A Novel Random Counts Estimation Method for PET Using a Symmetrical Delayed Window Technique and Random Single Event Acquisition

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***Abstract*– We have developed a novel logic scheme for the estimation of the random count distribution based on a dual symmetrical delayed window technique. The solution has been applied to a dual head PET case. We have also implemented a new method for noise variance reduction in the random count distribution.**

I. Introduction

To obtain quantitative data in PET it is necessary to estimate the rate of random coincidences in the measured data in each LOR to obtain the sum of the true and scattered coincidences count rate.

Any random count correction technique consists in estimating the distribution of the LOR's generated by random counts [1]. The technique usually implemented for random correction is based on the delayed window technique. Alternatively, the random coincidences are estimated from singles count rate. The estimated random distribution may be subtracted from the prompt signal on-line, or stored as a separate sinogram or planogram for later processing. For example this distribution can be either subtracted from the measured data or used in a MLEM algorithm. In the standard delayed window technique both signals from the detectors involved in the "random" event are acquired. In systems where an event is acquired only when a coincidence is detected (singles are not acquired) some timing constraint should be applied to the position signals (e.g. Anger signals) from the PMT's (figure 1):

- To acquire the signals from the modules involved in a prompt coincidence a time delay $\Delta T_{acq.}$ should be applied to the position signals to wait for the system to be ready for the coincidence trigger.
- The time delay $\Delta T_{uncorr.}$ should be larger than the dead time (DT_{CFD}) of the constant fraction discriminator (if applicable) to avoid statistical biases on the delayed event distribution (e.g. if $\Delta T_{uncorr.} < DT_{CFD}$ no random coincidences can occur after a prompt coincidence).
- To acquire the signals from the "delayed" side an additional $\Delta T_{sign.}$ should be summed to $\Delta T_{uncorr.}$ to delay the position signal, i.e., $\Delta T_{sign.} > \Delta T_{uncorr.} + \Delta T_{acq.}$.

The first two constraints are also valid for the acquisition of prompt coincidences while the latter is valid for the acquisition of delayed coincidences only. In general minimizing $\Delta T_{sign.}$ means minimizing the pile-up and dead time.

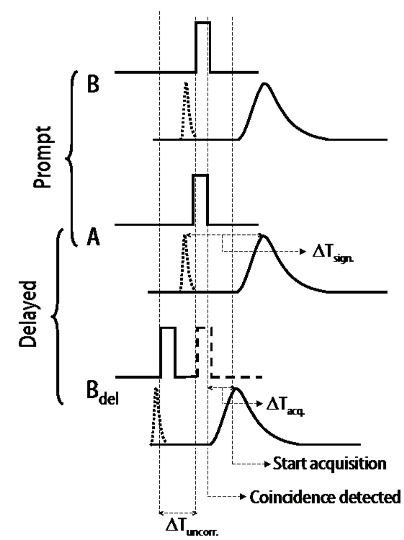

Fig. 1. Timing scheme for the standard acquisition of the Anger signals in a prompt and in a delayed (A vs. B_{del}) coincidence (opposing detectors A and B are considered). To be able to acquire B side signals in a delayed coincidence $\Delta T_{sign.}$ should be larger than $\Delta T_{uncorr.} + \Delta T_{acq.}$.

With the delayed window technique timing signals from one detector are delayed by a time significantly greater than the coincidence resolving time τ of the circuitry (figure 1). A random LOR is measured when a delayed coincidence is detected, i.e., a delayed time signal in one detector is in time coincidence with a non delayed signal in a second detector. This technique has the advantage of a low systematic error (the random counts are measured by the same acquisition chain of the coincidence events with the same dead time) but the random distribution is usually affected by strong noise due to the relatively low statistics [2]. This noise also affects the reconstructed image. In the singles-based technique the

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random count rate in each LOR connecting element *i* on head A and element *j* on head B is given by [3]:

$$r_{ij} = c_i \cdot c_j \cdot 2\tau \qquad (1)$$

Where c_i and c_j are the singles count rate in elements *i* on head A and *j* on head B, respectively and τ is the width of the coincidence time window. Due to the high count rate statistics for the singles this method is usually characterized by a lower noise but it can be subjected to a systematic error due to the a-priori estimation of τ.

