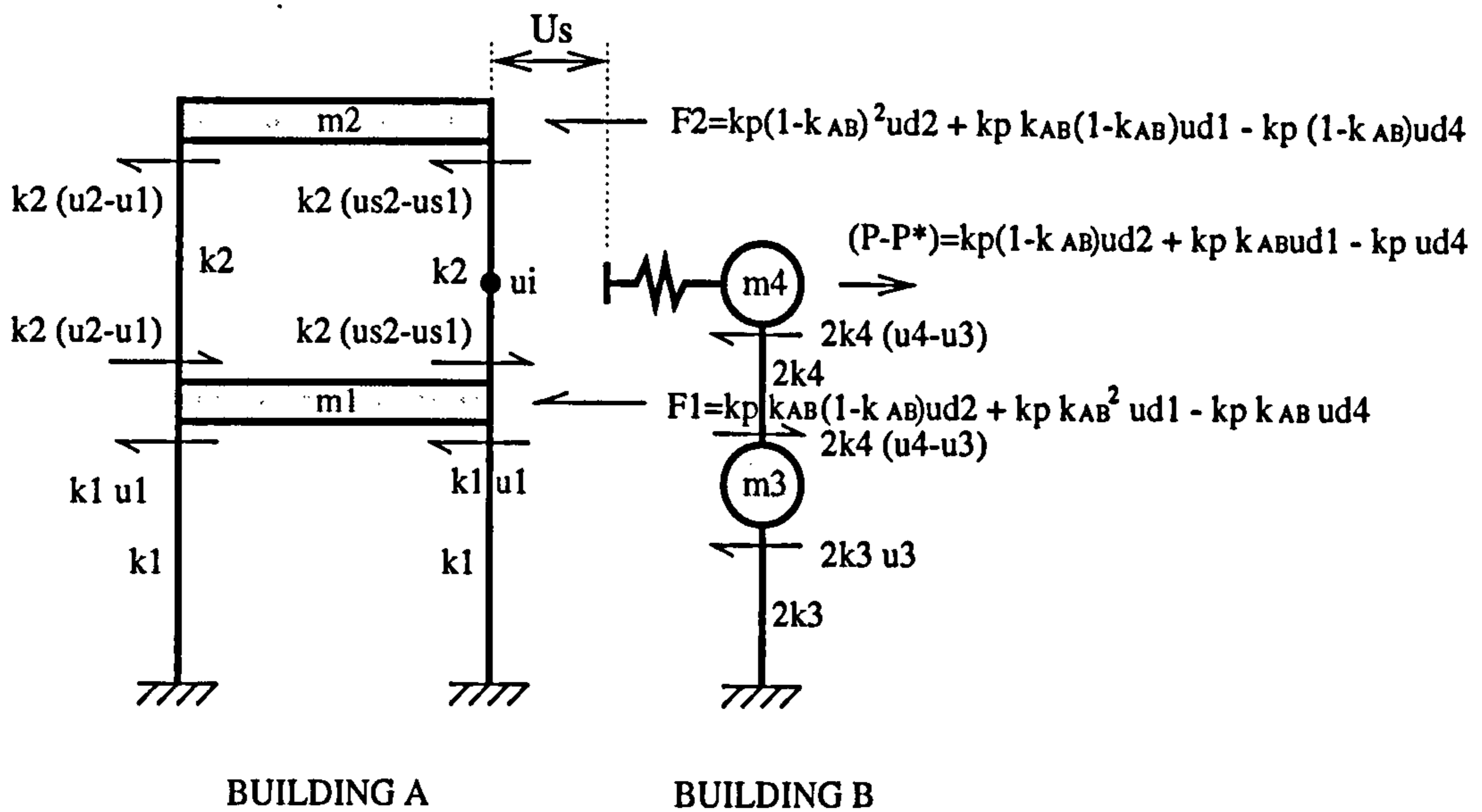


Figure 5.10 : Shear forces diagram of two MDOF systems (contact stage).

Thus the equations of motion for the contact stage can be written as below:

#### BUILDING A

$$\begin{aligned}
 m_2 \ddot{u}_2 &= -k_2(u_2 - u_1) - k_2(us_2 - us_1) - kp(1 - k_{AB})^2 ud_2 - kp k_{AB}(1 - k_{AB})ud_1 + kp(1 - k_{AB})ud_4 \\
 m_1 \ddot{u}_1 &= k_2(u_2 - u_1) + k_2(us_2 - us_1) - kp k_{AB}(1 - k_{AB})ud_2 - kp k_{AB}^2 ud_1 + kp k_{AB}ud_4 - 2k_1 u_1
 \end{aligned} \quad (5.56)$$

#### BUILDING B

$$\begin{aligned}
 m_4 \ddot{u}_4 &= -2k_4(u_4 - u_3) + kp(1 - k_{AB})ud_2 + kp k_{AB}ud_1 - kpud_4 \\
 m_3 \ddot{u}_3 &= 2k_4(u_4 - u_3) - 2k_3 u_3
 \end{aligned} \quad (5.57)$$

In matrix form these can be written as:

$$\begin{bmatrix} m_2 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_4 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} k_2 + kp(1 - k_{AB})^2 & -k_2 + kp k_{AB}(1 - k_{AB}) & -kp(1 - k_{AB}) & 0 \\ -k_2 + kp k_{AB}(1 - k_{AB}) & k_2 + kp k_{AB}^2 + 2k_1 & -kp k_{AB} & 0 \\ -kp(1 - k_{AB}) & -kp k_{AB} & kp + 2k_4 & -2k_4 \\ 0 & 0 & -2k_4 & 2k_4 + 2k_3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \\ u_4 \\ u_3 \end{Bmatrix} + \begin{bmatrix} -k_2 + kp(1 - k_{AB})^2 & k_2 + kp k_{AB}(1 - k_{AB}) & -kp(1 - k_{AB}) & 0 \\ k_2 + kp k_{AB}(1 - k_{AB}) & -k_2 + kp k_{AB}^2 & -kp k_{AB} & 0 \\ -kp(1 - k_{AB}) & -kp k_{AB} & kp & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} us_2 \\ us_1 \\ us_4 \\ us_3 \end{Bmatrix} \quad (5.58)$$

In general these can be written in the form of :

$$m \ddot{u}_i + \bar{k} u_i = \bar{b} \quad (5.59)$$

The solutions of these equations are the same as in Equation (5.15), where

$$\begin{aligned} y_{sii} &= \frac{Q_i^T \bar{b}}{\omega_{ic}^2} \\ &= \frac{(\alpha_{i1}^T + \alpha_{i2}^T + \alpha_{i3}^T) \bar{b}}{\omega_{ic}^2} \quad (i = 1,2,3,4) \end{aligned} \quad (5.60)$$

The total displacements and velocities response are given by Equation (5.6) and the maximum contact force,  $(P-P^*)$  is equal to Equation (5.52). The displacement at the point of contact,  $u_i$  is given by Equation (5.48).

### 5.3.1 Change of state rules

The contact stage starts when the relative displacements and velocities are such:

$$\begin{aligned} \text{Aligned floor case: } & (u_2 - u_4) = Us \text{ and } (\dot{u}_2 - \dot{u}_4) \geq 0 \\ \text{Unaligned floor case: } & (u_i - u_4) = Us \text{ and } (\dot{u}_i - \dot{u}_4) \geq 0 \end{aligned} \quad (5.61)$$

And the contact stage is finished when the relative displacements and velocities are such:

$$\begin{aligned} \text{Aligned floor case: } & (u_2 - u_4) = Us \text{ and } (\dot{u}_2 - \dot{u}_4) \leq 0 \\ \text{Unaligned floor case: } & (u_i - u_4) = Us \text{ and } (\dot{u}_i - \dot{u}_4) \leq 0 \end{aligned} \quad (5.62)$$

### 5.3.4 Numerical Examples of MDOF systems

The analysis of two flexible buildings in contact at floor level (aligned case) and between floor levels (unaligned case) induced by a snap back analysis are presented. Two examples for the aligned floor ( $a = 0$  and  $a = L1$ ) and four examples for the unaligned floor ( $a = L1/4$ ,  $L1/3$ ,  $L1/2$  and  $3L1/4$ ) are considered in this research. In this section, only two cases ( $a=L1$ ) and ( $a=L1/3$ ) are shown as numerical examples.

Consider two storey shear frame as shown in Figure (5.8) and having the physical properties as in Table 5.1 below:

BUILDING A ( 2 storey building)	BUILDING B ( 2 storey building)
$m_2, m_1$ = building masses 1.0, 1.5, kips-sec <sup>2</sup> /in. (175197 and 262795 kg )	$m_4, m_3$ = building masses 1.5, 2.0 kips-sec <sup>2</sup> /in. (262795 and 350394 kg )
$2k_2, 2k_1$ = building stiffness of 1200 and 1800 kips/in. ( $210.24 \cdot 10^3$ and $315.35 \cdot 10^3$ kN/m)	$2k_4, 2k_3$ = building stiffness of 1200, 1800 kips/in. ( $210.24 \cdot 10^3$ and $315.35 \cdot 10^3$ kN/m)
$ks_2 = ks_1 = 50,000$ kips/in. ( $8.76 \cdot 10^6$ kN/m)	$ks_4 = ks_3 = 50,000$ kips/in. ( $8.76 \cdot 10^6$ kN/m)
Initial $u_2$ and $u_1$ displacements : ( -0.0254 m and -0.0127 m)	not applicable
Building separation, $U_s = 0.008$ m	
Ground acceleration, ( $\ddot{u}_{gt} = 0$ )	
Undamped free vibration, $c = 0$	
Height between floors, $L1 = 72$ in. (1.8288 m)	Height between floors, $L2 = 36$ in. (0.9144 m) for ( $a=L1$ ) $L2 = 60$ in. (1.524 m) for ( $a=L1/3$ )
Young Modulus of steel, $E = 30 \cdot 10^3$ kip/in <sup>2</sup> ( $205 \cdot 10^6$ kN/m <sup>2</sup> )	

Table 5.1 : Physical properties of the building A and B.



### 5.3.4.1 Aligned floor case ( $a = L1$ )

The first case where pounding occurs at the second floor level of Building A is illustrated in Figure 5.11 below:

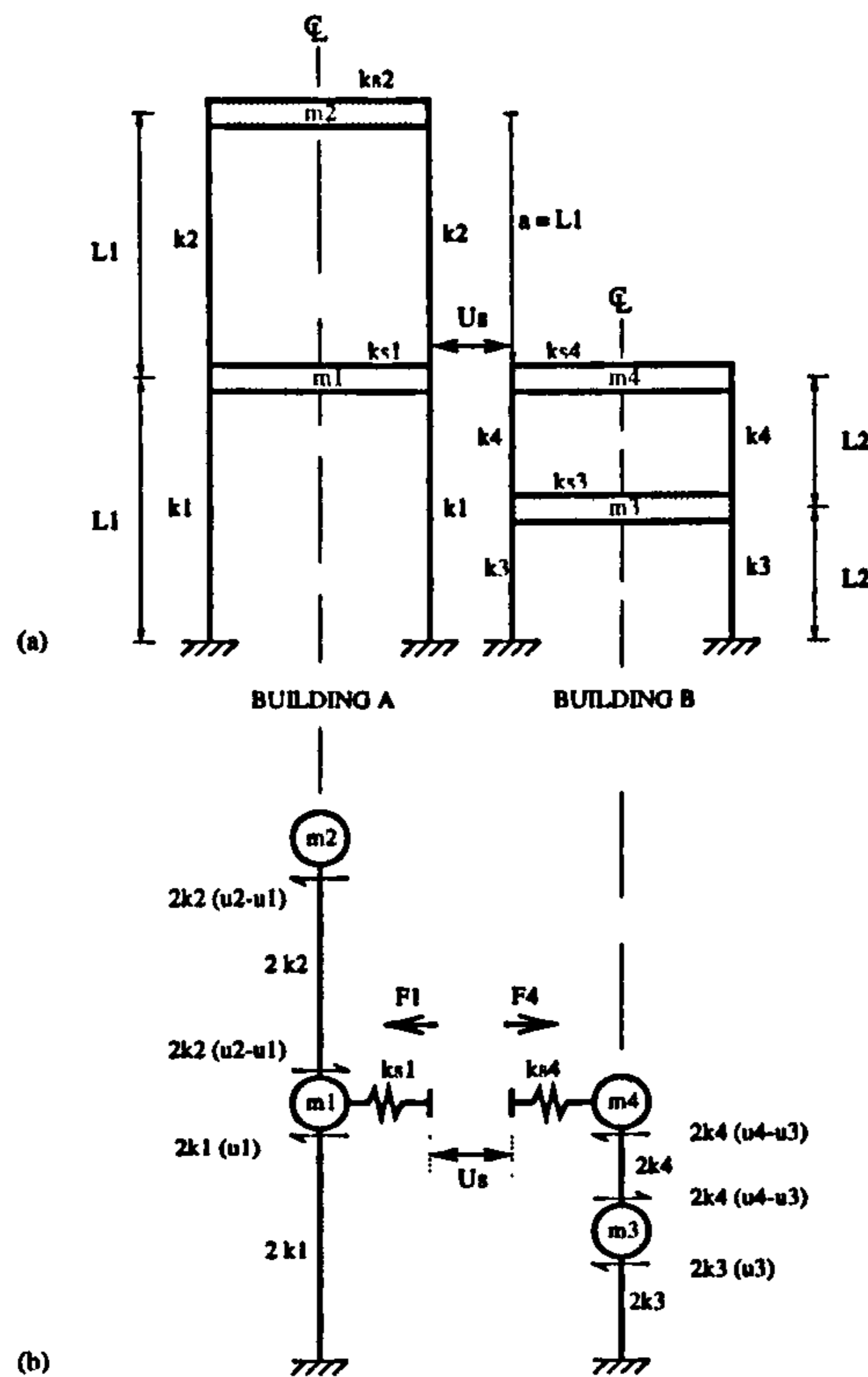


Figure 5.11: (a) Pounding of two MDOF systems at ( $a=L1$ ) (b) The model idealization.

#### 5.3.4.1.1 No contact stage

The equations of motion during the no contact stage is given by Equation (5.35). Substituting the physical properties given in Table 5.1 into Equation (5.35) gives:

$$\begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_4 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 1200 & -1200 & 0 & 0 \\ -1200 & 3000 & 0 & 0 \\ 0 & 0 & 1200 & -1200 \\ 0 & 0 & -1200 & 3000 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \\ u_4 \\ u_3 \end{Bmatrix} = 0$$

In general, these can be represented by Equation (5.36) and the solutions are the

same as in Equation (5.5). Total displacements and velocities response are defined in Equation (5.6).

### 3.5.4.1.2 Contact stage

The contact stage starts when the conditions of relative displacements and velocities are such:

$$(u_1 - u_4) = U_s \quad \text{and} \quad (\dot{u}_1 - \dot{u}_4) \geq 0$$

Then, the change from the no contact stage to the contact stage takes place. The contact forces  $F1$  and  $F4$  (Equation 5.39 and 5.40) at the floor level becomes:

$$F1 = -k(u_1 - U_s - u_4) \quad \text{and} \quad F4 = k(u_1 - U_s - u_4)$$

where constant  $k$  is defined by Equation (5.41). In this example,  $ks2=ks1=ks4=ks3$ , thus:

$$k = \frac{ks1}{2} = 25\,000 \text{ kips/in.}$$

And substitutes into the equations of motion (Equation 5.44) giving

$$\begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_4 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 1200 & -1200 & 0 & 0 \\ -1200 & 28000 & -25000 & 0 \\ 0 & -25000 & 26200 & -1200 \\ 0 & 0 & -1200 & 3000 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \\ u_4 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ 25000 \\ -25000 \\ 0 \end{bmatrix} U_s$$

In general, these equations are in the form of Equation (5.45). The solution of these equation are same as in Equation (5.15) where the  $ysii$  term is expressed:

$$\begin{aligned} y_{sii} &= \frac{Q_i^T \bar{b}}{\omega_{ic}^2} \\ &= \frac{(\alpha_{i2}^T - \alpha_{i3}^T) (25000 U_s)}{\omega_{ic}^2} \quad (i = 1,2,3,4) \end{aligned}$$

The total displacements and velocities response are given by Equation (5.6) and the contact force,  $P$  is equal to  $F1$ .

### 5.3.4.2 Unaligned floor case ( $a = L/3$ )

For this case, consider a buildings illustrated in Figure 5.8 where the contact occurs at  $a=L/3$  from the top of building A.

#### 5.3.4.2.1 No contact stage

The same equations derived for the aligned floor as in section (5.3.4.1.1). The displacement at the point of contact is given by Equation (5.48) where:

$$u_i = \frac{20}{27} u_2 + \frac{7}{27} u_1$$

#### 5.3.4.2.2 Contact stage

The contact stage starts when the relative displacements and velocities given by Equation (5.61) for the unaligned floor case, are satisfied. The  $k_{AB}$  term from Equation (5.20) and the column displacement at the contact point (Equation 5.48), are become:

$$\begin{aligned} k_{AB} &= \frac{7}{27} \quad (1 - k_{AB}) = \frac{20}{27} \\ \therefore u_i &= \frac{20}{27} u_2 + \frac{7}{27} u_1 \\ u_{di} &= \frac{20}{27} u_{d2} + \frac{7}{27} u_{d1} \end{aligned}$$

The lateral stiffness of the top columns are given by:

$$\begin{aligned} \frac{12 E I}{L^3} &= 600 \text{ kips/in.} \\ \therefore E I &= 18662400 \text{ kips in.}^2 \end{aligned}$$

The net force, at point of contact is defined by Equation (5.52)

$$P - P^* = k_p \frac{20}{27} u_{d2} + k_p \frac{7}{27} u_{d1} - k_p u_{d4}$$

where  $k_p$  is calculated using Equation (5.53).

$$k_p = 13668.75 \text{ kips/in.}$$

Substituting the physical properties given in Table 5.1 into Equation (5.58) gives:

$$\begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 1.5 & 0 \\ 0 & 0 & 0 & 2.0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_4 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 8100 & 2025 & -10125 & 0 \\ 2025 & 3318.75 & -3543.75 & 0 \\ -10125 & -3543.75 & 14868.75 & -1200 \\ 0 & 0 & -1200 & 3000 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \\ u_4 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 6900 & 3225 & -10125 & 0 \\ 3225 & 318.75 & -3543.75 & 0 \\ -10125 & -3543.75 & 13668.75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_{s2} \\ u_{s1} \\ u_{s4} \\ u_{s3} \end{Bmatrix}$$

In general these can be represented by Equation (5.59). The solutions are given by Equation (5.15) where  $y_{sij}$  as in Equation (5.60).

The summary of the parameters calculated for other cases of  $a$  (0, L/4, L/2, 3L/4, ) are listed in Table 5.2.

PARAMETERS (units)	$k_{AB}$ , ( $1-k_{AB}$ )	$k_s$ , slab stiffness $k_p$ , column rigidity (kips/in)	natural frequencies, $\omega_i$ (rad/sec)
Aligned floor $a = 0$	0, 1	$k_s = 50,000$	21.007, 40.584, 48.496, 206.70
Unaligned floor $a = L/4$	5/32, 27/32	$k_p = 22755.56$	19.611, 40.500, 42.819, 179.95
Unaligned floor $a = L/3$	7/27, 20/27	$k_p = 13668.75$	19.637, 40.844, 42.522, 133.65
Unaligned floor $a = L/2$	1/2, 1/2	$k_p = 9600.0$	19.708, 39.415, 42.739, 105.51
Unaligned floor $a = 3L/4$	27/32, 5/32	$k_p = 22755.56$	19.831, 34.224, 43.408, 166.06
Aligned floor $a = L$	1, 0	$k_s = 50,000$	22.090, 37.096, 46.940, 186.46

Table 5.2 : Summary of parameters during contact stage of MDOF systems.

The FORTRAN programs called TFLEX1.FOR, TFLEX2.FOR, TFLEX3.FOR, TFLEX4.FOR, TFLEX5.FOR and TFLEX6.FOR respectively, are written in Appendix H.

## **5.4 RESULTS AND DISCUSSIONS**

Analyses for various values of  $a$  ( 0,  $L/4$ ,  $L/3$ ,  $L/2$ ,  $3L/4$  and  $L$ ) are performed and the results are shown in Figure 5.12 - 5.20. A snap back analysis is performed at building A for 1.0 second. Within this time, it produces four or more contact stages, but for discussion, only the first three contact forces are tabulated in Table 5.3.

Figure 5.12 shows the displacements curves and velocity curves for a free vibration analysis. Figure 5.19 shows that when pounding occurs, the displacement at point of contact  $u_i$ , of Building A, moves beyond the building separation,  $U_s$ . This is because there is no restriction on this building to move compared to the one that hits a rigid adjacent building (chapter 3), where  $u_i$  forms a plateau which is equal to the building separation value,  $U_s$ .

Figures (5.13 - 5.18 ) show that Building B start to move when it is hit by building A at the first contact. Then both buildings move in or out of phase towards each other, which produces the consecutive contacts. The velocity curves, during the contact stage suddenly decrease until they reach a zero velocity (maximum  $u_i$  displacement) and then become negative within this contact region.

Figure 5.20a shows the value of the maximum contact force for various value of  $a$ . It is found that the first and the second contacts behave in similar trends where the value of this force decreases from ( $a=0$  to  $L/2$ ) and increases from ( $a=L/2$  to  $L$ ). This is due to the lower column rigidity,  $k_p$  at ( $a=L/2$ ) than other positions of  $a$  (Table 5.2). The value of the starting time of the contact stage increases as the value of  $a$  increases as shown in Figure 5.20b. The duration of the contact stage is longer at  $a=L/2$  than other positions of  $a$ .

The results are verified using DYNA3D as shown in Figure 5.14b - 5.19b.

DISTANCE "a"		1st CONTACT			2nd CONTACT			3rd CONTACT		
		FORTRAN	DYNA3D	FORTRAN	DYNA3D	FORTRAN	DYNA3D	FORTRAN	DYNA3D	
0	Contact time (sec)	0.08110-0.09596 (0.01486)	0.08088-0.10002 (0.01914)	0.29216-0.30674 (0.01458)	0.29229-0.31000 (0.01771)	0.50271-0.51711 (0.01440)	0.49980-0.52113 (0.02133)			
	Contact forces (kN)	10,508.234	9,289.860	13,203.320	12,764.100	10,280.921	9,568.650			
L/4	Contact time (sec)	0.08240-0.09945 (0.01705)	0.08401-0.10500 (0.02099)	0.29385-0.31097 (0.01712)	0.29800-0.31801 (0.02001)	0.50274-0.51963 (0.01689)	0.50300-0.51990 (0.01690)			
	Contact forces (kN)	10,153.956	10,524.200	13,779.043	10,745.600	10,770.796	12,711.900			
L/3	Contact time (sec)	0.08340-0.10613 (0.02273)	0.08302-0.10701 (0.02399)	0.29802-0.32119 (0.02317)	0.29801-0.31802 (0.02001)	0.51096-0.53375 (0.02279)	0.51001-0.53001 (0.02000)			
	Contact forces (kN)	7,699.389	7,788.190	11,128.616	12,383.000	8,891.833	9,025.700			
L/2	Contact time (sec)	0.08610-0.11441 (0.02831)	0.08801-0.12001 (0.03200)	0.30590-0.33537 (0.02947)	0.31000-0.34302 (0.03302)	0.52625-0.55574 (0.02949)	0.53301-0.56703 (0.03402)			
	Contact forces (kN)	5,937.488	6,259.380	9,583.371	9,429.780	8,505.232	9,005.800			
3L/4	Contact time (sec)	0.09120-0.10933 (0.01813)	0.09100-0.11201 (0.02101)	0.31152-0.32983 (0.01831)	0.31401-0.33701 (0.02300)	0.53951-0.55654 (0.01703)	0.54299-0.56601 (0.02302)			
	Contact forces (kN)	7,459.018	8,070.150	11,641.980	11,005.600	8,301.418	7,560.550			
L	Contact time (sec)	0.09420-0.11036 (0.01616)	0.09400-0.10700 (0.01300)	0.31715-0.33311 (0.01596)	0.31700-0.32900 (0.01200)	0.56050-0.57268 (0.01218)	0.56099-0.57099 (0.01000)			
	Contact forces (kN)	6,546.880	6,209.650	9,370.416	9,336.730	17,252.498	15,142.200			

Table 5.3 : Overall results for the contact stages of the pounding of two flexible buildings (MDOF) systems.

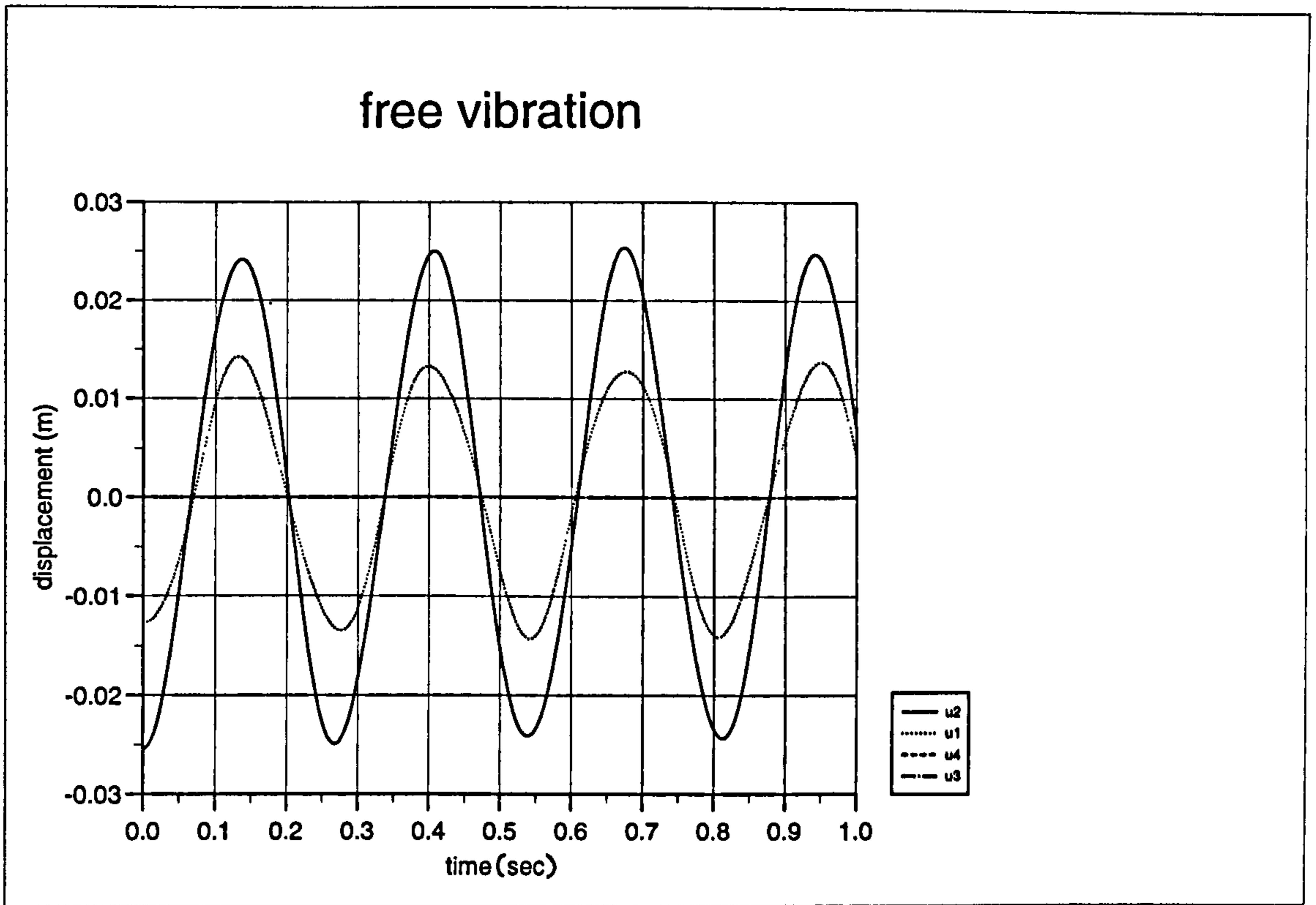


Figure 5.12a) : Displacement curves for free vibration of two flexible buildings.

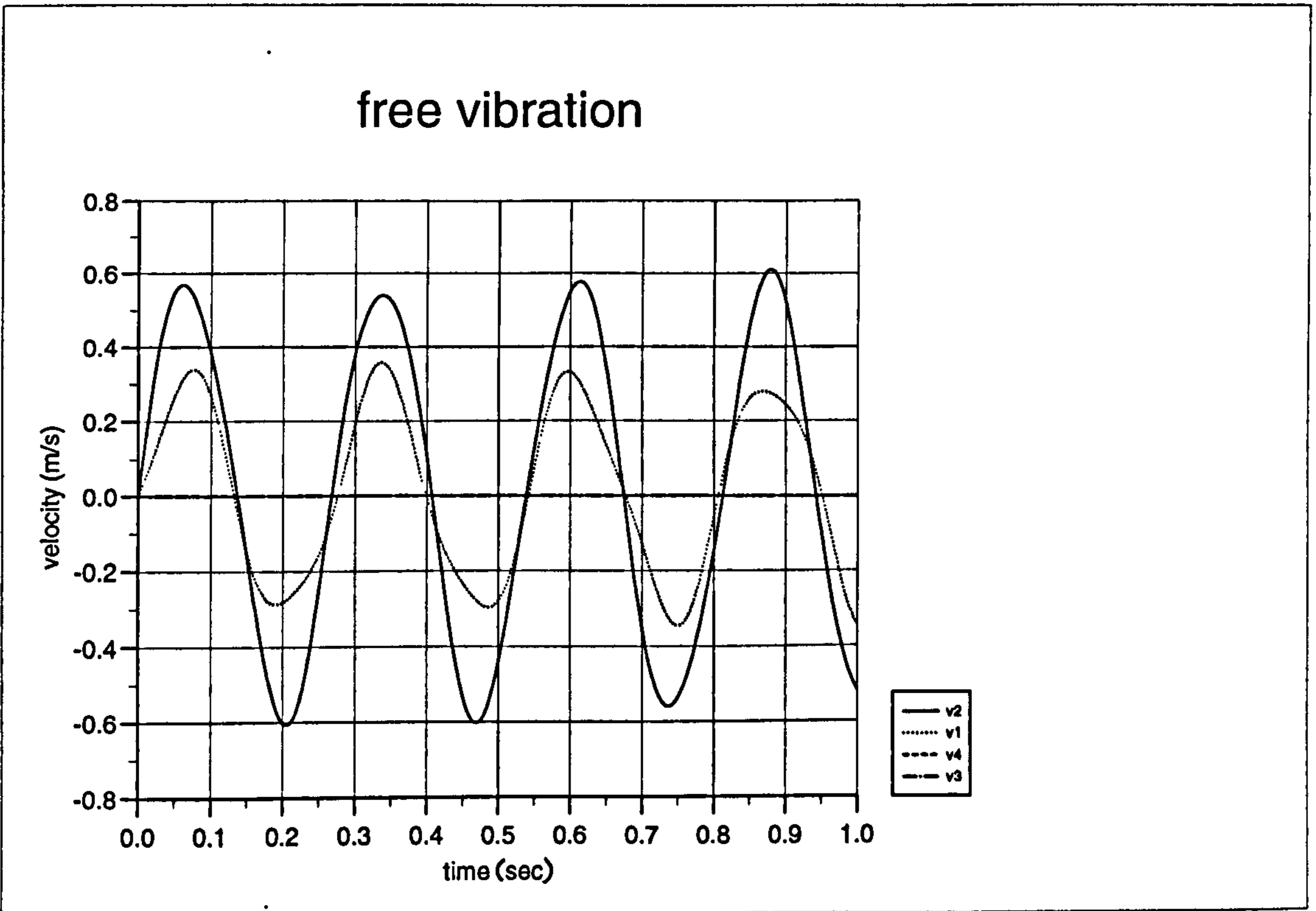


Figure 5.12b) : Velocity curves for free vibration of two flexible buildings.

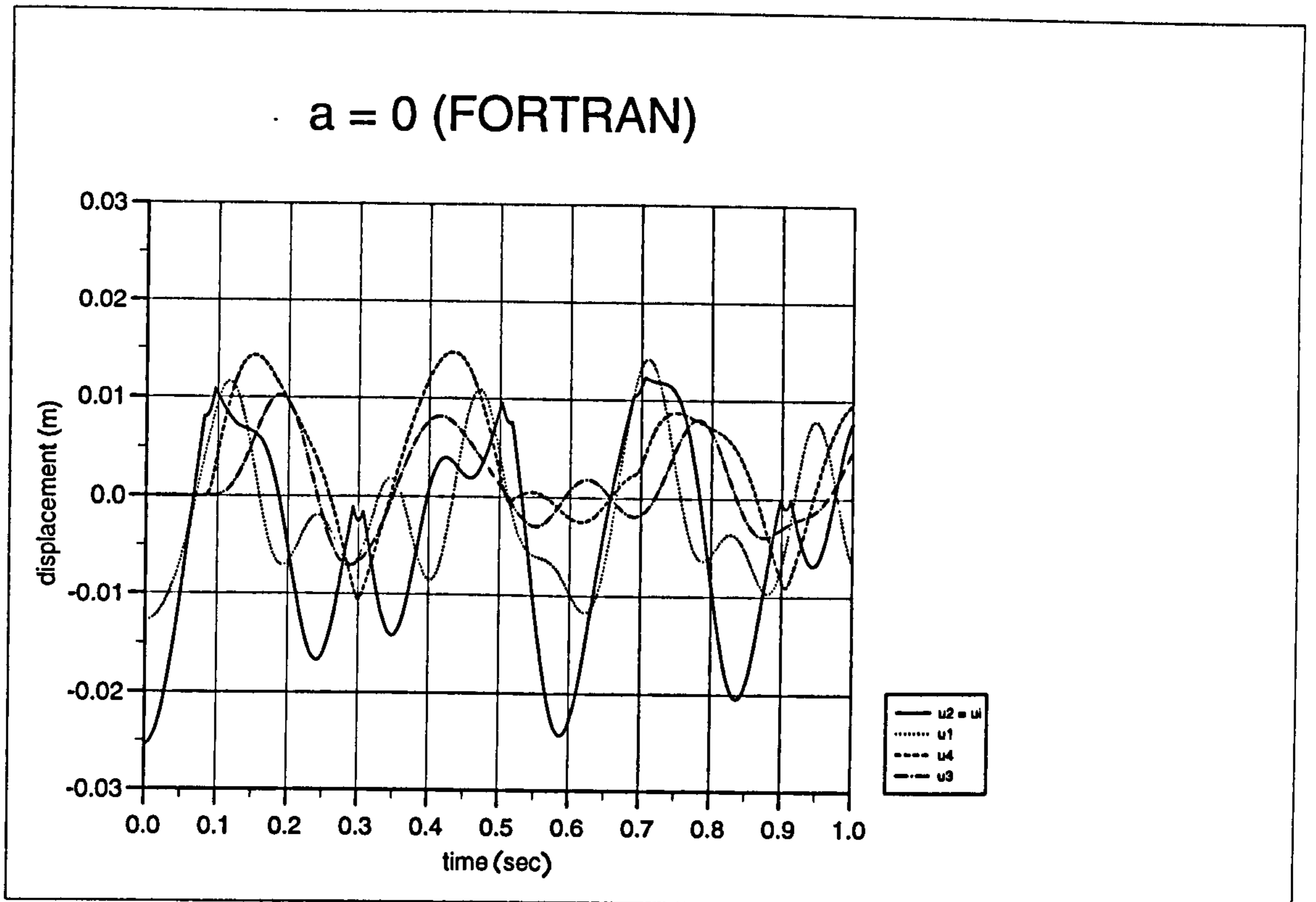


Figure 5.13a) : Displacement curves for pounding case at  $a = 0$  (FORTRAN)

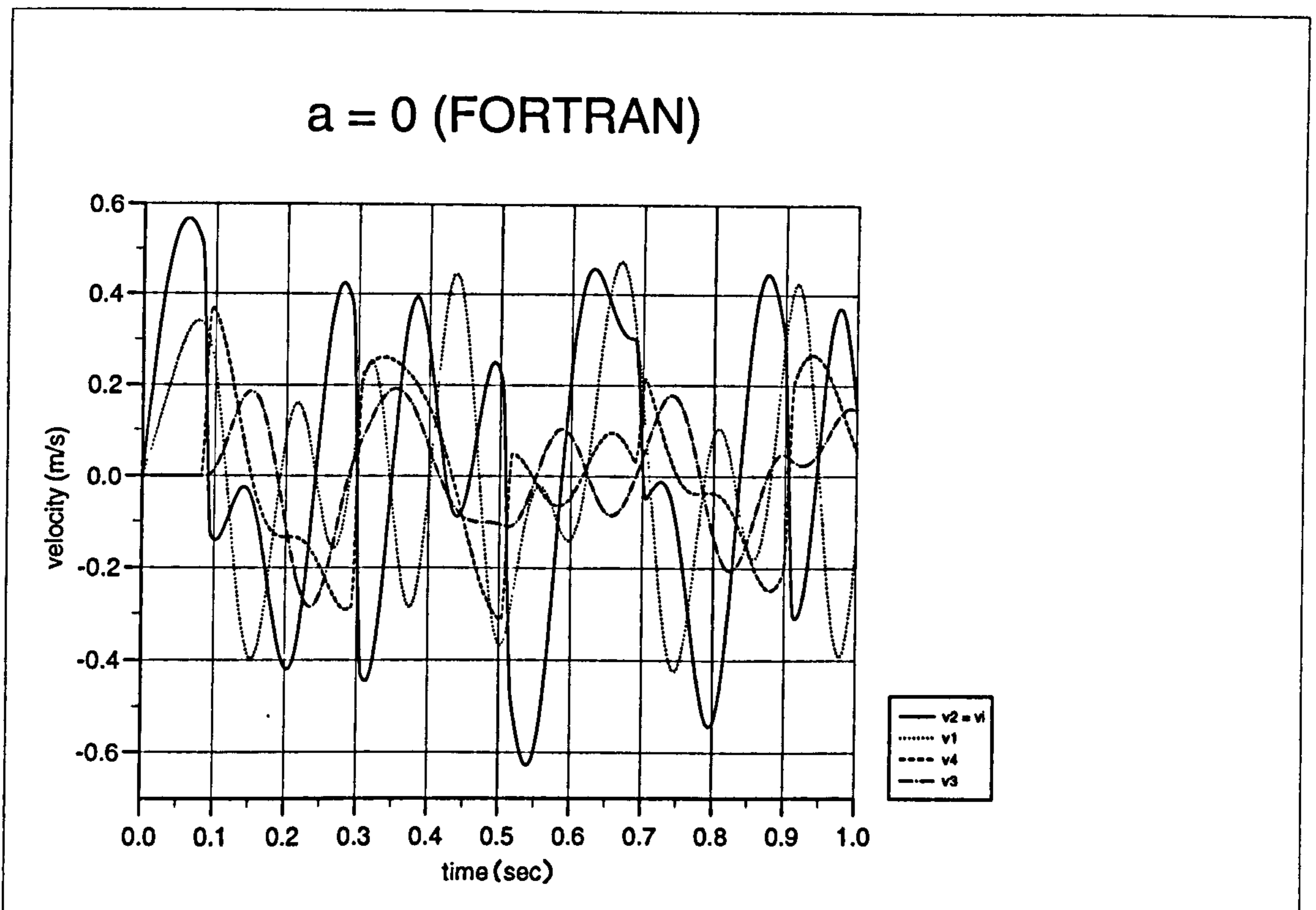


Figure 5.13b) : Velocity curves for pounding case at  $a = 0$  (FORTRAN)



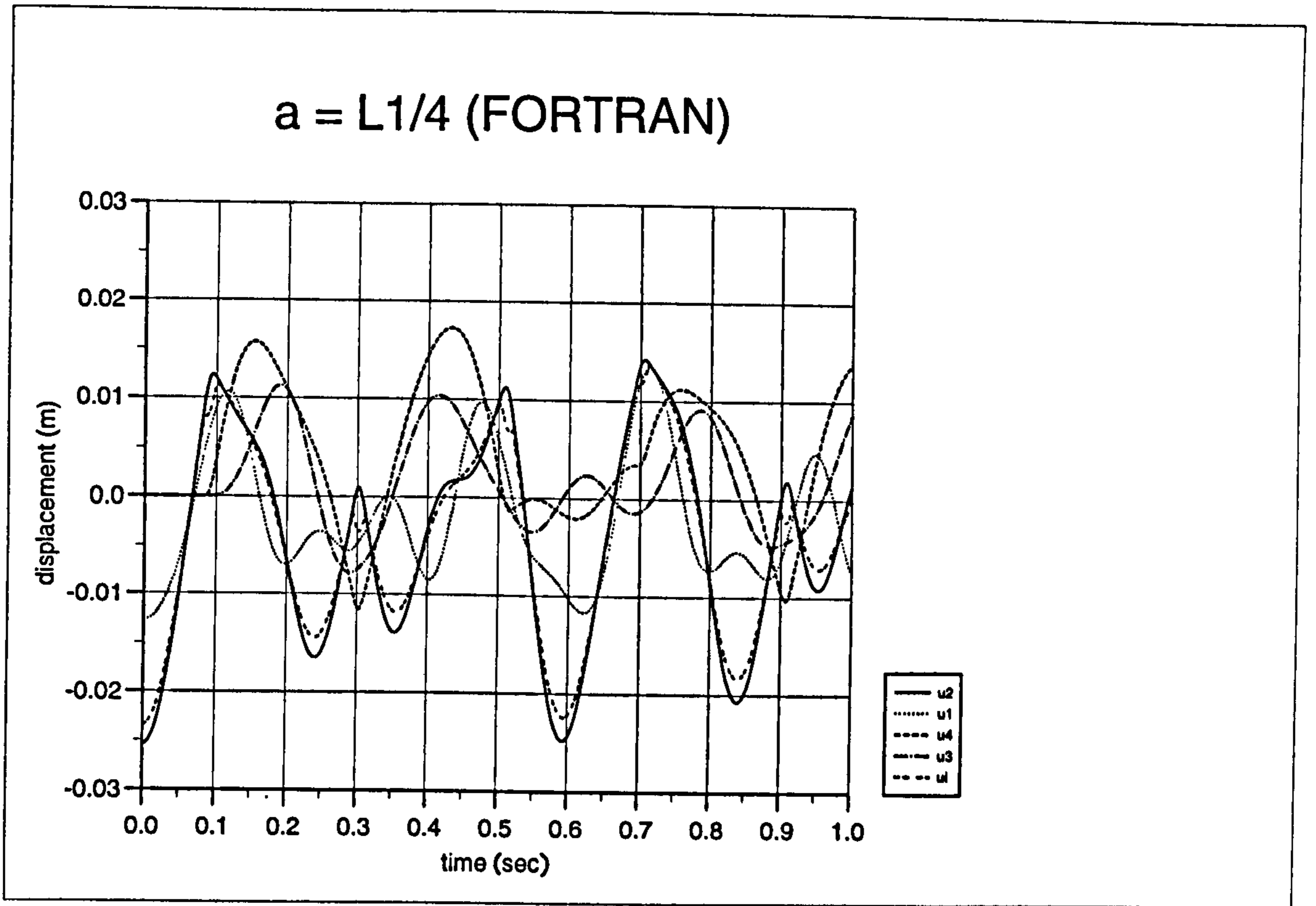


Figure 5.14a) : Displacement curves for pounding case at  $a = L/4$  (FORTRAN)

