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Resonance frequency shift in a cavity with a thin conducting film near a conducting wall

C. Braggio [a], G. Bressi [b], G. Carugno [c], A.V. Dodonov [d], V.V. Dodonov [e,*], G. Galeazzi [f], G. Ruoso [f], D. Zanello [g]

[a] *Dipartimento di Fisica, Università di Ferrara and INFN, Via del Paradiso 12, 44100 Ferrara, Italy*
[b] *INFN, Sezione di Pavia, Via Bassi 6, 27100 Pavia, Italy*
[c] *INFN, Sezione di Padova, Via F. Marzolo 8, 35131 Padova, Italy*
[d] *Departamento de Física, Universidade Federal de São Carlos, Via Washington Luiz, Km 235, São Carlos 13565-905, SP, Brazil*
[e] *Instituto de Física, Universidade de Brasília, Caixa Postal 04455, 70910-900 Brasília, DF, Brazil*
[f] *INFN, LNL, Viale dell'Università 2, 35020 Legnaro, Italy*
[g] *INFN, Sezione di Roma, Piazzale A. Moro 2, 00185 Roma, Italy*

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Abstract

We show that a very thin conducting film (whose thickness can be much smaller than the skin depth), placed nearby a wall of an electromagnetic cavity, can produce the same shift of the resonance frequency as a bulk conducting slab, provided the displacement of the film from the wall is much bigger than the skin depth. We derive a simple analytical formula for the frequency shift and compare it with exact numerical calculations and experimental data.

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1. Introduction

The problem of shift of resonance frequencies of electromagnetic cavities due to small variations of geometry and material properties of the walls or internal parts of the cavities was considered long ago [1–3], and some results can be found in monographs or textbooks on classical electrodynamics [4,5]. Various applications were considered, e.g., in Refs. [6–10]. Our interest to the problem of frequency shift in cavities originated from the planned experimental verification of the dynamical Casimir effect (DCE) [11,12], i.e., the phenomenon (not observed up to now) of photon creation from the initial vacuum state due to the motion of boundaries.

The main idea of the experiment is to simulate the motion of the cavity wall by means of periodical creation of an effective electron–hole "plasma mirror" at the surface of a thin semiconductor slab, attached to a cavity wall and illuminated by a sequence of short laser pulses. Another scheme of simulating the dynamical Casimir effect, where periodical changes of the cavity eigenfrequency can be achieved by changing the surface impedance of a *superconducting film* illuminated by laser pulses, was proposed recently in [13].

The feasibility of experiment [11] depends (beside many other factors) on the thickness G of a highly conducting "plasma mirror" which must be created. At first glance, one could suppose that this thickness must exceed the skin depth

$$\delta_s = c/\sqrt{2\pi\sigma\omega} \tag{1}$$

corresponding to the conductivity σ of the "mirror" (we use the CGS units; ω is the frequency of the electromagnetic field in

* Corresponding author.
E-mail addresses: caterina.braggio@lnl.infn.it (C. Braggio), vdodonov@fis.unb.br (V.V. Dodonov).

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rad/s). On the other hand, it is clear that $G \sim \alpha^{-1}$, where α is the absorption coefficient of the laser beam inside the semiconductor slab. For typical values $\alpha \sim 10^3$–10^4 cm^{-1} [11] we have $G \sim 10$–1 µm, that is of the same order of magnitude as the skin depth of good metals (such as Cu, which has $\sigma \sim 5 \times 10^{17}$ s^{-1} or, equivalently, the specific resistance $\rho \sim 2$ µΩ cm) for frequencies belonging to the GHz band, which correspond to cavities with dimensions of a few centimeters. To create a film of such a thickness with metallic concentration of carriers ($n \sim 10^{22}$ cm^{-3}) in a semiconductor material with energy gap $E_g \sim 1$ eV and total area $S \sim 10$ cm^2 [12] one needs laser pulses with energy $W \sim n\delta_s S E_g > 1$ J, which is, of course, unrealistic.

