# Possible Conservation of the $\boldsymbol{K}$-Quantum Number in Excited Rotating Nuclei 

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#### Abstract

The $\gamma$ cascades feeding into low- $K$ and high- $K$ bands in ${ }^{163} \mathrm{Er}$ are investigated analyzing variances and covariance of the spectrum fluctuations. From a large data set of $10^{9}$ triple coincidences, $\gamma-\gamma$ coincidence spectra gated by resolved low-lying rotational bands are analyzed. Low- $K$ bands are found to be fed by a much larger effective number of cascades than high- $K$ bands. The covariance between pairs of gated spectra shows that the cascades feeding low- $K$ bands are different from those feeding the high- $K$ bands. The persistence of the $K$-selection rules for the excited rotational bands within the angular momentum region $30 \hbar \leq I \leq 40 \hbar$ is suggested as explanation.


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In well deformed rare earth nuclei, up to about 20 individual rotational bands are now identified, yielding information on the interplay between rotation, superfluity, and quantum numbers of the intrinsic configurations. Additional information about rotation at thermal energy higher than that covered by the resolved bands has been obtained recently through the study of the overall shape [1] and fluctuations [2] of the unresolved quasicontinuum in coincident $\gamma-\gamma$ energy spectra. These studies have shown unambiguously that the rotational transition strength gets progressively fragmented with increasing excitation energy above yrast. This fragmentation of the rotational strength is a manifestation of a damping of the rotational motion [3,4].

The $\gamma$ decay among the cold discrete rotational bands is governed by selection rules associated with the quantum numbers of the intrinsic structure. However, at excitation energies high above the yrast line, such selection rules may be lost. Here the very large level density implies a strong fragmentation of the rotational decay [5], and the fragmented rotational states might have intrinsic structures that are a random combination of the available configurations [6].

In the present Letter a first attempt is made to investigate the validity of selection rules for unresolved rotational bands, and for $\gamma$-ray transitions in the damped region. We shall concentrate the discussion on the projection $K$ of the angular momentum on the nuclear symmetry axis. Recently, the persistence of the $K$-quantum number for neutron resonance states has attracted renewed interest $[7,8]$. Our study addresses a complementary problem, namely, the $K$-quantum number for nuclear states at higher rotational frequency and lower thermal energy.

In ${ }^{163} \mathrm{Er}$ two deformation aligned bands with $K=\frac{19}{2}$ have been identified recently, denoted $K 1$ and $K 2$ in Ref. [9]. (The interpretation in [9] of the nature of $K 2$ is changed with the present data set [10].) The two bands have an excitation energy above yrast varying between 0.8 and 1.4 MeV , thus extending into the region where rotational damping should be dominating. This makes the ${ }^{163} \mathrm{Er}$ nucleus an attractive case to study the interaction between high $K$ and low $K$ and in more detail the problem of rotational band mixing (rotational damping) and its configuration dependence. The present study is based on a new measurement of triple coincidences of $\gamma$ transitions emitted by ${ }^{163} \mathrm{Er}$. Count fluctuations of $E_{\gamma 1} \times$ $E_{\gamma 2}$ spectra gated by selected low lying rotational bands are analyzed. For the first time, convariance between spectra derived from different gates are analyzed. The covariance information provides a measure of how the cascades feeding into different selected bands are similar. The high- $K$ bands are found to be preceded by cascades originated from rotational bands that are different from those feeding into low- $K$ bands.

Threefold and higher folds of $\gamma$-ray coincidences were collected using the GA.SP multidetector array at the Tandem Accelerator Laboratory of Legnaro (Italy). The detector system consists of 40 Compton suppressed Ge detectors, surrounding a $4 \pi$ multiplicity filter and $\gamma$-ray calorimeter of 80 BGO crystals. The reaction employed was ${ }^{18} \mathrm{O}+{ }^{150} \mathrm{Nd}$ with a bombarding energy $E_{\text {beam }}=$ 87 MeV leading to the population of ${ }^{162,163} \mathrm{Er}$ as the main evaporation residua. A total of $10^{9}$ triplefold and higher fold events were collected. In order to minimize the contribution from delayed $\gamma$ rays and neutron induced events an energy dependent gate on the time of flight measured relative to the multiplicity filter
was applied. A partial separation of the ${ }^{163} \mathrm{Er}$ and ${ }^{162} \mathrm{Er}$ channels was obtained by placing appropriate gates on the coincidence fold and sum energy distributions measured by the multiplicity filter. Making use of the RADWARE programs [11] a large number of new transitions were identified and the extended level schemes of ${ }^{162,163} \mathrm{Er}$ will be discussed in Ref. [10].

