International Journal of Modern Physics D,

ON-LINE CONSISTENCY TESTS FOR BAR DETECTORS

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Received (received date)
Revised (revised date)

In order to detect gravitational wave signals with resonant bar detectors using Wiener-Kolmogorov (WK) filters, both a model for the power spectrum density (PSD) of the noise and a signal template should be provided. As the analysis is not meant to handle non-gaussian data, we have to discriminate (and constrain to) time periods where the noise follows a quasi-stationary gaussian model. Within these periods, candidate events are selected in the WK filter output, and their fundamental parameters (time of arrival and amplitude) are computed. A necessary and sufficient condition for the reliability of such estimates is the consistency of the signal shape with the template. This is done performing a goodness-of-the-fit test

1. A model for noise and signal in resonant bar detectors

Bar detectors now in operation[1] consist basically of a resonant mass (the bar) coupled to a electromechanical resonant transducer (capacitive, microwave, inductive) with the same free resonance frequency. The output of the detector –after a linear (or linearized) amplification stage– is acquired with an ADC, either directly (as in AURIGA) or through a band-pass amplifier. For our purposes the entire system can be conveniently modelled by two coupled damped harmonic oscillators. The solution of the dynamical equations for the normal modes is a doublet of resonances, that for presently working detectors are well separated (∼20Hz at ∼1kHz).

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Close to the k-th resonance frequency ω_k $(k = 1, 2)$, the transfer function of the system is proportional to that of a harmonic oscillator, namely $\left(\omega^2 - \frac{i}{Q_k}\omega_k\omega - \omega_k^2\right)^{-1} \equiv \left[(i\omega - p_k)(i\omega - p_k^*)\right]^{-1}$, where Q_k is the mechanical quality factor of the the k-th mode and the pole $p_k \equiv i\omega_k - \Delta\omega_k$ is in the negative real axis semiplane of the complex plane (this is a consequence of the system having a causal impulse response). The total transfer function for gravitational wave signals can therefore be modelled by:

$$G(\omega) = \alpha \prod_{k=1}^{2} \frac{i\omega}{(i\omega - p_k)(i\omega - p_k^*)} \tag{1}$$

where α is a calibration constant to be experimentally determined.

When computing the output power spectrum density (PSD) due to the *thermal noise* sources associated with the dissipative elements (through the Nyquist theorem), terms proportional to $\left|\omega^2 - \frac{i}{Q_k}\omega_k\omega - \omega_k^2\right|^{-2} = \frac{1}{(i\omega-p_k)(i\omega+p_k)\left(i\omega-p_k^*\right)\left(i\omega+p_k^*\right)}$ arise, i.e. the PSD for frequencies close to the fundamental modes is fitted by a Lorentzian curve. Note that each pole p_k in the PSD has also its positive real axis counterparts (this is a standard feature of a PSD). The output admittance of the system shares the same poles p_k so the noise due to back action of the SQUID amplifier's noise has an output spectrum again Lorentzian, acting as a contribution in excess of thermal noise. The total output noise PSD $S(\omega)$ is made by a linear combination of Lorentzian terms, one for each resonance, plus a constant *wide band noise* S_0 introduced by the preamplifier. Arranging all terms, and using symmetry properties of the noise PSD, we conclude that the general expression for $S(\omega)$ is

$$S(\omega) = S_0 \prod_{k=1}^{2} \frac{(i\omega - q_k)(i\omega + q_k)(i\omega - q_k^*)(i\omega + q_k^*)}{(i\omega - p_k)(i\omega + p_k)(i\omega - p_k^*)(i\omega + p_k^*)}. \tag{2}$$

The complex zeros can be written as $q_k \equiv i\omega_k - \delta\omega_k$ where $\delta\omega_k/\pi$ is the effective post-filtering bandwith, and $\mathrm{Im}\, q_k \approx \omega_k$ in the approximation $\Delta\omega_k \ll \delta\omega_k$. It is remarkable that these are all the parameters we need in order to to build the WK filter for an impulsive signal. This approach gives consistent results only when the detector real noise PSD reasonably follows the model (e.g. it doesn't exhibits extra noise resonances within 10Hz about the modes of the system).

Once the model is established, it is straightforward to build up the optimal WK filter for it.[†] Thinking to the filter as a linear system, with the raw data stream as input, its transfer function would be $\sigma^2 S(\omega)^{-1} G^*(\omega)$ where $\sigma^{-2} \equiv \int S(\omega)^{-1} |G(\omega)|^2$. For the specific model we are discussing here, more details can be found in [5]. We want to report here just the main result: when a GW burst with amplitude A impinges

[†]Though it is handy to describe the effect of the WK filter in the frequency domain, yet the real implementation is better done in the discrete time domain with an equivalent autoregressive and moving average (ARMA) type filter implementation, see [2]

on the bar, the Fourier transform of the signal after the WK filter is

$$\tilde{f}(\omega) = A\sigma^2[S(\omega)]^{-1}\,|G(\omega)|^2 = A\sigma^2 S_0^{-1}\prod_k \frac{\omega^2}{(i\omega - q_k)\,(i\omega + q_k)\,(i\omega - q_k^*)\,(i\omega + q_k^*)}. \tag{3}$$

Note how the decay times of the signal changes from $\Delta\omega_k^{-1}$ to $\delta\omega_k^{-1}$.