Fortunately, these naive estimations are not correct. As was shown in [11], one needs much smaller (by several orders of magnitude) energy to create a conducting film, whose electrodynamical properties are indistinguishable from the properties of metallic mirrors. Another important (and perhaps unexpected) discovery is the fact that a thin film behaves as an ideal mirror (in the sense of the value of the frequency shift), even if its thickness is several orders of magnitude *smaller* than the skin depth, provided the film is displaced from the cavity wall by the distance D which is much bigger than δ_s (being, nonetheless, much smaller than the dimensions of the cavity or resonance wavelength). The frequency shift depends, as a matter of fact, not on the ratios G/δ_s and D/δ_s as independent parameters, but on their product $A = 2GD/\delta_s^2$. The film becomes a “mirror” if $A > 1$. This phenomenon seems to be not reported in the available literature [1–10]. Therefore the aim of our Letter is to give a simple theory and to support it by experimental data.

2. Calculation of the frequency shift

The formulas for the frequency shift given in [1–5] (and used, e.g., in [8–10]) cannot be applied directly to the case concerned, because they were derived under the assumption that changes of parameters or fields are small, while in our case the dielectric constant inside a thin film can vary by many orders of magnitude. Nonetheless, the frequency change is small due to presence of other small parameters, and for cavities with simple geometrical shapes this shift can be calculated by solving the field equations.

We consider a cylindrical cavity of length L with an arbitrary cross section and the axis parallel to the x-direction, supposing that the main part of the cavity is empty, except for a thin slab of a thickness $D \ll L$, which consists of two parts: a conducting thin film of thickness G with a complex dielectric constant $\tilde{\varepsilon}_s = \varepsilon_1 + i\varepsilon_2$ (which obey the conditions $\varepsilon_2 \gg 1$ and $\varepsilon_1 \sim 1$) and a transparent background of thickness $D - G$ with a real dielectric constant ε_b. We assume that the dielectric permeability of the slab does not depend on the transverse coordinate $\mathbf{r}_\perp$. Thus the dielectric function inside the cavity depends on the longitudinal coordinate x as follows,

$$\varepsilon(x) = \begin{cases} 1 & \text{for } -L < x < 0, \\ \tilde{\varepsilon}_s & \text{for } 0 < x < G, \\ \varepsilon_b & \text{for } G < x < D. \end{cases} \tag{2}$$

Let us suppose first that the cavity walls are made from an ideal conductor, so that we can use the ideal boundary conditions $E_t|_{\text{wall}} = 0$ for the tangential component of the electric field.

We consider the fundamental TE mode with the only component of the electromagnetic field parallel to the slab surface. It satisfies the three-dimensional scalar Helmholtz equation (we assume that the magnetic permeability of the slab is the same as in the vacuum)

$$\Delta E + (\Omega/c)^2 \varepsilon(x) E = 0, \tag{3}$$

where Ω is the field eigenfrequency to be found. The solution to Eq. (3) can be factorized as $E(x, \mathbf{r}_\perp) = \psi(x)\Phi(\mathbf{r}_\perp)$, where the function $\Phi(\mathbf{r}_\perp)$ obeys the two-dimensional Helmholtz equation

$$\Delta_\perp \Phi + k_\perp^2 \Phi = 0, \qquad \Phi|_{\text{wall}} = 0, \tag{4}$$

so the problem is reduced to solving the one-dimensional Helmholtz equation

$$\psi'' + \left[(\Omega/c)^2 \varepsilon(x) - k_\perp^2\right]\psi = 0 \tag{5}$$

with the boundary conditions

$$\psi(-L) = \psi(D) = 0. \tag{6}$$

In the case of the dielectric function (2), the function $\psi(x)$ can be written as follows,

$$\psi(x) = \begin{cases} F_1 \sin[k(x + L)] & \text{for } -L < x < 0, \\ F_2 \sin(k_2 x + \phi_2) & \text{for } 0 < x < G, \\ F_3 \sin[k_3(x - D)] & \text{for } G < x < D, \end{cases} \tag{7}$$

where

$$k_2^2 = \left(k^2 + k_\perp^2\right)\tilde{\varepsilon}_s - k_\perp^2, \qquad k_3^2 = \left(k^2 + k_\perp^2\right)\varepsilon_b - k_\perp^2, \tag{8}$$