The excitation energy relative to a rigid rotor of the most intensely observed bands of ${ }^{163} \mathrm{Er}$ is given in Fig. 1, as a function of spin $I$. For the present analysis, the data are sorted into a number of two-dimensional $E_{\gamma 1} \times$ $E_{\gamma 2}$ spectra gated by low lying transitions in bands of both low- $K$ and high- $K$ quantum numbers, as indicated in Fig. 1. Since $K 2$ interacts with $K 4$ at $I \approx 21.5 \hbar$ $\left(E_{\gamma} \approx 600 \mathrm{keV}\right)$ the feeding into $K 2$ is mainly from the more strongly populated $K 4$ [10]. Each spectrum is then corrected for the background under the gate-selected peaks, by subtracting properly normalized background spectra chosen from narrow gates around the peaks. The Compton and other uncorrelated events are reduced by the standard COR treatment [12].

The fluctuations of counts in each channel of these 2D spectra, expressed as variance and covariance, are evaluated by the program statFit [4] and stored in 2D spectra. One additional option is applied: All pairs of


FIG. 1. Excitation energy vs spin for the most strongly populated bands in ${ }^{163} \mathrm{Er}$, relative to a rigid reference with $\mathscr{F}_{2}=63.3 \mathrm{MeV}^{-1} \hbar^{2}$. The gates used for the low lying parts of the cascades are shown by filled symbols. Nilsson labels are valid at low spin only.
resolved transitions are removed in the triangular sector $E_{\gamma 1} \geq E_{\gamma 2}$ with the proper intensity from the $\gamma-\gamma$ spectra, before the fluctuations are extracted, because the fluctuations are severely affected by the low lying intense transitions [4]. Since each $\gamma$ cascade on the average contributes one count in each $4 \hbar^{2} / \mathfrak{F}$ interval, the statistical moments are evaluated over sectors of $4 \hbar^{2} / \mathfrak{F} \times 4 \hbar^{2} / \mathfrak{F}$, corresponding to $60 \mathrm{keV} \times 60 \mathrm{keV}$ intervals for rare earth nuclei around ${ }^{163} \mathrm{Er}$.

The correlations in fluctuations between two spectra are expressed by the covariance of counts, defined as

$$
\begin{align*}
\mu_{2, \mathrm{cov}}(A, B) \equiv & \frac{1}{N_{\mathrm{ch}}} \sum_{j}\left[M_{j}(A)-\tilde{M}_{j}(A)\right] \\
& \times\left[M_{j}(B)-\tilde{M}_{j}(B)\right] \tag{1}
\end{align*}
$$

where $M(A)$ and $M(B)$ refer to spectra gated by transitions from two different bands, $A$ and $B$. The sum is over a region spanning the $N_{\text {ch }}$ channels (in this case 169) in the two-dimensional $60 \mathrm{keV} \times 60 \mathrm{keV}$ window, and $\tilde{M}$ denotes an average spectrum (which in our case is found by the routine STATFIT as a numerical smoothed third order approximation to the 2 D spectrum). To normalize the covariance and thereby determine the degree of correlation between the two spectra, the correlation coefficient $r(A, B)$ is calculated:

$$
\begin{equation*}
r(A, B) \equiv \frac{\mu_{2, \operatorname{cov}}(A, B)}{\sqrt{\left[\mu_{2}(A)-\mu_{1}(A)\right]\left[\mu_{2}(B)-\mu_{1}(B)\right]}} \tag{2}
\end{equation*}
$$

Here $\mu_{2}$ denotes the second moment defined for the same region $N_{\mathrm{ch}}$, related to the expression for the covariance by $\mu_{2}(A)=\mu_{2, \operatorname{cov}}(A, A)$. The first moment $\mu_{1}$ is the average of $M$ over the region $N_{\mathrm{ch}}$. The subtraction of the first moments in the denominator of (2) corrects for the direct contribution to $\mu_{2}$ from counting statistics, which is linear in the number of counts. The more interesting fluctuations are due to the nature of the finite number of transitions available to each cascade, and their contribution to $\mu_{2}$ is quadratic in the number of events.