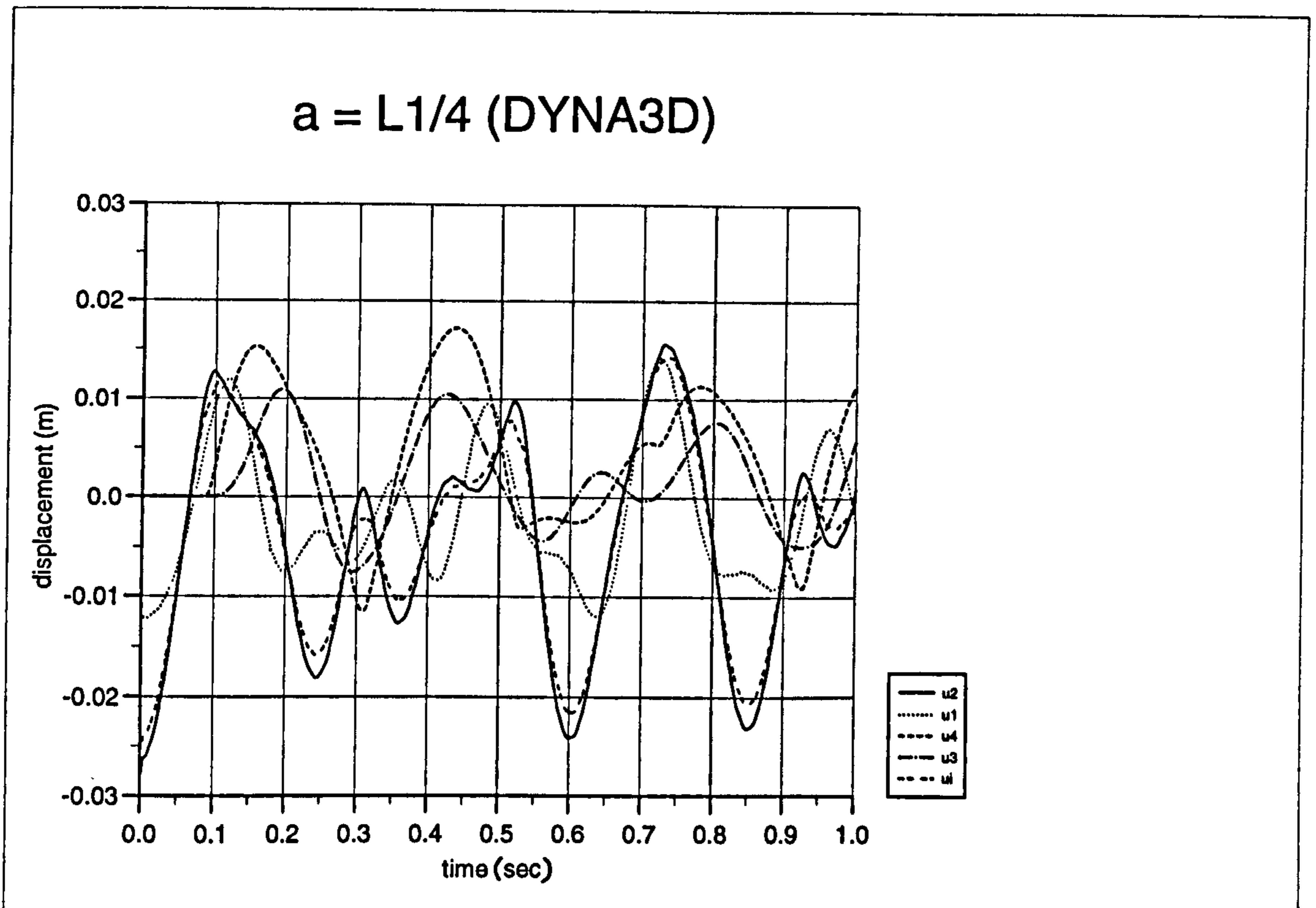


Figure 5.14b) : Displacement curves for pounding case at  $a = L/4$  (DYNA3D)

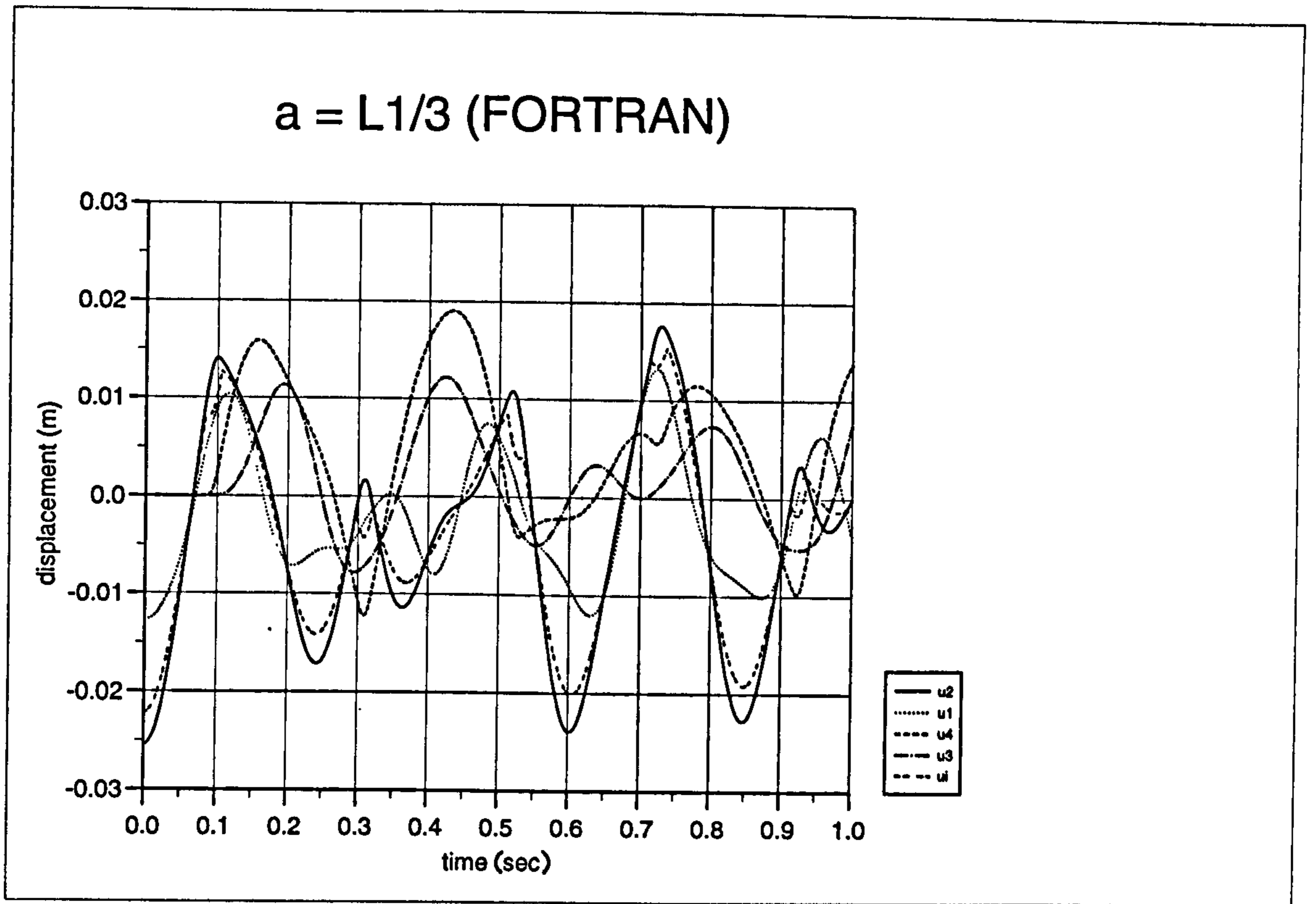


Figure 5.15a) : Displacement curves for pounding case at  $a = L1/3$  (FORTRAN)

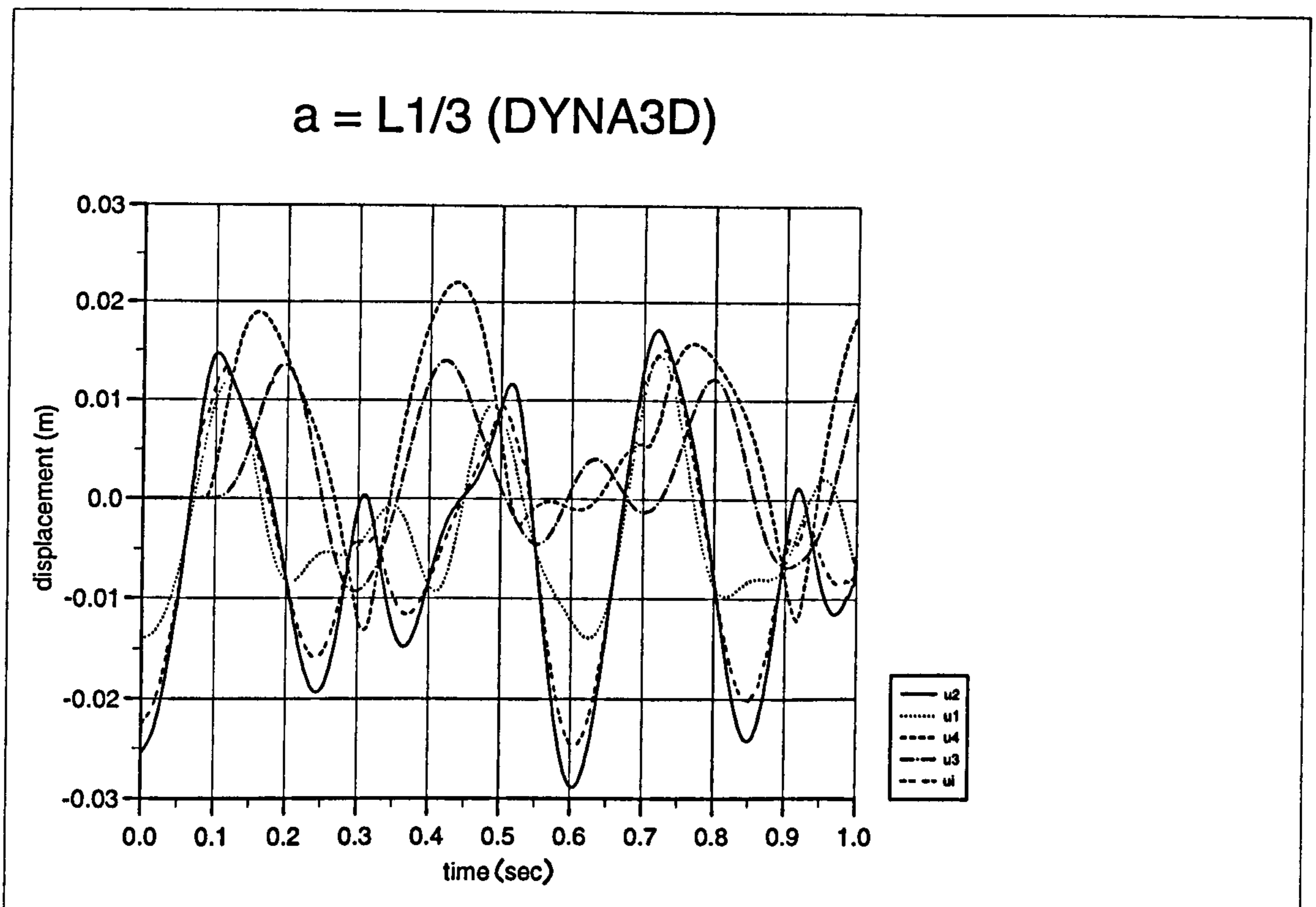


Figure 5.15b) : Displacement curves for pounding case at  $a = L1/3$  (DYNA3D)

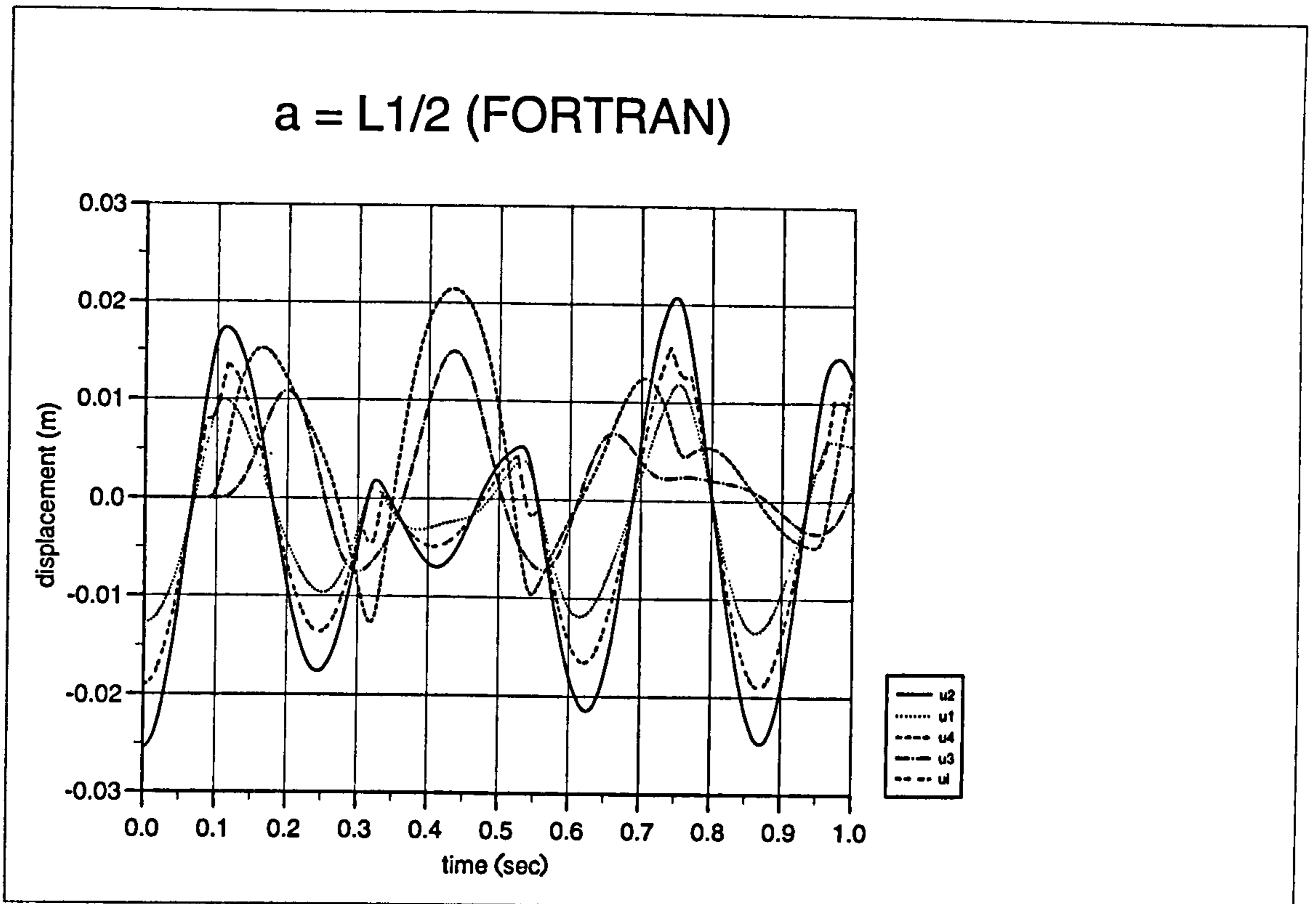


Figure 5.16a) : Displacement curves for pounding case at  $a = L1/2$  (FORTRAN)

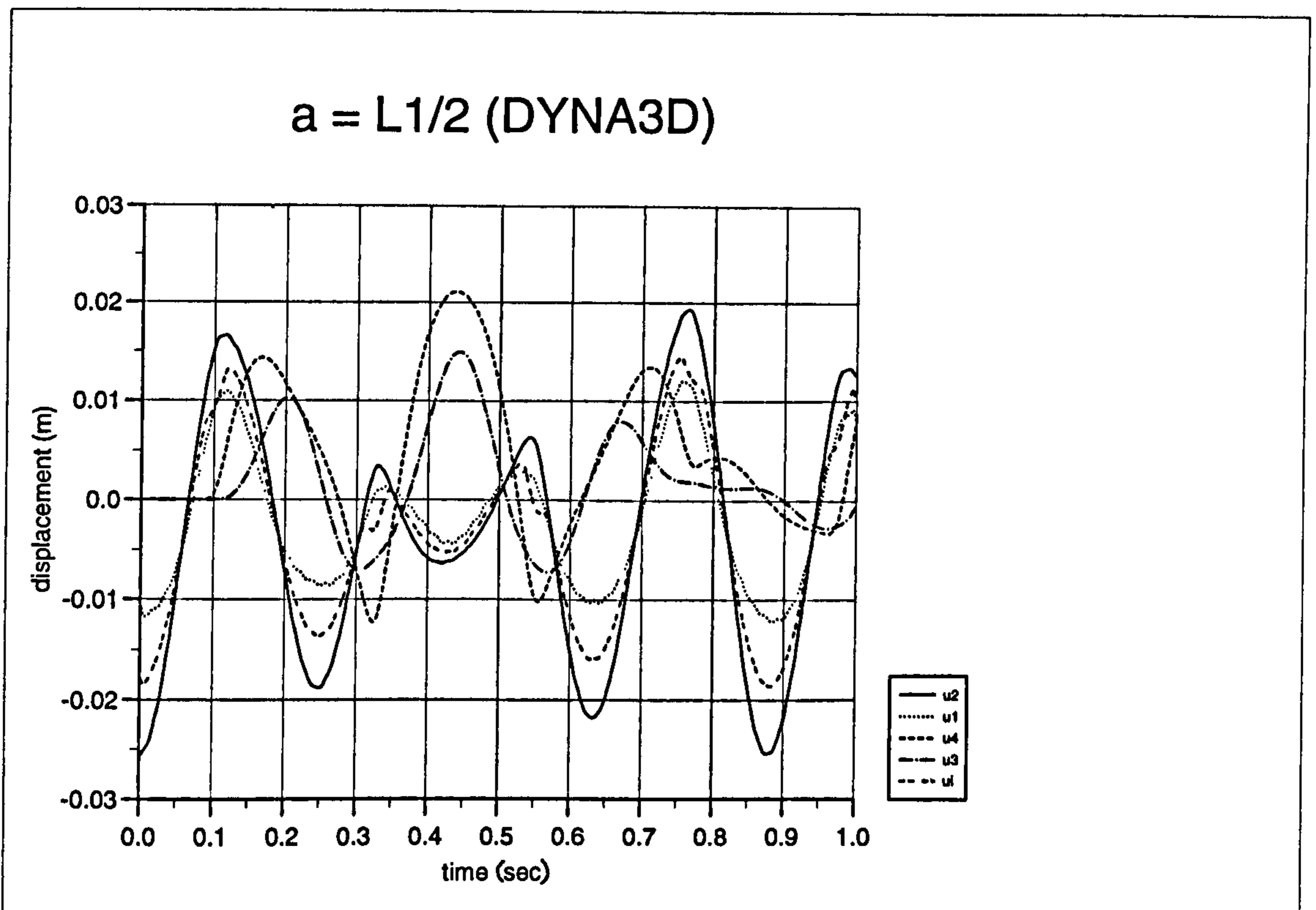


Figure 5.16b) : Displacement curves for pounding case at  $a = L1/2$  (DYNA3D)

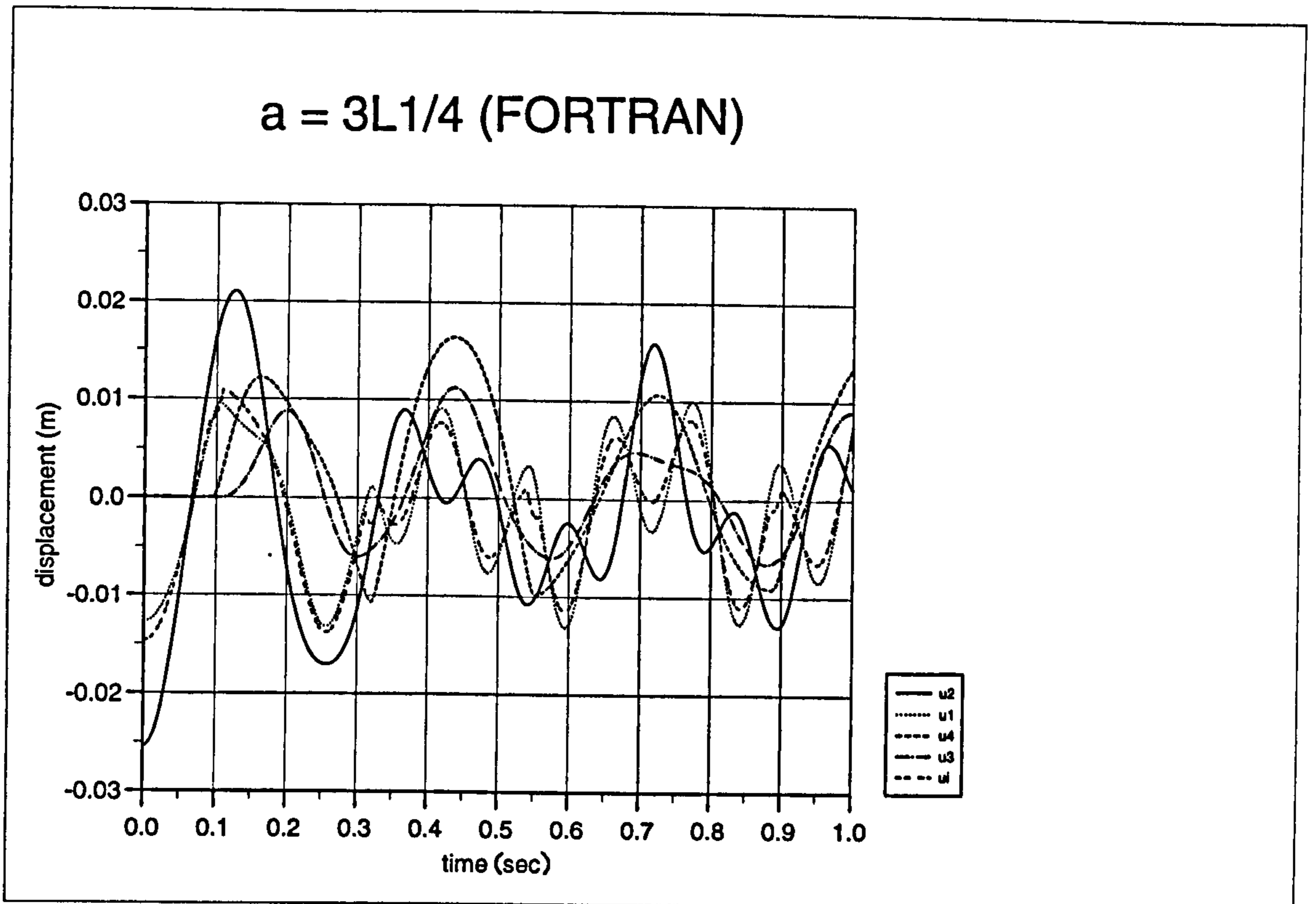


Figure 5.17a) : Displacement curves for pounding case at  $a = 3L1/4$  (FORTRAN)

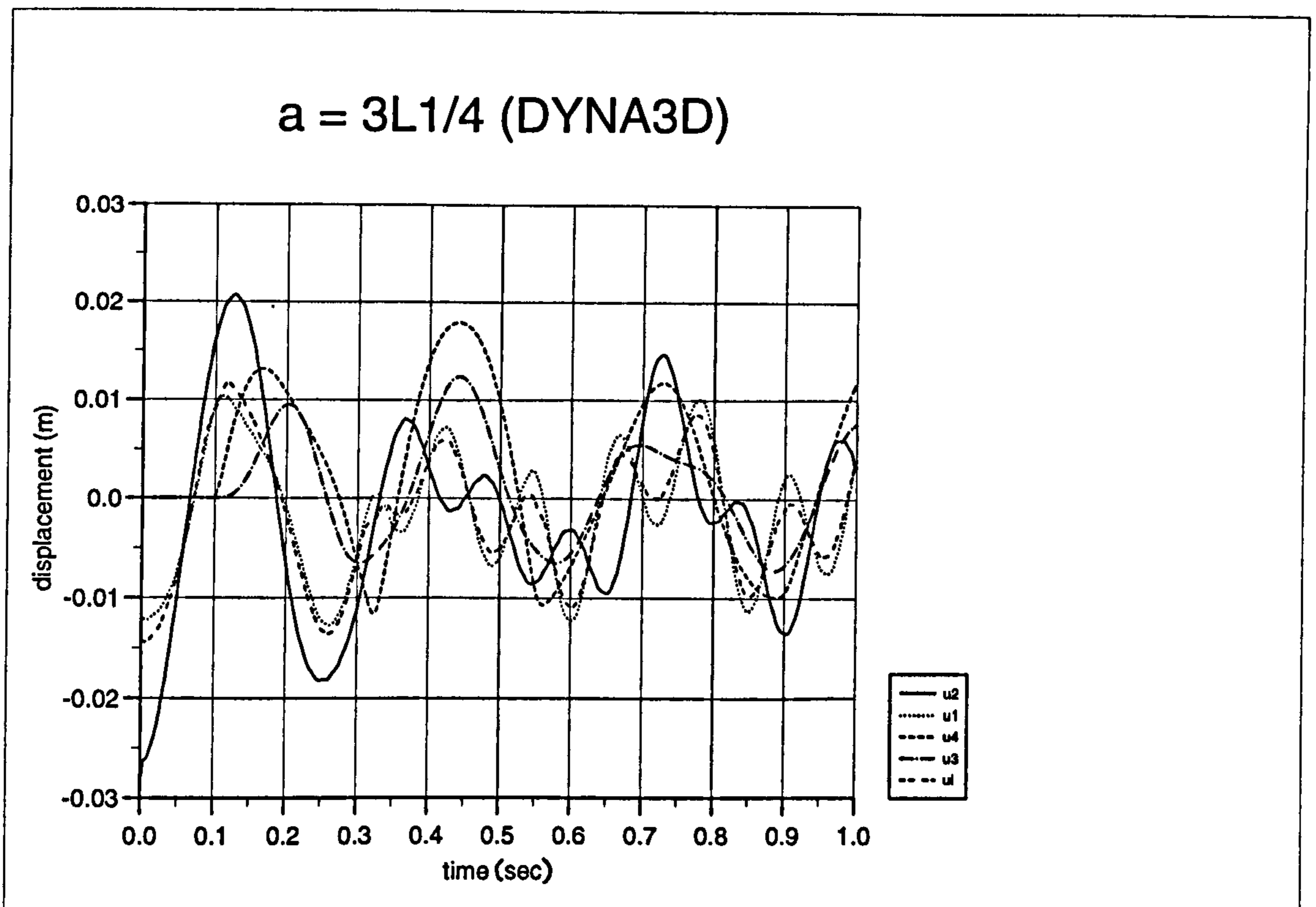


Figure 5.17b) : Displacement curves for pounding case at  $a = 3L1/4$  (DYNA3D)

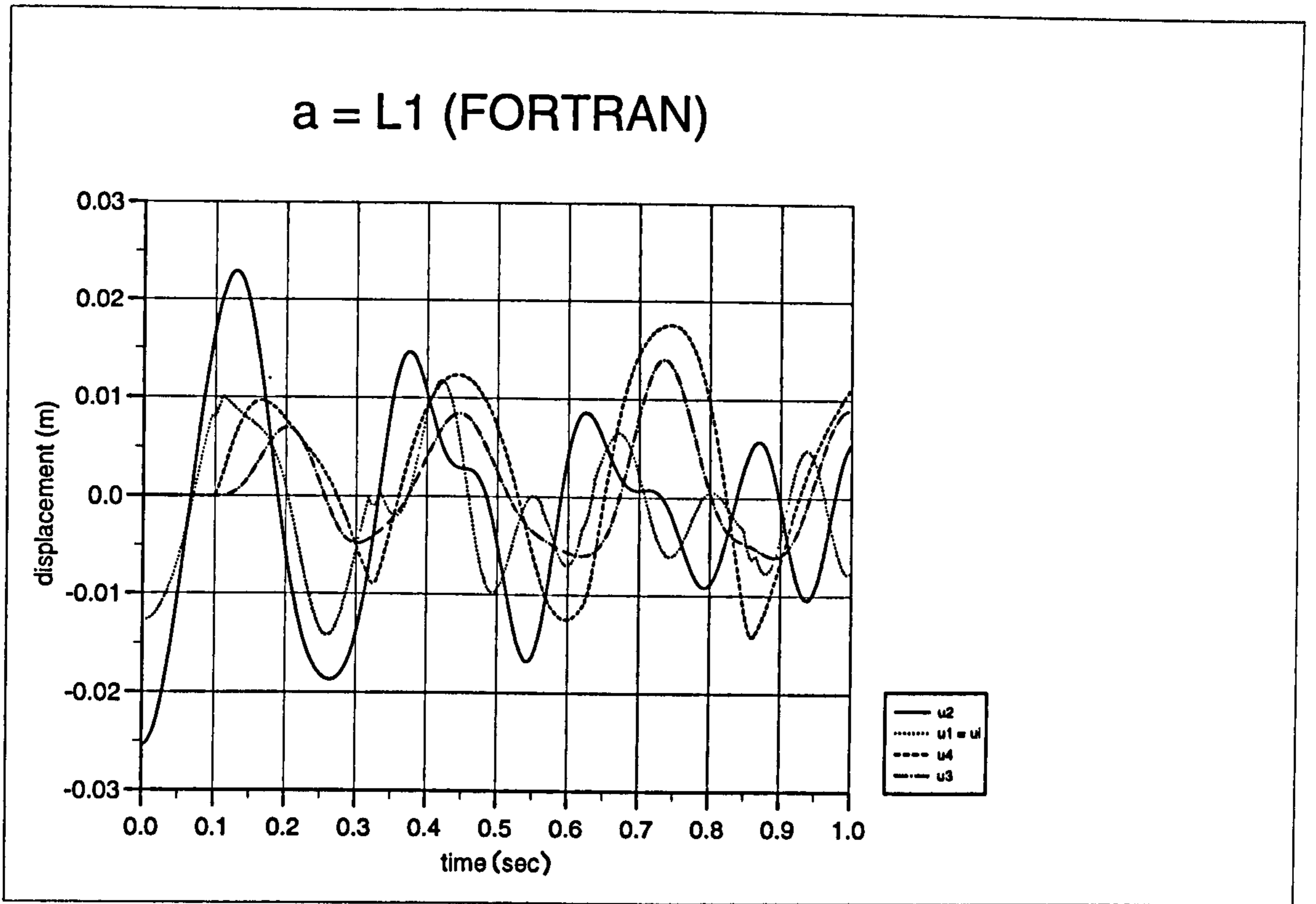


Figure 5.18a) : Displacement curves for pounding case at a= L1 (FORTRAN)

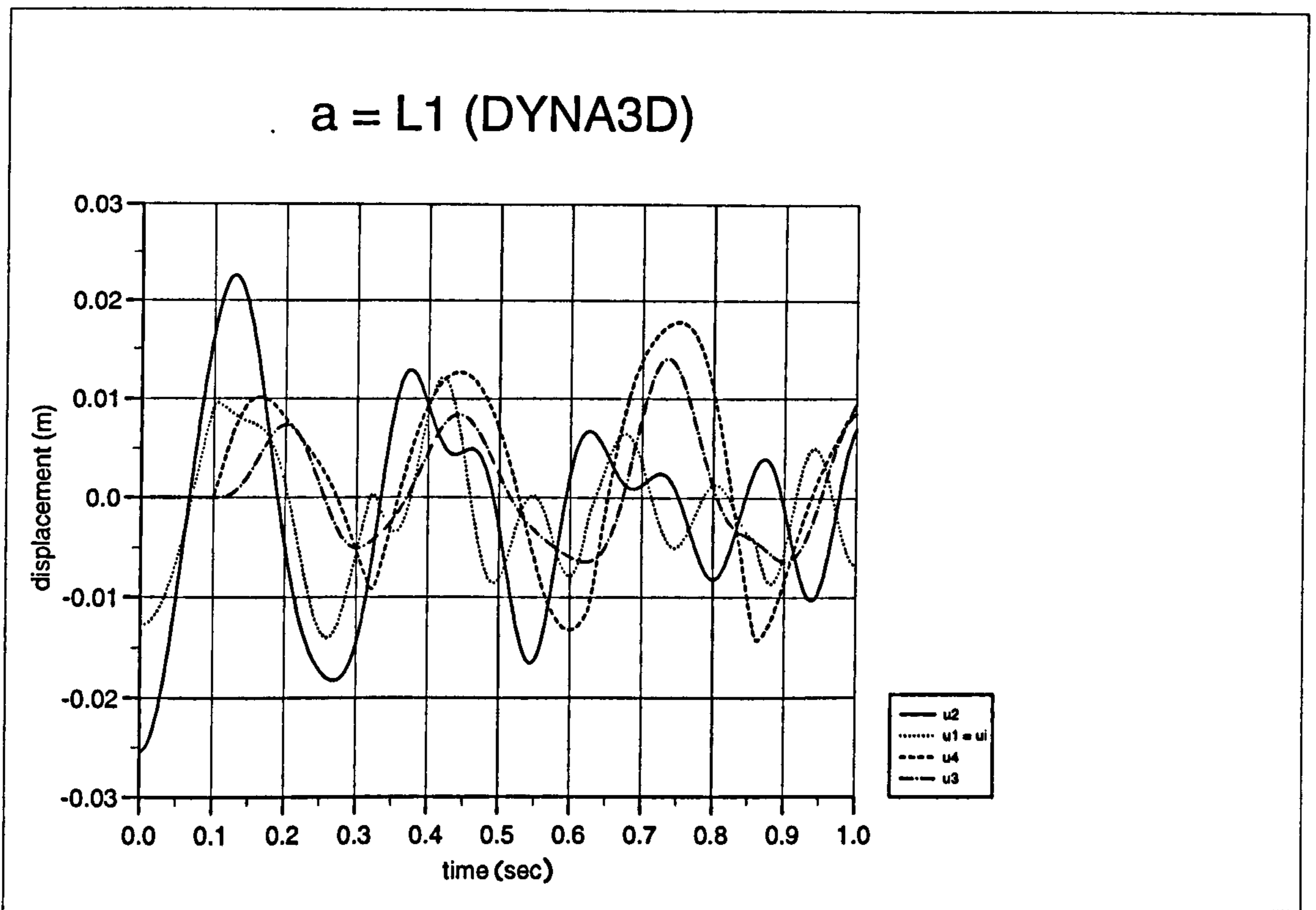


Figure 5.18b) : Displacement curves for pounding case at a= L1 (DYNA3D)

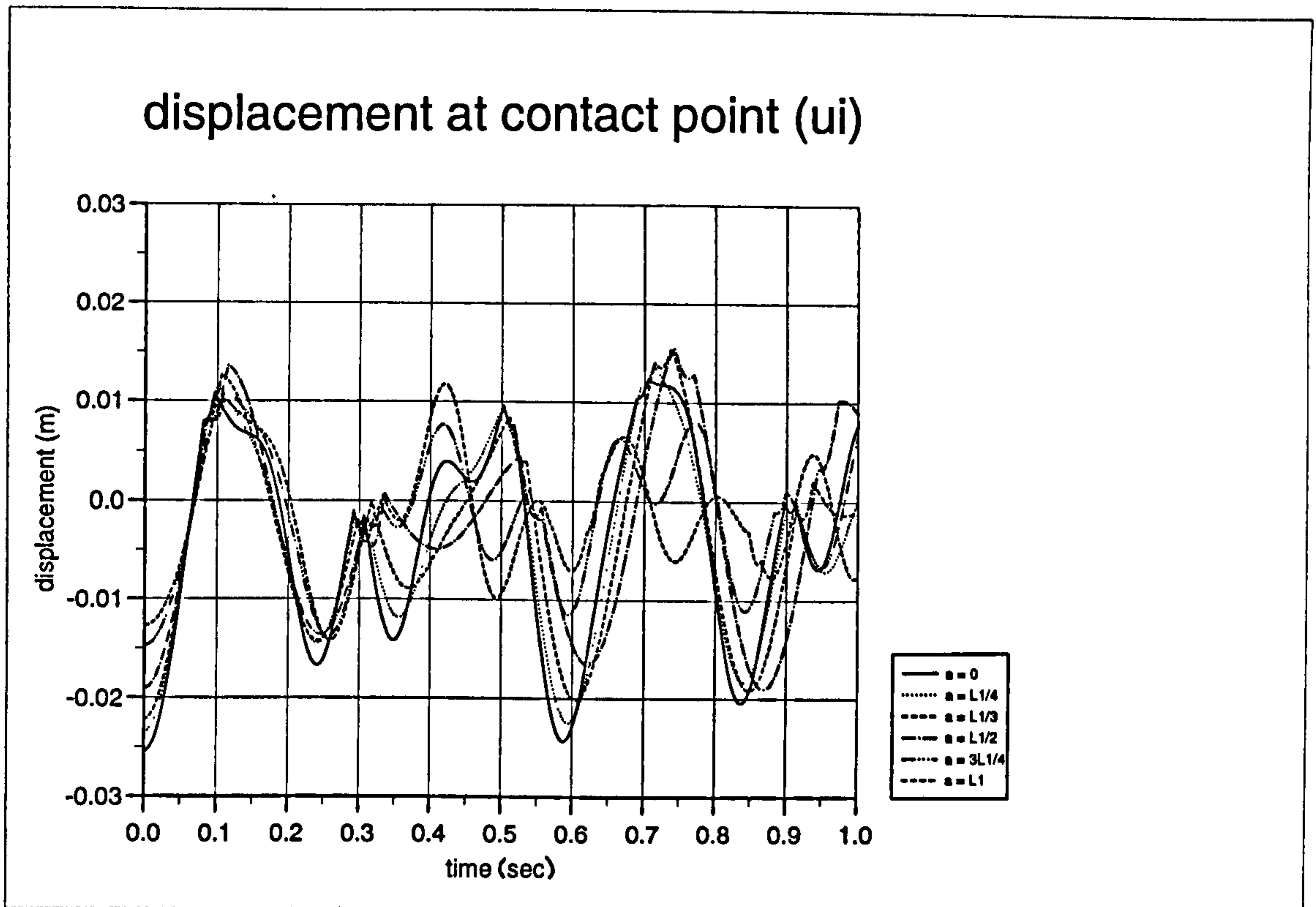


Figure 5.19a) : Displacement curves of  $u_i$  for different "a" values (FORTRAN)

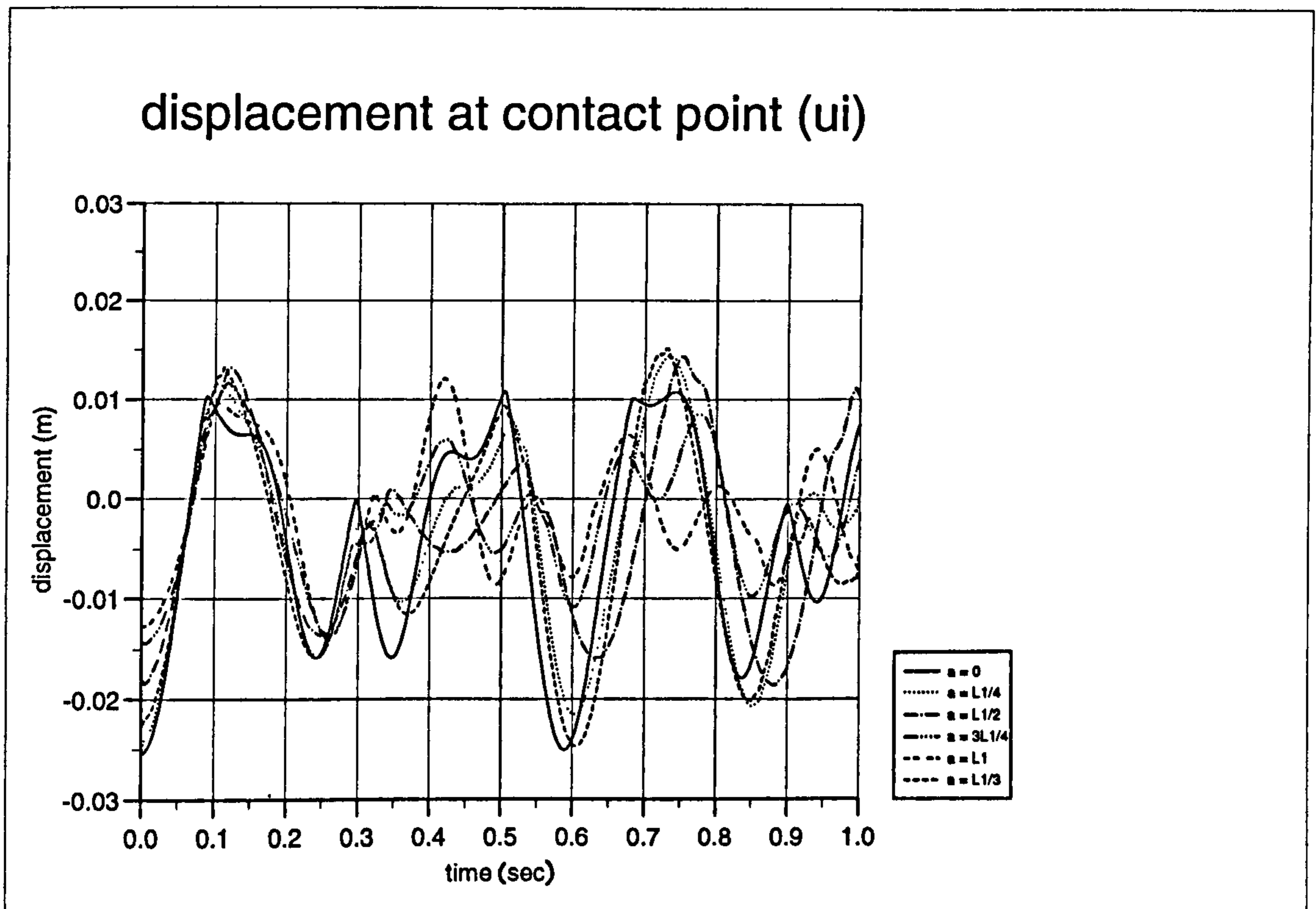


Figure 5.19b) : Displacement curves of  $u_i$  for different "a" values (DYNA3D)