II. Description of the New Method

With the delayed window method the determination of a random line-of-response (LOR) consists in measuring both LOR vertices, i.e., the points of interaction of the single events both in the delayed and non-delayed side. In this case the two detectors involved in the delayed coincidence are triggered to acquire the position signals, when a delayed coincidence is detected.

In principle, the correlation between the single events generating the delayed coincidence and those used for the random LOR is not required. Any randomly chosen single counts can be used for the generation of a random LOR. The only limitation is given by the fact that the random count rate for each LOR should be measured on-line in the exact conditions due to the time dependency of the random rate distribution.

With this assumption, to avoid the problem of acquiring the signals produced by the event in the delayed side, we have followed a different approach for estimating a random LOR.

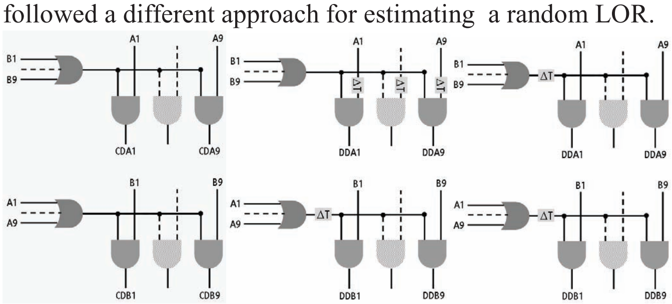

Fig. 2. Implementation of the new delayed window approach in a 9 vs. 9 modules dual head system. A*i* and B*j* signals are the timing signals from modules *i* and *j* on A and B side, respectively. *Left*: trigger scheme for a prompt coincidence. The acquisition of signals of a module *i* of the A side is triggered by CDA*i* when a logic AND between A*i* and the OR of all the B side timing signals is detected. Simultaneously, the acquisition of the module *j* on the B side is triggered by the corresponding signal CDB*j*. *Center*: standard delayed window scheme. Here, when a logic AND between the timing signal of a module B*j* and the delayed (by ΔT) timing signal A*j* the acquisition of signals on both A and B sides are triggered by DDA*i* and DDB*j*. *Right*: the new symmetrical delayed window approach. Here the delay ΔT is applied only at the OR of the timing signals on both sides. In this way only one side, the non-delayed, is triggered when a delayed coincidence is detected.

The implementation of the new method has been specifically designed for a dual head modular PET system we are developing, that is suitable for Positron Emission Mammography (PEM) and for in-beam PET monitoring in hadrontherapy [ ]. However, the proposed technique can be also applied in more general cases. In our case the PEM comprises up to 9 + 9 modules arranged in a 3×3 matrix for each head. The electronics system is designed so that the 9 modules of one head are in time coincidence with the 9 modules of the other head.

Figure 2 shows the coincidence schemes for prompt coincidences and delayed coincidences in both the standard and the new approach. In the new scheme, to measure the random counts, we have symmetrized the system by generating a delayed timing signal on both sides (figure 2, right). The logic OR of the delayed signals on one side is in coincidence with the non-delayed signal of the other side and vice versa. Once a delayed coincidence is detected, only the signal on the non delayed side is acquired. In this way only a single vertex of the LOR is measured. We call this event a "random single".

Since the system is completely symmetric we can assume to measure an equal number of "random singles" on both sides. The rate of these "random singles" will be equal to the actual system random count rate. The fluctuation of the numbers of random singles on both sides will be only related to the count statistics.

Once the single random events are acquired, a random LOR can be generated by the coordinates of the first random singles available on each side.