and the constant coefficient k is related to the field eigenfrequency Ω as

$$\Omega = c\left(k^2 + k_\perp^2\right)^{1/2}. \tag{9}$$

The value of the longitudinal wave number k can be found from the equations which are the consequences of the continuity conditions for the functions $\psi(x)$ and $\psi'(x)$ at the surfaces $x = 0$ and $x = G$:

$$\tan(kL) = \frac{k}{k_2}\tan(\phi_2), \tag{10}$$

$$\tan(k_2 G + \phi_2) = \frac{k_2}{k_3}\tan\left[k_3(G - D)\right]. \tag{11}$$

Solving Eq. (11) with respect to phase ϕ_2 and putting the solution into Eq. (10), we arrive at the equation

$$\tan(kL) = \frac{k}{k_2}\,\frac{k_2\tan[k_3(G - D)] - k_3\tan(k_2 G)}{k_3 + k_2\tan[k_3(G - D)]\tan(k_2 G)}. \tag{12}$$

In the most general case, the transcendental algebraic equation (12) should be solved numerically. However, in the case of a *thin* slab, it is possible to obtain a simple approximate explicit analytical solution. For this purpose, we make the following transformations. First, we introduce dimensionless parameters

$$\Delta = 2D/\lambda, \qquad g = G/D, \qquad \eta = \lambda/(2L), \tag{13}$$

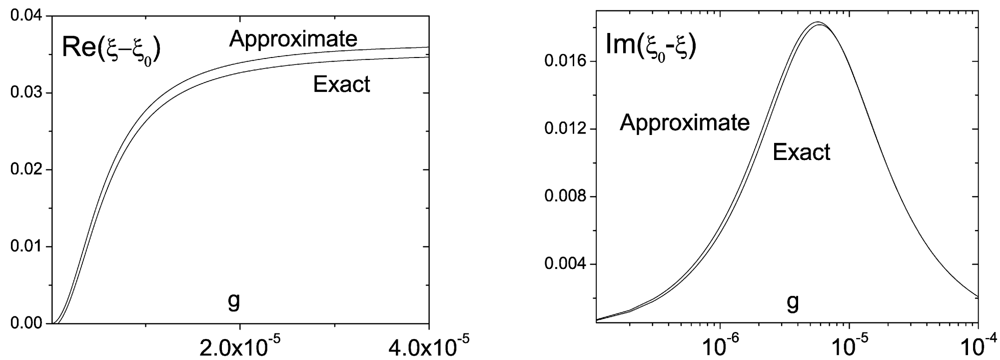

Fig. 1. The real and imaginary parts of the shift of the longitudinal wave number $\xi - \xi_0$ versus the dimensionless thickness of the conducting layer $g = G/D$. Note that $\mathrm{Re}(\xi - \xi_0) > 0$, whereas $\mathrm{Im}(\xi - \xi_0) < 0$.

where λ is the wavelength corresponding to the fundamental eigenfrequency of the ideal cavity of length L, i.e.,

$$\lambda = 2\pi\left[(\pi/L)^2 + k_\perp^2\right]^{-1/2}, \quad k_\perp = \frac{2\pi}{\lambda}\sqrt{1-\eta^2}.$$

Obviously, $g < 1$ and $\eta < 1$, and our main assumption is $\Delta \ll 1$ (thin slab). Since the dielectric constant of the *transparent* background slab ε_b cannot be very big, $k_3 D \sim \Delta \ll 1$. Thus we can replace the function $\tan[k_3(G-D)]$ by its argument. After this simplification, the coefficient k_3 drops out, together with parameter ε_b. Now we write the longitudinal wave number k as

$$k = (\pi/L)(1+\xi). \tag{14}$$

Obviously, if $\Delta \ll 1$, then also $|\xi| \ll 1$. Therefore we write $\tan(\pi\xi) = \pi\xi$ *in the left-hand side* of Eq. (12), putting at the same time $\xi = 0$ in the right-hand side (because this side is small even for $\xi = 0$). Besides, we suppose that the conductivity of the *thin film* is high enough, so that we can neglect the parameters ε_1 and $1-\eta^2$ in the coefficient k_2, replacing it by the value $k_2(\xi = 0) = (2\pi/\lambda)\sqrt{i\varepsilon_2}$. After these simplifications we arrive at a simple formula