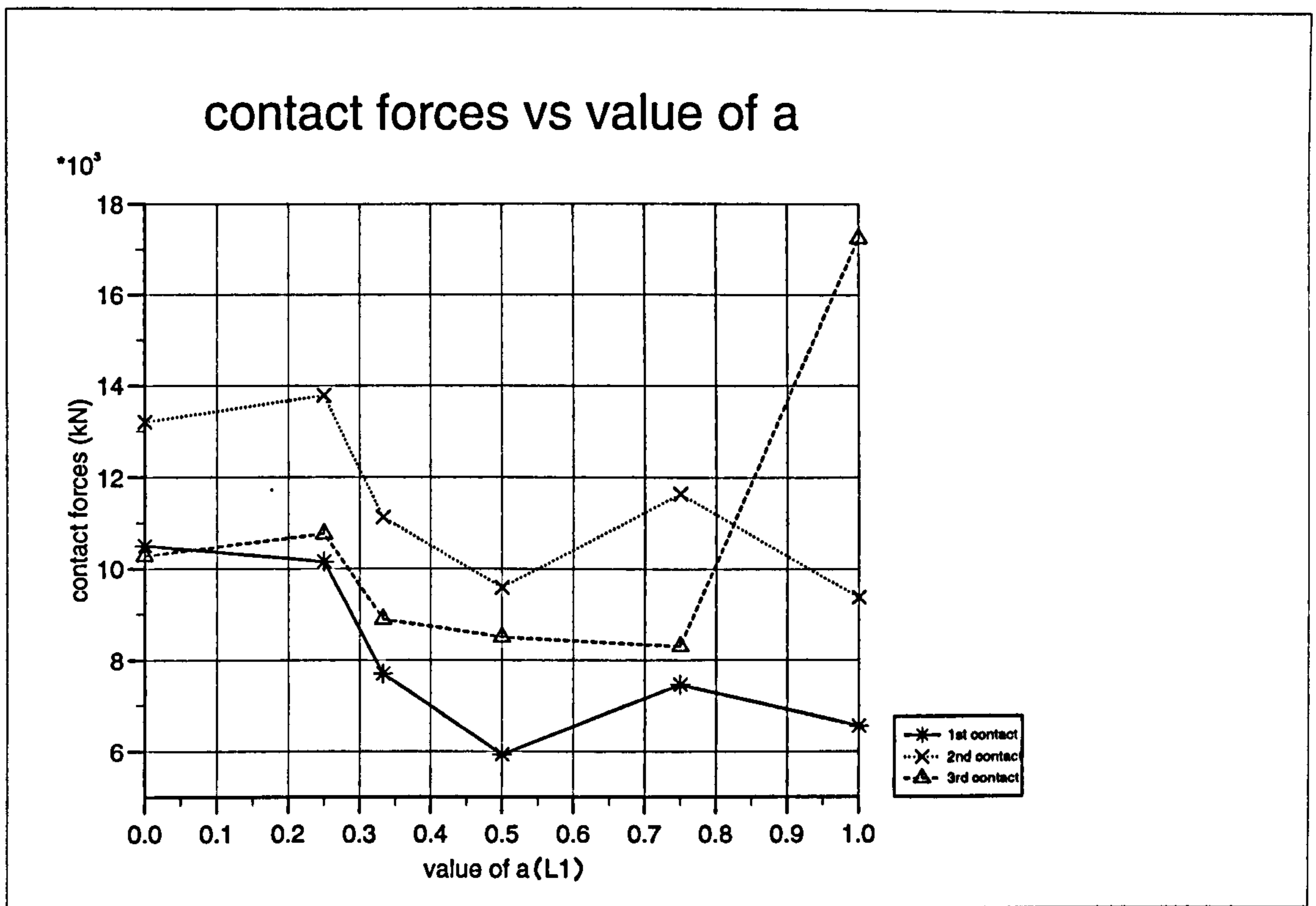


Figure 5.20a) : Maximum contact force against variation of "a" (FORTRAN)

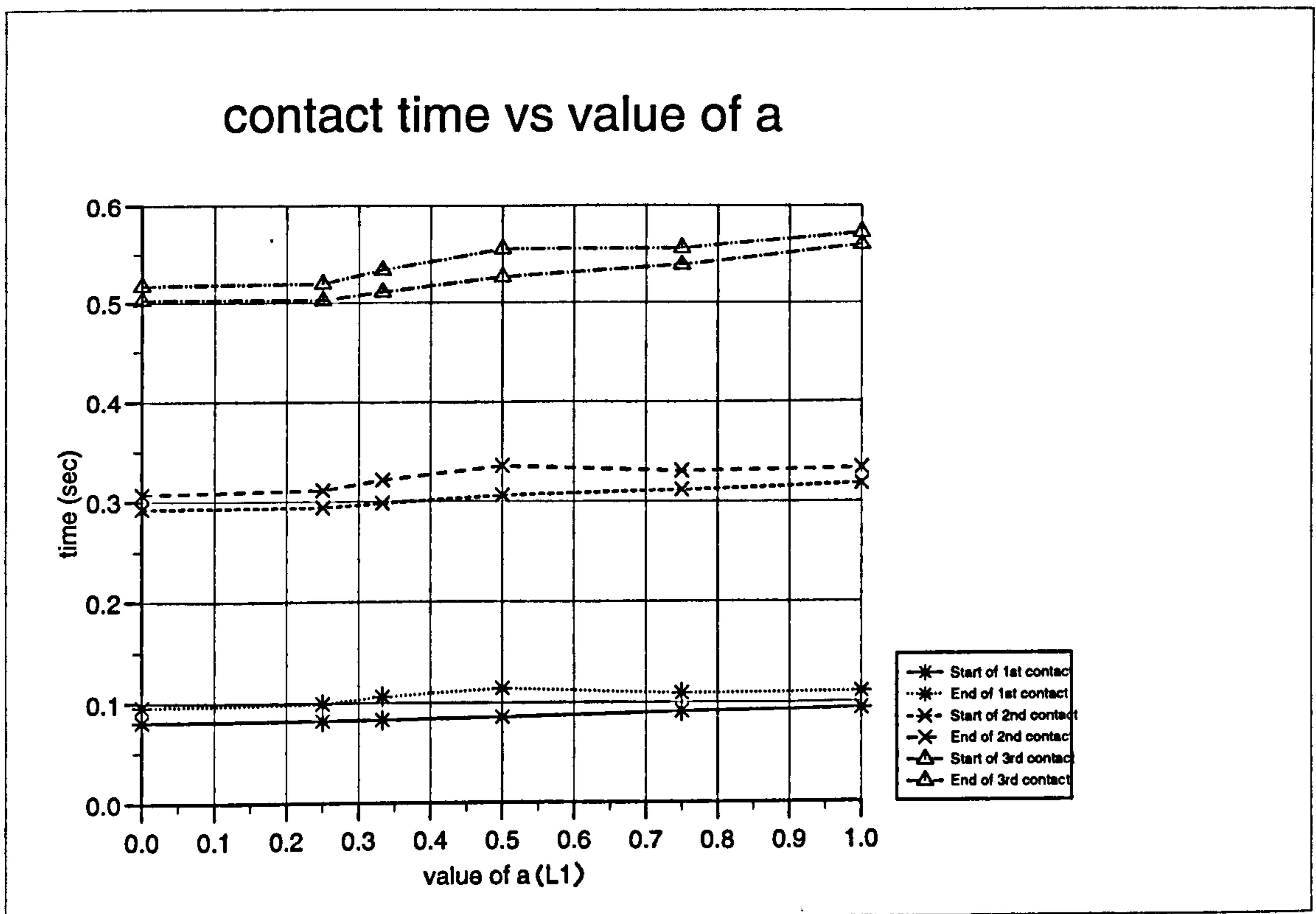


Figure 5.20b) : Contact stage time against variation of "a" (FORTRAN)

## **CHAPTER 6**

### **COMPUTER MODELLING (DYNA3D)**

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#### **6.1 INTRODUCTION**

Nowadays, computer packages for finite elements modelling and analysis of structural systems are widely available. In this research, the models are created and analyzed using two main packages, namely PATRAN Release 2.3 and OASYS-DYNA3D Version 5.1 [35, 36].

The computer analysis was carried out in the Department of Civil Engineering of University of Liverpool, using UNIX System V Release 3 as the operating system, available on a wide range of computer systems, including personal computers and SUN 4/490 workstations.

PATRAN is a pre-processing graphical program and is designed to generate a mesh of finite elements for a model or a structure. Its advantages are, that it gives an option for adjusting, editing the mesh and can be used repeatedly with minimum modifications where necessary.

OASYS-DYNA3D is a three-dimensional finite element package for analysing the dynamic response of the model, where sufficiently accurate boundary conditions can be applied to the structure and hence produce sufficiently accurate results. It also offers a plots program which is very user friendly in the graphical form of the presentation.

There are three main stages involved in this computer modelling, which are pre-processing, computer analysis and post-processing as shown in Figure 6.1 below.

Figure 6.1 : Sequence of computer analysis.



NO. OF STAGE	SEQUENCE OF COMPUTER ANALYSIS	OUTPUT FILE
STAGE 1 Pre-processor	<pre> graph TD     PATRAN --&gt; N_CODE[N/CODE]     N_CODE --&gt; DYNA3D     DYNA3D --&gt; D3PLOT[D3PLOT or T/HIS]     D3PLOT --&gt; UNIGRAPH           </pre>	patran.dat.1 patran.out.1
STAGE 2 Analysis Data Input Boundary Conditions	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> </div> Oasys_5 DYNA3D	xxxx.inf  xxxx.thf xxxx.xtf xxxx.ptf xxxx.otf
STAGE 3 Post-processor		disp.cur force.cur post001.out this001.psc  xx.plt xx.fold xx.plot

### 6.1.1 Pre-processing using PATRAN

PATRAN is a pre-processor which is designed to take the drudgery out of the process of a finite element mesh generation. There are three main stages to generate the finite element model. First, the geometry stage, where the model is generated using lines and coordinates position. The next stage is analysis model where the nodal numbers and element numbers are assigned to the model. In this stage the boundary conditions, restraints, loads and forces are introduced at the nodal points. Finally, the interface stage, where a "neutral file", namely (patran.out.1) file is created that can be used later in section (6.1.2). The data file is called (patran.dat.1). This data file can be re-opened and edited when necessary.

### **6.1.2 Computer Analysis using OASYS-DYNA3D**

N/CODE is a sub-package of the OASYS-DYNA3D computer program. At this stage, N/CODE reads a "neutral file" (patran.out.1) by changing PATRAN to a finite element code and creating a DYNA3D input file namely (xxxx.inf). All file names are made up of a four characters, followed by a three characters extension as the output file (Figure 6.1).

Data such as termination times, material properties, displacements boundary condition curves, spring and lumped mass properties must be prepared prior to the creation of (xxxx.inf) file. In the first analysis, these values are keyed in and this existing file can be used as the material data file in the next analysis.

After the N/CODE session is finished, the (xxxx.inf) file is submitted for the computer analysis using DYNA3D and it produces 'loads' of files for post processing and (xxxx.otf) is the only output file in which details of the calculations at each time step is shown. By default the time step is 1/100 of the time of analysis as in time history file (xxxx.thf) and the complete state file (xxxx.ptf) store 10 states of images. More frequent time intervals can be obtained during the contact stage in the pounding analysis by restarting the complete analysis at the occurrence of the contact. The (xxxx.xtf) file stores all the contact forces data.

### **6.1.3 Post-processing using D3PLOT, T/HIS and UNIGRAPH**

Most existing post-processor only to produce one image at a time. D3PLOT is used in transient analyses and is capable of producing a set of results at a time. This make much more sense so that one is able to see how the results change with time using (xxxx.ptf) file. Results in an animated form can be displayed from this file.

OASYS T/HIS is an x-y plotting program, specifically designed to produce time-history plots from transient analyses, such as those performed using OASYS DYNA3D and to plot any form of x-y data produced either by a program or by directly typing the values into the computer. The response quantities of interest are the lateral displacement, velocity and contact force. The time history file

(xxxx.thf) produces a displacement curve (disp.cur), velocity curve (vel.cur) and acceleration curve (acce.cur). The contact force value against time (force.cur) for the discrete elements can be obtained from the (xxxx.xtf) via T/HIS.

To produce a standard hardcopy of the results, UNIGRAPH package is used to plots all the time history results from DYNA3D using the (disp.cur) and theoretical analysis from FORTRAN program.

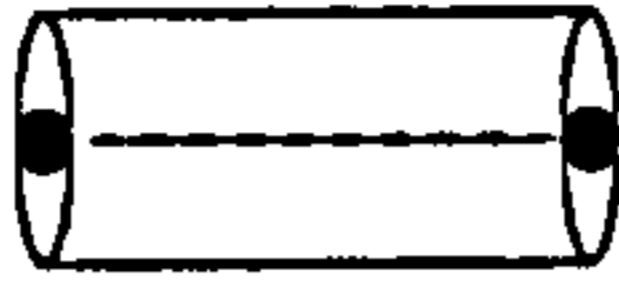


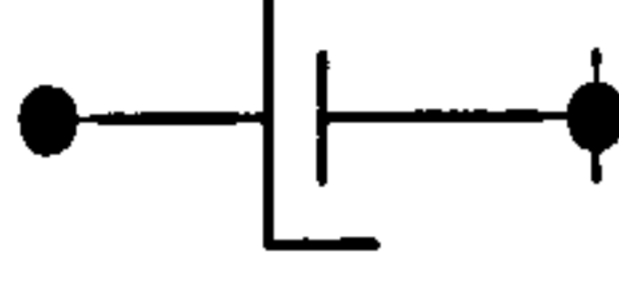
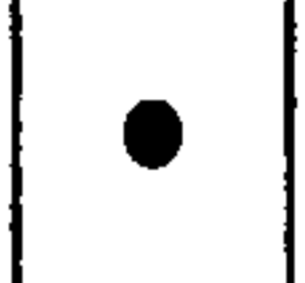
## **6.2 ELEMENT TYPE**

### **6.2.1 Introduction**

DYNA3D offers a wide range of element types. There are solids, shells and beams. Modelling can be with solid elements only, or with shell elements only, or either both of these along with beam elements. In addition there are discrete elements like masses, springs and dampers which are available.

In this research, the combination of beam and discrete elements are found to be the most simplest and efficient way to analyze the shear frame and the simple lumped mass model idealization. Some of the elements used in this modelling are shown in Figure 6.2.

The consistency of units used through out the computer modelling is very important. The units are N, m, sec, and kg. The elements personal identification (PID) and configuration (CONFIG) numbers will determine the material and element type, respectively. For example, from Figure 6.2, the damper element is assigned by PID number 2 and Configuration number 20. This is done in the pre-processor stage (PATRAN). The input data and the material type are assigned in the computer analysis stage.

ELEMENT TYPE	SYMBOLS	ELEMENT ID	INPUT DATA	MATERIAL TYPE
BEAMS		PID = 1 CONFIG = 0	Density, ( $\text{kg/m}^3$ ) Young's Modulus, E Poisson's ratio, $\nu$ Cross-sectional area, A $I_{yy}$ , $I_{zz}$ , $I_{xx}$ .	Material Type 1 (Elastic)
SPRING		PID = 1 CONFIG = 6	Elastic stiffness, $k_s$ (N/m) (force/disp.)	Property Type 1 (Linear Elastic)
GAP SPRING		PID = 1 CONFIG = 6	Elastic stiffness, $k_s$ (N/m) (force/disp.) Clearance / gap (m)	Property Type 1 (Linear Elastic)
DAMPER		PID = 2 CONFIG = 20	Damping constant, $c$ (force/disp. rate)	Property Type 2 (Linear Viscous)
LUMPED MASS		PID = 1 CONFIG = 7	mass (kg)	-

NOTE : ● REPRESENT NODAL POINT

Figure 6.2 : Element and Material types of DYNA3D

### 6.2.2 Beam Element

In this research , the columns of the shear frame model are represented by beam elements of a circular cross section. DYNA3D provide beams with two formulations they are; Hughes-Liu (H-I) and Belytschko-Schwer (B-S). Using Hughes-Liu beam, requires the actual cross-section dimensions, whilst for the Belytschko-Schwer beam, the moment of inertia and shear area are required. In this research, later beam is used. The floor of the frame is represent by the spring element as discussed in next section.

### 6.2.3 Discrete Element

Spring, damper and lumped mass are categorised as discrete elements in DYNA3D. The spring elements and lumped masses represent the slab stiffness,  $k_s$  and mass of the floor, respectively. In DYNA3D, spring elements generate a force which depends on displacement (i.e. change of length of the element). This force is applied along the element axis. The stiffness of the spring is given by:

$$k_s = \frac{E A}{L} \quad (6.1)$$

where  $k_s$  = spring stiffness  
 $E$  = floor stiffness  
 $A$  = cross sectional area of the floor  
 $L$  = the length of the floor.

To allow only the horizontal motion of the floor, the translational spring with linear elastic properties is selected in this research. In addition to that, the "clearance or gap" option gives a required property to represent the building separation. This spring works in compression only and is very useful in modelling a building hitting an adjacent building for the aligned floor case.

The same concept is applied to the damper element except the damping constant,  $c$  is required instead of the spring stiffness,  $k_s$ . The lumped mass is defined in PATRAN using a bar element with configuration number 7.

## **6.3 MATERIAL TYPE**

Not all the 28 available materials in DYNA3D can be used for analysis with a beam element, because certain material types, are only suitable for certain element. In this research, the steel shear frame structure is represented by material type no 1 (elastic).

Material data in OASYS DYNA3D has certain preset values. However, these can be altered to values required for a particular model.

### **6.3.1 Beam Element - Material Type No. 1 (elastic)**

For material type No. 1 in beam element, data input of the following properties are required:

- a. Density,  $\rho$  ( $\text{kg/m}^3$ )
- b. Young's Modulus,  $E$  ( $\text{N/m}^2$ )
- c. Poisson's Ratio,  $\nu$  (0.3)

Additional data are required for Belytschko-Schwer beams such as:

- a. Cross-sectional area,  $A = \pi d^2 / 4$
- b.  $I_{yy} = \pi d^4 / 64$
- c.  $I_{zz} = \pi d^4 / 64$
- d. Torsion Constant,  $I_{xx} = \pi d^4 / 32$
- e. Shear area,  $A_s$  (default same as  $A$ )

### **6.3.2 Spring Element - Property Type No. 1 (Linear Elastic)**

In DYNA3D, the property type numbers must be consecutive, starting from 1 if more than one springs stiffness is required. The only data input needed for this property is the spring stiffness (floor),  $ks$  in term of (force/disp.) . The gap-spring element can be achieved by assigning the clearance value in the N/CODE to represent the building separation.

### 6.3.3 Damper Element - Property Type No.2 (Linear Viscous)

In the previous chapters, the solution of the equation of the motion, for damping analysis requires the value of the damping coefficient,  $\xi$ . This value can be assigned directly. In DYNA3D, it requires the value of the damping constant,  $c$  instead of the damping coefficient,  $\xi$ . Therefore, in this section, a value of  $c$  is derived separately for SDOF and MDOF damping systems.

#### 6.3.3.1 SDOF damping system

For a damped SDOF system in Figure 6.3, the value of  $c$  can directly be expressed as:

$$c = \xi c_c \quad (6.2)$$

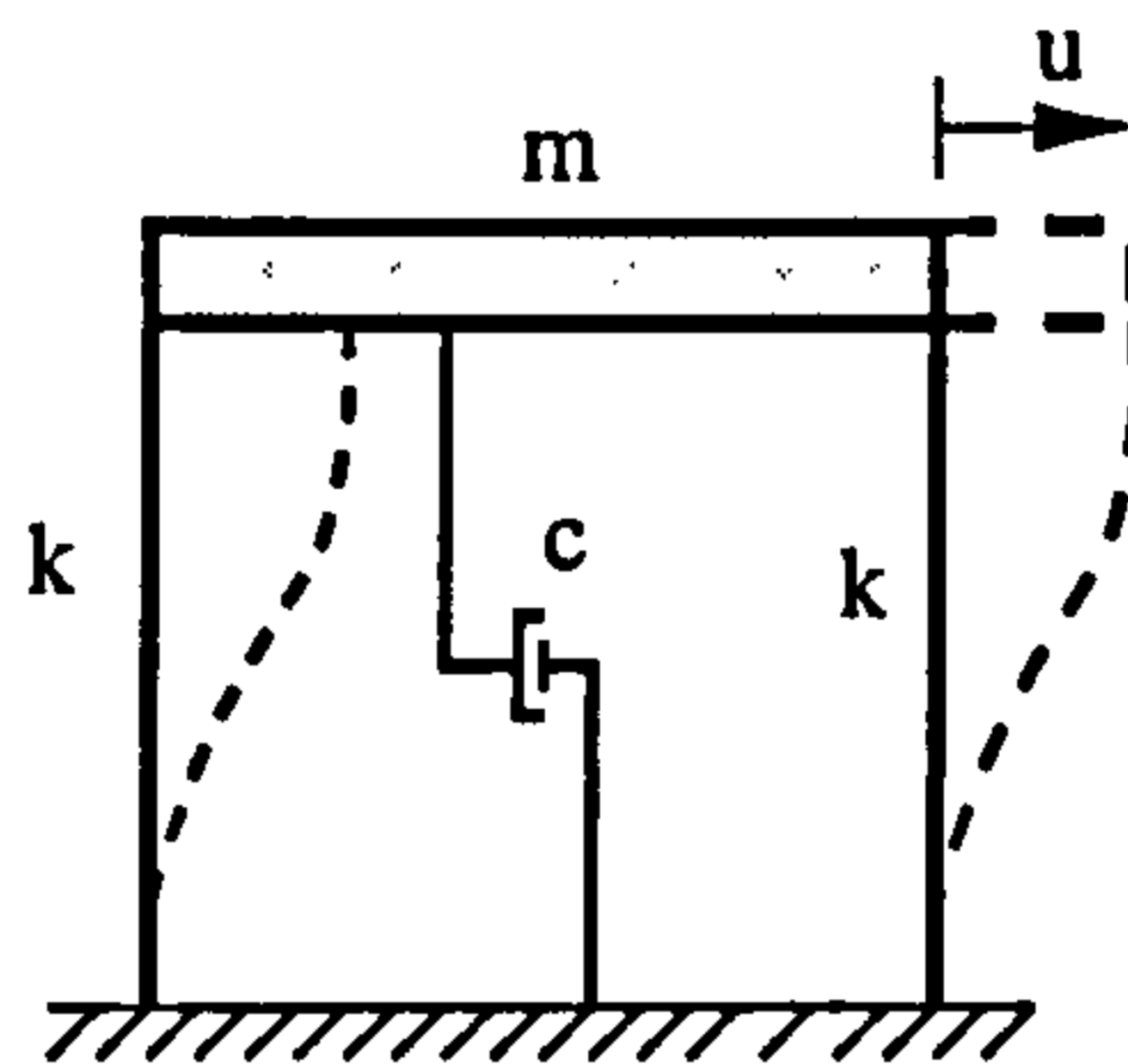


Figure 6.3 : SDOF damping system

where  $\xi$  = damping coefficient (eg. for steel say 2%)

$c_c$  = critical damping (  $2 m \omega$  )

$m$  = floor mass (N. sec<sup>2</sup>/m)

$\omega$  = natural frequency ( rad/sec)

### 6.3.3.2 MDOF damping system

For a damped MDOF system, Rayleigh damping is usually assumed. Consider the 2DOF system in Figure 6.4 below.

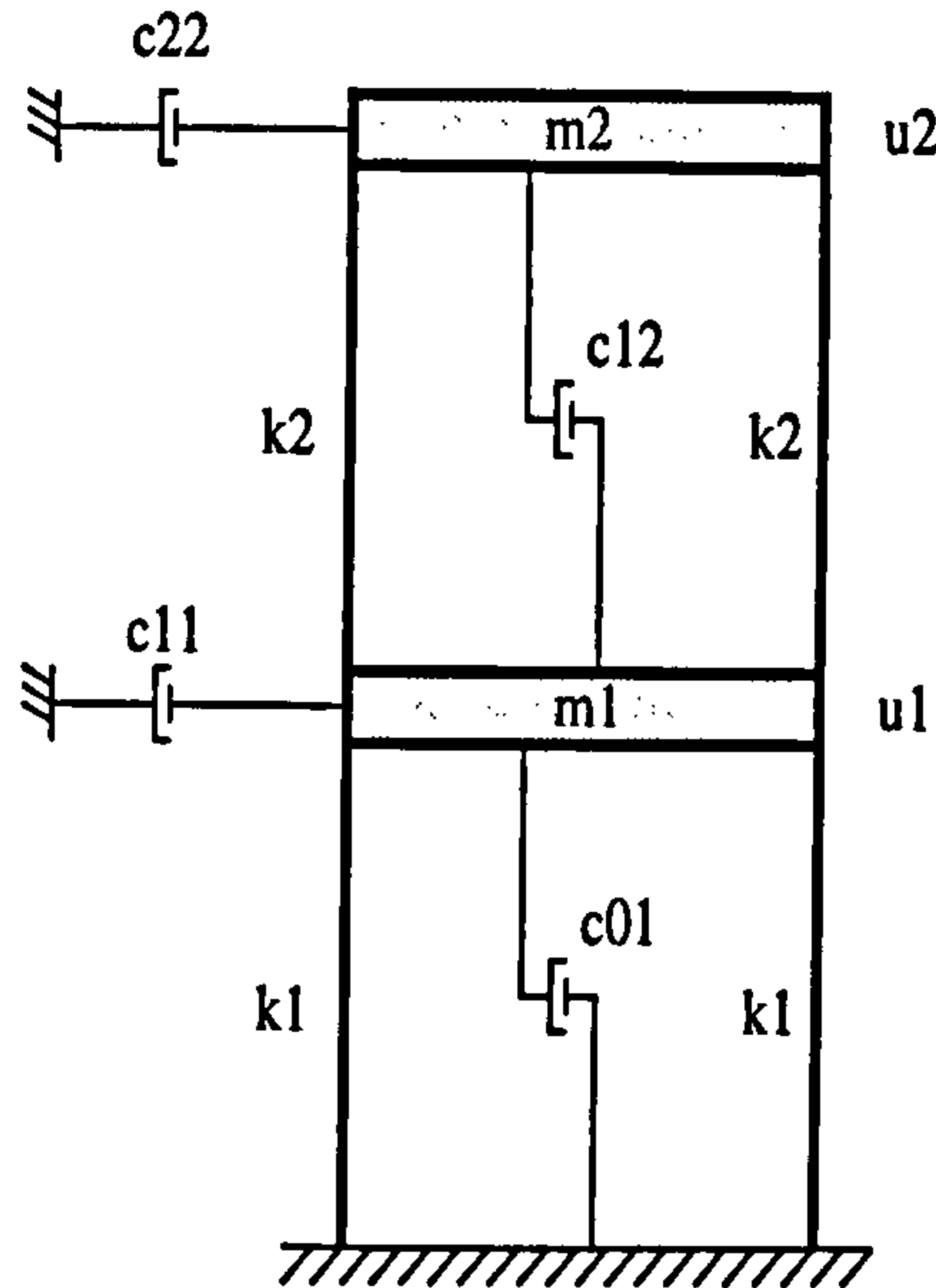


Figure 6.4 : MDOF damped system

For a damped system, the basic equations of motion are :

$$\begin{bmatrix} m2 & 0 \\ 0 & m1 \end{bmatrix} \begin{Bmatrix} \ddot{u}_2 \\ \ddot{u}_1 \end{Bmatrix} + \begin{bmatrix} c12+c22 & -c12 \\ -c12 & c11+c01+c12 \end{bmatrix} \begin{Bmatrix} \dot{u}_2 \\ \dot{u}_1 \end{Bmatrix} + \begin{bmatrix} 2k2 & -2k2 \\ -2k2 & 2k2+2k1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} = 0 \quad (6.3)$$

In general

$$m \ddot{u}_i + c u_i + k u_i = 0 \quad (6.4)$$

where  $i$  is the number of modes. Rearrange this equation will gives:

$$\ddot{u}_i + 2 \xi_i \omega_i \dot{u}_i + \omega^2 u_i = 0 \quad (6.5)$$

Introduce the Rayleigh damping,

$$c = C1 [ m ] + C2 [ k ] \quad (6.6)$$

where  $C1$  and  $C2$  are constant. Equation (6.6) can be expressed in usual notation as :



$$2 \xi_i \omega_i = C1 + C2 \omega_i^2 \quad (6.7)$$

To find the values of C1 and C2, consider two modes of MDOF system:

$$\begin{array}{l} \text{1st mode} \quad 2 \xi_1 \omega_1 = C1 + C2 \omega_1^2 \\ \text{2nd mode} \quad 2 \xi_2 \omega_2 = C1 + C2 \omega_2^2 \end{array} \quad (6.8)$$

Equating values of C1 gives :

$$C2 = \frac{2 \xi_2 \omega_2 - 2 \xi_1 \omega_1}{(\omega_2^2 - \omega_1^2)} \quad (6.9)$$

And

$$C1 = \frac{2 \omega_1 \omega_2 (\xi_1 \omega_2 - \xi_2 \omega_1)}{(\omega_2^2 - \omega_1^2)} \quad (6.10)$$

But some damping is always associated with mass,  $\underline{m}$  and stiffness  $\underline{k}$  of the model as in Equation (6.6). where for a system in Figure 6.4 above,

$$\underline{m} = \begin{vmatrix} m2 & 0 \\ 0 & m1 \end{vmatrix} \quad \text{and} \quad \underline{k} = \begin{vmatrix} 2k2 & -2k2 \\ -2k2 & 2k2 + 2k1 \end{vmatrix} \quad (6.11)$$

Substituting these values into Equation (6.6) gives:

$$\begin{aligned} c &= C1 [ \underline{m} ] + C2 [ \underline{k} ] \\ c &= \begin{vmatrix} (C1 m2 + C2 2k2) & -C2 2k2 \\ -C2 2k2 & (C1 m1 + C2 2k1 + C2 2k2) \end{vmatrix} \end{aligned} \quad (6.12)$$

Equating damping terms in Equation (6.3) and (6.12) gives:

$$\begin{aligned} c22 &= C1 m2 \\ c12 &= C2 2k2 \\ c11 &= C1 m1 \\ c01 &= C2 2k1 \end{aligned} \quad (6.13)$$

These terms are required as data input in the N/CODE stage.

## 6.4 EXAMPLES OF COMPUTER MODELLING

Two examples of the computer modelling from the previous chapter are illustrated in this section. First, the pounding of undamped MDOF system at aligned floor which hits at the top floor ( $a = 0$ ) as in section (3.5.2) and secondly the pounding of damped MDOF system of unaligned floors, which hits at the column level ( $a=L/2$ ) as in section (3.5.4.2).

In both analyses, some assumptions are made about the behaviours of the shear frame models:

- The horizontal members (floors) are assumed to be infinitely rigid compared with the vertical members (columns) which are assumed to be massless.
- Rotation of joints is assumed not to occur and the structure is assumed to sway only on its plane.

### 6.4.1 Undamped MDOF system hits a rigid body (aligned floor, $a=0$ )

To show how the computer modelling is performed using PATRAN and OASYS-DYNA3D, the same model and physical properties of section (3.4.1.5.2) as in Table 3.2 and shown in Table 6.1 below:

$m_3, m_2, m_1$ = building masses of 1.0, 1.5, 2.0 kips-sec <sup>2</sup> /in. = 175197, 262795 , and 350394 kg (N-sec <sup>2</sup> /m)
$2k_3, 2k_2, 2k_1$ = building stiffness of 600, 1200, 1800 kips/in. = $105.12 \cdot 10^3$ , $210.24 \cdot 10^3$ , and $315.35 \cdot 10^3$ (kN/m)
$k_s$ = slab stiffness of 50,000 kips/in. ( $8.76 \cdot 10^6$ kN/m)
Initial top displacement = -1.0 in. (-0.0254 m)
Building separation, $U_s = 0.5$ in. (0.0127 m)
Ground acceleration, ( $\ddot{u}_{gt} = 0$ )
Undamped free vibration, $c = 0$
Height between floors, $L = 72$ in. (1.8288 m)
Young Modulus of steel, $E = 30 \cdot 10^3$ kip/in <sup>2</sup> ( $205 \cdot 10^6$ kN/m <sup>2</sup> )

Table 6.1 : Physical properties of the building. (\*\* 1 kip = 4450N, 1 in=0.0254m)

The lumped mass model consists of 6 nodes, 3 beam elements, 3 lumped mass elements and 1 gap-spring element. In the pre-processor stage (using PATRAN) the following lines, nodal points and elements and boundary conditions are generated (Figure 6.5). The list of commands used in PATRAN to generate the model is in Appendix I. The node number 6 is referred as the initial orientation of the beam.

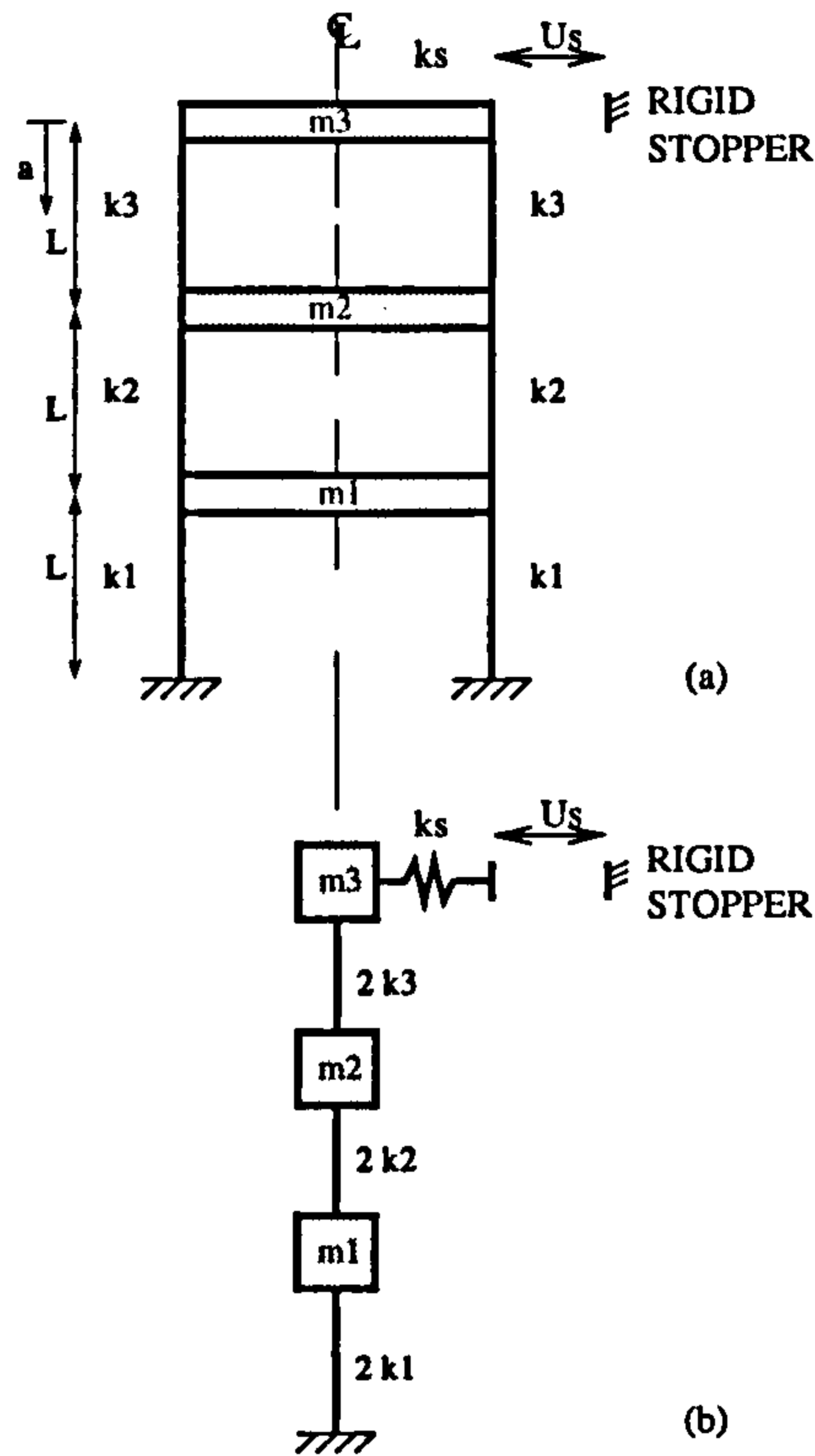


Figure 6.5 : (a) Pounding of MDOF system (b) The lumped mass model.

From the Figure 6.5 above, the explanation of the pre-processor stage are summarised in Table 6.2 below:

ELEMENTS	DESCRIPTION	ELEMENT ID	COMPUTER MODEL		
LI1, LI2, LI3, LI4, LI5	Number of lines created in the geometry stage	-			
E1, E2, E3	Lumped mass elements	PID=1 CONFIG=7		PID=2 CONFIG=7	PID=3 CONFIG=7
E4, E5, E6	Beam elements	PID=1 CONFIG=0		PID=2 CONFIG=0	PID=3 CONFIG=0
E7	Gap-spring elements	PID=1 CONFIG=6			
NODAL POINT	CO-ORDINATE (x,y,z)	BOUNDARY CONDITION			
N1	(1.0, 0, -1.0)	Moved in x direction only (D: /0/0) and no rotation (R: 0/0/0)			
N2	(1.0, 0, -2.8288)	BC_DISP_1 (at N1), BC_DISP_2 (at N2) BC_DISP_3 (at N3)			
N3	(1.0, 0, -4.6576)				
N4	(1.0, 0, -6.4864)	Fixed ended column (D: 0/0/0) and no rotation (R: 0/0/0)			
N5	(2.0, 0, -1.0)				
N6	(1.5, 0, -6.4864)	Only as beam orientation			

Table 6.2 : Data assigned in the Pre-processor (PATRAN) stage.

Table 6.3a and 6.3b lists the data required along with the default N/CODE values which represent a mild steel material and the boundary condition values.

BEAM PROPERTY	VALUE		
Density Young's Modulus Poisson's Ratio	7890 kg/m <sup>3</sup> 205 E+9 N/m <sup>2</sup> 0.3		
Belytschko-Schwer Beam	E4	E5	E6
Cross-sectional Area, $A=\pi d^2/4$ $I_{yy} = \pi d^4/64$ $I_{zz} = \pi d^4/64$ Torsion Constant, $I_{xx}=\pi d^4/32$ Shear Area, $A_s$ (default same as A)	5.704E-2 m <sup>2</sup> 2.589E-4 m <sup>4</sup> 2.589E-4 m <sup>4</sup> 5.178E-4 m <sup>4</sup> -	8.070E-2 m <sup>2</sup> 5.179E-4 m <sup>4</sup> 5.179E-4 m <sup>4</sup> 1.037E-3 m <sup>4</sup> -	9.875E-2 m <sup>2</sup> 7.768E-4 m <sup>4</sup> 7.768E-4 m <sup>4</sup> 1.552E-3 m <sup>4</sup> -

Table 6.3a : Beam Elements properties

DISPLACEMENT CURVES	3 POINT VALUES
BC_DISP_1 (at N1) BC_DISP_2 (at N2) BC_DISP_3 (at N3)	(0,0) , (0.001, -0.02540) , (1.0, -0.02540) (0,0) , (0.001, -0.01283) , (1.0, -0.01283) (0,0) , (0.001, -0.00366) , (1.0, -0.00366)
GAP - SPRING	ELEMENT E7
Spring stiffness, ks Clearance / gap, Us	8.76 E9 N/ m 0.0127 m
LUMPED MASS	VALUE
Mass for element E1 Mass for element E2 Mass for element E3	175197 kg 262795 kg 350394 kg

Table 6.3b : Load curves and discrete element properties.

The displacement curves at three nodal points are described by 3 points curve where their values are listed in the above table. These values ( -0.0254, -0.01283 and -0.00366) are taken from the free vibration mode shapes, and the coordinate of the second point (0.001) is introduced in order to avoid sudden changes in the displacement value. This is explained in Figure 6.6 below.

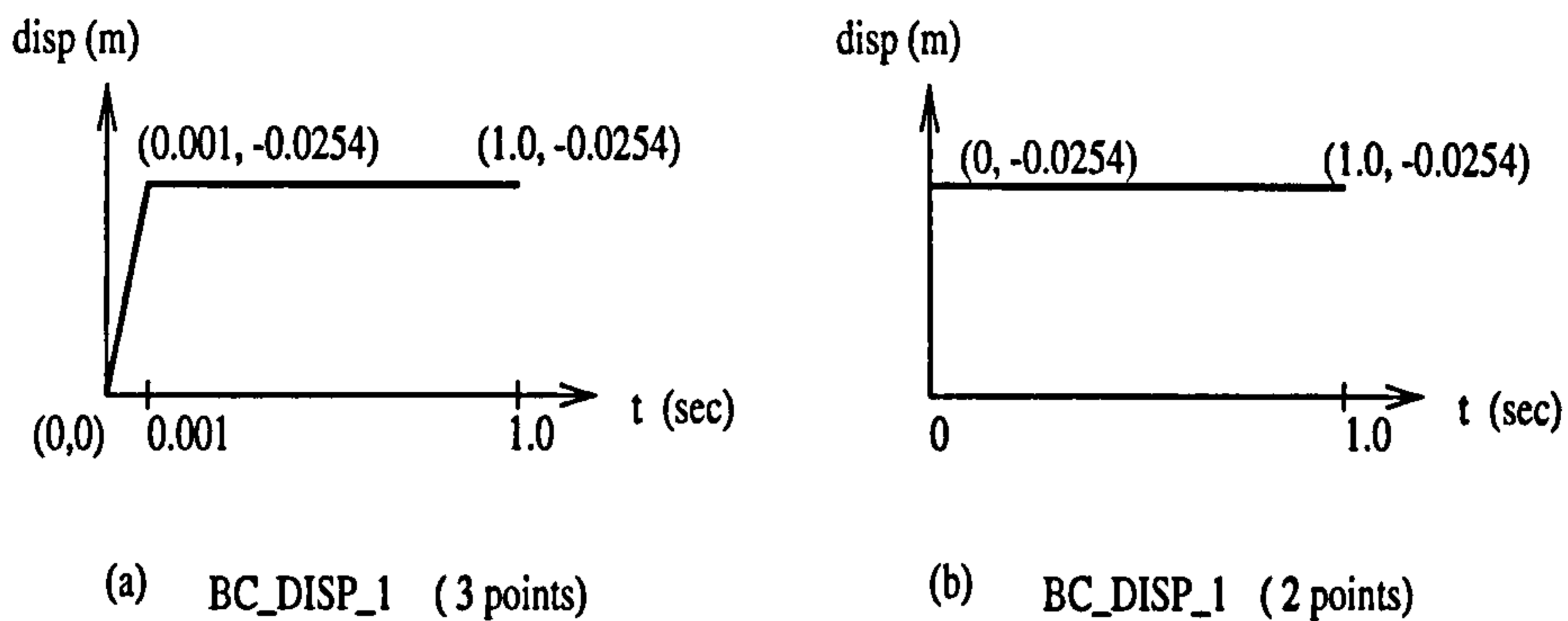


Figure 6.6 : Displacement curves (a) 3 points - correct (b) 2 points - wrong.

It is important to remember that in DYNA3D the displacement curve must be smoothed in describing the points along the curve. Figures 6.6a) and b) above show the correct "smooth" displacement curve and incorrect displacement curve respectively. This is not necessarily needed for force/load , velocity or acceleration curve, because DYNA3D will read the load curve data at each nodal point of the finite element mesh and then it calculates the acceleration, velocity and finally displacement. On the other hand, when the displacement curve is assigned, DYNA3D will read the displacement data and convert them into the forces at each nodal point before start calculating the acceleration, velocity and displacement values. This 'conversion' will cause an error if input data not entered properly . Details of command used in N/CODE, to submit job, restart a job, display output using D3PLOT and read time history file using T/HIS are listed in Appendix I.

### 6.4.2 Damped MDOF system hits a rigid body (unaligned floor, $a = L/2$ )

The pounding of damped MDOF systems of unaligned floor ( $a=L/2$ ) from section (3.4.2.2) is used in this analysis which is the second example of the computer modelling. The reason of choosing this example is to illustrate the use of the damper elements and the stone wall elements (representing the rigid adjacent building).

All the physical properties are the same as in previous section (Table 6.1) except the building separation is now 0.0126 m. The lumped mass model is shown in Figure 6.7 below. There are 11 nodes, 5 beams elements, 2 springs, 6 damper elements and 3 lumped mass elements in the model.