A. *Test system setup*

For testing purposes both the standard and the new modality were implemented in a simple dual head system. Each module modules is comprised of a square 64 anodes photomultiplier tube (Hamamatsu H8500) coupled to a matrix of 23 × 23 LYSO scintillating crystals (1.9 mm × 1.9 mm × 16 mm pixel dimensions, with a 2.0 mm pitch). A multiplexed setup based on Symmetric Charge Division (SCD) resistive networks has been chosen for the readout of each individual PMT as the best compromise between performances and simplicity. The SCD resistive network reduces the 8×8 signals of each PMT to 8+8 signals. The 8+8 signals enter a passive resistive chain that further reduces the number of signals to 2 (x) + 2 (y). The signals are then pre-amplified and sent to a DAQ board [4]. The timing signal for the coincidence is generated with a constant fraction discriminator mounted on each module.

The acquisition system is based on a pair of DAQ boards plugged into a mainboard. Each DAQ board is managed by a Cyclone II FPGA (Altera Corp., San Jose CA) and converts the four signals from the PMT's with four 12 bit ADCs. The results of the conversion are stored in an interfacing FIFO accessible by the mainboard. The mainboard hosts the two DAQ boards and an Opal Kelly XEM3005 board for system management and data transfer to PC. The XEM3005 is a fast prototyping module based on a Spartan-3E FPGA (Xilinx Corp., San Jose CA) and an EZ-USB FX2LP USB2.0 controller. The module is capable of on-line reconfiguration thus allowing seamless firmware interchange and feature exploration. At the current state, this system can work at an acquisition count rate up to 1 MHz. The coincidence network is simplified here since only two modules are used. The prompt and delayed coincidences are coded with three signals indicating the detection of: CD ≡ A AND B (prompt); DDA ≡ A_{del} AND B (delayed A); DDB≡A AND B_{del} (delayed B) as shown in figure 3. These signals are sent to the decoder inputs of the FPGA. The decoder send triggers to the required the DAQ boards and marks data with the appropriate flag.

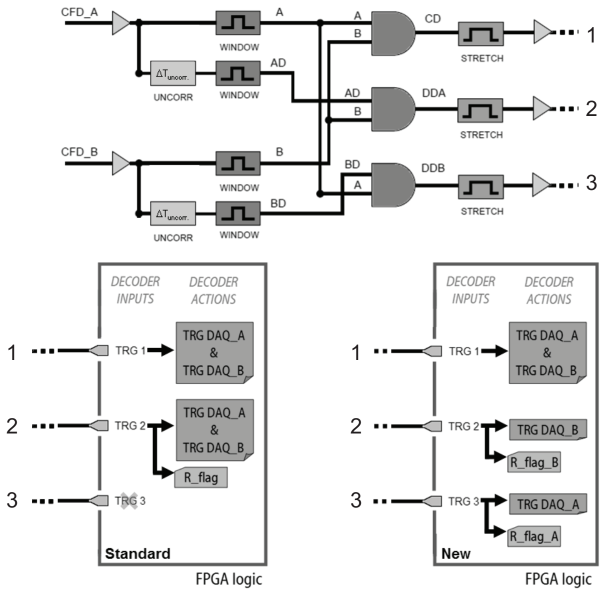

Fig. 3. Implementation of the new delayed window approach in a 1 vs. 1 modules dual head system. In this case the coincidence network scheme is fixed (top). The outputs of the coincidence network are connected to the inputs of the FPGA as numbered. The switching between the standard (bottom left) and the new (bottom right) delayed window approach is coded in the FPGA logic.

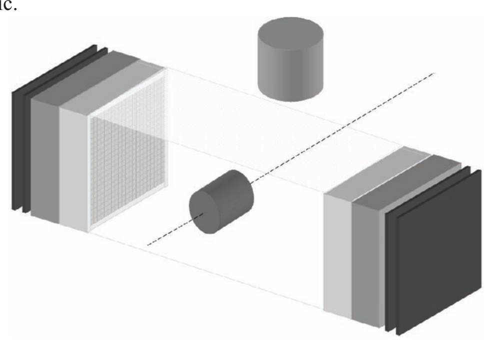

Fig. 4. Arrangement of the detector heads and the distribution of the sources in the FOV. Head-to-head distance is 14 cm. A cylinder approximately 2.5 cm × 2.5 cm ∅ filled with a ^{18}F solution is positioned along the scanner axis in the first half of the FOV. Total activity in this cylinder is approximately 100 μCi. A larger cylinder mimicking an external background source is positioned outside the FOV, slightly off-axis. The total activity in the background is about 1 mCi.