Typical $\mu_{1}, \mu_{2}$, and $\mu_{2, \text { cov }}$ spectra obtained from the matrices gated by low- $K$, high- $K$ and yrast transitions are shown in Fig. 2. One observes that the ridgevalley structure, namely, the typical pattern characterizing coincidence spectra of $\gamma$ transitions among rotational bands [1], is strongly enhanced in the fluctuation spectrum $\mu_{2}$ relative to the original average intensity spectrum $\mu_{1}$ for both low- $K$ and high- $K$ bands. On the other hand, a remarkable difference exists between the low- $K$-low$K$ covariance, which displays pronounced ridges, and the low- $K$-high- $K$ covariance, which varies in a much more random way.

From the fluctuation spectra shown by the examples in Fig. 2, we first extract the effective number of decay paths, which eventually feed into the gate-selected band. The number of decay paths $N_{\text {path }}^{(2)}$ having two $\gamma$ transition with energy lying in a chosen $60 \mathrm{keV} \times 60 \mathrm{keV}$ window


Fig. 2. Spectra of $\mu_{1}$ (left hand side) and $\mu_{2}$ (middle) obtained from $\gamma-\gamma$ coincidences gated by three different bands (named in each panel). They were obtained projecting $E_{\gamma 1} \times E_{\gamma 2}$ spectra along the $\left(E_{\gamma 1}+E_{\gamma 2}\right) / 2$ direction onto the $E_{\gamma 1}-E_{\gamma 2}$ axis, with a width of 60 keV . The average transition energy is chosen as $\left(E_{\gamma 1}+E_{\gamma 2}\right) / 2=940 \mathrm{keV}$. The right hand side shows the covariance between pairs of gated spectra, also evaluated in two dimensions, before projection. The spectra are not symmetric about 0 because on the right hand side all known discrete transitions were subtracted before applying the fluctuation analysis.
in the $\gamma-\gamma$ coincident spectrum is obtained from the following simple expression:

$$
\begin{equation*}
N_{\text {path }}^{(2)}=\frac{N}{\mu_{2} / \mu_{1}-1} \tag{3}
\end{equation*}
$$

where $N$ is the number of events in a sector $4 \hbar^{2} / \mathfrak{F} \times$ $4 \hbar^{2} / \mathfrak{F}$ corrected for the finite resolution of the detectors $[2,4]$. The superscript (2) indicates that the number of paths was derived using up to the second moment of the path detection probability. Figure 2 shows that the ridges in the spectra of $\mu_{1}, \mu_{2}$, and $\mu_{2 \text {,cov }}$ are superimposed on a background, from which they can be separated. For the valley, corrections for the fluctuations from the counting statistics of the original raw spectrum as well as from the background must be taken into account [13].

The numbers of paths obtained from the analysis of the first ridge gated by the different bands are shown in Figs. 3(a) and 3(b). They measure the effective number of discrete, yet unresolved, bands, which eventually will feed into the gate-selected band. For each of the seven cases, about 10 to 20 such unresolved bands are found to exist and the sum of these results is different than the value $\approx 40$ that was obtained from the ungated data. This fact suggests that the $\gamma$ cascades selected with the different gates have some common paths that, as it will be discussed below, will give nonzero convariance.

In contrast to the results of the ridge analysis the number of paths obtained by analyzing the valley region is found to depend significantly on the nuclear configuration. This result is shown in Fig. 3(c) together with the number of paths deduced from the ungated (total) $E_{\gamma 1} \times E_{\gamma 2}$ spectrum. As in previous studies for the same region of mass and deformation, a number of paths of the order of $\approx 10^{4}$ is found for the total spectrum. This number


FIG. 3. The quantity $N_{\text {path }}^{(2)}$ extracted from the ridge, (a) and (b), and the valley (c) analyzes as a function of $E_{\gamma}=$ $\left(E_{\gamma 1}+E_{\gamma 2}\right) / 2$, for each selected configuration. The chosen configurations are labeled in the legend.
was reproduced by a schematic calculation assuming a rotational damping width of the order of 100 keV and that the onset of rotational damping starts at $\approx 800 \mathrm{keV}$ above yrast [2]. The number of paths in the valley of spectra gated by high- $K$ bands is different than that of low- $K$ bands and about 5 times smaller. A possible explanation could be related to the fact that different $\gamma$-cascade pathways might be sampled when the gating transitions of the different bands do not correspond to the same spin (energy) intervals. However, the results corresponding to gates on transitions of low- $K$ bands at low spins [labeled A in Fig. 3(c)] and to high spins [labeled $A B C+F$ in Fig. 3(c)] are very similar and therefore do not support this explanation. Another possibility is that high- and low- $K$ states do not mix together in the region of unresolved transitions. In addition, since the number of paths in the valley is expected to be larger in the presence of rotational damping [2], the smaller number of paths in the valley of high- $K$ bands suggests a reduced mixing among high- $K$ bands as compared to that of low- $K$ bands.