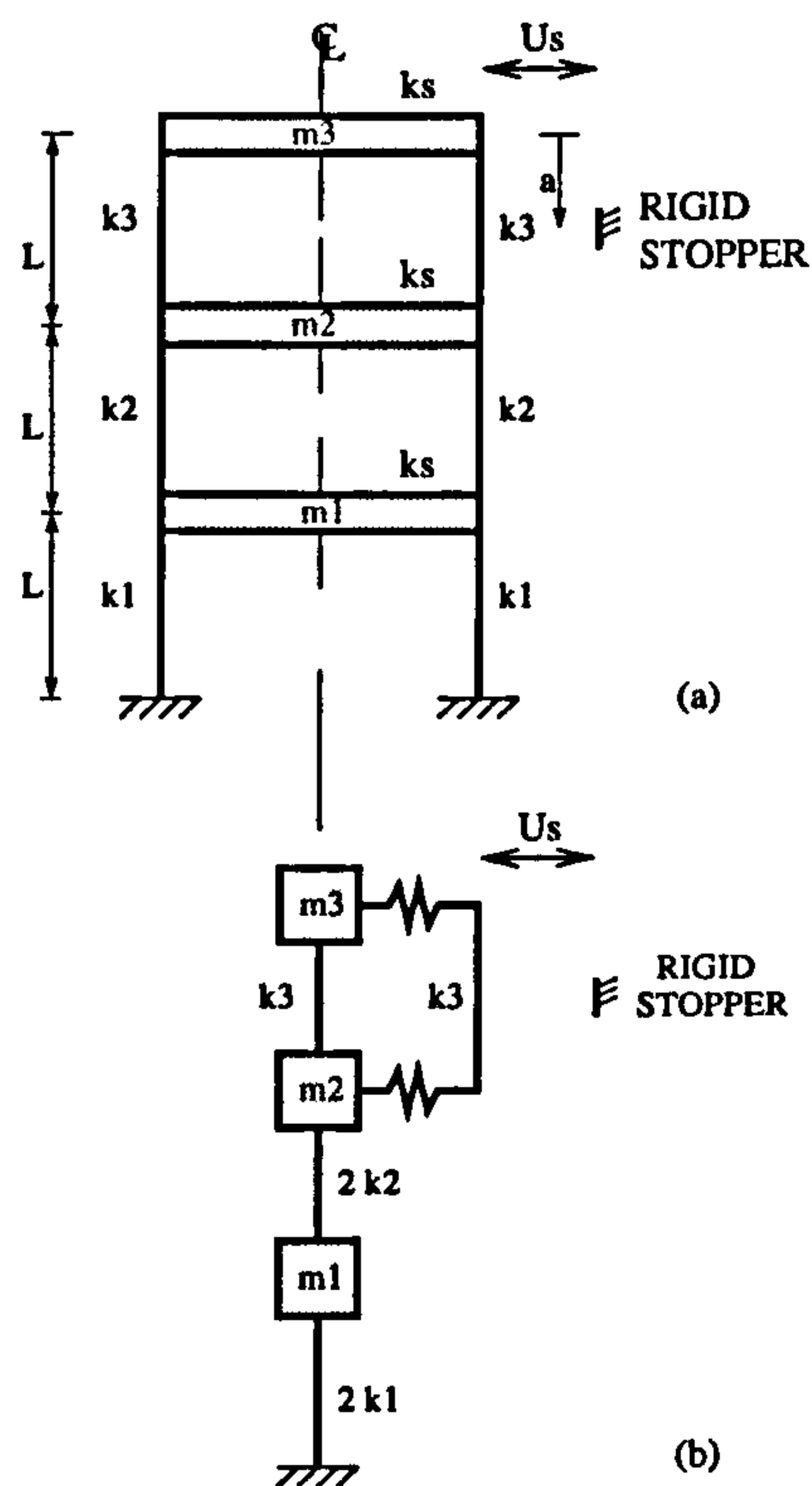


Figure 6.7 : (a) Pounding of MDOF system (b) The lumped mass model.

Table 6.4 below contains all the data required for the pre-processor stage.

ELEMENTS		DESCRIPTION	ELEMENT ID		COMPUTER MODEL	
LI1, - LI14		Number of lines created in the geometry stage	-			
E1, E2, E3		Lumped mass elements	PID=1 CONFIG=7	PID=2 CONFIG=7		PID=3 CONFIG=7
E4, E7, E8	E5	Beam elements	PID=1 CONFIG=0	PID=2 CONFIG=0		PID=3 CONFIG=0
E9, E10		Gap-spring elements	PID=1, CONFIG=6			
E11, E12, E13		Damper elements	PID=2 CONFIG=20	PID=3 CONFIG=20		PID=4 CONFIG=20
E14, E15, E16			PID=5 CONFIG=20	PID=6 CONFIG=20		PID=7 CONFIG=20
NODAL POINT		CO-ORDINATE (x,y,z)	BOUNDARY CONDITION			
N1		(1.0, 0, -1.0)	Moved in x direction only (D: /0/0) and no rotation (R: 0/0/0)			
N2		(1.0, 0, -2.8288)	BC_DISP_1 (at N1), BC_DISP_2 (at N2)			
N3		(1.0, 0, -4.6576)	BC_DISP_3 (at N3)			
N4		(1.0, 0, -6.4864)	Fixed ended column (D: 0/0/0) and no rotation (R: 0/0/0)			
N5		(2.0, 0, -1.0)	Moved in x direction only (D: /0/0) and no rotation (R: 0/0/0)			
N6		(2.0, 0, -1.9144)	STONE_1 (at node N6)			
N7		(2.0, 0, -2.8288)				
N8		(0, 0, -1.0)	Fixed ended beam (D: 0/0/0) and no rotation (R: 0/0/0)			
N9		(0, 0, -2.8288)				
N10		(0, 0, -4.6576)				
N11		(1.5, 0, -6.4864)	Only as beam orientation			

Table 6.4 : Data assigned in the Pre-processor (PATRAN) stage.



The properties of the beam are the same as in Table 6.3a except for material number 1 (PID=1) where the Belytschko-Schwer Beam data is different due to the stiffness of the single top column (k3) only and not (2k3) as previous. Table 6.5a and 6.5b lists the data required along with the default N/CODE values to represent a mild steel material and the boundary condition values required.

BEAM PROPERTY	VALUE		
Density Young's Modulus Poisson's Ratio	7890 kg/m <sup>3</sup> 205 E+9 N/m <sup>2</sup> 0.3		
Belytschko-Schwer Beam	E4,E7,E8	E5	E6
Cross-sectional Area, $A=\pi d^2/4$ $I_{yy} = \pi d^4/64$ $I_{zz} = \pi d^4/64$ Torsion Constant, $I_{xx}=\pi d^4/32$ Shear Area, $A_s$ (default same as A)	4.033E-2 m <sup>2</sup> 1.295E-4 m <sup>4</sup> 1.295E-4 m <sup>4</sup> 2.589E-4 m <sup>4</sup> -	8.070E-2 m <sup>2</sup> 5.179E-4 m <sup>4</sup> 5.179E-4 m <sup>4</sup> 1.037E-3 m <sup>4</sup> -	9.875E-2 m <sup>2</sup> 7.768E-4 m <sup>4</sup> 7.768E-4 m <sup>4</sup> 1.552E-3 m <sup>4</sup> -

Table 6.5a : Beam Elements properties

DISPLACEMENT CURVES	3 POINT VALUES
BC_DISP_1 (at N1) BC_DISP_2 (at N2) BC_DISP_3 (at N3)	(0,0) , (0.001, -0.02540) , (1.0, -0.02540) (0,0) , (0.001, -0.01283) , (1.0, -0.01283) (0,0) , (0.001, -0.00366) , (1.0, -0.00366)
STONE WALL	RIGID ELEMENT (STONE_1) at N6
Tail Co-ordinates (x,y,z) Head Co-ordinates (x,y,z) X' Vector (x,y,z)	(2.0126, -0.25, -2.1644) (0, -0.25, -2.1644) (2.0126, -0.25, -1.6644)
SPRING	ELEMENT E7
Spring stiffness, ks	8.76 E9 N/ m
DAMPER	DAMPING CONSTANT c (N sec/m)
C1 = 0.395768 , C2 = 0.0008777 ( E11 ), c23 = C2 2k3 ( E12 ), c12 = C2 2k2 ( E13 ), c01 = C2 2k1 ( E14 ), c33 = C1 m3 ( E15 ), c22 = C1 m2 ( E16 ), c11 = C1 m1	0.92269 E 5 1.84539 E 5 2.76808 E 5 0.69337 E 5 1.04005 E 5 1.38674 E 5
LUMPED MASS	VALUE
Mass for element E1 Mass for element E2 Mass for element E3	175197 kg 262795 kg 350394 kg

Table 6.5b : Load curves, stone wall and discrete element properties.

The stonewall element is used to model the adjacent rigid building with a gap or building separation,  $U_s$ . The gap is assigned by the position (co-ordinates) of the tail, head and  $X'$  vector of the stone wall. These values are taken from the co-ordinates as explained in Figure 6.8 below.

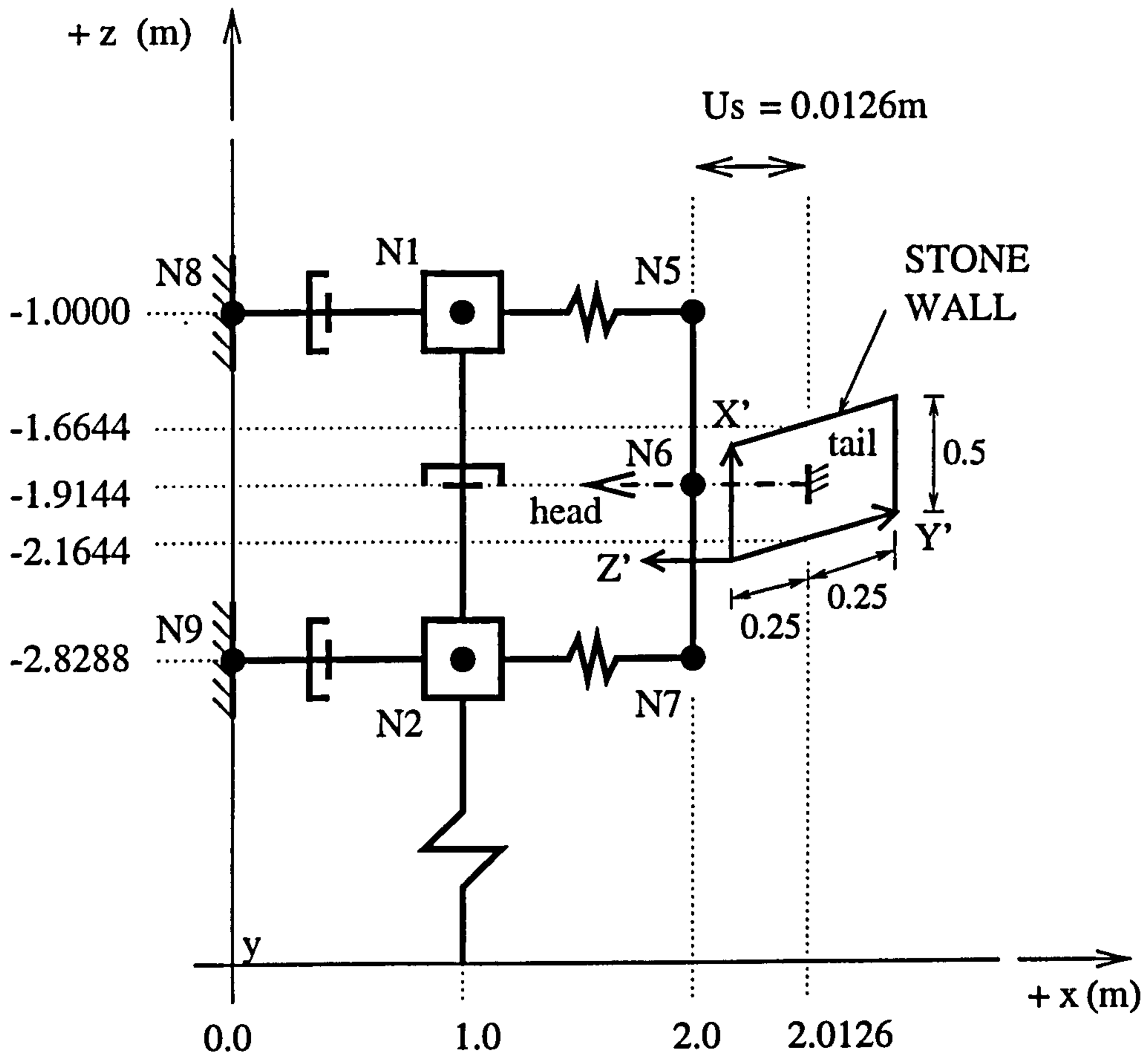


Figure 6.8 : Coordinates of stone wall element.

The damping constants are calculated from Equation (6.13).

## **6.5 DISCUSSIONS**

A finite element package, such as DYNA3D which is used in this study is a very powerful and modern computational tool in solving structural dynamics problem. From the previous chapters, DYNA3D is able to verify all the results from the theoretical formulation using the FORTRAN program.

However, in this chapter it is shown that DYNA3D is not a straight forward finite element package in solving the structural pounding analysis. Some knowledge such as computing system, finite element mesh generation, type of element and boundary conditions are pre-requisite.

On the other hand, by considering the basic formulation and solving the equations of motion using the direct integration method as in previous chapters, produces very good results.

# **CHAPTER 7**

## **DISCUSSIONS AND CONCLUSIONS**

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### **7.1 INTRODUCTION**

The work presented in this thesis represents a comprehensive formulation of the equations of motion on structural pounding using a displacement compatibility method for both aligned and unaligned floor cases. First, this chapter gives an overview of several types of pounding analysis as mentioned in previous chapters and it summarises the results. The final section of the chapter covers suggestions for future work.

### **7.2 DISCUSSIONS**

The study derives a formulation for the equations of motion using the displacement compatibility method for the analysis of structural pounding. The solutions of the equations of motion are uncoupled using the mode superposition technique and direct integration method over a linear variation time step.

The formulations are idealized as having two linear states, buildings not in contact and buildings in contact. A complete response time history is achieved as the response continues to change from one state to another. In this study, the application of the formulation of the equations of motion is applied to three main cases:

**Case 1 : Pounding of adjacent rigid building with rigid slab**

As explained in Chapter 3.

**Case 2 : Pounding of adjacent rigid building with flexible slab**

As explained in Chapter 4.

**Case 3 : Pounding of two flexible buildings with rigid slabs**

As explained in Chapter 5.

All the results are verified by finite element package PATRAN and DYNA3D as mentioned in Chapter 6. To illustrate each case, a typical two storey frame as shown in Figure (4.11) is taken as a numerical example having the same physical properties as in Table (4.4) for case 1 and 2 and additional properties for Building B as in Table (5.1) for case 3. Two aligned floor cases (  $a=0$ ,  $a=L$  ) and four unaligned cases (  $a=L/4$ ,  $L/3$ ,  $L/2$ ,  $3L/4$  ) are considered in this study.

Only the results for Case 1 are summarised in this section and shown in Table 7.1. Detailed results can be found in chapters 4 (Table 4.6) and 5 (Table 5.3) for case 2 and 3 respectively.

Overall variation of the contact force,  $P$  over a range of (  $0 \leq a \leq L$  ) is due to the slab stiffness,  $k_s$  for aligned floor case and both  $k_s$  and column rigidity,  $k_p$ , for the unaligned floor case. This variation is summarized in Table 7.2. A typical displacement at the contact point,  $u_i$  and contact forces,  $P$  time histories for various value of  $a$  are shown in Table 7.3 and Table 7.4 respectively.

This section compares and discusses the effect of the following parameters on the contact forces, initial time of contact, the duration of contact, time interval between two successive contacts, top floor displacement, damping factor, number of contacts and number of Degree of Freedom system by referring to Case 1, 2 and 3.

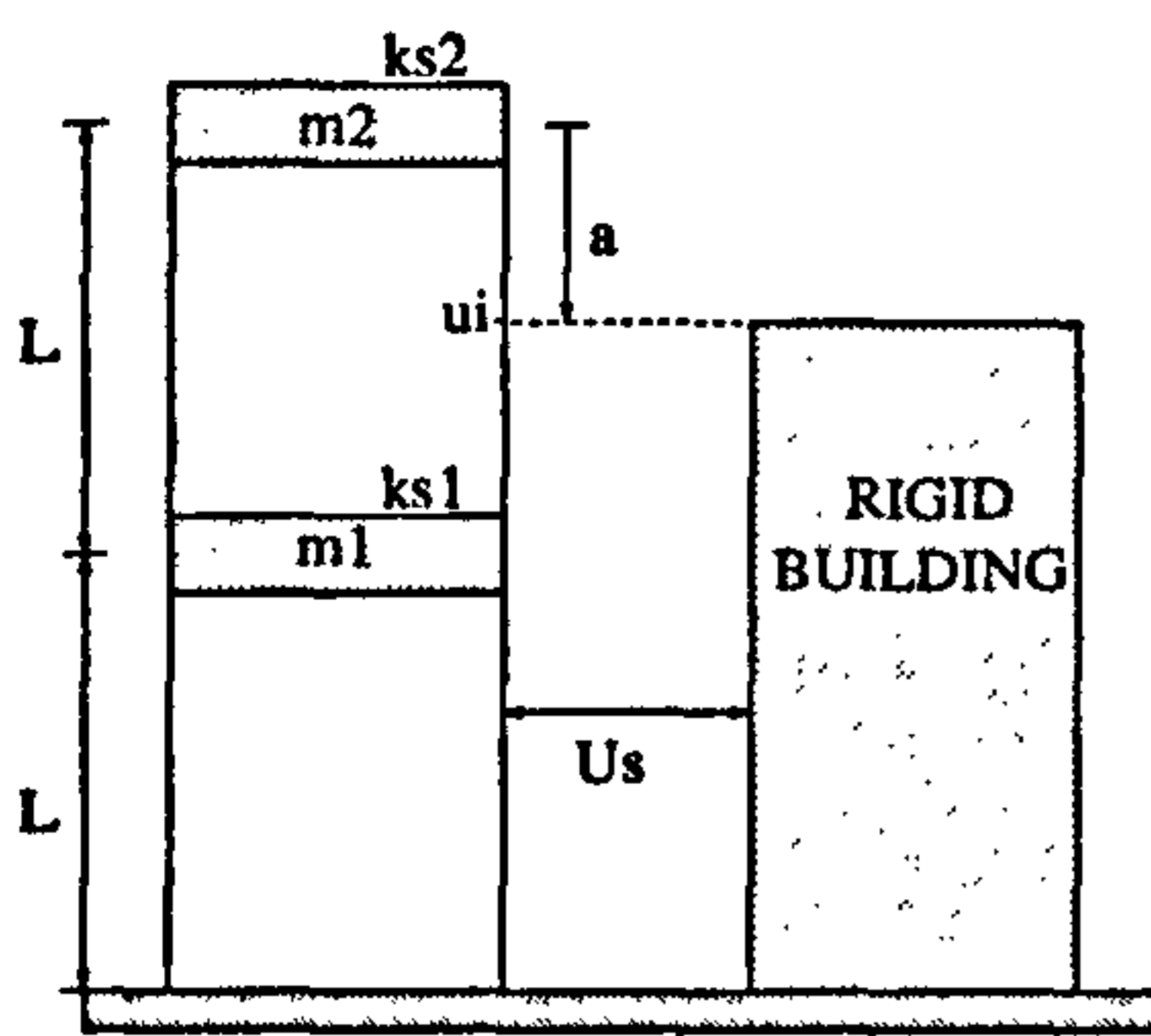
### 7.2.1 Effect of pounding location, $a$ on contact forces, $P$

The pounding location,  $a$  is measured from the floor above the point of contact. That is (  $0 \leq a \leq L$  ) and (  $a + b = L$  ). For all three cases shown in Table 7.4, the contact force,  $P$  decreases as the pounding location increases for a range of (  $0 \leq a \leq L/2$  ). Then  $P$  increases as the value of  $a$  increases from (  $L/2 \leq a \leq L$  ) for Case 1 (first contact) and Case 3. The contact forces  $P$  continue decreases for Case 1 (second contact) and Case 2. In general, the lowest value of  $P$  occurs at  $a = L/2$  and is due to the relative effects of the column rigidity,  $k_p$  and slab stiffness,  $k_s$  which is explained in section (7.2.8).

CASE 1 DISTANCE "a"		1st CONTACT	2nd CONTACT
		FORTRAN	FORTRAN
0	Contact time (sec)	0.08110-0.09480 (0.01370)	0.28280-0.29579 (0.01290)
	Contact forces (kN)	19,391.138	19,632.356
L/4	Contact time (sec)	0.08240-0.10580 (0.02340)	0.28480-0.30750 (0.02270)
	Contact forces (kN)	13,687.077	14,544.884
L/3	Contact time (sec)	0.08340-0.11550 (0.03210)	0.28900-0.32069 (0.03169)
	Contact forces (kN)	10,709.496	11,325.491
L/2	Contact time (sec)	0.08610-0.12900 (0.04290)	0.30150-0.34379 (0.04229)
	Contact forces (kN)	8,651.723	8,652.834
3L/4	Contact time (sec)	0.09120-0.11860 (0.02740)	0.31560-0.33790 (0.02230)
	Contact forces (kN)	10,695.105	7,794.655
L	Contact time (sec)	0.09420-0.11050 (0.01630)	0.32600-0.33500 (0.00900)
	Contact forces (kN)	13,133.550	1,764.129

Table 7.1 : Overall results for the contact stages of flexible slab of 2DOF systems.

### CASE 1



$ks1 = ks2 = 50,000$  kips/in (rigid)

$$P = ks2 (u2 - Us) \quad (1)$$

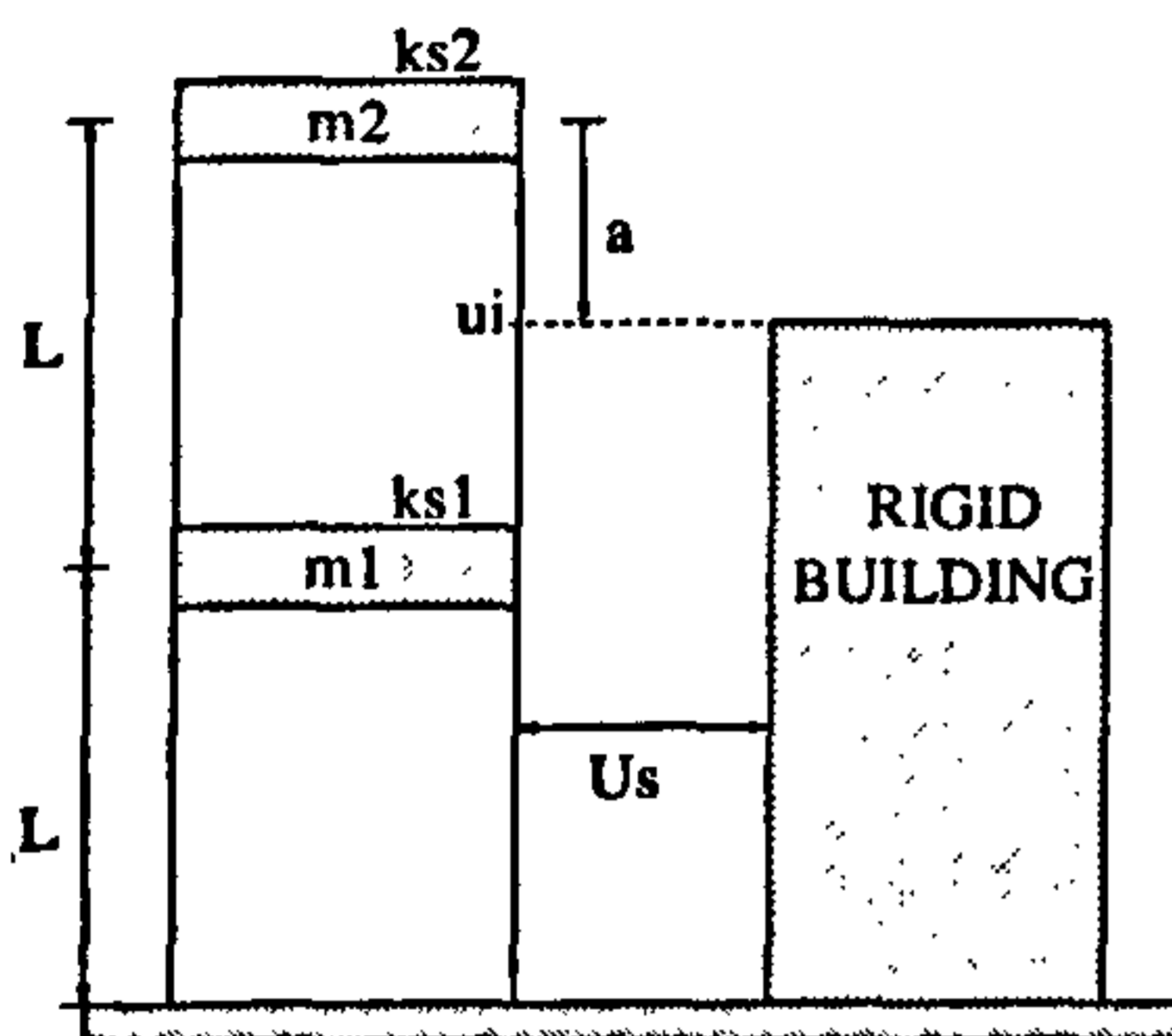
$$P = kp [(1 - k_{AB}) ud2 + k_{AB} ud1] \quad (2)$$

$$kp = \frac{3L^3 EI}{b^3 a^3}$$

$$P = ks1 (u1 - Us) \quad (3)$$

Equation (1), (2) and (3) represent  $a = 0$ ,  $0 < a < L$  and  $a = L$  respectively.

### CASE 2



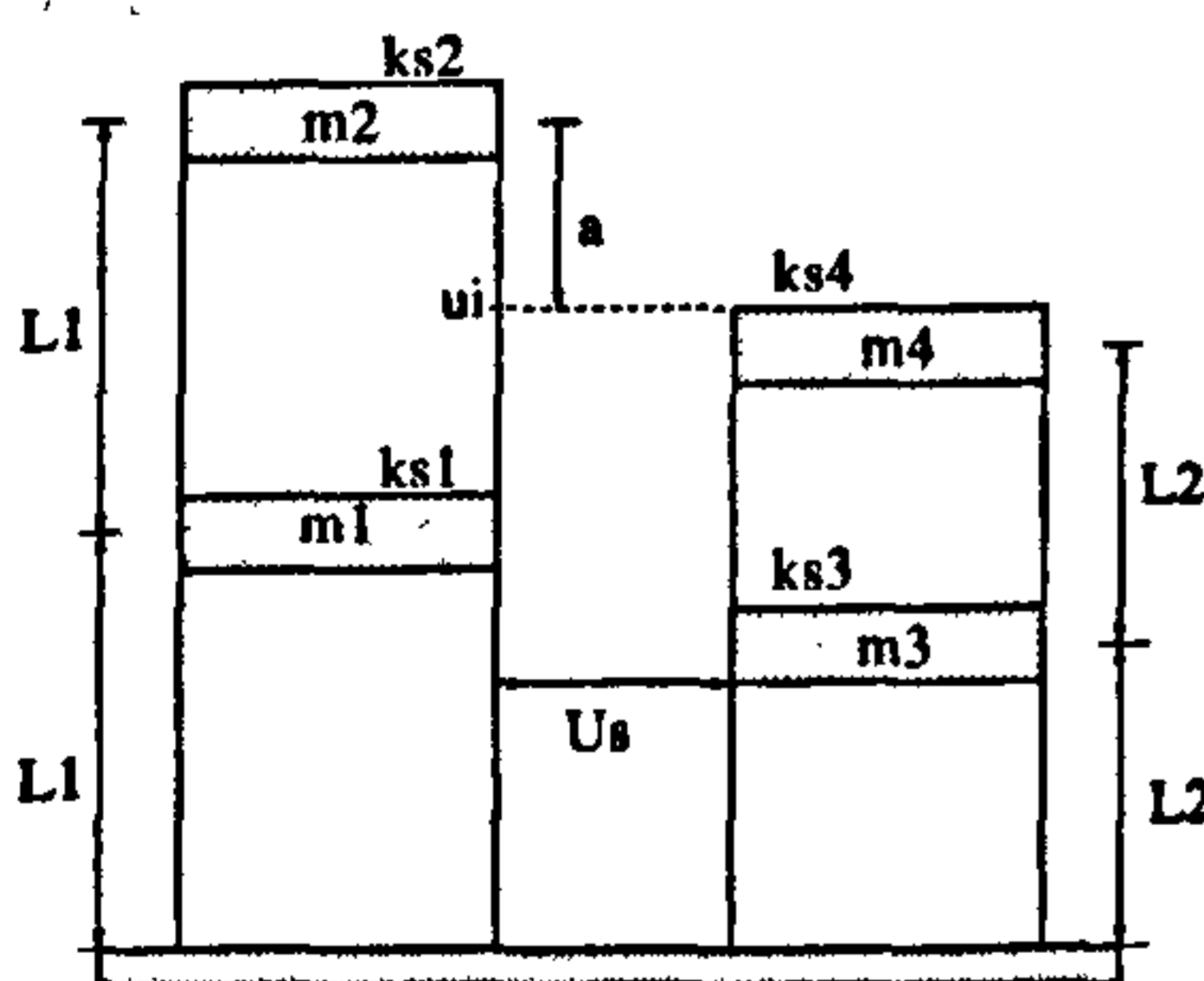
$ks1 = ks2 = 5,000$  kips/in (flexible)

$$P = kp [(1 - k_{AB}) ud2 + k_{AB} ud1] \quad (4)$$

$$kp = \frac{1}{\frac{a^3 b^3}{3L^3 EI} + \frac{1}{k_D} [(1 - k_{AB})^2 (ks1 + k1) + k2 + k_{AB}^2 ks2]}$$

This equation valid for ( $0 \leq a \leq L$ ).

### CASE 3



$ks1 = ks2 = ks3 = ks4 = 50,000$  kips/in (rigid)

$$P = \frac{ks2 ks4}{ks2 + ks4} (u2 - u4 - Us) \quad (5)$$

$$(P - P^*) = kp [(1 - k_{AB}) ud2 + k_{AB} ud1 - ud4] \quad (6)$$

$$kp = \frac{3L^3 EI}{b^3 a^3}$$

$$P = \frac{ks2 ks4}{ks2 + ks4} (u1 - u4 - Us) \quad (7)$$

Equation (5), (6) and (7) represent  $a = 0$ ,  $0 < a < L1$  and  $a = L1$  respectively.

Table 7.2 : Overall contact forces,  $P$  for case 1, 2 and 3 respectively.





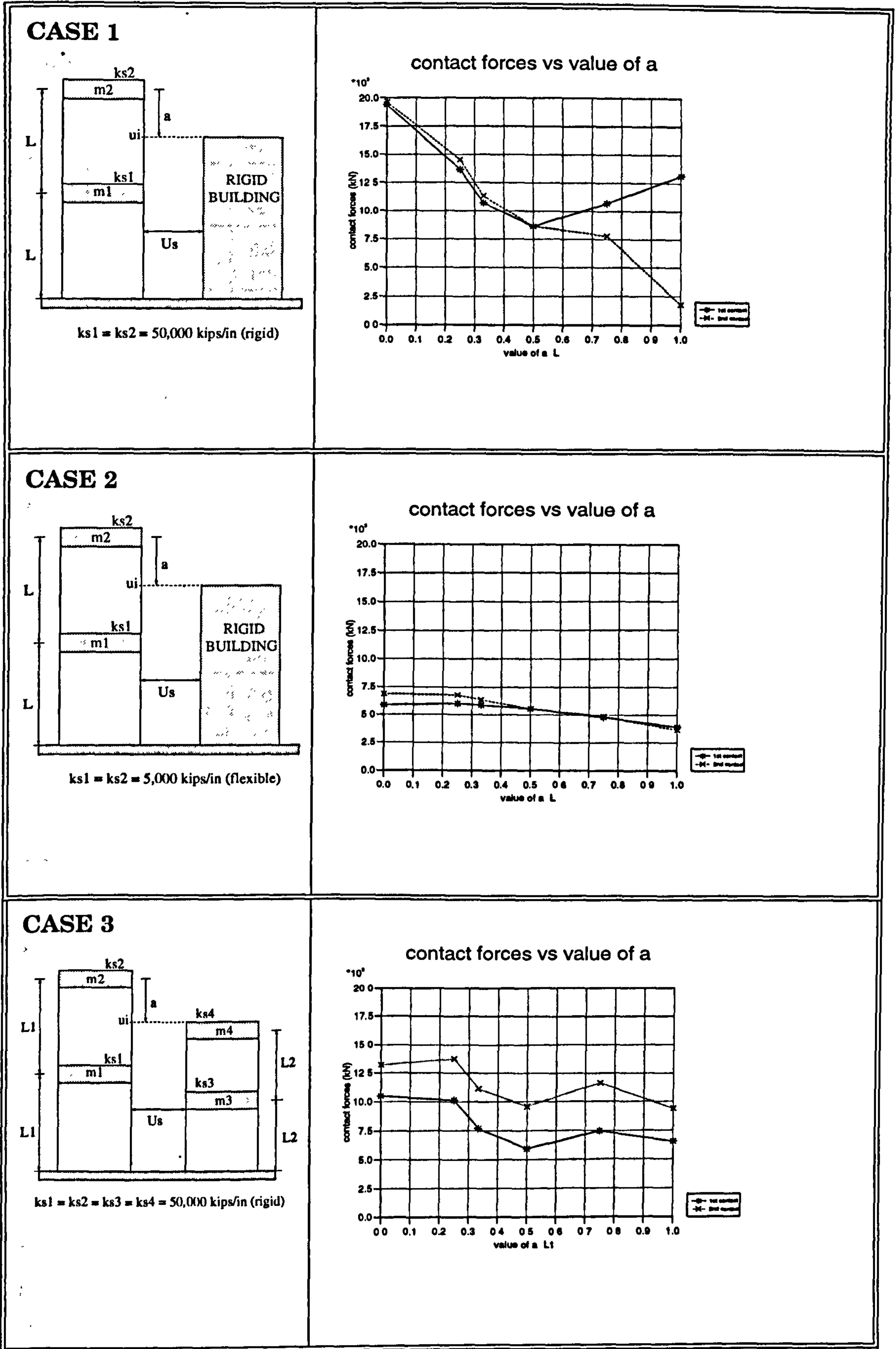


Table 7.4 : Contact forces against variation of a for case 1, 2 and 3 respectively.

DISTANCE "a"		1st CONTACT		2nd CONTACT	
		u2 (m)	u2 <sub>max</sub> (m)	u2 (m)	u2 <sub>max</sub> (m)
C A S E  1	a = 0	0.00800 t=0.08780 sec	0.00800 t=0.08780 sec	0.00800 t=0.28920 sec	0.00800 t=0.28920 sec
	a = L/4	0.01220 t=0.09400 sec	0.01221 t=0.09350 sec	0.01390 t=0.29590 sec	0.01390 t=0.29620 sec
	a = L/3	0.01383 t=0.09940 sec	0.01385 t=0.09840 sec	0.01632 t=0.30460 sec	0.01632 t=0.30490 sec
	a = L/2	0.01688 t=0.10750 sec	0.01688 t=0.10770 sec	0.01653 t=0.32250 sec	0.01653 t=0.32260 sec
	a = 3L/4	0.01798 t=0.10460 sec	-	0.00478 t=0.32640 sec	-
	a = L	0.01752 t=0.10230 sec	-	0.00252 t=0.33040 sec	-
C A S E  2	a = 0	0.00800 t=0.10380 sec	0.00800 t=0.10320 sec	0.00800 t=0.33030 sec	0.00800 t=0.33030 sec
	a = L/4	0.01574 t=0.10770 sec	0.01577 t=0.10630 sec	0.01901 t=0.33480 sec	0.01901 t=0.33490 sec
	a = L/3	0.01675 t=0.11110 sec	0.01680 t=0.10940 sec	0.02034 t=0.34030 sec	0.02034 t=0.34030 sec
	a = L/2	0.01886 t=0.11770 sec	0.01888 t=0.11630 sec	0.02082 t=0.35289 sec	0.02082 t=0.35220 sec
	a = 3L/4	0.02150 t=0.12120 sec	0.02184 t=0.12800 sec	0.01658 t=0.36590 sec	0.01942 t=0.39059 sec
	a = L	0.02224 t=0.12180 sec	0.02329 t=0.13480 sec	0.01503 t=0.36880 sec	-
C A S E  3	a = 0	0.01091 t=0.08845 sec	0.01119 t=0.09155 sec	0.00053 t=0.29940 sec	0.00054 t=0.29903 sec
	a = L/4	0.01200 t=0.09088 sec	0.01237 t=0.09475 sec	0.00085 t=0.30238 sec	0.00086 t=0.30201 sec
	a = L/3	0.01355 t=0.09470 sec	0.01409 t=0.10034 sec	0.00161 t=0.30957 sec	0.00161 t=0.30957 sec
	a = L/2	0.01598 t=0.10019 sec	0.01728 t=0.11241 sec	0.00146 t=0.32065 sec	0.00183 t=0.32613 sec
	a = 3L/4	0.01655 t=0.10023 sec	-	-0.00372 t=0.32061 sec	-
	a = L	0.01749 t=0.10222 sec	-	-0.00359 t=0.32502 sec	-

Table 7.5 : Overall top displacements during contact stages for case 1, 2 and 3 respectively. (-) means the  $u2_{max}$  does not occur in the contact stage.

The effect of  $kp$  for case 1 and 3 is inversely proportional to the cube of  $a$  and  $b$  as defined in Equation (2) in Table 7.2. Any increment in  $a$  produces a reduction in  $kp$  and therefore, value of  $P$  is decreased.

In addition to the factor  $kp$ , the constants  $(1 - k_{AB})$  and  $k_{AB}$  also affect the value of  $P$ . These constants, the coefficient of  $ud2$  and  $ud1$  respectively, affect the value of  $P$  because the amplitude of  $ud2$  is usually greater than  $ud1$  as shown in Figure 7.1 below. As  $a$  increases, value of  $k_{AB}$  also increases but  $(1 - k_{AB})$  decreases. Thus at  $a=L/2$  the value of contact forces  $P$  is the lowest compared to other values of  $a$ .

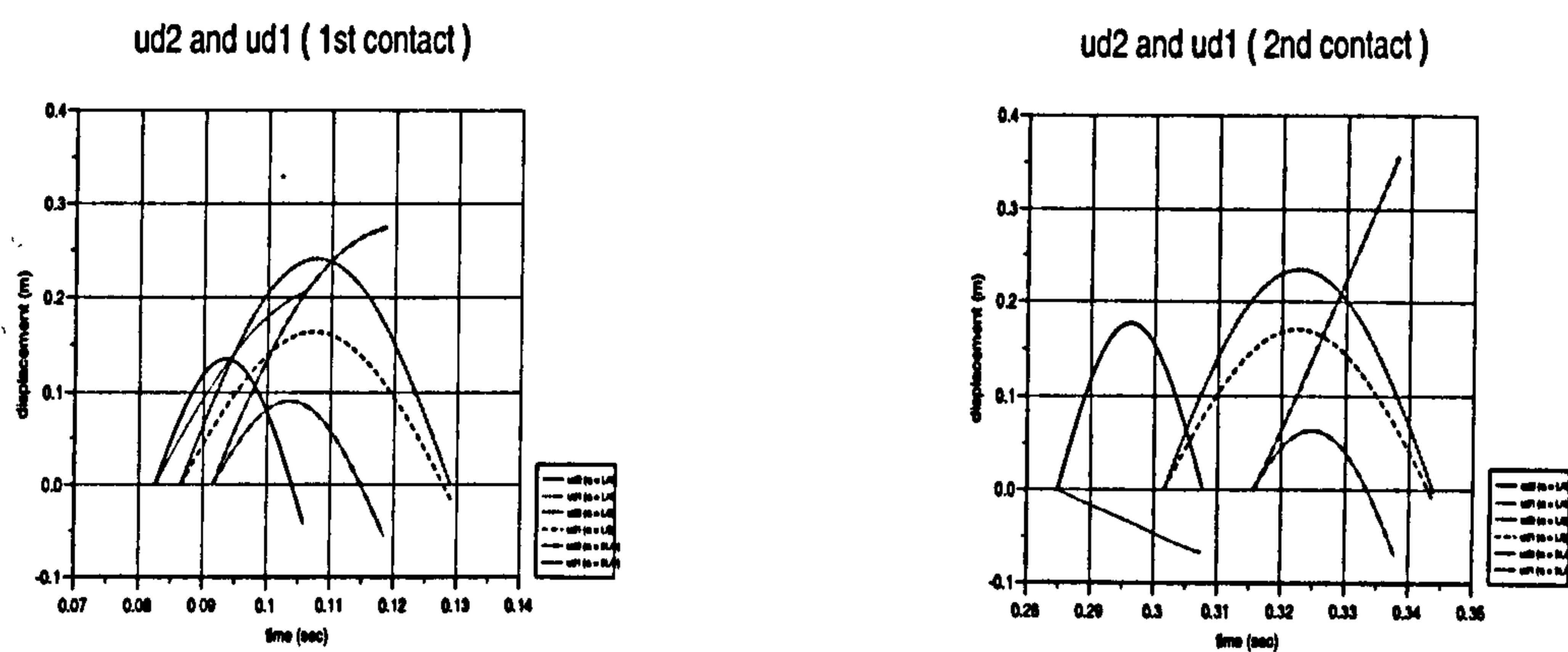


Figure 7.1 : The  $ud2$  and  $ud1$  values during the contact stage for case 1.

For example, case 1, at  $a=0$ , the contact forces  $P$  is equal to 19391 kN and at  $a=L/2$ , the contact forces  $P$  is equal to 8652 kN. A 50% increment in  $a$  produces 55.38% reduction in  $P$ . (Table 7.1)

### 7.2.2 Effect of pounding location, $a$ on initial contact time, $t_i$

The initial time for pounding to occur,  $t_i$  increases as the value of  $a$  increases for all three cases. The results can be found in Tables 7.1, 4.6 and 5.3 for case 1, 2 and 3 respectively. For examples, case 1, at  $a=0$ , initial contact time is equal to 0.08110 sec and at  $a=L$ , the initial contact time is 0.09420 sec. Increased  $a$  by 100%, resulting a 16.15% increased in the initial contact time.

### 7.2.3 Effect of pounding location, $a$ on contact stage duration, $dt_c$

The duration of contact stage,  $dt_c$  increases as the value of  $a$  is increased from  $a=0$  to  $a=L/2$  and it decreases from  $a=L/2$  to  $a=L$ . The maximum duration of contact stage occurs when pounding location at  $a=L/2$  as shown in Tables 7.1, 4.6 and 5.3 for case 1, 2 and 3 respectively. For example, in case 2, the duration of contact stage for  $a=0$  is 0.04250 sec, and at  $a=L/2$ , it is equal to 0.05770 sec. It means, 50% in  $a$  produces 35.76% increased in the contact stage duration.