$$\xi = \eta\frac{\Delta(g-1) - \tan(\Delta g R)/R}{1 + \Delta(g-1)R\tan(\Delta g R)}, \tag{15}$$

where

$$R = \pi\sqrt{i\varepsilon_2}. \tag{16}$$

In the CGS units we have $\varepsilon_2 = 4\pi\sigma/\omega$, where σ is the conductivity of the film and ω is the real frequency of the field. The skin depth is given by the formula $\delta_s = c/\sqrt{2\pi\sigma\omega}$. Consequently,

$$\Delta\,\mathrm{Re}(R) = D/\delta_s, \qquad \Delta g\,\mathrm{Re}(R) = G/\delta_s. \tag{17}$$

Actually, we need the difference between (15) and the value

$$\xi_0 = -\frac{D}{D+L} = -\frac{\eta\Delta}{1+\eta\Delta} \approx -\eta\Delta, \tag{18}$$

corresponding to the empty cavity of length $D+L$. Neglecting corrections of the order of Δ^2, we obtain the following formula for the shift of the longitudinal wave number after inserting a slab with a thin conducting film:

$$\xi - \xi_0 = \eta\,\frac{\Delta g - [(\Delta R)^2(1-g)+1]\tan(\Delta g R)/R}{1 + \Delta(g-1)R\tan(\Delta g R)}. \tag{19}$$

The corresponding small shift of the resonance frequency equals

$$\chi_\Omega \equiv (\Omega - \Omega_0)/\Omega_0 = \eta^2(\xi - \xi_0). \tag{20}$$

Now let us analyze various special cases. If $g = 1$ (i.e., the conducting film is put on the cavity wall without any intermediate background), then

$$\xi - \xi_0 = \eta\left[\Delta - \tan(\Delta R)/R\right], \tag{21}$$

and we see that a reasonable frequency shift can be observed only under the condition $D/\delta_s > 1$, which seems quite natural. If $D/\delta_s \ll 1$, then the conductivity of the slab does not manifest itself, and the frequency shift is extremely small, being proportional to Δ^3. A similar dependence on the slab thickness takes place in the case of a thin slab with not very big *real* dielectric constant, considered in Ref. [14]. Using the formula

$$\tan(a+ib) = \frac{\tan(a) + i\tanh(b)}{1 - i\tan(a)\tanh(b)}, \tag{22}$$

one can easily see that the frequency shift tends to the limit value $\chi_{\max} = \eta^3\Delta$ (corresponding to the ideal conducting plate of the thickness D put on the wall) if $D/\delta_s \gg 1$.

A more interesting special case is $g \ll 1$ and $G/\delta_s \ll 1$. Then the tangent function in (19) can be replaced by its argument, and we arrive at the formulas

$$\mathrm{Re}(\xi - \xi_0) = \frac{\eta\Delta A^2}{A^2+1}, \qquad \mathrm{Im}(\xi - \xi_0) = -\frac{\eta\Delta A}{A^2+1}, \tag{23}$$

where

$$A = \varepsilon_2 g(\pi\Delta)^2 \equiv 2GD/\delta_s^2. \tag{24}$$

In Fig. 1 we show the dependence of real and imaginary parts of the shift $\xi - \xi_0$ on the relative thickness of the conducting film with respect to the total thickness of the slab $g = G/D$ for the following values of other parameters:

$$\Delta = 1/15, \qquad \eta = 0.55,$$
$$\varepsilon_b = 2, \qquad \varepsilon_1 = 10, \qquad \varepsilon_2 = 4\times 10^6.$$

These numerical values correspond to the TE_{101} mode of the rectangular cavity with dimensions 110 × 71 mm, (resonance frequency 2.5 GHz), the thickness of transparent slab $D =$ 4 mm, and resistivity of the film $\rho = 200$ μΩ cm (10 times the resistivity of bulk Pb at the room temperature). The value $g = 10^{-6}$ corresponds to $G = 4$ nm, $g = 10^{-5}$—to $G = 40$ nm and $g = 10^{-4}$—to $G = 400$ nm. We compare the approximate expressions (23) with the difference between the exact numerical solution of Eq. (12) and exact value in Eq. (18). We see that the analytical approximation is very good.