2. Coping with non-stationary noise

It can't be blindly assumed that the noise characteristics are stable for real detectors, because of non-stationarities due to different sources, either external (maintenance activity, seismic excitations, etc.) or intrinsic (discharge in electromechanical transducer, temperature drifts, gain drift of amplifiers, etc). Three general classes of non-stationarities can be sketched, according to the speed of their evolution: the "*slow*" ones (drifts that take place in a long time compared to relaxation time,), which are easily followed by an adaptive algorithm; the "*fast*" ones, which evolve in a time of the order or less the relaxation time, so to make it impossible to estimate the noise PSD; the "*impulsive*" ones, that we may call *candidate events*. The latters are what our χ^2 test aims at (see following section).
While we cannot correct the WK filter when fast variation of the noise parameters occur (in other words, when they are ill-defined), the estimate of the noise is in principle simple when only very fast or very slow non-stationarities are present.[3]
To get rid of the slow ones we use moving averages to smooth the parameters estimates over time scales of the order of $\Delta\omega_k^{-1} \sim 1000s$, much longer than the Wiener time $\delta\omega_k^{-1} \sim 1s$. Everything happening on shorter timescales is considered a candidate event.
In other words: we call "*non-stationary noise*" what drifts slowly, candidate events everything else.
To be consistent with this paradigm, we have to measure noise parameters only when high amplitude events are absent, as they are likely to spoil our estimates. To do so, we collect the data stream in buffers, and for each of them we monitor the noise statistics: higher moments of the distribution (skewness and kurtosis), whitened noise correlation, tails of the filtered data above three times the standard deviation. We require all these statistics to be compatible with Gaussian statistic within specific thresholds, otherwise the noise parameters are kept "*frozen*".
If the candidate events rate is estremely high, maybe only a few buffers in a hundred are kept for parameters estimates. It can be argued that when Gaussian noise remains hidden for the most of the time, one can doubt it is there at all. Indeed, for AURIGA data validation it is defined a second (or *a posteriori*) level of vetoes[‡] by selecting only periods of time when excess events are scarce and the noise fits well with the model, and is consistent with Gaussian statistics.
Second level vetoes take into account also whether the noise parameters estimators

[‡] The first level vetoes are simply the record of known malfunctionig of the detector, like maintenance operations, electric power failure, earthquakes, etc.

has converged, basically by checking the level of the residual color at the output of a *whitening filter* for the detector noise.[3]

3. Event search and χ^2 test[§]

The output of the WK filter matched to δ-like signals is physically relevant as it will work for all impulsive phenomena, like supernova explosions or binary neutron stars coalescence. Such signals are described by (3) and appear in the time domain as the superposition of two decay patterns (one for each mode), both forward and backward in time, characterized by a carrier oscillation and beats between the modes. Event search in the stream of filtered data is performed by a maximum-hold algorithm, i.e. the oscillations extrema are precisely located by analytical interpolation of the carrier, and a local maximum is found per each time interval of the order of the reciprocal of the post-filtering bandwidth (that is the Wiener time $t_W = 1/\max_{k=1,2}\{\delta\omega_k\}$). This quantity represents the decay time of the filtered signal (and also the noise autocorrelation time), and it fixes a lower bound on the separability of two temporally close pulses. In other words, the Wiener filter introduces a "dead time" t_W around each event, because we have to wait that the signal has decayed into the noise before searching for next event.
Of course, when a signal has an intrinsic finite duration longer than the decay time of the Wiener filter it is erroneusly classified as a sequence of impulsive events (but they are readily rejected as spurious by the χ^2 test).
The χ^2 test is computed in the time domain[¶] from the squared residuals of the sampled noisy data after subtracting the reference template derived from (3) with the best estimated parameters of the gravitational wave burst –arrival time and amplitude. Provided that all self-consistency tests passed, this variable should follow a χ^2 statistic with $\sim$ 100 degrees of freedom. It has proved to be able to characterize electromagnetic burst interference on the readout electronics with apparent signal-to-noise ratio as low as 5 with an effective bandwith of 4Hz.[5]

References

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[§] More details on event search for the AURIGA data analysis can be found in [3,4].
[¶] A good choice for the time span over which to compute the χ^2 is something between one and three times t_W, depending on signal amplitude.