### 7.2.4 Effect of pounding location, $a$ on $u_2$ and $u_{2_{max}}$ displacement

In general, the  $u_2$  (top floor displacement) increases as the pounding location  $a$  increases as tabulated in Table 7.5. This is governed by the fundamental mode shapes, as shown in Figure 7.2 below:

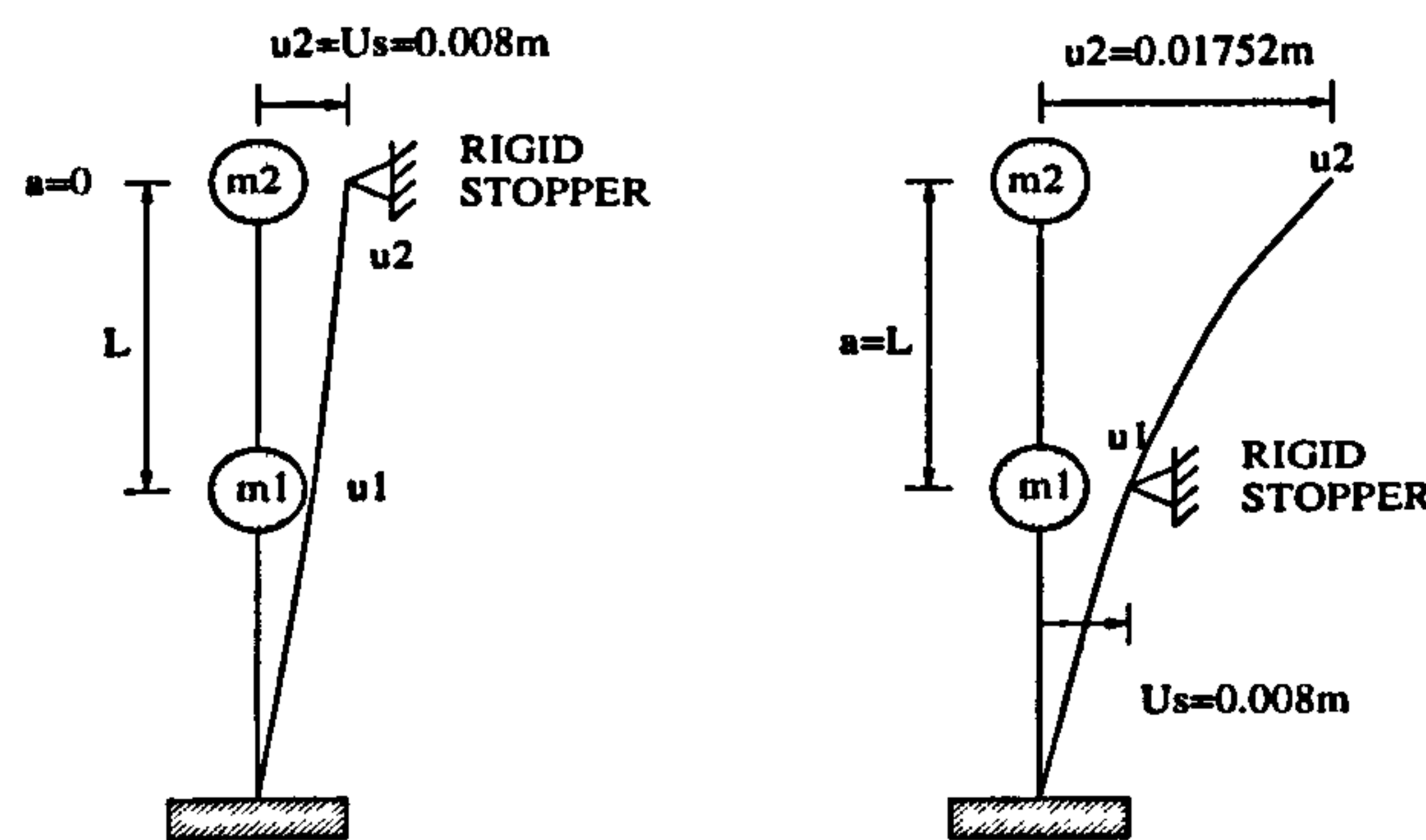


Figure 7.2 :  $u_2$  displacements for  $a = 0$  and  $a = L$  of case 1.

The value of  $u_2$  is corresponding to the maximum value of contact forces,  $P$ . Table 7.5 shows that the value of  $u_{2_{max}}$  may occurs at different time of the maximum contact forces, and it is not necessarily occurs during the contact stage. The variation of  $u_{2_{max}}$  is following the same pattern as for  $u_2$ . That is  $u_{2_{max}}$  increases as the value of  $a$  increases.

### **7.2.5 Effect of pounding location, $a$ on number of contacts**

Table 3.4 shows the overall results for the analysis of case 1 with 3DOF systems for a duration of 1.0 second. The number of contacts for  $a=0$ ,  $L/2$ ,  $3L/4$  and  $L$  is 3, whereas for  $a=L/4$  and  $L/3$ , the number of contacts is only 2. This shows that, there is no general pattern on how many times the pounding will occur and each case must be evaluated individually.

This becomes more obvious in the earthquake induced pounding analysis (section 3.4.3) where the Mexico earthquake, 1985 N90-W component is used in the analysis and is run for 30 secs. When pounding occurs at  $a=L/2$ , the number of contacts is 14 and when pounding occurs at  $a=0$ , the number of contacts reduces to only 9. This means, increased  $a$  by 50% results an increment of 55.55% in number of contacts.

### **7.2.6 Effect of damping on contact forces, $P$**

The contact force,  $P$  is reduced as the damping value is increased as tabulated in Table 3.6. For the case of  $a=L/2$ , with the damping coefficient,  $\xi=0.02$ , the value of  $P$  is equal to 1947 kN compared to those obtained without damping,  $P = 2337$  kN. This means that, increased  $\xi$  as small as 2% produces a reduction of  $P$  by 16.69%.

### **7.2.7 Effect of number of Degree of Freedom on contact forces, $P$**

Two studies on 2DOF systems and 3DOF systems for case 1 (rigid slab) are compared. Table 7.4 and Figure (3.27a) shows that the general pattern of the first contact force reduces for a range of  $(0 < a < L/2)$  and it increases for a range of  $(L/2 < a < L)$  as the value of  $a$  increases. For the second contact force, there are no similarities for 2DOF and 3DOF systems which is due to their differences in physical properties.

The magnitude of contact forces in the 2DOF systems (Table 7.1) is higher than in the 3DOF systems (Table 3.4) . For example, at  $a=L/2$  the contact forces  $P$  for 2DOF systems is equal to 8652 kN and for 3DOF systems, the value of  $P$  is reduced by 73.3 % (2307 kN).

### **7.2.8 Effect of slab stiffness, $ks$ on contact forces, $P$**

The effect of slab stiffness,  $ks$  on  $P$  is best illustrated by comparing Case 1 and Case 2. In case 1, the slab is assumed rigid in plane, with  $ks=50,000$  kips/in. and in case 2, the slab is assumed not rigid in plane with  $ks=5,000$  kips/in. Table 7.4 clearly shows that value of  $P$  decreases as  $ks$  decreases.

For example, at  $a=0$ , in case 1 (rigid slab), the value of contact force  $P$  is equal to 19391 kN and in case 2 (non rigid slab), the value of  $P = 5870$  kN. This shows that the slab stiffness reduced by a factor of 10, reduces the contact force by a factor of 3.3.

### **7.2.9 Effect of slab stiffness, $ks$ on initial contact time, $t_i$**

The initial contact time,  $t_i$  increases as the value of  $ks$  decreases. From Table 7.1 for case 1 (rigid slab) , at  $a=L$ , the initial contact time is 0.09420 sec and from Table 4.6 for case 2 (non rigid slab), at  $a=L$ , the initial contact time is 0.09850 sec. A reduction of  $ks$  by a factor of 10, resulting in an increment of 0.0043 sec to the initial contact time,  $t_i$ .

### **7.2.10 Effect of slab stiffness, $ks$ on contact stage duration, $dt_c$**

The duration of contact stage,  $dt_c$  increases as the value of  $ks$  decreases. Again, the longest duration occurs at  $a=L/2$  and for example, in case 1(rigid slab), Table 7.1 shows that at  $a=L/2$  the duration of contact stage is 0.04290 sec and for case 2 (non rigid slab) , at  $a=L/2$  the duration of contact stage is 0.05770 sec (Table 4.6).

This shows that, a reduction by a factor of 10 in  $ks$ , produces a longer contact stage by 0.0148 sec.

### 7.2.11 Effect of slab stiffness, $ks$ on $u_2$ and $u_{2_{max}}$ displacement

The top displacement ( $u_2$ ) at the time of the maximum contact forces, is higher for case 2 (non rigid slab) than in case 1 (rigid slab). The reduction in slab stiffness,  $ks$  produces an increase in the top displacement. This is associated with the presence of the axial slab displacements due to its flexibility,  $uk_2$  and  $uk_1$  as defined in Equation (4.57). The  $u_2$  displacement in this case represents the left hand side of the frame. The right hand side displacements are defined by  $u_2^r$ , which is always less by a factor given by Equation (4.36) than the value of  $u_2$ .

Hence, the  $u_{2_{max}}$  value in case 2 also increases compared to those obtained in case 1. For example, in Table 7.5, for case 1, at  $a=L/2$ , the  $u_{2_{max}} = 0.01688m$  and for case 2, at  $a=L/2$  the  $u_{2_{max}} = 0.02082m$ . That means, a reduction by a factor of 10 in the value of  $ks$  results in an increase of 0.00394m in the  $u_{2_{max}}$  value. Again, the  $u_{2_{max}}$  value does not necessarily occur during the contact stage.

### 7.2.12 Effect of slab stiffness, $ks$ on number of contacts

The analysis for both case 1 (rigid) and case 2 (non rigid) was run for 1.0 sec. The results are tabulated in Table 7.6 below at  $a=L/2$ .

CASE	No. of contacts	Maximum contact forces (kN)
CASE 1	5	$P=8661.778$ occurs at 3rd contact, at $t=0.53730$ sec
CASE 2	4	$P=5583.941$ occurs at 3rd contact, at $t=0.58790$ sec
CASE 3	5	$P=9583.362$ occurs at 2nd contact, at $t=0.32065$ sec

Table 7.6 : Number of contacts for case 1 , 2 and 3.

It shows that, for the case 1 (rigid slab), the number of contacts is 5 with maximum contact force of 8662 kN occurs in the third contact, and when the slab stiffness is reduced in case 2, the number of contacts reduces to 4 with the maximum contact force of 5584 kN. Case 3 will be discussed in section (7.2.17).

### **7.2.13 Effect of adjacent building rigidity on contact forces, $P$**

In this study, the shear frames having the same slab stiffness,  $k_s = 50,000$  kips/in but the adjacent building is assumed rigid in case 1 and not rigid (flexible) as in case 3. Table 5.3 (case 3) and Table 7.1 (case 1) show that the adjacent rigid building (case 1) produces higher contact forces than those obtained in case 3.

This is because, from (Table 7.3), for case 3, the contact forces,  $P$  is not only dependent on  $u_2$  (since  $u_2 = u_{s2} + u_{d2}$ ), but it also depends on the relative displacement at point of contact, for example, at  $a=0$ , the relative displacement of  $(u_2 - u_4)$  is used in case 3 instead of only  $u_2$  is used in case 1. The contact forces,  $P$  is equal to 10,508 kN compared to  $P = 19,391$  kN in case 1.

### **7.2.14 Effect of adjacent building rigidity on time interval between two successive contacts, $t_{IC}$**

The time interval between two successive contacts,  $t_{IC}$  is higher for case 3 than in case 1. For example, at  $a=0$ , the time interval between two successive contacts for case 1 is 0.20170 sec and for case 3 it is 0.21106 sec.

### **7.2.15 Effect of adjacent building rigidity on contact stage duration, $dt_c$**

In general, the duration of contact stage,  $dt_c$  is longer in case 1 than in case 3 (Tables 7.1 and 5.3). For example, at  $a=L/2$ , for case 1, the contact stage duration is 0.0429 sec and in case 3 it is only 0.02831 sec. That is almost 1.5 times of case 3 compared to case 1 (adjacent rigid building).



### **7.2.16 Effect of adjacent building rigidity on $u_2$ and $u_{2_{max}}$ displacement**

Table 7.5 shows that, in general, the  $u_2$  displacement of the building of interest for case 3 is less than those obtained in case 1 (adjacent rigid building). On the other hand, the  $u_{2_{max}}$  displacement for the first contact for case 3 is higher than those obtained in case 1. For example when pounding occurs at  $a=L/2$ ,

Case 1 :  $u_2 = u_{2_{max}} = 0.01688$  m

Case 3 :  $u_2 = 0.01598$  m, and  $u_{2_{max}} = 0.01728$  m

The difference in  $u_2$  is 0.0009m but almost half in  $u_{2_{max}}$  ( 0.0004 m).

### **7.2.17 Effect of adjacent building rigidity on number of contacts**

From Table 7.6 in section (7.2.12), it shows that the number of contacts for case 1 and case 3 are the same, that is equal to 5. The maximum contact forces for 1.0 sec analysis in case 1 (adjacent rigid building) is obtained from the third contact with a value of 8662 kN and in case 3, it is obtained from the second contact with higher value of 9583 kN.

### **7.3 CONCLUSIONS**

This study has developed the use of the displacement compatibility method for the analysis of structural pounding, for both the aligned floor and the unaligned floor case. The analyses consider both rigid slabs and flexible slabs as well as variation of adjacent building rigidity. Initially, a snap back analysis was performed to investigate the response behaviour of the structure, such as lateral displacement, velocity and contact forces. Later, the Mexico, 1985 earthquake was applied to the base of the building. The theoretical analyses are compared with the results using the DYNA3D computer system. From the results obtained, the following general conclusions can be drawn:

1) An essential step towards the solution of a dynamic problem, especially in structural pounding analysis is the establishment of the equations of the motion. In this study, the formulation of the equations of motion is presented in a very simple and straight forward manner, using the displacement compatibility method which considered both aligned and unaligned floor cases. This simplicity is an advantage over the Finite Element (DYNA3D) and Lagrange multiplier method.

The effects of pounding location,  $a$ , slab stiffness,  $k_s$  and adjacent building rigidity discussed in previous section can be concluded as :

2) The magnitude of the contact force,  $P$  over a range of ( $0 \leq a \leq L$ ) is dependent on the position of contact,  $a$ , the slab stiffness,  $k_s$  for aligned floor case and both  $k_s$  and column rigidity,  $k_p$  for the unaligned floor case as listed in Table 7.2. The value of  $P$  decreases as the pounding location increases for a range of ( $0 < a < L/2$ ) and it increases as  $a$  increases for a range of ( $L/2 < a < L$ ) in case 1 and 3.

The lowest value of  $P$  occurs at  $a=L/2$  which is associated to the lowest value of  $k_p$  which is inversely proportional to the cube of  $a$  as defined in Equation (2) of Table 7.2. The contact force,  $P$  decreases as the slab stiffness,  $k_s$  decreases.

In addition, the adjacent building rigidity will also affect the value of  $P$ . That is the value of  $P$  decreases as the adjacent building rigidity reduces. For example, in cases 1, 2 and 3, the normalised value of  $P$  are shown in Table 7.7 below:

$a = L/2$	Contact Forces, $P$ (kN)	Normalised $P$
CASE 1	8652	1.0
CASE 2	5550	0.64
CASE 3	5937	0.69

Table 7.7 : Contact forces for case 1, 2 and 3 at  $a=L/2$ .

It clearly shows that the contact force,  $P$  is reduced by a factor of 0.64 as the slab stiffness,  $ks$  is reduced by a factor of 10 (case 2) and the same contact force is reduced by a factor of 0.69 if the adjacent building rigidity is decreased (case 3).

3) The initial time,  $t_i$  for the first pounding increases as the value of  $a$  is increased. This is because, in the dynamic response analysis, the first mode is the predominant vibration. Although the second and higher modes are also present, particularly on deflection, their effect is very small compared with the first mode, hence they are neglected.

The initial contact time,  $t_i$  also increases as the value of  $ks$  decreases as shown in Table 7.8 below:

$a = L/2$	Initial contact time, $t_i$ (sec)	Normalised $t_i$
CASE 1	0.08610	1.0
CASE 2	0.08900	1.03

Table 7.8 : Initial contact time,  $t_i$  for case 1, 2 and 3 at  $a=L/2$ .

4) The duration of the contact stage,  $dt_c$  increases as the value of  $a$  increases for a range of  $(0 < a < L/2)$  and it decreases from  $a=L/2$  to  $L$ . The maximum duration of contact stage occurs when  $a=L/2$ . Comparing the results for three cases, 1, 2 and 3, the longest duration of contact stage is when the slab stiffness  $ks$  is reduced as

in case 2 (Table 7.9). The duration of contact,  $dt_c$  is increased by a factor of 1.34 when the slab stiffness is reduced by a factor of 10.

$a = L/2$	Contact stage duration, $dt_c$ (sec)	Normalised $dt_c$
CASE 1	0.04290	1.0
CASE 2	0.05770	1.34
CASE 3	0.02831	0.66

Table 7.9 : Contact stage duration for case 1, 2 and 3 at  $a=L/2$ .

The reduction of adjacent building rigidity as in Case 3, produces a reduction by a factor of 0.66 in the duration of the contact stage.

5) The time interval,  $t_{IC}$  between two successive contacts is the highest for case 2 than in case 3 with respect to Case 1 as shown in Table 7.10 below.

$a = L/2$	Time interval between two successive contacts, $t_{IC}$ (sec)	Normalised $t_{IC}$
CASE 1	0.20170	1.0
CASE 2	0.22800	1.13
CASE 3	0.21106	1.05

Table 7.10 : Contact stage duration for case 1, 2 and 3 at  $a=L/2$ .

6) The top floor lateral displacement,  $u_2$  and the maximum lateral top floor displacement,  $u_{2_{max}}$ , both increase as the pounding location,  $a$  increases. This is due to the predominant first mode shapes but still less than the no pounding case. The  $u_2$  displacement also increases as the slab stiffness decreases (case 2) and the adjacent building rigidity decreases (case 3) as shown in Table 7.11 below:

$a = L/2$	$u_2$ (m)	Normalised $u_2$	$u_{2_{max}}$ (m)	Normalised $u_{2_{max}}$
CASE 1	0.01688	1.0	0.01688	1.0
CASE 2	0.01886	1.12	0.02082	1.23
CASE 3	0.01598	0.95	0.01728	1.02

Table 7.11 :  $u_2$  and  $u_{2_{max}}$  for case 1, 2 and 3 at  $a=L/2$ .

The reduction in slab stiffness,  $k_s$  is more pronounced than a reduction in adjacent building rigidity to the  $u_2$  and  $u_{2_{max}}$ .

7) The number of contacts is dependent on pounding location,  $a$ . For example, in the earthquake induced pounding analysis (section 3.4.3) where the Mexico earthquake, 1985 N90-W component is used for 30.0 sec, the number of contacts is 14 at  $a=L/2$  and at  $a=0$ , the number of contacts reduces to only 9. This means, increased  $a$  by 50% results an increment of 55.55% in number of contacts.

8) The contact force,  $P$  decreases as the damping coefficient,  $\xi$  increases. Table 3.6 shows the case of  $a=L/2$ , with the damping coefficient,  $\xi=0.02$ , the value of  $P$  is equal to 1947 kN compared to those obtained without damping,  $P = 2337$  kN. This means that, increased  $\xi$  as small as 2% produces a reduction of  $P$  by 16.69%.

9) The contact force,  $P$  decreases as the number of degree of freedom increases. For examples, at  $a=L/2$  the contact force is 8652 kN for 2DOF systems (Table 7.1) and decreases by 73.3 % for 3DOF systems (Table 3.4) with  $P$  equal to 2307 kN. The variation of  $P$  in the first contact is similar in both systems, with a minimum contact forces at  $a=L/2$ . For the second contact forces, there no similarities for 2DOF and 3DOF systems and as expected, this is due to their differences in physical properties or in other words, their flexibility.

10) All the results obtained are based on linear elastic behaviour. However, the pounding forces on the column could produce inelastic behaviour, and this possibility needs to be considered when looking at the results.

Finally, the present study on pounding of adjacent building, for both aligned and unaligned floor cases, gives better understanding of pounding failure such as Hotel de Carlo during the 1985 Mexico City earthquake as shown in Figure 2.1.

#### **7.4 FUTURE WORK**

The present study has provided simple solutions to the formulation of the equations of motion for structural pounding, for both aligned floor and unaligned floor cases. However, many aspects have not been explored herein and therefore further suggestions are put forward for the continuation of this study:-

- 1) The variation of other parameters such as the building separation, could be carried out based on the present study.
- 2) The two-dimensional structures analysed in this study can be expanded to cover three dimensional structures. However it could possibly introduce eccentric pounding with one structure hitting another eccentric to its centre of mass and stiffness. Therefore the equations developed in this study would have to be expanded to cover any torsional deformations and forces.
- 3) The present study assumed only the lateral movement of the floor without any rotation of the column. Therefore, more complex analysis could be done by considering the column rotation in each interstorey column.
- 4) The analysis of structural pounding which has been undertaken in this study are based on linear dynamic response analysis. Further work on non-linear variation of the damping and stiffness of structure could be employed.
- 5) Similar analysis mentioned in (1) to (3) could be done using the flexible slab.
- 6) Steel frame models could be tested for further verification of the results.

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## APPENDIX A

### DIRECT INTEGRATION METHOD FOR SDOF SYSTEM

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The general equation of motion for the SDOF system is as below

$$m \ddot{u} + c \dot{u} + k u = - m \ddot{u}_g \quad (1)$$

Rearranging this equation will gives :

$$\ddot{u} + 2 \xi \omega \dot{u} + \omega^2 u = - \beta \ddot{u}_g \quad (2)$$

where  $\beta$  is the participation factor, and equal to 1 for SDOF system and equal to  $L_i / m_i^*$  for MDOF system.

Assumed the variation of the ground motion is linear over a small time step, as shown in Figure 1 below:

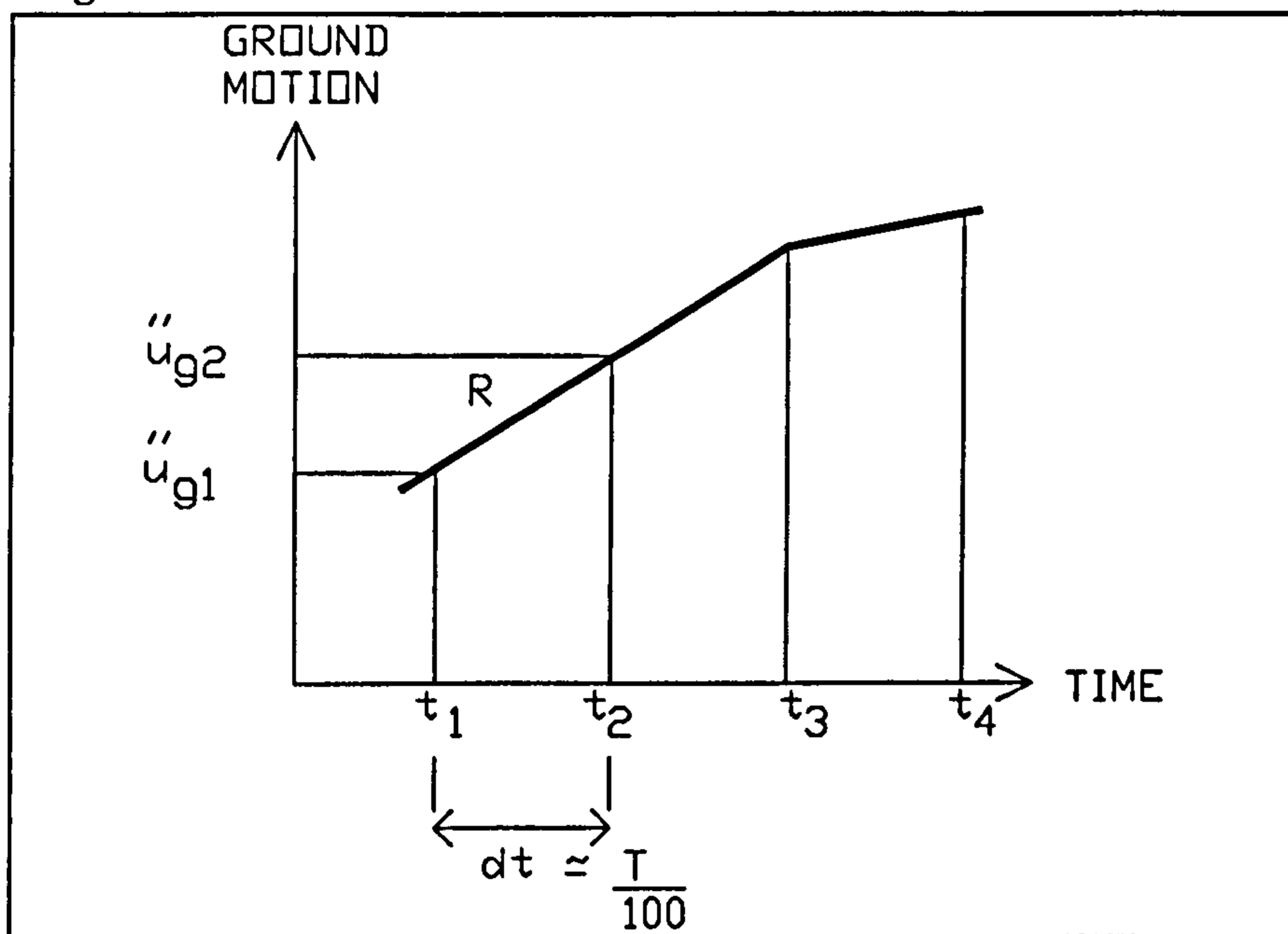


Figure 1: Idealized ground acceleration.

where  $R$  is the rate of change of the ground acceleration.

$$\begin{aligned} R &= \frac{\ddot{u}_{g2} - \ddot{u}_{g1}}{t_2 - t_1} \\ &= \frac{\ddot{u}_{g2} - \ddot{u}_{g1}}{dt} \end{aligned} \quad (3)$$

Re-arrange Equation (3) :

$$\ddot{u}_{g2} = \ddot{u}_{g1} + R dt \quad (4)$$

Substituting Equation (4) into Equation (2)

$$\ddot{u} + 2 \xi \omega \dot{u} + \omega^2 u = - \beta ( \ddot{u}_{g1} + R dt ) \quad (5)$$

The solution for the above second order of differential equation (5), are consists of Complementary Function (C.F) and the Particular Integral (P.I). i.e The general solution = C.F + P.I

a) Complementary Function

The Complementary Function is the general solution of the Left hand side of Equation (5). These are done by letting it equal to zero.

$$\ddot{u} + 2 \xi \omega \dot{u} + \omega^2 u = 0 \quad (6)$$

And assumed the auxiliary equation in the form of :

$$\begin{aligned} u &= A e^{st} \\ \dot{u} &= A e^{st} s \\ \ddot{u} &= A e^{st} s^2 \end{aligned} \quad (7)$$

Substituting Equation (7) into Equation (6),

$$A e^{st} [ s^2 + 2 \xi \omega s + \omega^2 ] = 0 \quad (8)$$

Since  $A e^{st}$  can not equal to zero , then the only solution is when [ ] equal to zero.

$$\begin{aligned} [ s^2 + 2 \xi \omega s + \omega^2 ] &= 0 \\ s &= \frac{- 2 \xi \omega \pm \sqrt{( 2 \xi \omega )^2 - 4 ( 1 ) ( \omega^2 )}}{2 ( 1 )} \\ &= - \xi \omega \pm \frac{\sqrt{( 2 \omega )^2 ( \xi^2 - 1 )}}{2} \\ &= - \xi \omega \pm \omega \sqrt{( \xi^2 - 1 )} \\ &= - \xi \omega \pm \omega \sqrt{( 1 - \xi^2 )} i^2 \\ &= - \xi \omega \pm \omega_d i \end{aligned} \quad (9)$$

where  $\omega_d$  is the damped natural frequency and is equal to  $\sqrt{( 1 - \xi^2 )}$  and the complex root ,  $i^2 = -1$ .

Since the roots of the auxiliary equation are two complex roots, thus the general equation for this Complementary Function is :

$$u = e^{- \xi \omega dt} [ C1 \sin \omega_d dt + C2 \cos \omega_d dt ] \quad (10)$$

where C1 and C2 are constants.

b) Particular Integral

The Particular Integral is depends on the function on the Right hand side of Equation (5).

$$- \beta \ddot{u}_{g1} - \beta R dt \quad (11)$$

The general equation for above equation which consists of a constant -  $\beta\ddot{u}_{g1}$  and -  $\beta R$  varies with time, t are in the form :

$$u = \lambda dt + \mu \quad (12)$$

Hence their derivatives are :

$$\begin{aligned} \dot{u} &= \lambda \\ \ddot{u} &= 0 \end{aligned} \quad (13)$$

Substitute into Equation (5) gives:

$$0 + \lambda (2 \xi \omega) + (\lambda dt + \mu) \omega^2 = - \beta \ddot{u}_{g1} - \beta R dt \quad (14)$$

Comparing and equate both dt and a constant term together:

( i) dt term :

$$\begin{aligned} \omega^2 \lambda &= - \beta R \\ \lambda &= - \beta \frac{R}{\omega^2} \end{aligned} \quad (15)$$

( ii) constant term, and substitute value of  $\lambda$ :

$$\begin{aligned} \lambda (2 \xi \omega) + \omega^2 \mu &= - \beta \ddot{u}_{g1} \\ \mu &= \left[ \frac{-\beta \ddot{u}_{g1} - \lambda (2 \xi \omega)}{\omega^2} \right] \\ \mu &= - \beta \frac{\ddot{u}_{g1}}{\omega^2} - \frac{(- \beta R)}{\omega^2} \frac{(2 \xi \omega)}{\omega^2} \\ \mu &= - \beta \frac{\ddot{u}_{g1}}{\omega^2} + \frac{\beta (2 \xi)}{\omega^3} R \end{aligned} \quad (16)$$

The general equation of Particular Integral is

$$u = - \beta \frac{R dt}{\omega^2} - \beta \frac{\ddot{u}_{g1}}{\omega^2} + \beta \frac{(2 \xi)}{\omega^3} R \quad (17)$$

c) General Solution

General solution of Equation (5) will be in the form of : C.F + P.I.

$$u = e^{-\xi \omega dt} [ C1 \sin \omega_d dt + C2 \cos \omega_d dt ] - \beta \frac{R dt}{\omega^2} - \beta \frac{\ddot{u}_{g1}}{\omega^2} + \beta \frac{(2 \xi)}{\omega^3} R \quad (18)$$

This represent the displacement's equation, and the velocity's equation is the derivatives of u :

$$\dot{u} = e^{-\xi \omega dt} \omega_d [ C1 \cos \omega_d dt - C2 \sin \omega_d dt ] - \xi \omega e^{-\xi \omega dt} [ C1 \sin \omega_d dt + C2 \cos \omega_d dt ] - \beta \frac{R}{\omega^2} \quad (19)$$

d) Calculation of constants C1 and C2

To calculate the values of constants C1 and C2 , it is important to know the initial conditions say at the beginning of the time step i.e at time t1 (refer to Figure 1). The displacement and the velocity at time t1 are :

$$\begin{aligned} \text{at } t = t1 , dt = 0 \\ u = u_1 \\ \dot{u} = \dot{u}_1 \end{aligned} \quad (20)$$

Substituting these initial conditions into Equations (18 - 19) give

$$\begin{aligned} u_1 &= C2 - \beta \frac{\ddot{u}_{g1}}{\omega^2} + \beta \frac{(2 \xi R)}{\omega^3} \\ \therefore C2 &= u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2 \xi R)}{\omega^3} \end{aligned} \quad (21)$$

Also

$$\begin{aligned} \dot{u}_1 &= \omega_d C1 - \xi \omega C2 - \beta \frac{R}{\omega^2} \\ \therefore C1 &= \frac{1}{\omega_d} ( \dot{u}_1 + \xi \omega C2 + \beta \frac{R}{\omega^2} ) \end{aligned} \quad (22)$$

Substituting Equation (21) into Equation (22) :

$$\begin{aligned}
C1 &= \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega \left( u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{2 \xi R}{\omega^3} \right) + \beta \frac{R}{\omega^2} \right] \\
&= \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} - \xi \beta \frac{2 \xi R}{\omega^2} + \beta \frac{R}{\omega^2} \right] \quad (23) \\
&= \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} - \frac{(2 \xi^2 - 1)}{\omega^2} \beta R \right]
\end{aligned}$$

The general equation of displacement at the beginning of time step is given by direct substitution of C1 (Equation 23) and C2 (Equation 21) into Equation (18):

$$\begin{aligned}
u_1 &= e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} - \frac{(2 \xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt \\
&+ e^{-\xi \omega dt} \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2 \xi R)}{\omega^3} \right] \cos \omega_d dt \quad (24) \\
&- \beta \frac{R dt}{\omega^2} - \beta \frac{\ddot{u}_g}{\omega^2} + \beta \frac{(2 \xi)}{\omega^3} R
\end{aligned}$$

The general equation of velocity at the beginning of time step is given by direct substitution of C1 (Equation 23) and C2 (Equation 21) into Equation (19):

$$\begin{aligned}
\dot{u}_1 &= e^{-\xi \omega dt} \omega_d \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} \right. \\
&\dots \left. - \frac{(2 \xi^2 - 1)}{\omega^2} \beta R \right] \cos \omega_d dt \\
&- e^{-\xi \omega dt} \omega_d \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2 \xi R)}{\omega^3} \right] \sin \omega_d dt \\
&- \xi \omega e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} \right. \\
&\dots \left. - \frac{(2 \xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt \quad (25) \\
&- \xi \omega e^{-\xi \omega dt} \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2 \xi R)}{\omega^3} \right] \cos \omega_d dt \\
&- \beta \frac{R}{\omega^2}
\end{aligned}$$

The equation of acceleration is given by rearranged of Equation (6)

$$\ddot{u} = -2 \xi \omega \dot{u} - \omega^2 u \quad (26)$$

e) At the end of time step

The boundary conditions for displacement and velocity at the end of time step are

$$\begin{aligned} \text{at } t = t_2, dt = t_2 - t_1 \\ d\ddot{u} = \ddot{u}_{g2} - \ddot{u}_{g1} \end{aligned} \quad (27)$$

Substituting these values into Equation (24), i.e the general equation for displacement where  $u_1$  becomes  $u_2$  :

$$\begin{aligned} u_2 = & e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} - \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt \\ & + e^{-\xi \omega dt} \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2\xi R)}{\omega^3} \right] \cos \omega_d dt \\ & - \beta \frac{R dt}{\omega^2} - \beta \frac{\ddot{u}_g}{\omega^2} + \beta \frac{(2\xi)}{\omega^3} R \end{aligned} \quad (28)$$

The equation of velocity at the end of time step is given by Equation (25) where the term  $\dot{u}_1$  becomes  $\dot{u}_2$ .

$$\begin{aligned} \ddot{u}_2 = & e^{-\xi \omega dt} \omega_d \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} \right. \\ & \dots \left. - \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right] \cos \omega_d dt \\ & - e^{-\xi \omega dt} \omega_d \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2\xi R)}{\omega^3} \right] \sin \omega_d dt \\ & - \xi \omega e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} \right. \\ & \dots \left. - \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt \\ & - \xi \omega e^{-\xi \omega dt} \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2\xi R)}{\omega^3} \right] \cos \omega_d dt \\ & - \beta \frac{R}{\omega^2} \end{aligned} \quad (29)$$

These can be further simplified by rearrange above equation into two main parts which consists of  $\sin \omega_d dt$  and  $\cos \omega_d dt$ .

(i) the  $\cos \omega_d dt$  coefficients



$$\begin{aligned}
\dot{u}_2 &= e^{-\xi \omega dt} \omega_d \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} \right. \\
&\quad \left. - \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right] \cos \omega_d dt \\
&\quad \dots \\
&\quad - \xi \omega e^{-\xi \omega dt} \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2\xi R)}{\omega^3} \right] \cos \omega_d dt \\
&= e^{-\xi \omega dt} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} - \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right. \\
&\quad \left. - \xi \omega u_1 - \xi \omega \beta \frac{\ddot{u}_{g1}}{\omega^2} + \beta \frac{(2\xi^2 \omega R)}{\omega^3} \right] \cos \omega_d dt \\
&= e^{-\xi \omega dt} \left[ \dot{u}_1 + \beta \frac{R}{\omega^2} \right] \cos \omega_d dt
\end{aligned} \tag{30}$$

(ii) the  $\sin \omega_d dt$  coefficients

$$\begin{aligned}
\dot{u}_2 &= - e^{-\xi \omega dt} \omega_d \left[ u_1 + \beta \frac{\ddot{u}_{g1}}{\omega^2} - \beta \frac{(2\xi R)}{\omega^3} \right] \sin \omega_d dt \\
&\quad - \xi \omega e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \dot{u}_1 + \xi \omega u_1 + \xi \beta \frac{\ddot{u}_{g1}}{\omega} \right. \\
&\quad \left. - \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt \\
&= - e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \omega_d^2 u_1 + \omega_d^2 \beta \frac{\ddot{u}_{g1}}{\omega^2} - \omega_d^2 \beta \frac{(2\xi R)}{\omega^3} \right. \\
&\quad \left. + \xi \omega \dot{u}_1 + \xi^2 \omega^2 u_1 + \xi^2 \omega \beta \frac{\ddot{u}_{g1}}{\omega} - \xi \omega \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt
\end{aligned} \tag{31}$$

Further assumptions are made to simplified above equation where for a light damped system, the damping coefficient,  $\xi$  lies between  $0.01 \leq \xi \leq 0.1$

$$\omega_d = \omega \sqrt{1 - \xi^2}$$

$$\text{If } \xi = 0.01, \quad \xi^2 = 0.001 \quad \text{then } \omega_d = 0.99995 \omega$$

$$\text{If } \xi = 0.1, \quad \xi^2 = 0.01 \quad \text{then } \omega_d = 0.99499 \omega$$

For structural systems, we may assumed :

$$\xi^2 \approx 0 \quad \therefore \quad \omega_d \approx \omega \tag{32}$$

Thus Equation (31) becomes :

$$\begin{aligned}
\dot{u}_2 &= - e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \omega_d^2 u_1 + \omega_d^2 \beta \frac{\ddot{u}_{g1}}{\omega^2} - \omega_d^2 \beta \frac{(2 \xi R)}{\omega^3} \right. \\
&\quad \left. + \xi \omega \dot{u}_1 + \xi^2 \omega^2 u_1 + \xi^2 \omega \beta \frac{\ddot{u}_{g1}}{\omega} - \xi \omega \frac{(2\xi^2 - 1)}{\omega^2} \beta R \right] \sin \omega_d dt \\
&= - e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \omega^2 u_1 + \beta \ddot{u}_{g1} \right. \\
&\quad \left. - \frac{2 \beta \xi R}{\omega} + \xi \omega \dot{u}_1 + 0 + 0 + \frac{\beta \xi R}{\omega} \right] \sin \omega_d dt \\
&= - e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ \omega^2 u_1 + \beta \ddot{u}_{g1} + \xi \omega \dot{u}_1 - \frac{\beta \xi R}{\omega} \right] \sin \omega_d dt \\
&= e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ - \omega^2 u_1 - \beta \ddot{u}_{g1} - \xi \omega \dot{u}_1 + \frac{\beta \xi R}{\omega} \right] \sin \omega_d dt
\end{aligned} \tag{33}$$

(iii) the constant term

This is referred to  $-\beta R / \omega^2$  from Equation (29)

(iv) the final velocity's equation at the end of time step

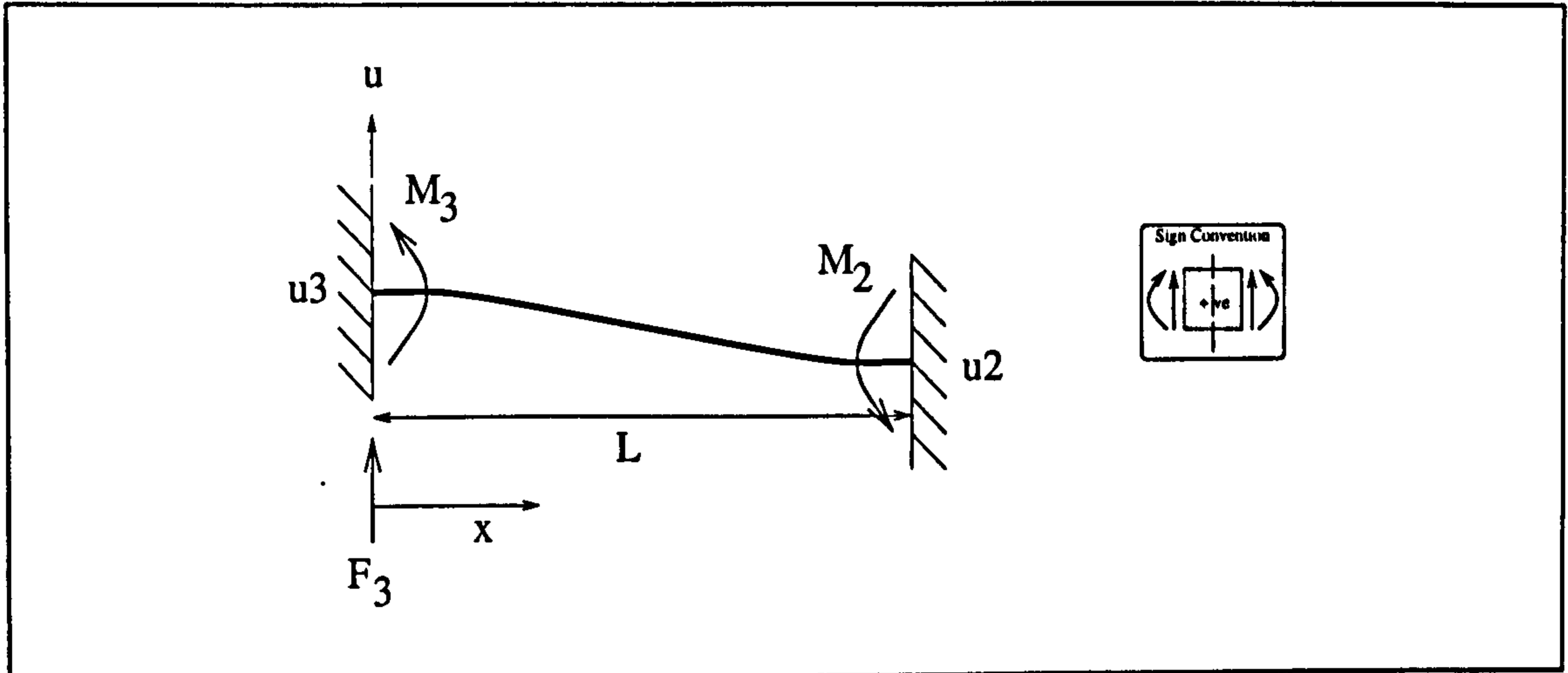
Collecting all the terms in Equation (30) and Equation (33) plus the constant term  $-\beta R / \omega^2$ , will give the equation of velocity at the end of time step,

$$\begin{aligned}
\dot{u}_2 &= e^{-\xi \omega dt} \left[ \dot{u}_1 + \beta \frac{R}{\omega^2} \right] \cos \omega_d dt + e^{-\xi \omega dt} \frac{1}{\omega_d} \left[ - \omega^2 u_1 \right. \\
&\quad \left. - \beta \ddot{u}_{g1} - \xi \omega \dot{u}_1 + \frac{\beta \xi R}{\omega} \right] \sin \omega_d dt \\
&\quad - \beta \frac{R}{\omega^2}
\end{aligned} \tag{34}$$

## APPENDIX B

### DETERMINATION OF $u_i$ , $u_{di}$ , $u_p$ AND $k_{AB}$

The basic analysis is by using the differential equations of the deflection curve for the statically indeterminate beams as shown below.