The influence of a finite (but high) conductivity of the cavity walls can be taken into account with the aid of the known formula [2–5] for the complex frequency shift

$$\delta\Omega_w \equiv \Omega - \Omega_{\rm id} = -\frac{ic}{2\sqrt{\tilde{\varepsilon}_w}}\frac{\oint |\mathbf{H}_t|^2\,dS}{\int |\mathbf{H}|^2\,dV}, \tag{25}$$

where $\tilde{\varepsilon}_w = \varepsilon_{1w} + i\varepsilon_{2w}$ is the complex dielectric constant of the material of the walls, $\Omega_{\rm id}$ is the eigenfrequency of the ideal cavity, and the magnetic field vector $\mathbf{H}(x, \mathbf{r}_\perp)$ corresponds to the ideal cavity. Eq. (25) holds if $\varepsilon_{2w} \gg 1$. The presence of the slab has a very small influence on the value of $\delta\Omega_w$, because the magnetic field remains practically the same, except for the small region occupied by the slab (only if $|\xi| \ll |\xi_0|$). Consequently, the relative change of the volume integral is of the order of Δ (actually, even smaller). The same is true for the surface integrals over all sides of the cavity, except for the side $x = L + D$ behind the slab with the area S_1. Thus the relative change of the value of the surface integral cannot exceed, by an order of magnitude, the ratio $S_1/S_{\rm tot}$, where $S_{\rm tot}$ is the total area of the cavity surface. Actually, this change is even smaller, if one takes into account the real magnetic field distribution (only a few percent for the geometry used in the experiment). Therefore we can neglect the variation of $\delta\Omega_w$. Then, using Eqs. (20) and (23), we arrive at the following formulas for the real and imaginary parts of the relative shift of the frequency $\Omega = \omega - i\gamma$ with respect to the frequency ω_0 of the empty cavity:

$$\chi \equiv \frac{\omega - \omega_0}{\omega_0} = \frac{\eta^3 \Delta A^2}{A^2 + 1}, \qquad \kappa \equiv \frac{\gamma - \gamma_0}{\omega_0} = \frac{\eta^3 \Delta A}{A^2 + 1}. \tag{26}$$

Obviously, γ/ω_0 is half the inverse quality factor of the cavity. Thus the total inverse quality factor depends on A as follows,

$$\frac{1}{Q} = \frac{1}{Q_0} + \frac{2\eta^3 \Delta A}{A^2 + 1}, \tag{27}$$

where Q_0 is the quality factor of the empty cavity.

3. Experimental results

We used a rectangular copper cavity with dimensions $112 \times 71 \times 22$ mm, so that $L + D = 11.2$ cm and $\eta = 0.54$. The measured resonance frequency of the TE_{101} cavity fundamental mode was $\nu_0 = \omega_0/(2\pi) = 2.47935$ GHz and the unloaded quality factor $Q_0 \approx 7000$ (with an accuracy δQ about 30–40). To study the problem of the frequency shift we prepared 4 mm-thick plexiglass slabs (so that $\Delta = 1/15$) and evaporated copper films of different thicknesses. The slab obtained with this procedure was set over the 71×22 mm cavity wall, where the electric field of the fundamental mode is approximately zero. Note that plexiglass has a dielectric constant between 2.2 and 3.4 and it can be considered transparent in our problem. To measure the frequency shift and quality factor of the cavity we used the experimental setup shown in Fig. 2. First the cyclic frequency ν is found through the critical coupling condition, where frequency and position of the loop are adjusted in such a way that the power reflected by the cavity is minimum. When this condition is satisfied, the relation $Q_{\rm exp} = 2\pi\tau_E\nu$ is valid and the quality factor can be measured through the time of the electric field τ_E stored in the cavity.