B. Experimental measurements

To make a comparison between the two modalities we have measured the distribution of both prompt and random counts. A positron emitting source (a cylinder filled with ^{18}F) is positioned along the scanner axis, while an additional high activity source is positioned outside the FOV to produce only random counts (figure 4). Two measurements were performed, one with the standard and one with the new delayed window modality. The two acquisitions were obtained in the same activity and source distribution conditions. For this measurement the width of the coincidence timing window was set to 6 ns.

C. Noise reduction technique with double blind random distribution estimation

The random distribution obtained with the delay window technique is usually affected by a strong noise due to the low counting statistics. Several techniques have been proposed for random variance reduction in PET [2]. To this aim, we have applied a new technique for random noise reduction. The implementation presented here is specifically dedicated to dual head PET systems where all of the detector elements on one side are in time coincidence with all of the opposing head elements. In this case the method does not require any "a priori" estimation of τ and/or single count rate measurement, being directly measured from random count rates.

In general, the expected random count rate in LOR connecting element i on head A and element j on head B is given by equation (1). When all of the elements i on head A are in coincidence with all of the elements j on head B we can define:

$$R_j = \Sigma_i r_{ij} = \Sigma_i c_i \cdot c_j \cdot 2\tau = C_A \cdot c_j \cdot 2\tau \quad \text{and}$$
$$R_i = \Sigma_j r_{ij} = \Sigma_j c_j \cdot c_i \cdot 2\tau = C_B \cdot c_i \cdot 2\tau$$

Where R_j and R_i are the "double blind random distributions", i.e., the spatial distribution of the single counts involved in a delayed coincidence (random single) on heads B and A, respectively, while C_A and C_B are the total singles count rate on head A and B, respectively.
Then:

$$R_i \cdot R_j = C_B \cdot c_i \cdot 2\tau \cdot C_A \cdot c_j \cdot 2\tau =$$
$$=(c_i \cdot c_j \cdot 2\tau) \cdot (C_A \cdot C_B \cdot 2\tau) = r_{ij} \cdot R$$

Hence, n_{ij} can be written in terms of R_i, R_j and R as:

$$r_{ij} = (R_i \cdot R_j) / R \qquad (2)$$

That is, r_{ij} can be fully estimated from the measured random count distributions because the estimation of C_A, C_B and τ, is not necessary since R (=Σ_{ij} r_{ij}) is the total random count rate. Generally, R_i and R_j can be estimated from directly measured $r_{ij}^{meas.}$ values.

A potential advantage of the innovative approach resides on having the double blind distributions R_i and R_j readily available by analyzing the distribution of the random singles of the two heads separately.

III. RESULTS

Prompt and delayed coincidence (random) data was obtained with the experimental setup shown in figure 4, with both the standard and the new delayed window modalities. Prompt and random data were sorted into planograms using a 350-850 keV energy window as shown in figure 5.