Assumed the flexural rigidity of the beam is  $EI$  and for the small angle of rotation and the slope is equal to

$$EI \frac{d^2u}{dx^2} = M_x \quad (1)$$

where

$$\begin{aligned} M_x &= F_3 x - M_3 \\ &= \frac{(M_3 + M_2)}{L} x - M_3 \end{aligned} \quad (2)$$

Substituting Equation (2) into (1) and two successive integrations give the slope  $du/dx$  and deflection,  $u$  :

$$\begin{aligned} EI \frac{d^2u}{dx^2} &= \frac{(M_3 + M_2)}{L} x - M_3 \\ EI \frac{du}{dx} &= \frac{(M_3 + M_2)}{2L} x^2 - M_3 x + A \\ EI u &= \frac{(M_3 + M_2)}{6L} x^3 - \frac{M_3}{2} x^2 + Ax + B \end{aligned} \quad (3)$$

There are four unknown quantities in these equations ( $M_3$ ,  $M_2$ ,  $A$  and  $B$ ) and four boundary conditions:

$$\begin{aligned}
\text{at } x = 0 \quad \frac{du}{dx} &= 0 \quad \therefore A = 0 \\
\text{at } x = L \quad \frac{du}{dx} &= 0 \quad \therefore M_3 = M_2 \\
\text{at } x = 0 \quad u &= u_3 \quad \therefore B = E I u_3 \\
\text{at } x = L \quad u &= u_2 \quad \therefore E I u_2 = \frac{2 M_3 L^3}{6 L} - \frac{M_3 L^2}{2} + E I u_3 \quad (4) \\
\therefore \frac{M_3 L^2}{6} &= E I (u_3 - u_2) \\
\therefore M_3 &= \frac{6 E I}{L^2} (u_3 - u_2)
\end{aligned}$$

To find the slope and deflection at say  $x=a$  :

$$\begin{aligned}
E I \frac{du}{dx} &= \frac{12 E I}{L^2} (u_3 - u_2) \frac{a^2}{2L} - \frac{6 E I}{L^2} (u_3 - u_2) a \quad (5) \\
\frac{du}{dx} &= \frac{6}{L} \left[ \left( \frac{a}{L} \right)^2 - \left( \frac{a}{L} \right) \right] (u_3 - u_2)
\end{aligned}$$

Since ( $L=a+b$ ) this equation can be further modify as

$$\begin{aligned}
\frac{du}{dx} &= \frac{6}{L} \left[ \frac{a^2}{L^2} - \frac{a}{L} \right] (u_3 - u_2) \\
&= \frac{6}{L^3} [a^2 - aL] (u_3 - u_2) \quad (6) \\
&= - \frac{6 a b}{L^3} (u_3 - u_2)
\end{aligned}$$

and

$$\begin{aligned}
E I u &= \frac{12 E I}{L^2} (u_3 - u_2) \frac{a^3}{6L} - \frac{6 E I}{L^2} (u_3 - u_2) \frac{a^2}{2} = E I u_3 \quad (7) \\
u &= u_3 \left[ 1 - 3 \left( \frac{a}{L} \right)^2 + 2 \left( \frac{a}{L} \right)^3 \right] + u_2 \left[ 3 \left( \frac{a}{L} \right)^2 - 2 \left( \frac{a}{L} \right)^3 \right]
\end{aligned}$$

If we let

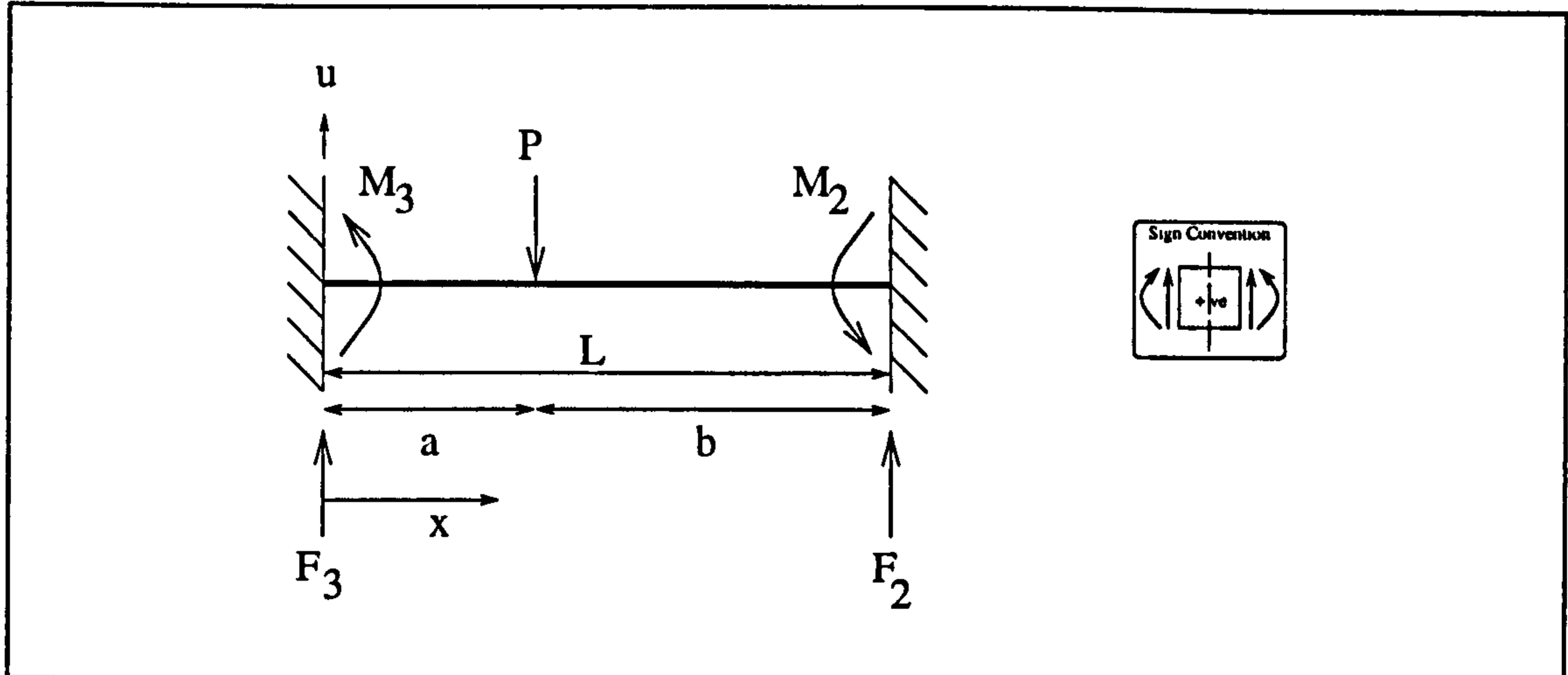
$$\begin{aligned}
k_{AB} &= \left[ 3 \left( \frac{a}{L} \right)^2 - 2 \left( \frac{a}{L} \right)^3 \right] \quad (8) \\
&= \frac{a^2}{L^3} [3b + a] \quad \text{where } L = (a + b)
\end{aligned}$$

Thus the equation of deflection  $u$  can be expressed in term of  $k_{AB}$  as:

$$u = u_3 [1 - k_{AB}] + u_2 [k_{AB}] \quad (9)$$

If  $u$  equal to  $u_i$ , thus  $u_i = u_3 [1 - k_{AB}] + u_2 [k_{AB}]$  and if  $u$  equal to  $u_{di}$ , by analogy to Equation (9), the  $u_{di} = u_{d3} [1 - k_{AB}] + u_{d2} [k_{AB}]$ .

A beam with a fixed ends is loaded by a force  $P$  acting at the position shown in figure below:



Assumed the flexural rigidity of the beam is  $EI$  and for the small angle of rotation and the slope is equal to Equation (1)

$$EI \frac{d^2u}{dx^2} = M_x$$

where

$$M_x = F_3 x - M_3 - P[x - a] \quad (10)$$

The reactions at the supports of a beam and their fixed end moments are given by:

$$\begin{aligned} F_3 &= \frac{Pb}{L} + \frac{M_3 + M_2}{L} \\ F_2 &= \frac{Pa}{L} - \frac{M_3 + M_2}{L} \end{aligned} \quad (11)$$

Substituting equation (11) into (10) and two successive integrations give the slope  $du/dx$  and deflection  $u$  :

$$\begin{aligned} EI \frac{d^2u}{dx^2} &= \frac{Pbx}{L} + \frac{(M_3 + M_2)x}{L} - M_3 - P[x - a] \\ EI \frac{du}{dx} &= \frac{Pbx^2}{2L} + \frac{(M_3 + M_2)x^2}{2L} - M_3 x - \frac{P[x - a]^2}{2} + A \\ EI u &= \frac{Pbx^3}{6L} + \frac{(M_3 + M_2)x^3}{6L} - \frac{M_3}{2} x^2 - \frac{P[x - a]^3}{6} + Ax + B \end{aligned} \quad (12)$$

The boundary conditions are:

$$\begin{aligned}
\text{at } x = 0 \quad \frac{du}{dx} &= 0 \quad \therefore A = 0 \\
\text{at } x = 0 \quad u &= 0 \quad \therefore B = 0 \\
\text{at } x = L \quad \frac{du}{dx} &= 0 \quad \therefore M_3 = M_2 + Pb - P\frac{b^2}{L} \\
\text{at } x = L \quad u &= 0 \quad \therefore M_3 = \frac{M_2}{2} + \frac{Pb}{2} - P\frac{b^3}{2L^2} \quad (13) \\
\therefore M_2 &= -\frac{Pba^2}{L^2} \\
\therefore M_3 &= \frac{Pab^2}{L^2}
\end{aligned}$$

At position of load, (  $x = a$  ) the slope ,  $du/dx$  :

$$\begin{aligned}
EI \frac{du}{dx} &= \frac{Pba^2}{2L} + \left( \frac{Pab^2}{L^2} - \frac{Pba^2}{L^2} \right) \frac{a^2}{2L} - \frac{Pab^2}{L^2} a \\
EI \frac{du}{dx} &= \frac{Pab}{2L^3} [a^2b - ab^2] \quad (14) \\
\frac{du}{dx} &= \frac{Pab}{EI 2L^3} [a^2b - ab^2]
\end{aligned}$$

And the deflection,  $u$  :

$$\begin{aligned}
EI u &= \frac{Pba^3}{6L} \left( \frac{Pab^2}{L^2} - \frac{Pba^2}{L^2} \right) \frac{a^3}{6L} - \frac{Pab^2}{L^2} \frac{a^2}{2} \\
EI u &= -\frac{Pb^3a^3}{3L^3} \quad (15) \\
u &= -\frac{Pb^3a^3}{EI 3L^3}
\end{aligned}$$

In this study, the displacement at the contact load is denoted as  $u_p$ , that is

$$u_p = -\frac{Pb^3a^3}{EI 3L^3} \quad (16)$$

# APPENDIX C

## FORTRAN PROGRAMS FOR CHAPTER 3

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```
c           MDOF1.FOR
c           CHAPTER 3 : UNDAMPED FREE VIBRATION
c           a = 0 (aligned floor case)
c
c All units are in kips,inch and convert to N,m at the end of programme
c matrix an & ac are normalised mode shapes for no contact & contact
c matrix ant & act are tranposed of an & ac
c matrix b is a mass matrix
c matrix cn & cc are the initial displacements & velocities
c -----
c           dimension u1(20000),v1(20000),u2(20000),v2(20000),u3(20000),
&v3(20000),a1(3),a2(3),a3(3),b1(3),b2(3),b3(3),b(3,3)
c           dimension an1(3),bn1(3)
c           dimension an(3,3),ant(3,3),cn(3,2),antb(3,3),antbcn(3,2)
c           dimension ac(3,3),act(3,3),cc(3,2),actb(3,3),actbcc(3,2)
c           dimension cm(3,2)
c           open(6,file='mdof1.out',status='unformatted')
c
c           data an /0.74265, 0.48164, 0.22417, -0.63577, 0.38566, 0.43168,
& 0.21037, -0.53475, 0.51323/
c           data ac /0.0076419, 0.63633, 0.44303, -0.0063228, -0.51155,
& 0.55111, 0.99995, -0.0080976, 0.000098942/
c
c           data ant /0.74265, -0.63577, 0.21037, 0.48164, 0.38566,
& -0.53475, 0.22417, 0.43168, 0.51323/
c           data act /0.0076419, -0.0063228, 0.99995, 0.63633, -0.51155,
& -0.0080976, 0.44303, 0.55111, 0.000098942/
c
c           data b /1.0, 0.0, 0.0, 0.0, 1.5, 0.0, 0.0, 0.0, 2.0/
c
c           tt=0.0
c           kn=2010
c           pi=3.141592654
c           cn(1,1)=-1.0000
c           cn(2,1)=-0.505106
c           cn(3,1)=-0.1440976
c           cn(1,2)=0.0
c           cn(2,2)=0.0
c           cn(3,2)=0.0
c           goto 7
c
c           5   tt=tt-dt
c           kn=6800
c           cn(1,1)=a1(1)
c           cn(2,1)=a1(2)
c           cn(3,1)=a1(3)
c           cn(1,2)=b1(1)
c           cn(2,2)=b1(2)
c           cn(3,2)=b1(3)
c           goto 7
c
c           c form a matrix ant*b(3,3)
c           -----
c           7   do 10 j=1,3
c               do 10 i=1,3
c                   antb(i,j)=0.0
c               do 10 k=1,3
c                   10 antb(i,j)=antb(i,j)+ant(i,k)*b(k,j)
c
c           c form a matrix ant*b*cn(3,2)
c           -----
c           do 20 j=1,2
c               do 20 i=1,3
c                   antbcn(i,j)=0.0
c               do 20 k=1,3
```

```

20 antbcn(i,j)=antbcn(i,j)+antb(i,k)*cn(k,j)
c
c form checking matrix cm = an*antbcn
c -----
do 25 j=1,2
do 25 i=1,3
cm(i,j)=0.0
do 25 k=1,3
25 cm(i,j)=cm(i,j)+an(i,k)*antbcn(k,j)
c
c print the final matrix antbcn(3,2)
c -----
print*, 'normalised disp vel checking init. disp vel'
do 30 i=1,3
write(6,40)(antbcn(i,j),j=1,2),(cm(i,j),j=1,2)
40 format (2(5x,f9.5,3x,f9.5,5x))
30 continue
c
wn1=14.522
wn2=31.048
wn3=46.099
u1(1)=antbcn(1,1)
u2(1)=antbcn(2,1)
u3(1)=antbcn(3,1)
v1(1)=antbcn(1,2)
v2(1)=antbcn(2,2)
v3(1)=antbcn(3,2)
dt=.0001
odt1=(wn1*dt)
odt2=(wn2*dt)
odt3=(wn3*dt)
print*, ' tt utop umid ubott vtop vmid
$ vbott ui vi'
do 100 n=1,kn
c write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
aa1=(v1(n)/wn1)*sin(odt1)
bb1=u1(n)*cos(odt1)
cc1=v1(n)*cos(odt1)
dd1=wn1*u1(n)*sin(odt1)
aa2=(v2(n)/wn2)*sin(odt2)
bb2=u2(n)*cos(odt2)
cc2=v2(n)*cos(odt2)
dd2=wn2*u2(n)*sin(odt2)
aa3=(v3(n)/wn3)*sin(odt3)
bb3=u3(n)*cos(odt3)
cc3=v3(n)*cos(odt3)
dd3=wn3*u3(n)*sin(odt3)
c u(n+1)=((v(n)/wn)*sin(odt))+u(n)*cos(odt)
c v(n+1)=(v(n)*cos(odt))-(wn*u(n)*sin(odt))
u1(n+1)=aa1+bb1
v1(n+1)=cc1-dd1
u2(n+1)=aa2+bb2
v2(n+1)=cc2-dd2
u3(n+1)=aa3+bb3
v3(n+1)=cc3-dd3
c now calculate the total mode displacement & velocity
do 500 l=1,3
b1(l)=an(l,1)*v1(n)
b2(l)=an(l,2)*v2(n)
b3(l)=an(l,3)*v3(n)
a1(l)=an(l,1)*u1(n)
a2(l)=an(l,2)*u2(n)
a3(l)=an(l,3)*u3(n)
500 continue
do 600 m=1,3
a1(m)=a1(m)+a2(m)+a3(m)
600 b1(m)=b1(m)+b2(m)+b3(m)
c
c convert from inches to m
c -----
do 650 m=1,3
an1(m)=a1(m)*0.0254
650 bn1(m)=b1(m)*0.0254
ui=an1(1)

```



```

vi=bn1(1)
write (6,400)tt,an1(1),an1(2),an1(3),bn1(1),bn1(2),bn1(3),ui,vi
c write (6,400)tt,ui
400 format(9(1x,f9.5))
c 400 format(2(1x,f9.5))
tt=tt+dt
c
c *****
if (tt.ge.1.0) goto 710
c
c *****
if (ui.ge.0.0127) then
goto 800
else
goto 100
endif
c *****
c
100 continue
c
c ~~~~~
800 cc(1,1)=a1(1)
cc(2,1)=a1(2)
cc(3,1)=a1(3)
cc(1,2)=b1(1)
cc(2,2)=b1(2)
cc(3,2)=b1(3)
c
c form a matrix act*b(3,3)
c -----
do 11 j=1,3
do 11 i=1,3
actb(i,j)=0.0
do 11 k=1,3
11 actb(i,j)=actb(i,j)+act(i,k)*b(k,j)
c
c form a matrix act*b*cc(3,2)
c -----
do 21 j=1,2
do 21 i=1,3
actbcc(i,j)=0.0
do 21 k=1,3
21 actbcc(i,j)=actbcc(i,j)+actb(i,k)*cc(k,j)
c form checking matrix cm = ac*actbcc
c -----
do 26 j=1,2
do 26 i=1,3
cm(i,j)=0.0
do 26 k=1,3
26 cm(i,j)=cm(i,j)+ac(i,k)*actbcc(k,j)
c
c print the final matrix actbcc(3,2)
c -----
print*, 'normalised disp vel checking init. disp vel'
do 31 i=1,3
write(6,41)(actbcc(i,j),j=1,2),(cm(i,j),j=1,2)
41 format (2(5x,f9.5,3x,f9.5,5x))
31 continue
c
tt=tt-dt
kn=4800
pi=3.141592654
wc1=25.263
wc2=45.353
wc3=224.96
u1(1)=actbcc(1,1)
u2(1)=actbcc(2,1)
u3(1)=actbcc(3,1)
v1(1)=actbcc(1,2)
v2(1)=actbcc(2,2)
v3(1)=actbcc(3,2)
dt=.00001
odt1=(wc1*dt)
odt2=(wc2*dt)

```

```

    odt3=(wc3*dt)
c
c calculate the value of "static preload" u1s, u2s & u3s
c -----
    u1s=0.299344659
    u2s=-0.076848857
    u3s=0.49397809
c
    print*, '----- contact -----'
c write (6,30)tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
    do 110 n=1,kn
        aa1=(v1(n)/wc1)*sin(odt1)
        bb1=(u1(n)-u1s)*cos(odt1)
        cc1=v1(n)*cos(odt1)
        dd1=wc1*(u1(n)-u1s)*sin(odt1)
        aa2=(v2(n)/wc2)*sin(odt2)
        bb2=(u2(n)-u2s)*cos(odt2)
        cc2=v2(n)*cos(odt2)
        dd2=wc2*(u2(n)-u2s)*sin(odt2)
        aa3=(v3(n)/wc3)*sin(odt3)
        bb3=(u3(n)-u3s)*cos(odt3)
        cc3=v3(n)*cos(odt3)
        dd3=wc3*(u3(n)-u3s)*sin(odt3)
c        u(n+1)=((v(n)/wn)*sin(odt))+u(n)*cos(odt)
c        v(n+1)=(v(n)*cos(odt))-(wn*u(n)*sin(odt))
    u1(n+1)=aa1+bb1+u1s
    v1(n+1)=cc1-dd1
    u2(n+1)=aa2+bb2+u2s
    v2(n+1)=cc2-dd2
    u3(n+1)=aa3+bb3+u3s
    v3(n+1)=cc3-dd3
c    now calculate the total mode displacement
    do 510 l=1,3
        b1(l)=ac(l,1)*v1(n)
        b2(l)=ac(l,2)*v2(n)
        b3(l)=ac(l,3)*v3(n)
        a1(l)=ac(l,1)*u1(n)
        a2(l)=ac(l,2)*u2(n)
        a3(l)=ac(l,3)*u3(n)
    510 continue
    do 610 m=1,3
        a1(m)=a1(m)+a2(m)+a3(m)
    610 b1(m)=b1(m)+b2(m)+b3(m)
c
c convert from inches to m
c -----
    do 615 m=1,3
        an1(m)=a1(m)*0.0254
    615 bn1(m)=b1(m)*0.0254
    ui=an1(1)
    vi=bn1(1)
c
c calculate contact force, P (kips), P2 (N)
c -----
    P=50000.0*(a1(1)-0.5)
    P2=P*4450.0
    write (6,410)tt,an1(1),an1(2),an1(3),bn1(1),bn1(2),bn1(3),ui,vi,P2
    tt=tt+dt
c
c *****
    if (tt.ge.1.0) goto 710
c
c *****
    if (ui.le.0.0127) then
        goto 5
    else
        goto 110
    endif
c *****
c
    110 continue
    410 format(9(1x,f9.5),1x,f15.5)
    710 stop
    end

```

```

c           MDOF2.FOR
c   CHAPTER 3 : UNDAMPED FREE VIBRATION
c           a = L/4 (unaligned floor case)
c
c   Similar to MDOF1.FOR , except the following parameters are changed :
c ~~~~~
c   open(6,file='mdof2.out',status='unformatted')
c
c   data an /0.74265, 0.48164, 0.22417, -0.63577, 0.38566, 0.43168,
& 0.21037, -0.53475, 0.51323/
c   data ac /-0.10022, 0.65519, 0.41596, 0.089021, -0.47490,
& 0.57174, 0.99097, 0.10892, -0.0092938/
c
c   data ant /0.74265, -0.63577, 0.21037, 0.48164, 0.38566,
& -0.53475, 0.22417, 0.43168, 0.51323/
c   data act /-0.10022, 0.089021, 0.99097, 0.65519, -0.47490,
& 0.10892, 0.41596, 0.57174, -0.0092938/
c
c   wn1=14.522
c   wn2=31.048
c   wn3=46.099
c
c   ui=((27.0*an1(1))+(5.0*an1(2)))/32.0
c   vi=((27.0*bn1(1))+(5.0*bn1(2)))/32.0
c
c   800 us1=a1(1)
c   us2=a1(2)
c
c   wc1=23.557
c   wc2=44.703
c   wc3=92.368
c
c calculate the value of "static preload" u1s, u2s & u3s
c ~~~~~
c   as1=ac(1,1)*((7800.0*us1)+(1800.0*us2))
c   bs1=ac(2,1)*((1800.0*us1)-(22.22*us2))
c   u1s=(as1+bs1)/(wc1**2.0)
c   as2=ac(1,2)*((7800.0*us1)+(1800.0*us2))
c   bs2=ac(2,2)*((1800.0*us1)-(22.22*us2))
c   u2s=(as2+bs2)/(wc2**2.0)
c   as3=ac(1,3)*((7800.0*us1)+(1800.0*us2))
c   bs3=ac(2,3)*((1800.0*us1)-(22.22*us2))
c   u3s=(as3+bs3)/(wc3**2.0)
c   write(6,*) 'u1s=', u1s, 'u2s=', u2s, 'u3s=', u3s
c
c calculate contact force, P (kips), P2 (N)
c ~~~~~
c   ud3=a1(1)-us1
c   ud2=a1(2)-us2
c   P=11377.778*(27.0*ud3+5.0*ud2)/32.0
c   P2=P*4450.0

```

```

c           MDOF3.FOR
c   CHAPTER 3 : UNDAMPED FREE VIBRATION
c           a = L/3 (unaligned floor case)
c
c   Similar to MDOF1.FOR , except the following parameters are changed :
c   ~~~~~
c   open(6,file='mdof3.out',status='unformatted')
c
c   data an /0.74265, 0.48164, 0.22417, -0.63577, 0.38566, 0.43168,
& 0.21037, -0.53475, 0.51323/
c   data ac /-0.18687, 0.63957, 0.41923, 0.22130, -0.45269,
& 0.56729, 0.95714, 0.22953, -0.049312/
c
c   data ant /0.74265, -0.63577, 0.21037, 0.48164, 0.38566,
& -0.53475, 0.22417, 0.43168, 0.51323/
c   data act /-0.18687, 0.22130, 0.95714, 0.63957, -0.45269,
& 0.22953, 0.41923, 0.56729, -0.049312/
c
c   wn1=14.522
c   wn2=31.048
c   wn3=46.099
c
c   ui=((20.0*an1(1))+(7.0*an1(2)))/27.0
c   vi=((20.0*bn1(1))+(7.0*bn1(2)))/27.0
c
c   800 us1=a1(1)
c   us2=a1(2)
c
c   wc1=24.179
c   wc2=44.484
c   wc3=65.520
c
c calculate the value of "static preload" u1s, u2s & u3s
c   ~~~~~
c   as1=ac(1,1)*((3450.0*us1)+(1612.5*us2))
c   bs1=ac(2,1)*((1612.5*us1)+(159.375*us2))
c   u1s=(as1+bs1)/(wc1**2.0)
c   as2=ac(1,2)*((3450.0*us1)+(1612.0*us2))
c   bs2=ac(2,2)*((1612.5*us1)+(159.375*us2))
c   u2s=(as2+bs2)/(wc2**2.0)
c   as3=ac(1,3)*((3450.0*us1)+(1612.5*us2))
c   bs3=ac(2,3)*((1612.5*us1)+(159.375*us2))
c   u3s=(as3+bs3)/(wc3**2.0)
c   write(6,*) 'u1s=', u1s, 'u2s=', u2s, 'u3s=', u3s
c
c calculate contact force, P (kips), P2 (N)
c   ~~~~~
c   ud3=a1(1)-us1
c   ud2=a1(2)-us2
c   P=6834.375*(20.0*ud3+7.0*ud2)/27.0
c   P2=P*4450.0

```

```

c           MDOF4.FOR
c   CHAPTER 3 : UNDAMPED FREE VIBRATION
c           a = L/2 (unaligned floor case)
c
c   Similar to MDOF1.FOR , except the following parameters are changed :
c ~~~~~
c   open(6,file='mdof4.out',status='unformatted')
c
c   data an /0.74265, 0.48164, 0.22417, -0.63577, 0.38566, 0.43168,
& 0.21037, -0.53475, 0.51323/
c   data ac /0.55099, -0.53326, -0.36733, 0.68599, -0.12760e-15,
& 0.51450, 0.47520, 0.61831, -0.31680/
c
c   data ant /0.74265, -0.63577, 0.21037, 0.48164, 0.38566,
& -0.53475, 0.22417, 0.43168, 0.51323/
c   data act /0.55099, 0.68599, 0.47520, -0.53326, -0.12760e-15,
& 0.61831, -0.36733, 0.51450, -0.31680/
c
c   ui=(an1(1)+an1(2))/2.0
vi=(bn1(1)+bn1(2))/2.0
c
c   800 us1=a1(1)
us2=a1(2)
c
c   wc1=25.079
wc2=38.730
wc3=51.682
c
c calculate the value of "static preload" u1s, u2s & u3s
c ~~~~~
c   as1=ac(1,1)*((900.0*us1)+(1500.0*us2))
bs1=ac(2,1)*((1500.0*us1)+(900.0*us2))
u1s=(as1+bs1)/(wc1**2.0)
c   as2=ac(1,2)*((900.0*us1)+(1500.0*us2))
bs2=ac(2,2)*((1500.0*us1)+(900.0*us2))
u2s=(as2+bs2)/(wc2**2.0)
c   as3=ac(1,3)*((900.0*us1)+(1500.0*us2))
bs3=ac(2,3)*((1500.0*us1)+(900.0*us2))
u3s=(as3+bs3)/(wc3**2.0)
write(6,*) 'u1s=', u1s, 'u2s=', u2s, 'u3s=', u3s
c
c calculate contact force, P (kips), P2 (N)
c ~~~~~
c   ud3=a1(1)-us1
ud2=a1(2)-us2
P=4800.0*(ud3+ud2)/2.0
P2=P*4450.0

```

```

c           MDOF5.FOR
c   CHAPTER 3 : UNDAMPED FREE VIBRATION
c           a = 3L/4 (unaligned floor case)
c
c   Similar to MDOF1.FOR , except the following parameters are changed :
c ~~~~~
c   open(6,file='mdof5.out',status='unformatted')
c
c   data an /0.74265, 0.48164, 0.22417, -0.63577, 0.38566, 0.43168,
& 0.21037, -0.53475, 0.51323/
c   data ac /0.97892, -.14094, -0.077226, 0.12960, 0.091126,
& 0.69669, 0.15788, 0.79906, -0.093072/
c
c   data ant /0.74265, -0.63577, 0.21037, 0.48164, 0.38566,
& -0.53475, 0.22417, 0.43168, 0.51323/
c   data act /0.97892, 0.12960, 0.15788, -.14094, 0.091126,
& 0.79906, -0.077226, 0.69669, -0.093072/
c
c   wn1=14.522
c   wn2=31.048
c   wn3=46.099
c
c   ui=((5.0*an1(1))+(27.0*an1(2)))/32.0
c   vi=((5.0*bn1(1))+(27.0*bn1(2)))/32.0
c
c   800 us1=a1(1)
c   us2=a1(2)
c
c   wc1=20.125
c   wc2=37.703
c   wc3=81.555
c
c calculate the value of "static preload" u1s, u2s & u3s
c ~~~~~
c   as1=ac(1,1)*((-22.222*us1)+(1800.0*us2))
c   bs1=ac(2,1)*((1800.0*us1)+(7800.0*us2))
c   u1s=(as1+bs1)/(wc1**2.0)
c   as2=ac(1,2)*((-22.222*us1)+(1800.0*us2))
c   bs2=ac(2,2)*((1800.0*us1)+(7800.0*us2))
c   u2s=(as2+bs2)/(wc2**2.0)
c   as3=ac(1,3)*((-22.222*us1)+(1800.0*us2))
c   bs3=ac(2,3)*((1800.0*us1)+(7800.0*us2))
c   u3s=(as3+bs3)/(wc3**2.0)
c   write(6,*) 'u1s=', u1s, 'u2s=', u2s, 'u3s=', u3s
c
c calculate contact force, P (kips), P2 (N)
c ~~~~~
c   ud3=a1(1)-us1
c   ud2=a1(2)-us2
c   P=11377.778 *(5.0*ud3+27.0*ud2)/32.0
c   P2=P*4450.0

```

```

c           MDOF6.FOR
c   CHAPTER 3 : UNDAMPED FREE VIBRATION
c           a = L (aligned floor case)
c
c   Similar to MDOF1.FOR , except the following parameters are changed :
c ~~~~~
c   open(6,file='mdof6.out',status='unformatted')
c
c   data an /0.74265, 0.48164, 0.22417, -0.63577, 0.38566, 0.43168,
c   & 0.21037, -0.53475, 0.51323/
c   data ac /0.99983, 0.011970, 0.0079168, -0.011500, 0.016973,
c   & 0.70691, -0.014423, 0.81623, -0.014816/
c
c   data ant /0.74265, -0.63577, 0.21037, 0.48164, 0.38566,
c   & -0.53475, 0.22417, 0.43168, 0.51323/
c   data act /0.99983, -0.011500, -0.014423, 0.01197, 0.016973,
c   & 0.81623, 0.0079168, 0.70691, -0.014816/
c
c   wn1=14.522
c   wn2=31.048
c   wn3=46.099
c
c   ui=an1(2)
c   vi=bn1(2)
c
c   wc1=24.348
c   wc2=38.543
c   wc3=185.89
c
c calculate the value of "static preload" u1s, u2s & u3s
c ~~~~~
c   u1s=0.504786316
c   u2s=0.285632479
c   u3s=0.590528017
c
c calculate contact force, P (kips), P2 (N)
c ~~~~~
c   P=50000.0*(a1(2)-0.5)
c   P2=P*4450.0

```

## APPENDIX D

### FORTRAN PROGRAMS FOR CHAPTER 3

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---

```
c          DAM1.FOR
c          CHAPTER 3 : DAMPED FREE VIBRATION-SDOF SYSTEM
c          a = 0 (aligned floor case)
c
c All units are in N, m and sec
c Damping coefficient, xi = 0.0, 0.5% and 2%
c ~~~~~
c          dimension u1(800000),v1(800000)
c          open(6,file='dam1.out',status='unformatted')
c
c          tt=0.0
c          kn=80000
c          pi=3.141592654
c          u1(1)=-0.020574
c          v1(1)=0.00
c          i=1
c          goto 7
c
5          tt=tt-dt
c          kn=80000
c          i=n
c          goto 7
c
7          w=5.547
c
c          enter value of xi
c ~~~~~
c          xi=0.0
c          dt=.001
c          odt=(w*dt)
c          wd=w*sqrt(1.0-xi**2.0)
c          wodt=(wd*dt)
c          print*, '  tt      utop      vtop'
c          do 100 n=i,kn
c          write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
c          aa1=exp(-xi*odt)*(v1(n)+(xi*w*u1(n)))/wd*sin(wodt)
c          bb1=exp(-xi*odt)*u1(n)*cos(wodt)
c          cc1=exp(-xi*odt)*v1(n)*cos(wodt)
c          dd1=exp(-xi*odt)*(w**2*u1(n)+(xi*w*v1(n)))/wd*sin(wodt)
c          u(n+1)=((v(n)/wn)*sin(odt))+(u(n)*cos(odt))
c          v(n+1)=(v(n)*cos(odt))-(wn*u(n)*sin(odt))
c          u1(n+1)=aa1+bb1
c          v1(n+1)=cc1-dd1
c          ui=(u1(n))/2.0
c          vi=(v1(n))/2.0
c          write (6,400)tt,u1(n),v1(n)
c          400 format(3(1x,f9.5))
c          tt=tt+dt
c
c          *****
c          if (tt.ge.4.0) goto 710
c          *****
c          if (u1(n).ge.0.01) then
```





```

c          DAM2.FOR
c          CHAPTER 3 : DAMPED FREE VIBRATION-MDOF SYSTEMS
c          a = L/2 (unaligned floor case)
c
c All units are in kips, inch and covert to N, m at the end of programme
c matrix an & ac are normalised mode shapes for no contact & contact
c matrix ant & act are tranposed of an & ac
c matrix b is a mass matrix
c matrix cn & cc are the initial displacements & velocities
c Damping coefficient, xi = 0.0, 0.5% and 2%
c -----
  dimension u1(20000),v1(20000),u2(20000),v2(20000),u3(20000),
&v3(20000),a1(3),a2(3),a3(3),b1(3),b2(3),b3(3),b(3,3)
  dimension an1(3),bn1(3)
  dimension an(3,3),ant(3,3),cn(3,2),antb(3,3),antbcn(3,2)
  dimension ac(3,3),act(3,3),cc(3,2),actb(3,3),actbcc(3,2)
  dimension cm(3,2)
  open(6,file='dam2.out',status='unformatted')
c
  data an /0.74265, 0.48164, 0.22417, -0.63577, 0.38566, 0.43168,
& 0.21037, -0.53475, 0.51323/
  data ac /0.55099, -0.53326, -0.36733, 0.68599, -0.12760e-15,
& 0.51450, 0.47520, 0.61831, -0.31680/
c
  data ant /0.74265, -0.63577, 0.21037, 0.48164, 0.38566,
& -0.53475, 0.22417, 0.43168, 0.51323/
  data act /0.55099, 0.68599, 0.47520, -0.53326, -0.12760e-15,
& 0.61831, -0.36733, 0.51450, -0.31680/
c
  data b /1.0, 0.0, 0.0, 0.0, 1.5, 0.0, 0.0, 0.0, 2.0/
c
  tt=0.0
  kn=2010
  pi=3.141592654
  cn(1,1)=-1.0000
  cn(2,1)=-0.505106
  cn(3,1)=-0.1440976
  cn(1,2)=0.0
  cn(2,2)=0.0
  cn(3,2)=0.0
  goto 7
5  tt=tt-dt
  kn=6800
  cn(1,1)=a1(1)
  cn(2,1)=a1(2)
  cn(3,1)=a1(3)
  cn(1,2)=b1(1)
  cn(2,2)=b1(2)
  cn(3,2)=b1(3)
  goto 7
c
c form a matrix ant*b(3,3)
c -----
7  do 10 j=1,3
  do 10 i=1,3
    antb(i,j)=0.0
  do 10 k=1,3
10 antb(i,j)=antb(i,j)+ant(i,k)*b(k,j)
c
c form a matrix ant*b*cn(3,2)
c -----
  do 20 j=1,2
  do 20 i=1,3
    antbcn(i,j)=0.0
  do 20 k=1,3
20 antbcn(i,j)=antbcn(i,j)+antb(i,k)*cn(k,j)
c
c form checking matrix cm = an*antbcn
c -----
  do 25 j=1,2
  do 25 i=1,3
    cm(i,j)=0.0
  do 25 k=1,3

```

```

25 cm(i,j)=cm(i,j)+an(i,k)*antbcn(k,j)
c
c print the final matrix antbcn(3,2)
c -----
print*, 'normalised disp vel checking init. disp vel'
do 30 i=1,3
write(6,40)(antbcn(i,j),j=1,2),(cm(i,j),j=1,2)
40 format (2(5x,f9.5,3x,f9.5,5x))
30 continue
c
c enter value of xi
c -----
xi=0.0
c assumed wdi=wni*sqrt(1-xi**2.0)=wni
wn1=14.522
wn2=31.048
wn3=46.099
u1(1)=antbcn(1,1)
u2(1)=antbcn(2,1)
u3(1)=antbcn(3,1)
v1(1)=antbcn(1,2)
v2(1)=antbcn(2,2)
v3(1)=antbcn(3,2)
dt=.0001
odt1=(wn1*dt)
odt2=(wn2*dt)
odt3=(wn3*dt)
print*, ' tt utop umid ubott vtop vmid
$ vbott ui vi'
do 100 n=1,kn
c write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
aa1=exp(-xi*odt1)*(v1(n)+(xi*wn1*u1(n)))/wn1*sin(odt1)
bb1=exp(-xi*odt1)*u1(n)*cos(odt1)
cc1=exp(-xi*odt1)*v1(n)*cos(odt1)
dd1=exp(-xi*odt1)*(wn1**2*u1(n)+(xi*wn1*v1(n)))/wn1*sin(odt1)
c
aa2=exp(-xi*odt2)*(v2(n)+(xi*wn2*u2(n)))/wn2*sin(odt2)
bb2=exp(-xi*odt2)*u2(n)*cos(odt2)
cc2=exp(-xi*odt2)*v2(n)*cos(odt2)
dd2=exp(-xi*odt2)*(wn2**2*u2(n)+(xi*wn2*v2(n)))/wn2*sin(odt2)
c
aa3=exp(-xi*odt3)*(v3(n)+(xi*wn3*u3(n)))/wn3*sin(odt3)
bb3=exp(-xi*odt3)*u3(n)*cos(odt3)
cc3=exp(-xi*odt3)*v3(n)*cos(odt3)
dd3=exp(-xi*odt3)*(wn3**2*u3(n)+(xi*wn3*v3(n)))/wn3*sin(odt3)
c
u1(n+1)=aa1+bb1
v1(n+1)=cc1-dd1
u2(n+1)=aa2+bb2
v2(n+1)=cc2-dd2
u3(n+1)=aa3+bb3
v3(n+1)=cc3-dd3
c now calculate the total mode displacement & velocity
do 500 l=1,3
b1(l)=an(1,1)*v1(n)
b2(l)=an(1,2)*v2(n)
b3(l)=an(1,3)*v3(n)
a1(l)=an(1,1)*u1(n)
a2(l)=an(1,2)*u2(n)
a3(l)=an(1,3)*u3(n)
500 continue
do 600 m=1,3
a1(m)=a1(m)+a2(m)+a3(m)
600 b1(m)=b1(m)+b2(m)+b3(m)
c convert from inches to m
c -----
do 650 m=1,3
an1(m)=a1(m)*0.0254
650 bn1(m)=b1(m)*0.0254
ui=(an1(1)+an1(2))/2.0
vi=(bn1(1)+bn1(2))/2.0
write (6,400)tt,an1(1),an1(2),an1(3),bn1(1),bn1(2),bn1(3),ui,vi
c write (6,400)tt,ui
400 format(9(1x,f9.5))

```

```

c 400 format(2(1x,f9.5))
   tt=tt+dt
c *****
   if (tt ge.1.0) goto 710
c *****
   if (ui.ge.0.0126) then
     goto 800
   else
     goto 100
   endif
c *****
100 continue
c ~~~~~
800 cc(1,1)=a1(1)
   cc(2,1)=a1(2)
   cc(3,1)=a1(3)
   cc(1,2)=b1(1)
   cc(2,2)=b1(2)
   cc(3,2)=b1(3)
   us1=a1(1)
   us2=a1(2)
c form a matrix act*b(3,3)
c ~~~~~
   do 11 j=1,3
     do 11 i=1,3
       actb(i,j)=0.0
     do 11 k=1,3
       11 actb(i,j)=actb(i,j)+act(i,k)*b(k,j)
c
c form a matrix act*b*cc(3,2)
c ~~~~~
   do 21 j=1,2
     do 21 i=1,3
       actbcc(i,j)=0.0
     do 21 k=1,3
       21 actbcc(i,j)=actbcc(i,j)+actb(i,k)*cc(k,j)
c form checking matrix cm = ac*actbcc
c ~~~~~
   do 26 j=1,2
     do 26 i=1,3
       cm(i,j)=0.0
     do 26 k=1,3
       26 cm(i,j)=cm(i,j)+ac(i,k)*actbcc(k,j)
c
c print the final matrix actbcc(3,2)
c ~~~~~
   print*, 'normalised disp   vel   checking init. disp   vel'
   do 31 i=1,3
     write(6,41)(actbcc(i,j),j=1,2),(cm(i,j),j=1,2)
   41 format (2(5x,f9.5,3x,f9.5,5x))
   31 continue
c
   tt=tt-dt
   kn=7000
   pi=3.141592654
c assumed wdi=wci*sqrt(1-xi**2.0)=wci
   wc1=25.079
   wc2=38.730
   wc3=51.682
   xi1=0.0*(wn1/wc1)
   xi2=0.0*(wn2/wc2)
   xi3=0.0*(wn3/wc3)
   u1(1)=actbcc(1,1)
   u2(1)=actbcc(2,1)
   u3(1)=actbcc(3,1)
   v1(1)=actbcc(1,2)
   v2(1)=actbcc(2,2)
   v3(1)=actbcc(3,2)
   dt=.00001
   odt1=(wc1*dt)
   odt2=(wc2*dt)
   odt3=(wc3*dt)
   write(6,*) 'xi1=',xi1, 'xi2=',xi2, 'xi3=',xi3
c Calculate the value of "static preload" u1s, u2s & u3s

```