The covariance $\mu_{2, \text { cov }}$ is illustrated by the two examples on the right hand side of Fig. 2. Correlation coefficients [Eq. (2)] have been extracted for the ridge for
combinations of selected configurations, and the result is shown in Fig. 4. For the valley, however, the counting statistics of the present experiment is not good enough for obtaining reliable results. The correlation coefficient measures directly how the spectrum $M(A)$ is similar to spectrum $M(B)$ and can be written in terms of the probabilities $W_{i}(A)$ and $W_{i}(B)$ of following path $i$ when gates on configurations $A$ and $B$ are imposed. Its expression was derived with a mathematical procedure equivalent to that described in Ref. [4], performing averages over random transition energies,

$$
\begin{equation*}
r(A, B) \equiv \frac{\sum_{i} W_{i}(A) W_{i}(B)}{\sqrt{\left[\sum_{i} W_{i}(A)^{2}\right]\left[\sum_{i} W_{i}(B)^{2}\right]}} \tag{4}
\end{equation*}
$$

Since the path probabilities are all positive, $r$ can only attain positive values, in the scheme of averaging over transition energies. One should be aware, however, that with unresolved spectra it is not possible to decide whether the correlations arise from the sharing of decay paths, or whether they alternatively are due to energy correlations between different decay paths. The available data on the distribution of transition energies of resolved rotational bands in the rare earth region support the present assumption of random transition energies, leading to the expressions of equations [Eqs. (3) and (4)].

The extracted values of the correlation coefficient for coincident transitions on the ridges are positive for similar configurations (low $K$ with low $K$, and high $K$ with high $K$ ) and approximately zero for the combination low $K$ with high $K$, showing that there are basically no cross transitions between the $\approx 15$ bands feeding high- $K$ bands [Fig. 3(a)] and the $\approx 15$ bands feeding low- $K$ bands


FIG. 4. The correlation coefficients extracted from the first ridge are plotted as a function of $E_{\gamma}=\left(E_{\gamma 1}+E_{\gamma 2}\right) / 2$. The data labeled "low- $K-$ low- $K$ " and "low- $K$-high- $K$ " refer to correlation between states with the same parity ( $\pi=+, \pi=$ + ), while the "high- $K-$ low- $K$ " points refer to correlations between $(K 1, \pi=+)$ and $(K 2, \pi=-)$ states. The error bars reflect the uncertainties associated to the quantities appearing in expression (2) and were evaluated assuming random transition energies.
[Fig. 3(b)]. This result can be interpreted as an effect of the persistence of a strong $K$ selection rule governing the rotational bands forming the ridges. These bands are characterized by energy up to $\approx 0.8 \mathrm{MeV}$ above yrast [2] and by $\gamma$-transition energies corresponding to angular momentum in the $(30-40) \hbar$ interval.

In summary, the results of the present new covariance analysis of the ridges and of the fluctuation analysis of the valley show consistently a different behavior of lowand high- $K$ bands. In particular, the covariance of the ridges indicates that high- $K$ bands are fed in the last cascade steps by transitions of high- $K$ bands, while the fluctuations in the valley, probing a warmer part of the decay cascades, suggest that high- $K$ and low- $K$ states may not mix together and that the band mixing due to residual interaction (rotational damping) is smaller for high- $K$ bands. Based on the fact that we obtain for the valley of low- $K$ bands the same results when we gate on low and on high spin transitions it does not seem very likely that the different findings for low- and high- $K$ bands could be due to possible changes in $\gamma$-cascade pathways induced by differences in spins of the selected $\gamma$ transitions.

The possible persistence of the $K$-quantum number suggested by this work cannot be explained by simply considering the Coriolis interaction effects that reproduce rather well the low energy spectra. Similar conclusions on $K$ conservation at high excitation energies were obtained from the analysis of neutron resonances [7], although that work motivated controversial discussions [14]. If indeed $K$ is conserved at high spins and internal energy, the reason is not yet understood and our new covariance analysis may provide a general tool to address experimentally this question when many decay paths are available. Furthermore, other aspects as, for example, the mixing between superdeformed and normally deformed states in the cascades which precede the feeding into superdeformed discrete bands, could be studied with this new technique.

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