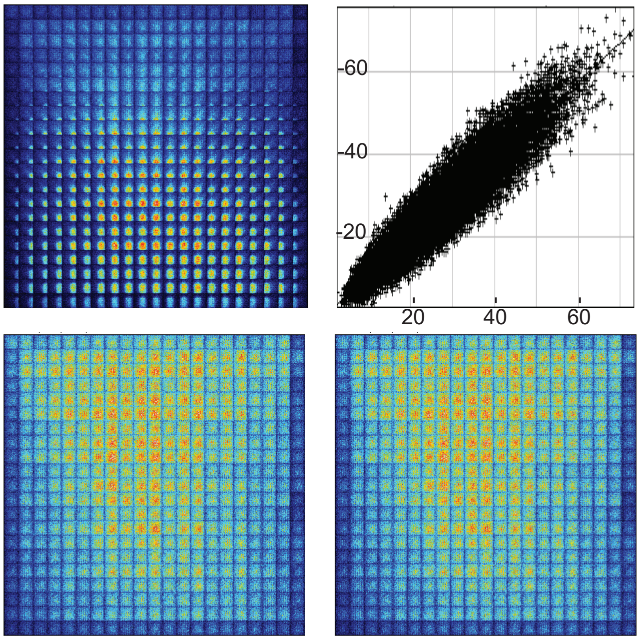

Fig. 5. Top left: full planogram of prompt coincidence data obtained with the dual head system with the source distribution described in figure 4. Energy window is 350-850 keV. Planograms are not normalized for LOR efficiency. Bottom row: planograms of delayed coincidence data for the standard (left) and new (right) methods. Top right: scatter plot composed of the LOR values of the new (*x*) and the standard (*y*) delayed coincidence planograms showing the correlation between the two planograms.

To obtain the image of the planogram as shown in figure 5, the number of counts in each LOR connecting a pixel *i* of head A and all of the N×N pixels of head B, are represented in a N×N matrix. Such matrices are then assembled in a N×N montage to obtain a larger (N×N) × (N×N) matrix showing the counts in each of the possible LOR. Here, LOR counts are not normalized for the LOR efficiency. As shown in figure 5, for the proposed source distribution, prompt and delayed counts distributions are well separated while the delayed coincidence distribution (bottom left) is very similar to that obtained with the new method (bottom right). To demonstrate the equivalence of the two planograms an image correlation plot was produced (figure 5, top right). Data show a good correlation being the Pearson product-moment correlation coefficient equal to 0.94. To reduce the noise level a 2×2 binning is applied to the planograms prior the correlation analysis.

To evaluate the effectiveness of the proposed noise reduction technique we have performed a additional measurement where the source distribution shown in figure 4 is replaced with a planar phantom (6 cm × 6 cm, 3 mm thick) filled with ^{18}F and placed in the mid plane between the two detectors, parallel to the detectors surface, thus occupying all of the possible LOR. For this measurement only the new random estimation technique is implemented.

Figure 6, left, shows the planogram obtained from the delayed window coincidences, where the number of counts in each LOR is obtained as usual. Equation (2) was applied to the R_i and R_j distribution directly obtained by analyzing the two heads separately. In this way, the noise reduced random distribution planogram was obtained (figure 6, right).

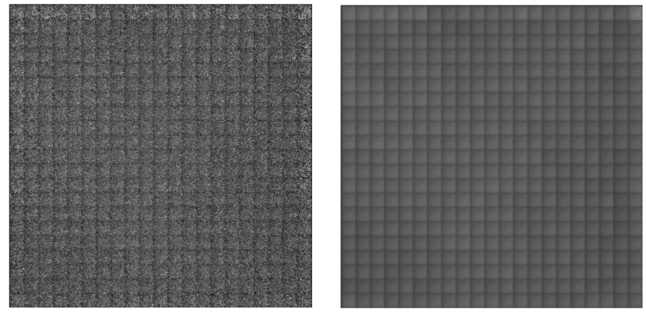

Fig. 6. Comparison of the delayed coincidence planograms obtained as directly measured (left) and by applying equation (2) (right). Planograms are not normalized for LOR efficiency.

Considering σ_N/N as the estimation of the noise in the planogram, where N is the value in each LOR and the σ_N is the standard deviation of the same value estimated in an uniform region of the same planogram, we have measured a noise reduction factor of 12.5.

IV. Conclusions

We have introduced a new random counts estimation method that eliminates some technical constraints in the design of some PET acquisition architectures and we have demonstrated that it is as accurate as the standard delayed window technique. The new solution allows a more flexible implementation and introduces no additional dead time.

We have also applied a new technique that strongly reduces the noise in the estimated random distribution. This method does not require any single counts measurements or a-priori estimation of the width (τ) of the coincidence timing window.

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