```

c -----
  as1=ac(1,1)*((900.0*us1)+(1500.0*us2))
  bs1=ac(2,1)*((1500.0*us1)+(900.0*us2))
  u1s=(as1+bs1)/(wc1**2.0)
  as2=ac(1,2)*((900.0*us1)+(1500.0*us2))
  bs2=ac(2,2)*((1500.0*us1)+(900.0*us2))
  u2s=(as2+bs2)/(wc2**2.0)
  as3=ac(1,3)*((900.0*us1)+(1500.0*us2))
  bs3=ac(2,3)*((1500.0*us1)+(900.0*us2))
  u3s=(as3+bs3)/(wc3**2.0)
c   write(6,*) 'u1s=', u1s, 'u2s=', u2s, 'u3s=', u3s
  print*, '----- contact -----'
  do 110 n=1,kn
    aal=exp(-xi1*odt1)*(v1(n)+(xi1*wc1*(u1(n)-u1s)))/wc1*sin(odt1)
    bb1=exp(-xi1*odt1)*(u1(n)-u1s)*cos(odt1)
    cc1=exp(-xi1*odt1)*v1(n)*cos(odt1)
    dd1=exp(-xi1*odt1)*wc1*((u1(n)-u1s)+(xi1*v1(n)/wc1))*sin(odt1)
c
    aa2=exp(-xi2*odt2)*(v2(n)+(xi2*wc2*(u2(n)-u2s)))/wc2*sin(odt2)
    bb2=exp(-xi2*odt2)*(u2(n)-u2s)*cos(odt2)
    cc2=exp(-xi2*odt2)*v2(n)*cos(odt2)
    dd2=exp(-xi2*odt2)*wc2*((u2(n)-u2s)+(xi2*v2(n)/wc2))*sin(odt2)
c
    aa3=exp(-xi3*odt3)*(v3(n)+(xi3*wc3*(u3(n)-u3s)))/wc3*sin(odt3)
    bb3=exp(-xi3*odt3)*(u3(n)-u3s)*cos(odt3)
    cc3=exp(-xi3*odt3)*v3(n)*cos(odt3)
    dd3=exp(-xi3*odt3)*wc3*((u3(n)-u3s)+(xi3*v3(n)/wc3))*sin(odt3)
c
    u1(n+1)=aal+bb1+u1s
    v1(n+1)=cc1-dd1
    u2(n+1)=aa2+bb2+u2s
    v2(n+1)=cc2-dd2
    u3(n+1)=aa3+bb3+u3s
    v3(n+1)=cc3-dd3
c   now calculate the total mode displacement
  do 510 l=1,3
    b1(l)=ac(1,1)*v1(n)
    b2(l)=ac(1,2)*v2(n)
    b3(l)=ac(1,3)*v3(n)
    a1(l)=ac(1,1)*u1(n)
    a2(l)=ac(1,2)*u2(n)
    a3(l)=ac(1,3)*u3(n)
  510 continue
  do 610 m=1,3
    a1(m)=a1(m)+a2(m)+a3(m)
  610 b1(m)=b1(m)+b2(m)+b3(m)
c convert from inches to m
c -----
  do 615 m=1,3
    an1(m)=a1(m)*0.0254
  615 bn1(m)=b1(m)*0.0254
    ui=(an1(1)+an1(2))/2.0
    vi=(bn1(1)+bn1(2))/2.0
c calculate contact force, P (kips), P2 (N)
c -----
    ud3=a1(1)-us1
    ud2=a1(2)-us2
    P=4800.0*(ud3+ud2)/2.0
    P2=P*4450.0
    write (6,410)tt,an1(1),an1(2),an1(3),bn1(1),bn1(2),bn1(3),ui,vi,P2
    tt=tt+dt
c *****
  if (tt.ge.1.0) goto 710
c *****
  if (ui.le.0.0126) then
    goto 5
  else
    goto 110
  endif
c *****
  110 continue
  410 format(9(1x,f9.5),1x,f15.5)
  710 stop
  end

```

# APPENDIX E

## FORTRAN PROGRAMS FOR CHAPTER 3

```
c          EQA1.FOR
c          CHAPTER 3 : UNDAMPED FORCED VIBRATION
c          a = 0 (aligned floor case)
c
c          USING THE MEXICO EARTHQUAKE, 1985, N90-W COMPONENT
c
c All units are in N,m, kg and sec
c matrix an & ac are normalised mode shapes for no contact & contact
c matrix ant & act are transposed of an & ac (N/m)
c matrix b is a mass matrix (N-sec2/m)
c matrix cn & cc are the initial displacements & velocities
c g originally is in 9.81 m/s2, ks=8.76e9 N/m
c ug in (g), vg in (cm/s) and dg in (cm)
c PF is the modal participant factor = phi trans * m * r
c r is the Pseudo static vector i.e ( 1 , 1, 1)
c -----
c          dimension u1(90000),v1(90000),u2(90000),v2(90000),u3(90000),
c          &v3(90000),a1(3),a2(3),a3(3),b1(3),b2(3),b3(3),b(3,3)
c          dimension ax1(3),bx1(3)
c          dimension an(3,3),ant(3,3),cn(3,2),antb(3,3),antbcn(3,2)
c          dimension ac(3,3),act(3,3),cc(3,2),actb(3,3),actbcc(3,2)
c          dimension cm(3,2)
c          dimension time(20000),ug(20000),vg(20000),dg(20000)
c          open(6,file='eqa1.out',status='unformatted')
c          open(10,file='eq1.out')
c
c          data an /0.0017743, 0.0011507, 0.00053557, -0.0015189, 0.00092139,
c          & 0.0010313, 0.00050260, -0.0012776, 0.0012262/
c
c          data ac /0.45572e-5, .0015188, 0.0010601, -0.36993e-5, -0.0012241,
c          & 0.0013153, 0.0023891, -0.47925e-5, 0.14442e-7/
c
c          data ant /0.0017743, -0.0015189, 0.00050260, 0.0011507, .00092139,
c          & -0.0012776, 0.00053557, 0.0010313, 0.0012262/
c
c          data act /0.45572e-5, -0.36993e-5, 0.0023891, .0015188, -.0012241,
c          & -0.47925e-5, 0.0010601, 0.0013153, 0.14442e-7/
c
c          data b /175197, 0.0, 0.0, 0.0, 262795, 0.0, 0.0, 0.0, 350394/
c
c          k=15350
c          do 1 i=1,k+1
c            read(10,*)time(i),ug(i),vg(i),dg(i)
1          continue
c
c          tt=0.0
c          kn=80000
c          pi=3.141592654
c          xi=0.0
c          g=9.81
c          cn(1,1)=0.0
c          cn(2,1)=0.0
c          cn(3,1)=0.0
c          cn(1,2)=0.0
c          cn(2,2)=0.0
c          cn(3,2)=0.0
c          n=1
c          goto 7
c
5          tt=tt-dt
c          kn=40000
c          n=n
c          cn(1,1)=a1(1)
c          cn(2,1)=a1(2)
c          cn(3,1)=a1(3)
```

```

cn(1,2)=b1(1)
cn(2,2)=b1(2)
cn(3,2)=b1(3)
goto 7
c form a matrix ant*b(3,3)
c -----
7 do 10 j=1,3
do 10 i=1,3
antb(i,j)=0.0
do 10 k=1,3
10 antb(i,j)=antb(i,j)+ant(i,k)*b(k,j)
c form a matrix ant*b*cn(3,2)
c -----
do 20 j=1,2
do 20 i=1,3
antbcn(i,j)=0.0
do 20 k=1,3
20 antbcn(i,j)=antbcn(i,j)+antb(i,k)*cn(k,j)
c form checking matrix cm = an*antbcn
c -----
do 25 j=1,2
do 25 i=1,3
cm(i,j)=0.0
do 25 k=1,3
25 cm(i,j)=cm(i,j)+an(i,k)*antbcn(k,j)
c print the final matrix antbcn(3,2)
c -----
c print*, 'normalised disp vel checking init. disp vel'
c do 30 i=1,3
c write(6,40)(antbcn(i,j),j=1,2),(cm(i,j),j=1,2)
c 40 format (2(5x,f9.5,3x,f9.5,5x))
c 30 continue
c
wn1=7.2608
wn2=15.524
wn3=23.050
wd1=wn1*SQRT(1.0-(xi**2))
wd2=wn2*SQRT(1.0-(xi**2))
wd3=wn3*SQRT(1.0-(xi**2))
PF1=ant(1,1)*b(1,1)+ant(1,2)*b(2,2)+ant(1,3)*b(3,3)
PF2=ant(2,1)*b(1,1)+ant(2,2)*b(2,2)+ant(2,3)*b(3,3)
PF3=ant(3,1)*b(1,1)+ant(3,2)*b(2,2)+ant(3,3)*b(3,3)
u1(n)=antbcn(1,1)
u2(n)=antbcn(2,1)
u3(n)=antbcn(3,1)
v1(n)=antbcn(1,2)
v2(n)=antbcn(2,2)
v3(n)=antbcn(3,2)
dt=.002
odt1=(wn1*dt)
odt2=(wn2*dt)
odt3=(wn3*dt)
c write(6,*)'odt1=',odt1, ' odt2=',odt2, ' odt3=',odt3
c write(6,*)'wd1=',wd1, ' wd2=',wd2, ' wd3=',wd3
c write(6,*)'PF1=',PF1, ' PF2=',PF2, ' PF3=',PF3
print*, ' tt u utop umid ubott vtop
$ vmid vbott gdisp'
do 100 n=n,kn
R=(ug(n+1)-ug(n))*g/dt
c write(6,*)'R=',R
c write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
c 1st mode
aa1=(exp(-xi*odt1)/wd1)*(v1(n)+xi*wn1*u1(n)+PF1*(xi*ug(n))*g/wn1
$ -R*(2*xi**2-1.0)/wn1**2)*Sin(wd1*dt)
bb1=exp(-xi*odt1)*(u1(n)+PF1*((ug(n))*g/wn1**2)
$ -(2.0*xi*R/wn1**3))*Cos(wd1*dt)
cons1=PF1*((-ug(n))*g/wn1**2)+(2.0*xi*R/wn1**3)-(R*dt/wn1**2)
cc1=exp(-xi*odt1)*((v1(n)+PF1*R/wn1**2))*cos(wd1*dt)
dd1=(exp(-xi*odt1)/wd1)*(-wn1**2*u1(n)-xi*wn1*v1(n)-PF1*ug(n))*g+
$ PF1*xi*R/wn1)*Sin(wd1*dt)
cons11=-PF1*R/wn1**2
c 2nd mode
aa2=(exp(-xi*odt2)/wd2)*(v2(n)+xi*wn2*u2(n)+PF2*(xi*ug(n))*g/wn2
$ -R*(2*xi**2-1.0)/wn2**2)*Sin(wd2*dt)

```





```

do 21 k=1,3
21 actbcc(i,j)=actbcc(i,j)+actb(i,k)*cc(k,j)
c
c form checking matrix cm = ac*actbcc
c -----
do 26 j=1,2
do 26 i=1,3
cm(i,j)=0.0
do 26 k=1,3
26 cm(i,j)=cm(i,j)+ac(i,k)*actbcc(k,j)
c
c print the final matrix actbcc(3,2)
c -----
c print*, 'normalised disp vel checking init. disp vel'
c do 31 i=1,3
c write(6,41)(actbcc(i,j),j=1,2),(cm(i,j),j=1,2)
c 41 format (2(5x,f9.5,3x,f9.5,5x))
c 31 continue
c
tt=tt-dt
kn=40000
pi=3.141592654
xi=0.0
g=9.81
wc1=12.653
wc2=22.685
wc3=223.94
wd1=wc1*SQRT(1.0-(xi**2))
wd2=wc2*SQRT(1.0-(xi**2))
wd3=wc3*SQRT(1.0-(xi**2))
PF1=act(1,1)*b(1,1)+act(1,2)*b(2,2)+act(1,3)*b(3,3)
PF2=act(2,1)*b(1,1)+act(2,2)*b(2,2)+act(2,3)*b(3,3)
PF3=act(3,1)*b(1,1)+act(3,2)*b(2,2)+act(3,3)*b(3,3)
u1(n)=actbcc(1,1)
u2(n)=actbcc(2,1)
u3(n)=actbcc(3,1)
v1(n)=actbcc(1,2)
v2(n)=actbcc(2,2)
v3(n)=actbcc(3,2)
dt=.002
odt1=(wc1*dt)
odt2=(wc2*dt)
odt3=(wc3*dt)
c
c calculate the value of "static preload" u1s, u2s & u3s
c -----
u1s=ac(1,1)*8.76e9*0.02/wc1**2.0
u2s=ac(1,2)*8.76e9*0.02/wc2**2.0
u3s=ac(1,3)*8.76e9*0.02/wc3**2.0
c write(6,*)'u1s=',u1s, ' u2s=',u2s, ' u3s=',u3s
c write(6,*)'odt1=',odt1, ' odt2=',odt2, ' odt3=',odt3
c write(6,*)'wd1=',wd1, ' wd2=',wd2, ' wd3=',wd3
c write(6,*)'PF1=',PF1, ' PF2=',PF2, ' PF3=',PF3
print*, '----- contact -----'
do 110 n=n,kn
R=(ug(n+1)-ug(n))*g/dt
c write(6,*)'R=',R
c write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
c 1st mode
aa1=(exp(-xi*odt1)/wd1)*(v1(n)+xi*wc1*u1(n)+PF1*(xi*ug(n))*g/wc1
$ -R*(2*xi**2-1.0)/wc1**2)-(xi*wc1*u1s))*Sin(wd1*dt)
bb1=exp(-xi*odt1)*(u1(n)+PF1*((ug(n))*g/wc1**2)
$ -(2.0*xi*R/wc1**3))-u1s)*Cos(wd1*dt)
cons1=PF1*((-ug(n))*g/wc1**2)+(2.0*xi*R/wc1**3)-(R*dt/wc1**2))
cc1=exp(-xi*odt1)*((v1(n)+PF1*R/wc1**2))*cos(wd1*dt)
dd1=(exp(-xi*odt1)/wd1)*(-wc1**2*u1(n)-xi*wc1*v1(n)-PF1*ug(n))*g+
$ PF1*xi*R/wc1+wc1**2*u1s)*Sin(wd1*dt)
cons11=-PF1*R/wc1**2
c 2nd mode
aa2=(exp(-xi*odt2)/wd2)*(v2(n)+xi*wc2*u2(n)+PF2*(xi*ug(n))*g/wc2
$ -R*(2*xi**2-1.0)/wc2**2)-(xi*wc2*u2s))*Sin(wd2*dt)
bb2=exp(-xi*odt2)*(u2(n)+PF2*((ug(n))*g/wc2**2)
$ -(2.0*xi*R/wc2**3))-u2s)*Cos(wd2*dt)
cons2=PF2*((-ug(n))*g/wc2**2)+(2.0*xi*R/wc2**3)-(R*dt/wc2**2))

```

```

cc2=exp(-xi*odt2)*((v2(n)+PF2*R/wc2**2))*cos(wd2*dt)
dd2=(exp(-xi*odt2)/wd2)*(-wc2**2*u2(n)-xi*wc2*v2(n)-PF2*ug(n)*g+
$ PF2*xi*R/wc2+wc2**2*u2s)*Sin(wd2*dt)
cons22=-PF2*R/wc2**2
c 3rd mode
aa3=(exp(-xi*odt3)/wd3)*(v3(n)+xi*wc3*u3(n)+PF3*(xi*ug(n)*g/wc3
$ -R*(2*xi**2-1.0)/wc3**2)-(xi*wc3*u3s))*Sin(wd3*dt)
bb3= exp(-xi*odt3)*(u3(n)+PF3*((ug(n)*g/wc3**2)
$ -(2.0*xi*R/wc3**3))-u3s)*Cos(wd3*dt)
cons3=PF3*((-ug(n)*g/wc3**2)+(2.0*xi*R/wc3**3)-(R*dt/wc3**2))
cc3=exp(-xi*odt3)*((v3(n)+PF3*R/wc3**2))*cos(wd3*dt)
dd3=(exp(-xi*odt3)/wd3)*(-wc3**2*u3(n)-xi*wc3*v3(n)-PF3*ug(n)*g+
$ PF3*xi*R/wc3+wc3**2*u3s)*Sin(wd3*dt)
cons33=-PF3*R/wc3**2
c
u1(n+1)=aa1+bb1+cons1+u1s
v1(n+1)=cc1+dd1+cons11
u2(n+1)=aa2+bb2+cons2+u2s
v2(n+1)=cc2+dd2+cons22
u3(n+1)=aa3+bb3+cons3+u3s
v3(n+1)=cc3+dd3+cons33
c write(6,*)n,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
c now calculate the total mode displacement
do 510 l=1,3
b1(l)=ac(1,1)*v1(n)
b2(l)=ac(1,2)*v2(n)
b3(l)=ac(1,3)*v3(n)
a1(l)=ac(1,1)*u1(n)
a2(l)=ac(1,2)*u2(n)
a3(l)=ac(1,3)*u3(n)
510 continue
do 610 m=1,3
a1(m)=a1(m)+a2(m)+a3(m)
610 b1(m)=b1(m)+b2(m)+b3(m)
do 705 j=1,3
ax1(j)=a1(j)+(dg(n)*0.01)
bx1(j)=b1(j)+(vg(n)*0.01)
705 continue
gdisp=dg(n)*0.01
c
c calculate contact force, P (N)
c -----
P=8.76e9*(a1(1)-0.02)
c write (6,410)tt,a1(1),ax1(1),ax1(2),ax1(3),bx1(1),bx1(2),bx1(3),
c &gdisp
write (6,410)tt,a1(1),ax1(1),ax1(2),ax1(3),bx1(1),bx1(2),bx1(3),
&gdisp,P
tt=tt+dt
c
c *****
if (tt.ge.30.0) goto 710
c
c *****
if (a1(1).le.0.02) then
goto 5
else
goto 110
endif
c
c *****
110 continue
c 410 format(9(1x,f9.5))
410 format(9(1x,f9.5),1x,f15.3)
710 stop
end

```

```

c          EQA2.FOR
c      CHAPTER 3 : UNDAMPED FORCED VIBRATION
c          a = L/2 (unaligned floor case)
c
c      USING THE MEXICO EARTHQUAKE, 1985, N90-W COMPONENT
c
c      Similar to EQA1.FOR , except the following parameters are changed :
c      ~~~~~
c      open(6,file='eqa2.out',status='unformatted')
c      open(10,file='eq1.out')
c
c      data an /0.0017743, 0.0011507, 0.00053557, -0.0015189, 0.00092139,
c      & 0.0010313, 0.00050260, -0.0012776, 0.0012262/
c
c      data ac /.13164e-2, -.12740e-2, -.87759e-3, .16389e-2,-.53662e-20,
c      & 0.12292e-2, 0.11353e-2, 0.14772e-2, -.75687e-3/
c
c      data ant /0.0017743, -0.0015189, 0.00050260, 0.0011507, .00092139,
c      & -0.0012776, 0.00053557, 0.0010313, 0.0012262/
c
c      data act /0.13164e-2, .16389e-2, .11353e-2,-.1274e-2, -.53662e-20,
c      & 0.14772e-2, -.87759e-3, .12292e-2, -0.75687e-3/
c
c      wn1=7.2608
c      wn2=15.524
c      wn3=23.050
c
c      ui=(ax1(1)+ax1(2))/2.0
c      vi=(bx1(1)+bx1(2))/2.0
c
c      800 us1=a1(1)
c      us2=a1(2)
c
c      wc1=12.540
c      wc2=19.365
c      wc3=25.841
c
c      Calculate the value of "static preload" u1s, u2s & u3s
c      ~~~~~
c      as1=ac(1,1)*((39419291.34*us1)+(65698818.9*us2))
c      bs1=ac(2,1)*((65698818.9*us1)+(39419291.34*us2))
c      u1s=(as1+bs1)/(wc1**2.0)
c      as2=ac(1,2)*((39419291.34*us1)+(65698818.9*us2))
c      bs2=ac(2,2)*((65698818.9*us1)+(39419291.34*us2))
c      u2s=(as2+bs2)/(wc2**2.0)
c      as3=ac(1,3)*((39419291.34*us1)+(65698818.9*us2))
c      bs3=ac(2,3)*((65698818.9*us1)+(39419291.34*us2))
c      u3s=(as3+bs3)/(wc3**2.0)
c
c
c      calculate the contact forces P (N)
c      ~~~~~
c      ud3=a1(1)-us1
c      ud2=a1(2)-us2
c      P=(1200.0*4450.0/0.0254)*(ud3+ud2)/2.0

```

# APPENDIX F

## FORTRAN PROGRAMS FOR CHAPTER 4

```
c          SFLEX1.FOR
c          CHAPTER 4 : UNDAMPED FREE VIBRATION-SDOF SYSTEM
c          a = 0 (aligned floor case)
c
c All units are in kips, inch and convert to N,m at the end of programme
c Building separation, Us = 0.008 m (0.314960629")
c Slab flexibility, ks = 10,000 kips/in (1.752e9 N/m)
c flexibility constant = (ks/k1+ks)
c -----
c          dimension u1(800000),v1(800000),u2(800000),uk1(800000)
c          open(6,file='sfex1.out',status='unformatted')
c
c          tt=0.0
c          kn=800000
c          pi=3.141592654
c          cons=(10000.0/(600.0+10000.0))
c          write(6,*) 'cons=', cons
c          u2(1)=cons*(-0.81)
c          u1(1)=-0.81
c          v1(1)=0.00
c          i=1
c          goto 7
c
5          tt=tt-dt
c          kn=800000
c          i=n
c          cons=(10000.0/(600.0+10000.0))
c          goto 7
c
c          7 w=5.4679446
c          dt=.001
c          odt=(w*dt)
c          print*, '  tt      utop    vtop    u2     ui'
c          do 100 n=i,kn
c          write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
c          aa1=(v1(n)/w)*sin(odt)
c          bb1=u1(n)*cos(odt)
c          cc1=v1(n)*cos(odt)
c          dd1=w*u1(n)*sin(odt)
c          u(n+1)=((v(n)/wn)*sin(odt))+u(n)*cos(odt)
c          v(n+1)=(v(n)*cos(odt))-(wn*u(n)*sin(odt))
c          u1(n+1)=aa1+bb1
c          v1(n+1)=cc1-dd1
c          u2(n+1)=cons*u1(n+1)
c
c          convert to m
c          -----
c          a1=u1(n)*0.0254
c          b1=v1(n)*0.0254
c          a2=u2(n)*0.0254
c          ui=a2
c          write (6,400)tt,a1,b1,ui
c          write (6,400)tt,ui
c          400 format(4(1x,f9.5))
c          400 format (2(1x,f9.5))
c          tt=tt+dt
c          *****
c          if (tt.ge.1.0) goto 710
c          *****
c          if (ui.ge.0.008) then
c          goto 800
c          else
c          goto 100
c          endif
```



```

c          SFLEX2.FOR
c    CHAPTER 4 : UNDAMPED FREE VIBRATION-SDOF SYSTEM
c          a = L/4 (unaligned floor case)
c
c    Similar to SFLEX1.FOR , except the following parameters are changed :
c ~~~~~
c    open(6,file='sflex2.out',status='unformatted')
c
c    7 w=5.4679446
c
c    ui=a2*27.0/32.0
c
c    800 uk1(1)=0.604473557*(u2(n)-us1)
c    x1kab=27.0/32.0
c
c    xkp=(72.0**3/4096.0)*27.0
c    xkp=x1kab/(xkp/(3.0*18662400.0)+(x1kab**2/10600.0))
c    uki=x1kab*xkp/(10600.0)
c
c Calculate the value of "static preload" u1s
c ~~~~~
c    u1s=(xk3*us1)/(39.0*wc**2)
c
c calculate contact force, P2 (N)
c ~~~~~
c    P2=xkp*(u2(n+1)-us1)*4450.0

```

```

c          SFLEX3.FOR
c    CHAPTER 4 : UNDAMPED FREE VIBRATION-SDOF SYSTEM
c          a = L/3 (unaligned floor case)
c
c    Similar to SFLEX1.FOR , except the following parameters are changed :
c ~~~~~
c    open(6,file='sflex3.out',status='unformatted')
c
c    7 w=5.4679446
c
c    ui=a2*20.0/27.0
c
c    800 uk1(1)=0.414360525*(u2(n)-us1)
c    x1kab=20.0/27.0
c
c    xkp=(72.0**3/729.0)*8.0
c    xkp=x1kab/(xkp/(3.0*18662400.0)+(x1kab**2/10600.0))
c    uki=x1kab*xkp/(10600.0)
c
c Calculate the value of "static preload" u1s
c ~~~~~
c    u1s=(xk3*us1)/(39.0*wc**2)
c
c calculate contact force, P2 (N)
c ~~~~~
c    P2=xkp*(u2(n+1)-us1)*4450.0

```

```

c           .           SFLEX4.FOR
c           CHAPTER 4 : UNDAMPED FREE VIBRATION-SDOF SYSTEM
c           a = L/2 (unaligned floor case)
c
c           Similar to SFLEX1.FOR , except the following parameters are changed :
c ~~~~~
c           open(6,file='sflex4.out',status='unformatted')
c
c           7 w=5.4679446
c
c           ui=a2/2.0
c           vi=b2/2.0
c
c           800 uk1(1)=0.184612831*(u2(n)-us1)
c           x1kab=1.0/2.0
c
c           xkp=x1kab/((72.0**3/64.0)/(3.0*18662400.0)+(x1kab**2/10600.17978))
c           uki=x1kab*xkp/(10600.17978)
c
c           Calculate the value of "static preload" u1s
c ~~~~~
c           u1s=(xk3*us1)/(39.0*wc**2)
c
c           calculate contact force, P2 (N)
c ~~~~~
c           P2=xkp*(u2(n+1)-us1)*4450.0

```

# APPENDIX G

## FORTRAN PROGRAMS FOR CHAPTER 4

---

---

```
c           MFLEX1.FOR
c           CHAPTER 4 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c           a = 0 (aligned floor case)
c
c All units are in kips,inch and convert to N,m at the end of programme
c Building separation, Us = 0.008 m (0.314960629")
c Slab flexibility, ks = 5,000 kips/in (8.76e8 N/m)
c flexibility constant = (ks/k1+ks)
c matrix an & ac are normalised mode shapes for no contact & contact
c matrix ant & act are tranposed of an & ac
c matrix b is a mass matrix
c matrix cn & cc are the initial displacements & velocities
c -----
c           dimension u1(800000),v1(800000),u2(800000),v2(800000)
c           dimension a1(2),a2(2),b1(2),b2(2),b(2,2)
c           dimension an(2,2),ant(2,2),cn(2,2),antb(2,2),antbcn(2,2)
c           dimension ac(2,2),act(2,2),cc(2,2),actb(2,2),actbcc(2,2)
c           dimension cm(2,2)
c           open(6,file='mflex1.out',status='unformatted')
c
c           data an /0.82642, 0.45974, -0.56306, 0.67477/
c           data ac /0.036678, 0.81595, 0.99933, -0.029947/
c
c           data ant /0.82642, -0.56306, 0.45974, 0.67477/
c           data act /0.036678, 0.99933, 0.81595, -0.029947/
c
c           data b /1.0, 0.0, 0.0, 1.5/
c
c           tt=0.0
c           kn=80000
c           pi=3.141592654
c           cons=(5000.0/36040000.0)
c           write (6,*) 'cons=', cons
c           cn(1,1)=-1.0000
c           cn(2,1)=-0.5000
c           cn(1,2)=0.0
c           cn(2,2)=0.0
c           goto 7
c
5           tt=tt-dt
c           kn=80000
c           cons=(5000.0/36040000.0)
c           cn(1,1)=a1(1)
c           cn(2,1)=a1(2)
c           cn(1,2)=b1(1)
c           cn(2,2)=b1(2)
c           goto 7
c form a matrix ant*b(2,2)
c -----
7           do 10 j=1,2
c           do 10 i=1,2
c           antb(i,j)=0.0
c           do 10 k=1,2
10          antb(i,j)=antb(i,j)+ant(i,k)*b(k,j)
c
c form a matrix ant*b*cn(2,2)
c -----
c           do 20 j=1,2
c           do 20 i=1,2
c           antbcn(i,j)=0.0
c           do 20 k=1,2
20          antbcn(i,j)=antbcn(i,j)+antb(i,k)*cn(k,j)
c
c form checking matrix cm = an*antbcn
```



```

c -----
  do 25 j=1,2
    do 25 i=1,2
      cm(i,j)=0.0
    do 25 k=1,2
      25 cm(i,j)=cm(i,j)+an(i,k)*antbcn(k,j)
c print the final matrix antbcn(2,2)
c -----
c print*, 'normalised disp vel checking init. disp vel'
c do 30 i=1,2
c write(6,40)(antbcn(i,j),j=1,2),(cm(i,j),j=1,2)
c 40 format (2(5x,f9.5,3x,f9.5,5x))
c 30 continue
c
  wn1=22.930
  wn2=48.051
  u1(1)=antbcn(1,1)
  u2(1)=antbcn(2,1)
  v1(1)=antbcn(1,2)
  v2(1)=antbcn(2,2)
  dt=.0005
  odt1=(wn1*dt)
  odt2=(wn2*dt)
  print*, '-----'
  print*, ' tt utop umid vtop vmid uf2'
  $ uf1 ui
  do 100 n=1,kn
c write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
  aa1=(v1(n)/wn1)*sin(odt1)
  bb1=u1(n)*cos(odt1)
  cc1=v1(n)*cos(odt1)
  dd1=wn1*u1(n)*sin(odt1)
  aa2=(v2(n)/wn2)*sin(odt2)
  bb2=u2(n)*cos(odt2)
  cc2=v2(n)*cos(odt2)
  dd2=wn2*u2(n)*sin(odt2)
c
  u1(n+1)=aa1+bb1
  v1(n+1)=cc1-dd1
  u2(n+1)=aa2+bb2
  v2(n+1)=cc2-dd2
c now calculate the total mode displacement & velocity
  do 500 l=1,2
  b1(l)=an(l,1)*v1(n)
  b2(l)=an(l,2)*v2(n)
  a1(l)=an(l,1)*u1(n)
  a2(l)=an(l,2)*u2(n)
  500 continue
  do 600 m=1,2
  a1(m)=a1(m)+a2(m)
  600 b1(m)=b1(m)+b2(m)
  uf2=cons*(600.0*a1(2)+6500.0*a1(1))
  uf1=cons*(600.0*a1(1)+5600.0*a1(2))
c convert to m
c -----
  ax1=a1(1)*0.0254
  bx1=b1(1)*0.0254
  ax2=a1(2)*0.0254
  bx2=b1(2)*0.0254
  af2=uf2*0.0254
  af1=uf1*0.0254
  ui=af2
c vi=b1(1)
  write (6,40)tt,ax1,ax2,bx1,bx2,af2,af1,ui
c write (6,40) tt,ui
  400 format(8(1x,f9.5))
c 400 format (2(1x,f9.5))
  tt=tt+dt
c *****
  if (tt.ge.0.5) goto 710
c *****
  if (ui.ge.0.008) then
  goto 800
  else

```



```

cc1=v1(n)*cos(odt1)
dd1=wc1*(u1(n)-u1s)*sin(odt1)
aa2=(v2(n)/wc2)*sin(odt2)
bb2=(u2(n)-u2s)*cos(odt2)
cc2=v2(n)*cos(odt2)
dd2=wc2*(u2(n)-u2s)*sin(odt2)
c
u1(n+1)=aa1+bb1+u1s
v1(n+1)=cc1-dd1
u2(n+1)=aa2+bb2+u2s
v2(n+1)=cc2-dd2
c  now calculate the total mode displacement
do 510 l=1,2
b1(l)=ac(l,1)*v1(n)
b2(l)=ac(l,2)*v2(n)
a1(l)=ac(l,1)*u1(n)
a2(l)=ac(l,2)*u2(n)
510 continue
do 610 m=1,2
a1(m)=a1(m)+a2(m)
610 b1(m)=b1(m)+b2(m)
uf2=cons*(6500.0*a1(1)+600.0*a1(2))
uf1=cons*(600.0*a1(1)+5600.0*a1(2))
c
c convert u1, v1, u2, v2, uf2, uf1 to m, m/s
c -----
ax1=a1(1)*0.0254
bx1=b1(1)*0.0254
ax2=a1(2)*0.0254
bx2=b1(2)*0.0254
af2=uf2*0.0254
af1=uf1*0.0254
c
ui=af2
c
ud2=uf2-us1
ud1=uf1-us2
uk2=1.0*ud2
uk1=0.092307692*ud2
xu2=uf2-uk2
xu1=uf1-uk1
c
c convert xu2, xu1 to m
c -----
c1=xu1*0.0254
c2=xu2*0.0254
c
c calculate contact force, P2 (N)
c -----
P=5544.615385*ud2
P2=P*4450.0
c write(6,410)tt,ax1,ax2,bx1,bx2,af2,af1,ui,c2,c1,P2
write(6,410)tt,ax1,ax2,bx1,bx2,c2,c1,ui
tt=tt+dt
c *****
c if (tt.ge.0.5) goto 710
c
c *****
c if (ui.le.0.008) then
c goto 5
c else
c goto 110
c endif
c
c *****
c 110 continue
c 410 format(10(1x,f9.5),1x,f13.5)
410 format(8(1x,f9.5))
710 stop
end

```

c MFLEX2.FOR  
c CHAPTER 4 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS  
c a = L/4 (unaligned floor case)

c Similar to MFLEX1.FOR , except the following parameters are changed :

```

c -----
open(6,file='mflex2.out',status='unformatted')
c
data an /0.82642, 0.45974, -0.56306, 0.67477/
data ac /-0.064595, 0.81479, 0.99791, 0.052742/
c
data ant /0.82642, -0.56306, 0.45974, 0.67477/
data act /-0.064595, 0.99791, 0.81479, 0.052742/
c
wn1=22.930
wn2=48.051
c
ui=(af2*27.0+af1*5.0)/32.0
vi=(b1(1)*27.0+b1(2)*5.0)/32.0
c
800 uk2=(0.723398826*ud2+0.133962745*ud1)
uk1=(0.179127328*ud2+0.033171727*ud1)
c
wc1=33.525
wc2=62.683
c
c Calculate the value of "static preload" u1s, u2s
c -----
as1=ac(1,1)*((3016.994131*us1)+(1269.813728*us2))
bs1=ac(2,1)*((1495.636642*us1)+(-1334.141363*us2))
u1s=(as1+bs1)/(wc1**2.0)
as2=ac(1,2)*((3016.994131*us1)+(1269.813728*us2))
bs2=ac(2,2)*((1495.636642*us1)+(-1334.141363*us2))
u2s=(as2+bs2)/(wc2**2.0)
c
c calculate contact force, P2 (N)
c -----
P=5539.372289*(ud2*27.0+ud1*5.0)/32.0
P2=P*4450.0

```

c MFLEX3.FOR  
c CHAPTER 4 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS  
c a = L/3 (unaligned floor case)

c Similar to MFLEX1.FOR , except the following parameters are changed :

```

c -----
open(6,file='mflex3.out',status='unformatted')
c
data an /0.82642, 0.45974, -0.56306, 0.67477/
data ac /-0.15126, 0.80710, 0.98849, 0.12351/
c
data ant /0.82642, -0.56306, 0.45974, 0.67477/
data act /-0.15126, 0.98849, 0.80710, 0.12351/
c
wn1=22.930
wn2=48.051
c
ui=(af2*20.0+af1*7.0)/27.0
vi=(b1(1)*20.0+b1(2)*7.0)/27.0
c
800 uk2=(0.540634903*ud2+0.189222216*ud1)
uk1=(0.206263092*ud2+0.072192082*ud1)
c
wc1=34.572
wc2=56.226
c Calculate the value of "static preload" u1s, u2s

```

```

c -----
as1=ac(1,1)*((2103.174518*us1)+(1546.111081*us2))
bs1=ac(2,1)*((1631.315464*us1)+(-1139.039588*us2))
u1s=(as1+bs1)/(wc1**2.0)
as2=ac(1,2)*((2103.174518*us1)+(1546.111081*us2))
bs2=ac(2,2)*((1631.315464*us1)+(-1139.039588*us2))
u2s=(as2+bs2)/(wc2**2.0)

```

```

c
c calculate contact force, P2 (N)

```

```

c -----
P=5292.171134*(ud2*20.0+ud1*7.0)/27.0
P2=P*4450.0

```

```

c
c MFLEX4.FOR
c CHAPTER 4 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c a = L/2 (unaligned floor case)

```

```

c Similar to MFLEX1.FOR , except the following parameters are changed :

```

```

c -----
open(6,file='mflex4.out',status='unformatted')

```

```

c data an /0.82642, 0.45974, -0.56306, 0.67477/
c data ac /-0.60980, 0.64712, 0.79255, 0.49790/

```

```

c data ant /0.82642, -0.56306, 0.45974, 0.67477/
c data act /-0.60980, 0.79255, 0.64712, 0.49790/

```

```

c wn1=22.930
c wn2=48.051

```

```

c ui=(af2+af1)/2.0

```

```

c 800 uk2=0.250735726*(ud2+ud1)
c uk1=0.218952324*(ud2+ud1)

```

```

c wc1=36.432
c wc2=46.210

```

```

c Calculate the value of "static preload" u1s, u2s

```

```

c -----
as1=ac(1,1)*((653.678634*us1)+(1853.678634*us2))
bs1=ac(2,1)*((1694.761624*us1)+(-405.238376*us2))
u1s=(as1+bs1)/(wc1**2.0)
as2=ac(1,2)*((653.678634*us1)+(1853.678634*us2))
bs2=ac(2,2)*((1694.761624*us1)+(-405.238376*us2))
u2s=(as2+bs2)/(wc2**2.0)

```

```

c calculate contact force, P2 (N)

```

```

c -----
P=5090.994703*0.5*(ud2+ud1)
P2=P*4450.0

```

```

c
c MFLEX5.FOR
c CHAPTER 4 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c a = 3L/4 (unaligned floor case)

```

```

c Similar to MFLEX1.FOR , except the following parameters are changed :

```

```

c -----
open(6,file='mflex5.out',status='unformatted')

```

```

c data an /0.82642, 0.45974, -0.56306, 0.67477/
c data ac /0.99409, -0.088627, 0.10855, 0.81167/

```

```

c data ant /0.82642, -0.56306, 0.45974, 0.67477/
c data act /0.99409, 0.10855, -0.088627, 0.81167/

```

```

c
  wn1=22.930
  wn2=48.051
c
  ui=(af2*5.0+af1*27.0)/32.0
  vi=(b1(1)*5.0+b1(2)*27.0)/32.0
c
800 uk2=(0.040390133*ud2+0.218106722*ud1)
   uk1=(0.127888267*ud2+0.690596646*ud1)
c
  wc1=29.144
  wc2=53.362
c
c Calculate the value of "static preload" u1s, u2s
c -----
  as1=ac(1,1)*((-398.0493307*us1)+(1690.533614*us2))
  bs1=ac(2,1)*((1239.4413389*us1)+(1952.98323*us2))
  u1s=(as1+bs1)/(wc1**2.0)
  as2=ac(1,2)*((-398.0493307*us1)+(1690.533614*us2))
  bs2=ac(2,2)*((1239.4413389*us1)+(1952.98323*us2))
  u2s=(as2+bs2)/(wc2**2.0)
c
c calculate contact force, P2 (N)
c -----
  P=6121.545275*(ud2*5.0+ud1*27.0)/32.0
  P2=P*4450.0

```

```

c
c MFLEX6.FOR
c CHAPTER 4 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c a = L(aligned floor case)

```

```

c Similar to MFLEX1.FOR , except the following parameters are changed :

```

```

c -----
  open(6,file='mflex6.out',status='unformatted')
c
  data an /0.82642, 0.45974, -0.56306, 0.67477/
  data ac /0.99871, 0.041444, -0.050758, 0.81544/
c
  data ant /0.82642, -0.56306, 0.45974, 0.67477/
  data act /0.99871, -0.050758, 0.041444, 0.81544/
c
  wn1=22.930
  wn2=48.051
c
  ui=af1
  vi=b1(2)
c
800 uk2=0.107142857*ud1
   uk1=1.0*ud1
c
  wc1=25.238
  wc2=59.978
c
c Calculate the value of "static preload" u1s, u2s
c -----
  as1=ac(1,1)*((-600.0*us1)+(1135.714286*us2))
  bs1=ac(2,1)*((600.0*us1)+(3500.0*us2))
  u1s=(as1+bs1)/(wc1**2.0)
  as2=ac(1,2)*((-600.0*us1)+(1135.714286*us2))
  bs2=ac(2,2)*((600.0*us1)+(3500.0*us2))
  u2s=(as2+bs2)/(wc2**2.0)
c
c calculate contact force, P2 (N)
c -----
  P=6435.71428*ud1
  P2=P*4450.0

```

# APPENDIX H

## FORTRAN PROGRAMS FOR CHAPTER 5

---

```
c          TFLEX1.FOR
c          CHAPTER 5 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c          a = 0 (aligned floor case)
c
c All units are in kips,inch and convert to N,m at the end of programme
c Building separation, Us = 0.008 m (0.314960629")
c Slab flexibility, ks = 50,000 kips/in (8.76e9 N/m)
c matrix an & ac are normalised mode shapes for no contact & contact
c matrix ant & act are tranposed of an & ac
c matrix b is a mass matrix
c matrix cn & cc are the initial displacements & velocities
c -----
c          dimension u1(800000),v1(800000),u2(800000),v2(800000)
c          dimension u3(800000),v3(800000),u4(800000),v4(800000)
c          dimension a1(4),a2(4),a3(4),a4(4),b1(4),b2(4),b3(4),b4(4),b(4,4)
c          dimension an(4,4),ant(4,4),cn(4,2),antb(4,4),antbcn(4,2)
c          dimension ac(4,4),act(4,4),cc(4,2),actb(4,4),actbcc(4,2)
c          dimension an1(4), bn1(4)
c          dimension cm(4,2)
c          open(6,file='tflex1.out',status='unformatted')
c
c          data an /0.83005, 0.45535, 0.0, 0.0, -0.55769, 0.67773, 0.0, 0.0,
c          &0.0, 0.0, 0.69544, 0.37050, 0.0, 0.0, -0.42782, 0.60227/
c
c          data ac /-0.16843, -.38175, -0.14709,0.60024, 0.77747, -0.015272,
c          &-.51322, 0.0074692, 0.52985, 0.27194, 0.53287, 0.30199, -.29401,
c          &0.66839, -0.31254, 0.22013/
c
c          data ant /0.83005, -0.55769, 0.0, 0.0, 0.45535, 0.67773, 0.0, 0.0,
c          &0.0, 0.0, 0.69544, -0.42782, 0.0, 0.0, 0.37050, 0.60227/
c
c          data act /-0.16843, 0.77747, 0.52985, -.29401, -.38175, -0.015272,
c          &0.27194, 0.66839, -0.14709, -0.51322, 0.53287, -0.31254, 0.60024,
c          &0.0074692, 0.30199, 0.22013/
c
c          data b/1.0, 0.0, 0.0, 0.0, 0.0, 1.5, 0.0, 0.0, 0.0, 0.0, 1.5, 0.0,
c          &0.0, 0.0, 0.0, 2.0/
c
c          tt=0.0
c          kn=80000
c          pi=3.141592654
c          cn(1,1)=-1.0000
c          cn(2,1)=-0.5
c          cn(3,1)=0.0
c          cn(4,1)=0.0
c          cn(1,2)=0.0
c          cn(2,2)=0.0
c          cn(3,2)=0.0
c          cn(4,2)=0.0
c          goto 7
5          tt=tt-dt
c          kn=80000
c          cn(1,1)=a1(1)
c          cn(2,1)=a1(2)
c          cn(3,1)=a1(3)
c          cn(4,1)=a1(4)
c          cn(1,2)=b1(1)
c          cn(2,2)=b1(2)
c          cn(3,2)=b1(3)
c          cn(4,2)=b1(4)
c          goto 7
c
c form a matrix ant*b(4,4)
```

```

c -----
7  do 10 j=1,4
   do 10 i=1,4
     antb(i,j)=0.0
     do 10 k=1,4
10  antb(i,j)=antb(i,j)+ant(i,k)*b(k,j)
c
c form a matrix ant*b*cn(4,2)
c -----
   do 20 j=1,2
   do 20 i=1,4
     antbcn(i,j)=0.0
     do 20 k=1,4
20  antbcn(i,j)=antbcn(i,j)+antb(i,k)*cn(k,j)
c
c form checking matrix cm = an*antbcn
c -----
   do 25 j=1,2
   do 25 i=1,4
     cm(i,j)=0.0
     do 25 k=1,4
25  cm(i,j)=cm(i,j)+an(i,k)*antbcn(k,j)
c
c print the final matrix antbcn(4,2)
c -----
c  print*, 'normalised disp  vel  checking init. disp  vel'
c  do 30 i=1,4
c  write(6,40)(antbcn(i,j),j=1,2),(cm(i,j),j=1,2)
c 40  format (2(5x,f9.5,3x,f9.5,5x))
c 30  continue
c
   wn1=23.274
   wn2=51.559
   wn3=19.334
   wn4=43.889
   u1(1)=antbcn(1,1)
   u2(1)=antbcn(2,1)
   u3(1)=antbcn(3,1)
   u4(1)=antbcn(4,1)
   v1(1)=antbcn(1,2)
   v2(1)=antbcn(2,2)
   v3(1)=antbcn(3,2)
   v4(1)=antbcn(4,2)
   dt=.0001
   odt1=(wn1*dt)
   odt2=(wn2*dt)
   odt3=(wn3*dt)
   odt4=(wn4*dt)
   print*, '  tt  u2  u1  u4  u3  v2
$vl  v4  v3  c1  c2  ui  P(N)'
   do 100 n=1,kn
c  write tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
   aa1=(v1(n)/wn1)*sin(odt1)
   bb1=u1(n)*cos(odt1)
   cc1=v1(n)*cos(odt1)
   dd1=wn1*u1(n)*sin(odt1)
c
   aa2=(v2(n)/wn2)*sin(odt2)
   bb2=u2(n)*cos(odt2)
   cc2=v2(n)*cos(odt2)
   dd2=wn2*u2(n)*sin(odt2)
c
   aa3=(v3(n)/wn3)*sin(odt3)
   bb3=u3(n)*cos(odt3)
   cc3=v3(n)*cos(odt3)
   dd3=wn3*u3(n)*sin(odt3)
c
   aa4=(v4(n)/wn4)*sin(odt4)
   bb4=u4(n)*cos(odt4)
   cc4=v4(n)*cos(odt4)
   dd4=wn4*u4(n)*sin(odt4)
c
   u1(n+1)=aa1+bb1
   v1(n+1)=cc1-dd1

```



```

u2(n+1)=aa2+bb2
v2(n+1)=cc2-dd2
u3(n+1)=aa3+bb3
v3(n+1)=cc3-dd3
u4(n+1)=aa4+bb4
v4(n+1)=cc4-dd4
c   now calculate the total mode displacement & velocity
do 500 l=1,4
b1(l)=an(l,1)*v1(n)
b2(l)=an(l,2)*v2(n)
b3(l)=an(l,3)*v3(n)
b4(l)=an(l,4)*v4(n)
a1(l)=an(l,1)*u1(n)
a2(l)=an(l,2)*u2(n)
a3(l)=an(l,3)*u3(n)
a4(l)=an(l,4)*u4(n)
500 continue
do 600 m=1,4
a1(m)=a1(m)+a2(m)+a3(m)+a4(m)
600 b1(m)=b1(m)+b2(m)+b3(m)+b4(m)
c
c convert from inches to m
c -----
do 650 m=1,4
an1(m)=a1(m)*0.0254
650 bn1(m)=b1(m)*0.0254
ui=an1(1)
c1=an1(1)-an1(3)
c2=bn1(1)-bn1(3)
c   write (6,400)tt,an1(1),an1(2),an1(3),an1(4),bn1(1),bn1(2),bn1(3),
c   &bn1(4),c1,c2,ui
c   write (6,400)tt,ui
c 400 format(12(1x,f8.5))
c 400 format (2(1x,f8.5))
tt=tt+dt

c *****
if (tt.ge.1.0) goto 710
c
c *****
if (c1.ge.0.008) then
goto 800
else
goto 100
endif
c *****
100 continue
c
c ~~~~~
800 cc(1,1)=a1(1)
cc(2,1)=a1(2)
cc(3,1)=a1(3)
cc(4,1)=a1(4)
cc(1,2)=b1(1)
cc(2,2)=b1(2)
cc(3,2)=b1(3)
cc(4,2)=b1(4)
us1=a1(1)
us2=a1(2)
us3=a1(3)
c
c form a matrix act*b(4,4)
c -----
do 11 j=1,4
do 11 i=1,4
actb(i,j)=0.0
do 11 k=1,4
11 actb(i,j)=actb(i,j)+act(i,k)*b(k,j)
c
c form a matrix act*b*cc(4,2)
c -----
do 21 j=1,2
do 21 i=1,4
actbcc(i,j)=0.0

```

```

do 21 k=1,4
21 actbcc(i,j)=actbcc(i,j)+actb(i,k)*cc(k,j)
c
c form checking matrix cm = ac*actbcc
c -----
do 26 j=1,2
do 26 i=1,4
cm(i,j)=0.0
do 26 k=1,4
26 cm(i,j)=cm(i,j)+ac(i,k)*actbcc(k,j)
c
c print the final matrix actbcc(4,2)
c -----
c print*, 'normalised disp vel checking init. disp vel'
c do 31 i=1,4
c write(6,41)(actbcc(i,j),j=1,2),(cm(i,j),j=1,2)
c 41 format (2(5x,f9.5,3x,f9.5,5x))
c 31 continue
c
tt=tt-dt
kn=80000
pi=3.141592654
wc1=40.584
wc2=206.70
wc3=21.007
wc4=48.496
u1(1)=actbcc(1,1)
u2(1)=actbcc(2,1)
u3(1)=actbcc(3,1)
u4(1)=actbcc(4,1)
v1(1)=actbcc(1,2)
v2(1)=actbcc(2,2)
v3(1)=actbcc(3,2)
v4(1)=actbcc(4,2)
dt=.00001
odt1=(wc1*dt)
odt2=(wc2*dt)
odt3=(wc3*dt)
odt4=(wc4*dt)
c
c Calculate the value of "static preload" u1s, u2s, u3s, u4s
c -----
as1=ac(1,1)*(25000*0.314960629)
bs1=ac(3,1)*(25000*0.314960629)
u1s=(as1-bs1)/(wc1**2.0)
as2=ac(1,2)*(25000*0.314960629)
bs2=ac(3,2)*(25000*0.314960629)
u2s=(as2-bs2)/(wc2**2.0)
as3=ac(1,3)*(25000*0.314960629)
bs3=ac(3,3)*(25000*0.314960629)
u3s=(as3-bs3)/(wc3**2.0)
as4=ac(1,4)*(25000*0.314960629)
bs4=ac(3,4)*(25000*0.314960629)
u4s=(as4-bs4)/(wc4**2.0)
c
c print*, '-----contact-----'
c write (6,30)tt,u1(n),v1(n),u2(n),v2(n),u3(n),v3(n)
do 110 n=1,kn
c
aa1=(v1(n)/wc1)*sin(odt1)
bb1=(u1(n)-u1s)*cos(odt1)
cc1=v1(n)*cos(odt1)
dd1=wc1*(u1(n)-u1s)*sin(odt1)
c
aa2=(v2(n)/wc2)*sin(odt2)
bb2=(u2(n)-u2s)*cos(odt2)
cc2=v2(n)*cos(odt2)
dd2=wc2*(u2(n)-u2s)*sin(odt2)
c
aa3=(v3(n)/wc3)*sin(odt3)
bb3=(u3(n)-u3s)*cos(odt3)
cc3=v3(n)*cos(odt3)
dd3=wc3*(u3(n)-u3s)*sin(odt3)
c

```

```

aa4=(v4(n)/wc4)*sin(odt4)
bb4=(u4(n)-u4s)*cos(odt4)
cc4=v4(n)*cos(odt4)
dd4=wc4*(u4(n)-u4s)*sin(odt4)
c
c      u(n+1)=((v(n)/wn)*sin(odt))+u(n)*cos(odt)
c      v(n+1)=(v(n)*cos(odt))-(wn*u(n)*sin(odt))
u1(n+1)=aa1+bb1+u1s
v1(n+1)=cc1-dd1
u2(n+1)=aa2+bb2+u2s
v2(n+1)=cc2-dd2
u3(n+1)=aa3+bb3+u3s
v3(n+1)=cc3-dd3
u4(n+1)=aa4+bb4+u4s
v4(n+1)=cc4-dd4
c      now calculate the total mode displacement
do 510 l=1,4
b1(l)=ac(l,1)*v1(n)
b2(l)=ac(l,2)*v2(n)
b3(l)=ac(l,3)*v3(n)
b4(l)=ac(l,4)*v4(n)
a1(l)=ac(l,1)*u1(n)
a2(l)=ac(l,2)*u2(n)
a3(l)=ac(l,3)*u3(n)
a4(l)=ac(l,4)*u4(n)
510 continue
do 610 m=1,4
a1(m)=a1(m)+a2(m)+a3(m)+a4(m)
610 b1(m)=b1(m)+b2(m)+b3(m)+b4(m)
c
c convert from inches to m
c -----
do 615 m=1,4
an1(m)=a1(m)*0.0254
615 bn1(m)=b1(m)*0.0254
c1=an1(1)-an1(3)
c2=bn1(1)-bn1(3)
uii=0.008+an1(3)
c
c calculate contact force, P (N)
c -----
P=(8.76e9/2.0)*(c1-0.008)
c
write (6,410)tt,an1(1),an1(2),an1(3),an1(4),bn1(1),bn1(2),
&bn1(3),bn1(4),c1,c2,uii,P
c write (6,410)tt,an1(1),an1(2),an1(3),an1(4),bn1(1),bn1(2),
c &bn1(3),bn1(4),c1,c2,uii
c write (6,410)tt,uii
tt=tt+dt
c
c *****
if (tt.ge.1.0) goto 710
c
c *****
if (c1.le.0.008) then
goto 5
else
goto 110
endif
c
c *****
c
110 continue
410 format(12(1x,f8.5),1x,f13.2)
c 410 format (12(1x,f8.5))
c 410 format (2(1x,f8.5))
710 stop
end

```

```

c          TFLEX2.FOR
c          CHAPTER 5 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c          a = L/4 (unaligned floor case)

```

```

c          Similar to TFLEX1.FOR , except the following parameters are changed :

```

```

c -----
c          open(6,file='tflex2.out',status='unformatted')
c
c          data an /0.83005, 0.45535, 0.0, 0.0, -0.55769, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, 0.37050, 0.0, 0.0, -0.42782, 0.60227/
c
c          data ac /-0.067354, -.54120, -.12083, 0.51683, 0.71182,
&0.081680, -.56749, 0.011025, 0.59400, 0.19651, 0.53241,
&0.28639, -.36871, 0.57313, -.21577, 0.38826/
c
c          data ant /0.83005, -0.55769, 0.0, 0.0, 0.45535, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, -0.42782, 0.0, 0.0, 0.37050, 0.60227/
c
c          data act /-0.067354, 0.71182, 0.59400, -.36871, -0.54120,
&0.081680, 0.19651, 0.57313, -0.12083, -0.56749, 0.53241,
&-0.21577, 0.51683, 0.011025, 0.28639, 0.38826/
c
c          wn1=23.274
c          wn2=51.559
c          wn3=19.334
c          wn4=43.889
c
c          ui=((an1(1)*27.0)+(an1(2)*5.0))/32.0
c          vi=((bn1(1)*27.0)+(bn1(2)*5.0))/32.0
c
c          wc1=40.500
c          wc2=179.95
c          wc3=19.611
c          wc4=42.819

```

```

c          Calculate the value of "static preload" u1s, u2s, u3s, u4s

```

```

c -----
c          as1=ac(1,1)*((15600.0*us1)+(3600.0*us2)-(19200.0*us3))
c          bs1=ac(2,1)*((3600.0*us1)-(44.44444445*us2)-(3555.555556*us3))
c          cs1=ac(3,1)*((-19200.0*us1)-(3555.555556*us2)+(22755.55556*us3))
c          u1s=(as1+bs1+cs1)/(wc1**2.0)
c          as2=ac(1,2)*((15600.0*us1)+(3600.0*us2)-(19200.0*us3))
c          bs2=ac(2,2)*((3600.0*us1)-(44.44444445*us2)-(3555.555556*us3))
c          cs2=ac(3,2)*((-19200.0*us1)-(3555.555556*us2)+(22755.55556*us3))
c          u2s=(as2+bs2+cs2)/(wc2**2.0)
c          as3=ac(1,3)*((15600.0*us1)+(3600.0*us2)-(19200.0*us3))
c          bs3=ac(2,3)*((3600.0*us1)-(44.44444445*us2)-(3555.555556*us3))
c          cs3=ac(3,3)*((-19200.0*us1)-(3555.555556*us2)+(22755.55556*us3))
c          u3s=(as3+bs3+cs3)/(wc3**2.0)
c          as4=ac(1,4)*((15600.0*us1)+(3600.0*us2)-(19200.0*us3))
c          bs4=ac(2,4)*((3600.0*us1)-(44.44444445*us2)-(3555.555556*us3))
c          cs4=ac(3,4)*((-19200.0*us1)-(3555.555556*us2)+(22755.55556*us3))
c          u4s=(as4+bs4+cs4)/(wc4**2.0)

```

```

c          calculate contact force, P (N)

```

```

c -----
c          F2=(19200.0*(a1(1)-us1)+(3555.555556*(a1(2)-us2))
c          F1=(22755.55556*(a1(3)-us3))
c          P=(F2-F1)*4450.0

```

```

c          TFLEX3.FOR
c          CHAPTER 5 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c          a = L/3 (unaligned floor case)

```

```

c          Similar to TFLEX1.FOR , except the following parameters are changed :

```

```

c -----
c          open(6,file='tflex3.out',status='unformatted')
c
c          data an /0.83005, 0.45535, 0.0, 0.0, -0.55769, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, 0.37050, 0.0, 0.0, -0.42782, 0.60227/
c
c          data ac /-0.16304, -.34232, -.17204, 0.61370, -0.65292,
&-0.14692, 0.60020, -.022008, 0.62104, 0.20635, 0.51445,

```

```

&0.27699, -.40177, 0.69665, -.11034, 0.21487/
c
data ant /0.83005, -0.55769, 0.0, 0.0, 0.45535, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, -0.42782, 0.0, 0.0, 0.37050, 0.60227/
c
data act /-0.16304, -.65292, 0.62104, -.40177, -0.34232,
&-0.14692, 0.20635, 0.69665, -0.17204, 0.60020, 0.51445,
&-0.11034, 0.61370, -.022008, 0.27699, 0.21487/
c
wn1=23.274
wn2=51.559
wn3=19.334
wn4=43.889
c
ui=((an1(1)*20.0)+(an1(2)*7.0))/27.0
vi=((bn1(1)*20.0)+(bn1(2)*7.0))/27.0
c
wc1=40.844
wc2=133.65
wc3=19.637
wc4=42.522
c
c Calculate the value of "static preload" u1s, u2s, u3s, u4s
c -----
as1=ac(1,1)*((6900.0*us1)+(3225.0*us2)-(10125.0*us3))
bs1=ac(2,1)*((3225.0*us1)+(318.75*us2)-(3543.75*us3))
cs1=ac(3,1)*((-10125.0*us1)-(3543.75*us2)+(13668.75*us3))
u1s=(as1+bs1+cs1)/(wc1**2.0)
as2=ac(1,2)*((6900.0*us1)+(3225.0*us2)-(10125.0*us3))
bs2=ac(2,2)*((3225.0*us1)+(318.75*us2)-(3543.75*us3))
cs2=ac(3,2)*((-10125.0*us1)-(3543.75*us2)+(13668.75*us3))
u2s=(as2+bs2+cs2)/(wc2**2.0)
as3=ac(1,3)*((6900.0*us1)+(3225.0*us2)-(10125.0*us3))
bs3=ac(2,3)*((3225.0*us1)+(318.75*us2)-(3543.75*us3))
cs3=ac(3,3)*((-10125.0*us1)-(3543.75*us2)+(13668.75*us3))
u3s=(as3+bs3+cs3)/(wc3**2.0)
as4=ac(1,4)*((6900.0*us1)+(3225.0*us2)-(10125.0*us3))
bs4=ac(2,4)*((3225.0*us1)+(318.75*us2)-(3543.75*us3))
cs4=ac(3,4)*((-10125.0*us1)-(3543.75*us2)+(13668.75*us3))
u4s=(as4+bs4+cs4)/(wc4**2.0)
c
c calculate contact force, P (N)
c -----
F2=(10125.0*(a1(1)-us1)+(3543.75*(a1(2)-us2))
F1=(13668.75*(a1(3)-us3))
P=(F2-F1)*4450.0

```

```

c
c TFLEX4.FOR
c CHAPTER 5 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS
c a = L/2 (unaligned floor case)
c
c Similar to TFLEX1.FOR , except the following parameters are changed :
c -----

```

```

open(6,file='tflex4.out',status='unformatted')
c
data an /0.83005, 0.45535, 0.0, 0.0, -0.55769, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, 0.37050, 0.0, 0.0, -0.42782, 0.60227/
c
data ac /-0.49271, 0.26010, -0.050938,0.57090, -0.45464, -0.32964,
&0.64656,-0.040278, 0.68942, 0.23146, 0.46190, 0.24932, 0.27431,
&-0.66089, -0.18078, 0.33209/
c
data ant /0.83005, -0.55769, 0.0, 0.0, 0.45535, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, -0.42782, 0.0, 0.0, 0.37050, 0.60227/
c
data act /-0.49271, -0.45464, 0.68942, 0.27431, 0.26010, -0.32964,
&0.23146, -0.66089, -0.050938,0.64656, 0.46190, -0.18078, 0.57090,
&-0.040278, 0.24932, 0.33209/
c
wn1=23.274
wn2=51.559
wn3=19.334
wn4=43.889

```



```

u1s=(a1+bs1+cs1)/(wc1**2.0)
as2=ac(1,2)*((-44.44444445*us1)+(3600.0*us2)-(3555.555556*us3))
bs2=ac(2,2)*((3600.0*us1)+(15600.0*us2)-(19200.0*us3))
cs2=ac(3,2)*((-3555.555556*us1)-(19200.0*us2)+(22755.55556*us3))
u2s=(as2+bs2+cs2)/(wc2**2.0)
as3=ac(1,3)*((-44.44444445*us1)+(3600.0*us2)-(3555.555556*us3))
bs3=ac(2,3)*((3600.0*us1)+(15600.0*us2)-(19200.0*us3))
cs3=ac(3,3)*((-3555.555556*us1)-(19200.0*us2)+(22755.55556*us3))
u3s=(as3+bs3+cs3)/(wc3**2.0)
as4=ac(1,4)*((-44.44444445*us1)+(3600.0*us2)-(3555.555556*us3))
bs4=ac(2,4)*((3600.0*us1)+(15600.0*us2)-(19200.0*us3))
cs4=ac(3,4)*((-3555.555556*us1)-(19200.0*us2)+(22755.55556*us3))
u4s=(as4+bs4+cs4)/(wc4**2.0)

```

c  
c calculate contact force, P (N)

```

c -----
F2=(3555.555556*(a1(1)-us1)+(19200.0*(a1(2)-us2))
F1=(22755.55556*(a1(3)-us3))
P=(F2-F1)*4450.0

```

c  
c **TFLEX6.FOR**  
c **CHAPTER 5 : UNDAMPED FREE VIBRATION-MDOF SYSTEMS**  
c **a = L (aligned floor case)**

c  
c Similar to TFLEX1.FOR , except the following parameters are changed :

```

c -----
open(6,file='tflex6.out',status='unformatted')
c
data an /0.83005, 0.45535, 0.0, 0.0, -0.55769, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, 0.37050, 0.0, 0.0, -0.42782, 0.60227/
c
data ac /0.57975, -.085100, -0.11611, -.56249, -.021006, 0.58757,
&-.56656, 0.010219, 0.65666, 0.38964, 0.39347, 0.23327, .48192,
&-.40294, -0.42115, 0.35928/
c
data ant /0.83005, -0.55769, 0.0, 0.0, 0.45535, 0.67773, 0.0, 0.0,
&0.0, 0.0, 0.69544, -0.42782, 0.0, 0.0, 0.37050, 0.60227/
c
data act /0.57975, -.021006, 0.65666, 0.48192, -.085100, 0.58757,
&0.38964, -.40294, -0.11611, -0.56656, 0.39347, -0.42115, -.56249,
&0.010219, 0.23327, 0.35928/
c
wn1=23.274
wn2=51.559
wn3=19.334
wn4=43.889
c
ui=an1(2)
c
wc1=37.096
wc2=186.46
wc3=22.090
wc4=46.940

```

c  
c Calculate the value of "static preload" u1s, u2s, u3s, u4s

```

c -----
as1=ac(2,1)*(25000*0.314960629)
bs1=ac(3,1)*(25000*0.314960629)
u1s=(as1-bs1)/(wc1**2.0)
as2=ac(2,2)*(25000*0.314960629)
bs2=ac(3,2)*(25000*0.314960629)
u2s=(as2-bs2)/(wc2**2.0)
as3=ac(2,3)*(25000*0.314960629)
bs3=ac(3,3)*(25000*0.314960629)
u3s=(as3-bs3)/(wc3**2.0)
as4=ac(2,4)*(25000*0.314960629)
bs4=ac(3,4)*(25000*0.314960629)
u4s=(as4-bs4)/(wc4**2.0)

```

c  
c calculate contact force, P (N)

```

c -----
P=(8.76e9/2.0)*(c1-0.008)

```

# APPENDIX I

## DATA INPUT FOR PATRAN AND DYNA3D

---

### THE LIST OF COMMANDS USED IN PATRAN

#### LOGON INTO PATRAN

DISPLAY=gum-10:0.0;export DISPLAY  
(gum-10 is the sun machine number)  
patran2  
xuxb (device mnemonic)  
GO  
Patran data file? 1 (New)  
( patran.dat.1 written )  
Mode ? SET,MENU,ON (now the cursor is active)  
PLOT (the origin axes display)  
VIEW > AXES ABSOLUTE  
> Input Axes Absolute theta x,y,z for screen 1  
> -90, 0, 0

#### 1) GEOMETRY

MENU MODE > LINE > MORE > MORE > MORE  
> BLANK

CHOOSE ANY OPTION TO MODIFY OR 'G' TO CREATE LINE		
x length :0.0000	x origin :10.000	GO
y length :0.0000	y origin :0.000	Next Id =1
z length :100.00	z origin :0.000	REJECT

> GO  
> END > END > END

#### 2) ANALYSIS MODEL

2.1 GENERATE MESH > NODES ONLY  
> INDIVIDUAL SELECT

PICK A LINE FOR NODE MESH LIST(1), OR ANY OPTION		
Entity = LINE	Mesh 1 = 2	Zoom list (optional)
Coord frame = 0		Node list (optional)
Node type = Q60		Ratio : 1.00
		REJECT

> ELEMENTS ONLY



```

PICK A LINE FOR ELEM MESH LIST(1), OR ANY OPTION

Entity = LINE          Elem Type= Bar      Elem list :(optional)
PID = 0*              No. Nodes = 2      Del list  :(optional)
Pattern = NA          Config   = 0*      Assc Data :(optional)      REJECT

```

```

* For Line (Bar)      > PID=1      CONFIG=0
  For Line (Spring)  > PID=1      CONFIG=6

                >      INDIVIDUAL SELECT
                >      CHOOSE MESH/LINE
(if line, the following data is required: PROCEED > XY PLANE)

```

```

DEFINE BEAM XY PLANE BY GRID, NODE OR VECTOR ? (G/N/V) N

ENTER NODE ID(S) : <RETURN>

```

```

>      END  >      END  >      END

```

## 2.2 EQUIVALENT NODES

```

Tolerance = 0.005000 Do you wish to override ? (Y/N) N

```

```

>      GEOMETRIC
>      ALL

```

```

Do you want the active set recreated to reflec these changes? (Y/N) Y

```

## 2.3 GENERATE LOADS/BCS

```

>      INDIVIDUAL SELECT

```

```

PICK A LINE FOR NODE MESH LIST(1), OR ANY OPTION

Entity = LINE          B.C = DISPL      INCL LIST :(optional)*
D (X,Y,Z) = // /      CID LIST :(optional)
R (X,Y,Z) = // /      SET ID : 1          REJECT

```

```

* INCL LIST : N1 (eg for node no 1 only)
  INCL LIST : N1T3 (eg for node no 1 to node no 3 only)

```

```

* D(X,Y,Z) = /0/0 } these will allow the x displacement only.
  R(X,Y,Z) = 0/0/0 }

```

```

* D(X,Y,Z) = 0/0/0 } these will restraint all displacements and
  R(X,Y,Z) = 0/0/0 } rotations.

```

> END

## 2.4 VERIFY MODEL

> LOADS/BC > DISP

Plot or show ? (P/S) P

> PROPERTIES > END  
> AUTO >PLOT/FILL HIDE/ETC

## 2.5 OPTIMIZE MODEL

> COMPACT NODE\_IDS  
> OPTIMIZE NODE\_IDS > BOTH >PROFILE  
> END

## 3) ADDITIONAL COMMANDS

Click mouse into the COMMAND menu

SET, ACTIVE, NONE  
NODE,1,PLOT  
NAME,BC\_DISP\_1  
SET, ACTIVE, NONE  
NODE,2,PLOT  
NAME,BC\_DISP\_2  
SET, ACTIVE, NONE  
NODE,3,PLOT  
NAME,BC\_DISP\_3  
SET, ACTIVE, ALL  
NAME, ELEMENT\_BLOCK  
NAME, NODE\_BLOCK  
END

## 4) INTERFACE

CREATE OUTPUT  
ENTIRE MODEL

Input the title card for neutral output **JOB1** <RETURN>

Do you wish to output phase - I data ? (Y/N) **Y**

Do you wish to output GFEG/CFEG TABLES ? (Y/N) **Y**

## 5) STOP

patran.out.1 written  
patran.dat.1 written

## THE LIST OF COMMANDS USED IN N/CODE

### 1) LOGIN INTO N/CODE

oa51 ( enter to OASYS DYNA3D )

Enter command [ h for help, m for menu ] ? **m**

option		
su	-	submit OASYS DYNA3D job
nc	-	OASYS N/CODE
pl	-	OASYS D3PLOT
th	-	OASYS T/HIS
etc		
etc		

**nc**

### 2) READING MODEL INFORMATION

Pre-processor type (ideas/patran/nisa/pafec/femgen) ? **patran**

Oasys N/CODE
<u>PATRAN to OASYS DYNA3D file translator</u>
(version 5.1a)
Liverpool University (T.M)
SUN SPARCstation

Neutral File name (patran.out) ? **patran.out.1**

OASYS DYNA3D input file name ? **job1**

OK to open new file job1.inf (YES) ? **<return>**

Reading Neutral file

Reading packet 25	-	Title Card
Reading packet 01	-	Node Data
Reading packet 02	-	Element Data
Reading packet 08	-	Node Displacements
Reading packet 21	-	NAMED Component Definition
Neutral File read	: 861 lines	

### 3) WRITING OASYS DYNA3D INPUT FILE

Writing title ...

JOB1

Writing Control cards ...

Termination time for the analysis ? **1.0**

Timing

id	- description .....	default
1	- termination time	1.0000E-01
2	- time between writing time history files	1.0000E-02
3	- time between writing complete state files	1.0000E-01
4	- initial time step (0-chosen by code)	0.0000E+00
5	- scale factor on time step	9.0000E-01
6	- minimum time step factor	5.0000E-02
7	- number of steps between restart dumps	5000
8	- number of steps between auxilliary dumps	1000
9	- load curve limiting time step	0

Timing ? **<return>**

Loading

1.	Number of loads curves	0
2.	Base acceleration in x-direction	no
3.	Base acceleration in y-direction	no
4.	Base acceleration in z-direction	no
5.	Angular velocity about x-axis	no
6.	Angular velocity about y-axis	no
7.	Angular velocity about z-axis	no
8.	Thermal effects option	0
	n	- temperature defined by load curve n
	0	- no thermal effects
	-n	- temperature option n (n=1 or 2)
	<-2	- TOPAZ3D temperature file

Loading ? **1 <return>**

...new value ? **3**

Loading ? **<return>**

Initialisation by dynamic relaxation (NO) ? **<yes>**

Dynamic relaxation

1.	Dynamic relaxation for initialization	yes
2.	Database written during relaxation	0
	0 - database not written	
	1 - written at convergence check	
	2 - written at PTF interval	
3.	Number of steps between convergence checks	250
4.	Convergence tolerance	1.0000E-03
5.	Relaxation factor	9.9500E-01

Dynamic relaxation ? **<return>**

Displacement control

1.	Node for disp controlled termination a node 0 means this option is not set	0
2.	Degree of freedom of disp	x
3.	Maximum limiting coordinate	0.0000E+00
4.	Minimum limiting coordinate	0.0000E+00

Displacement control ? **<return>**

General

1.	No. cycles for reports to standard output	100
2.	No. rigid body merges	0
3.	No. rigid body inertias	0
4.	No. rigid body velocities	0
5.	Compute hourglass energy	no
6.	Compute stonewall energy dissipation	yes
7.	Calculate accelerations from velocities	no
8.	Energy imbalance termination	0
	0 - not set	
	1 - check against initial kinetic energy	
	2 - as 2 but includes external work	
9.	Maximum % energy imbalance	0.0000E+00

General ? **<return>**

Writing materials ...

Please select the required type of material input :

sk	- skip
fi	- read from another file
in	- read from another OASYS DYNA3D input file
de	- set material defaults, use pre-processor thickness
dm	- set material defaults, input thickness
ky	- input all material data from keyboard

Enter choice ( L for material list ) : **ky**

Dyna material : 1  
 Material 1 : PATRAN Material Property : 1  
 Physical 1 : PATRAN Beam Property : 1

Material type number or material id ? **13**

Elastic

Density ? **7890**  
 Youngs Modulus ? **205e9**  
 Poissons ratio ? **0.3**

Material OK (YES) ? **<return>**

Element formulation for beam (B-S) ? **<return>**

Belytschko-Schwer Section data

1 Shear factor .....	1.0000E+00
2 cross sectional area .....	1.0000E+00
3 2nd moment of area, Iyy .....	1.0000E+00
4 2nd moment of area, Izz .....	1.0000E+00
5 2nd moment of area, Ixx .....	1.0000E+00
6 shear area, As .....	1.0000E+00

Belytschko-Schwer Section data ? **2**

... new value ? **0.057041125**

Belytschko-Schwer Section data ? **3**

... new value ? **2.58929e-4**

Belytschko-Schwer Section data ? **4**

... new value ? **2.58929e-4**

Belytschko-Schwer Section data ? **5**

... new value ? **5.17841e-4**

Belytschko-Schwer Section data ? **<return>**

Dyna material : 2  
 Material 2 : PATRAN Material Property : 2  
 Physical 2 : PATRAN Beam Property : 2

Material type number or material id ? **1**

Elastic

Density ? **7.89**

Youngs Modulus ? **205e9**

Poissons ratio ? **0.3**

Material OK (YES) ? **<return>**

Element formulation for shells (B-T) ? **<return>**

**< for example, enter data as in Table 7.3a for material type 1, 2 and 3>**

Writing nodes ...

Writing beam elements ...

Node for orienting beam 1 (1) ? **6**

Node for orienting beam 2 (6) ? **<return>**

Node for orienting beam 3 (6) ? **<return>**

Writing nodal time history blocks ...

Writing element time history blocks ...

Writing load curves ...

Select the required type of load input

sk - skip

fi - read all the curves from another file

ky - input data for each curve separately (keyboard/file)

Enter choice : **ky**

Load curve no. **1**

No of points in load curve or file with points (2)? **3**

Stress initialization by dynamic relaxation

0 - load curve used only in transient analysis

1 - load curve used in stress initialization only

2 - load curve used in both

Load curve option (0) ? **1**

Description of curve? **dis1**

Time, value ? **0,0**

Time, value ? **0.001, -0.0254**

Time, value ? **1.0, -0.0254**

Load curve no.2

**< for example, enter data as in Table 7.3b for disp. curve no.1, 2 and 3>**

Writing velocity/acceleration/displacement specification ...

Displacement boundary condition

Load curve : 1

Number of nodes: 1

DOF Specification (XT) ? **<return>** (i.e. x translation)

Scale factor on load curve (1.0) ? **<return>**

Displacement boundary condition

Load curve : 2

Number of nodes: 1

DOF Specification (XT) ? **<return>** (i.e. x translation)

Scale factor on load curve (1.0) ? **<return>**

Displacement boundary condition

Load curve : 3

Number of nodes: 1

DOF Specification (XT) ? **<return>** (i.e. x translation)

Scale factor on load curve (1.0) ? **<return>**

Rigid body velocity boundary conditions

Writing Springs,dampers and masses ...

Select the required type of spring property input

sk - skip

fi - read all the curves from another file

ky - input data for each curve separately (keyboard/file)

Enter choice : **ky**

Number of orientation vectors (0) ? **2** (i.e. orientation defined by 2 nodes)

Spring material : **1**

Type : **Translational Spring**

PATRAN Material Property : **1**

Spring property type [ m for menu ] ? **1** ( i.e. linear elastic)

Properties

Linear Elastic

1 Elastic Stiffness.....	1.0000E+00
2 Dynamic magnification factor.....	0.0000E+00
3 Test velocity .....	0.0000E+00
4 Clearance .....	0.0000E+00
5 Failure deflection in tension.....	0.0000E+00
6 Deflection limit in compression.....	0.0000E+00
7 Deflection limit in tension.....	0.0000E+00

Linear Elastic ? **1**

... new value ? **8.76e9**

Linear Elastic ? **4**

... new value ? **0.008**

Linear Elastic ? **<return>**

... orientation vectors

Select the required type of orientation vector input

sk - skip

fi - read all the curves from another file

ky - input data for each curve separately (keyboard/file)

Enter choice : **ky**

Define orientation vectors of one of the following types :

0	- orientation defined by vector
1	- orientation defined by plane normal to vector
2	- orientation defined by 2 nodes

Orientation option for vector 1 (0) ? **2**

Nodes for orientation ? **1, 5**

Orientation option for vector 2 (2) ? **<return>**

Nodes for orientation ? **1, 5**

... springs and dampers

Orienting option for spring/damper 7 (0) ? **2**

... lumped masses

Mass for element 1 at node 1 (0.000E+00)? **175197**

Mass for element 2 at node 2 (1.752E+05)? **262795**

Mass for element 3 at node 3 (2.628E+05)? **350394**

Data file written

OK to delete session file (YES) ? **<return>**

Normal termination of N/CODE

Enter command [ h for help, m for menu ] ? **exit**



## THE LIST OF COMMAND TO SUBMIT A JOB

### LOGIN INTO SUBMIT JOB OASYS DYNA3D

oa51 ( enter to OASYS DYNA3D )

Enter command [ h for help, m for menu ] ? **su** (i.e. submit OASYS DYNA3D job)

```
OASYS DYNA3D
Submission program

(Version 5.1)
```

Jobname [ls for list] ? **job1**

#### SUBMISSION STATUS

```
Job Name ..... JOB1
rs      - restart ..... no
cpu     - CPU limit ..... -
ctf     - write contact force file ..... no
asc     - subdirectory for ASCII output ..... -
tz      - use OASYS TOPAZ 3D temperature file.. no
sub     - submission [background/foreground]..... bg
ast     - analysis start time ..... now
asd     - analysis start day ..... -
```

- means that the option is not set

Submit ? **ctf**

Write a .ctf file [yes or no] ? **y**

Submit ? **go**

Writing control file and submitting to background

[1] 29329

Enter command [ h for help, m for menu ] ? **exit**

## THE LIST OF COMMAND TO RESTART A JOB

### 1) CREATE THE .IN1 FILE

It is necessary to create a restart input file ( .in1) to obtain more frequent intervals, before the occurrence of the contact of the two building at job1.d04 onward. For details of restart input deck, please refer to User Manual Version 5.1a rev 0 September 1992. The relevant restart cards that have been used in this computer modelling are **Title** and **Control**. Other quantities such as Displacement Termination, Deleted Contact Surfaces, Deleted Materials etc are irrelevant.

```
restart file job1
$2345678901234567890123456789012345678901234567890
0.0      2.8421E-07 0      0
```

where

<b>Columns</b>	<b>Quantity</b>	<b>Format</b>
1-10	New termination time 0.0:termination time remains unchanged	E10.0
11-20	New interval for writing time history data 0.0:interval remains unchanged	E10.0
21-30	New interval for writing complete state data 0.0:interval remains unchanged	E10.0
<b>31-65</b>	<b>can be ignored as far as this research is concerned.</b>	

### 2) LOGIN INTO RESTART JOB OASYS DYNA3D

oa51 ( enter to OASYS DYNA3D )

Enter command [ h for help, m for menu ] ? **su** (i.e. submit OASYS DYNA3D job)

```
OASYS DYNA3D
Submission program

(Version 5.1)
```

Jobname [ls for list] ? **job1**

SUBMISSION STATUS

```
Job Name ..... JOB1
rs      - restart ..... no
cpu     - CPU limit ..... -
ctf     - write contact force file ..... no
asc     - subdirectory for ASCII output ..... -
tz      - use OASYS TOPAZ 3D temperature file.. no
sub     - submission [background/foreground]..... bg
ast     - analysis start time ..... now
asd     - analysis start day ..... -
```

- means that the option is not set

Submit ? **rs**

Which dump to restart from (5) ? **<return>** where no 5 means job1.d04

OK to use input file job1.in1 (YES) ? **<return>**

Copying dump file job1.d04 to restart file job1.rtf ...

Warning - contact force file found, write option switched to yes

Submit ? **go**

Writing control file and submitting to background

[1] 29329

Enter command [ h for help, m for menu ] ? **exit**

## THE LIST OF COMMAND TO DISPLAY OUTPUT USING D3PLOT

### 1) LOGIN INTO D3PLOT DYNA3D

oa51 ( enter to OASYS DYNA3D )

Enter command [ h for help, m for menu ] ? **pl** (i.e. D3PLOT DYNA3D)

```
OASYS D3PLOT ( Version 5.1, September 1992 )
Graphical post-processing for OASYS DYNA3D
Compatible with      : OASYS DYNA3D 5.1
                    : OASYS DYNA3D 5.0
                    : OASYS DYNA3D 4.2 and before
For support from OASYS   Tel : (+44) 71 465 2229
                        Fax : (+44) 71 465 2312
```

```
Liverpool University (T.M)
Sun SPARCstation
```

Give terminal type (M for list) **xw**

{ place cursor on D3PLOT window }

Give input .PTF filename ? **job1**

D3PLOT\_MANAGER >>> [ H for help ] **rs**

Give angles in degrees : (0.0.0) **-90, 0, 0**

D3PLOT\_MANAGER >>> [ H for help ] **show**

D3PLOT\_MANAGER >>> [ H for help ] **ct**

D3PLOT\_MANAGER >>> [ H for help ] **mg**

Give magnification scale : **0.5**

D3PLOT\_MANAGER >>> [ H for help ] **ct**

D3PLOT\_MANAGER >>> [ H for help ] **animation**

SEQUENCE\_GENERATION [ H for help ] >>> **set**

Give start,interval,finish times or ALL : **all**

D3PLOT\_MANAGER >>> [ H for help ] **ct**

OK to continue ? (No) **Y**

SEQUENCE\_GENERATION [ H for help ] >>> **play**

PLAY\_IMAGES [ H for help ] >> **animation**

Repeat ? **y**

Repeat ? **n**

SEQUENCE\_GENERATION [ H for help ] >>> /

## 2) TO GET COLOUR PRINTOUT

```
D3PLOT_MANAGER >>> [ H for help ] display  
[ type m for help ]
```

```
laser_option : on  
label_plot    > colour_mode : <return>  
              > status  
              > /
```

```
D3PLOT_MANAGER >>> [ H for help ] re 1
```

```
D3PLOT_MANAGER >>> [ H for help ] ct  
Give file title : job1  
Give figure no : 1  
[ create post001.out file ]
```

```
D3PLOT_MANAGER >>> [ H for help ] exit
```

```
Enter command [ h for help, m for menu ] ? exit
```

## 3) SENT POSTSCRIPT FILE TO QMS PRINTER

```
$ lpr -Pqms -Fa4sb post001.out
```

## THE LIST OF COMMAND TO DISPLAY OUTPUT USING T/HIS

### 1) LOGIN INTO T/HIS DYNA3D

oa51 ( enter to OASYS DYNA3D )

Enter command [ h for help, m for menu ] ? **th** (i.e. T/HIS DYNA3D)

Time history processing package

( Version 5.1 )

Device type ? **xw**

{ place cursor on T/HIS window }

### 2) TO READ TIME HISTORY FILE

T/HIS > **read**

Read > **th**

OASYS DYNA3D THF filename ? **job1**

JOB1

	Total	In blocks
Nodes :	6	6
Solid elements:	0	0
Beam elements:	3	3
Shell elements:	0	0
Thick shell elemnets:	0	0
Number of materials:	3	
Number of times:	20680	
Final time :	9.9999E-1	

Strain tensor stored for shell elements

Read THF > **nodes**

Node number ? **1**

Nodal component ? **dx** (i.e. displacement in x direction)

Output curve ? **dx1**

Read THF > **nodes**

Node number ? **2**

Nodal component ? **dx** (i.e. displacement in x direction)

Output curve ? **dx2**

Read THF > **nodes**

Node number ? **3**

Nodal component ? **dx** (i.e. displacement in x direction)

Output curve ? **dx3**

Read THF > **nodes**

Node number ? **1**

Nodal component ? **vx** (i.e. velocity in x direction)

Output curve ? **vx1**

### 3) TO GET PRINT OUT OF TIME HISTORY FILE

Read THF > /

T/HIS > **read**

Read > **curve**

Curve file [ls & lc for list ] ? **dx1**

Curve file [ls & lc for list ] ? **dx2**

Curve file [ls & lc for list ] ? **<return>**

T/HIS > **plot**

T/HIS > **defaults**

Defaults > **grid**

Grid lines (on) ? **<return>**

Defaults > /

T/HIS > **plot**

T/HIS > **pf**

Picture formats (L) ? **<return>**

Postscript file ? **<return>**

[ Postscript file : this001.psc ]

Plot title ? **job1**

Figure number (eg 2.1) ? **1.1**

Parked X driver

Reactivated X driver

### 4) TO READ EXTRA TIME HISTORY FILE

T/HIS > **read**

Read > **xtf**

OASYS DYNA3D XTF filename ? **job1**

Read XTF > **spring**

Spring/damper element ? **7**

[ Translational spring ]

Spring data component ? **m**

FT - Force v time

ET - Elongation v time

EF - Elongation v force  
EN - Energy v time  
/ - Back up a level

Spring data component ? et  
Output curve ? e7

Read XTF > **spring**  
Spring/damper element ? 7  
[ Translational spring ]

Spring data component ? m  
FT - Force v time  
ET - Elongation v time

EF - Elongation v force  
EN - Energy v time  
/ - Back up a level

Spring data component ? ft  
Output curve ? f7

#### **5) TO GET PRINT OUT OF EXTRA TIME HISTORY FILE**

.. same as in 3) ...

Enter command [ h for help, m for menu ] ? **exit**

#### **6) SENT POSTSCRIPT FILE TO LASER PRINTER**

\$ lpr -Plaser this001.out