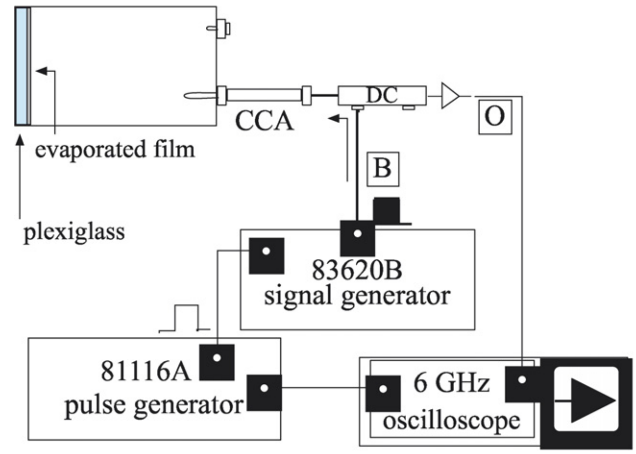

Fig. 2. Experimental setup for the measurement of the parameters of the cavity ν and $Q_{\rm exp}$.

For the chosen geometry, the numerical value of parameter A (24) depends on the film thickness G (expressed in nanometers) and the resistivity of deposited metal ρ (expressed in µΩ cm) as

$$A = 8G/\rho. \tag{28}$$

The maximal frequency shift according to formula (26) must be equal

$$(\nu - \nu_0)_{\max} = \eta^3 \Delta\nu_0 = 27.5 \text{ MHz}. \tag{29}$$

The measured frequency shift was 28 MHz for the thickness $G = 250$, 125, 75 nm, in total accordance with (29), because $A \gg 1$ for these values of G, even if ρ is bigger than the resistivity of bulk copper at room temperature $\rho = 1.7$ µΩ cm (which corresponds to the conductivity $\sigma = 5 \times 10^{17}$ s^{-1} in the CGS units). The expected frequency shift has been obtained for a minimum value of $G = 75$ nm, which is about 20 times smaller than the calculated skin depth at the resonance frequency ν_0 ($\delta_{sw} \approx 1.3$ µm).

For smaller values of G the applicability of formula (26) is questionable, because it is obtained within the frameworks of macroscopic electrodynamics, whereas the mean free path of electrons in copper at room temperature is about $l_{\rm Cu} \approx 40$ nm [15]. This was confirmed experimentally as with an evaporated 25 nm film the cavity mode TE_{101} resonance could not be detected, this showing partial transmission through the evaporated film of the electromagnetic field.

Note that the quality factor is more sensitive to variations of parameter A than the frequency shift. In Table 1 we compare the measured Q factor with theoretical values obtained from formula (27) for different film thicknesses. The resistivity ρ has been measured separately for each evaporated film as it appears

Table 1
Experimental and theoretical Q-factors for copper films

G (nm)	ρ (μΩ cm)	ν	Q_{ex}	Q_{th}
250	1.7	2.50863	6220	6350
120	1.8	2.50867	5770	5500
75	7.5	2.5086	2930	2470

that the conductivity of evaporated copper differs significantly from the bulk conductivity [16]. The agreement between experimental and theoretical data seems quite satisfactory. The difference can be explained by an insufficient knowledge of the exact value of resistivity of evaporated films, because the measurements of ρ were done by applying a constant voltage to the film, whereas ρ in formula (28) should be taken at the frequency 2.5 GHz.

It was interesting to repeat these measurements with films of a different metal, and we chose lead, that has bulk resistivity $\rho = 20$ μΩ cm. The measurements could be reproduced for $G = 250$ nm and $G = 75$ nm, but for smaller thicknesses of lead films the oxide already growing during the evaporation process prevented any further analysis. For example, with the $G = 250$ nm film, in which the measured resistivity was still close to the bulk value, $\rho = 28.4$ μΩ cm, the measured frequency shift was 28.5 MHz, in accordance with the value $A = 70$. Then formula (27) gives the theoretical value $Q_{th} = 2200$, in agreement with the experimental value $Q_{ex} = 2280$. The thickness of this film is 20 times smaller than the skin depth corresponding to the measured value of ρ.

4. Conclusion

We have demonstrated, both theoretically and experimentally, that a very thin conducting film, whose thickness is much less than the skin depth, can produce the same frequency shift in an electromagnetic cavity as a bulk conducting slab, provided the distance between the film and the cavity wall is much bigger than the skin depth. This result supports the idea of a possibility of an experimental study of the dynamical Casimir effect using semiconductor films with time-dependent conductivity.

A difference between the system studied here and the experimental scheme [12] concerns the inhomogeneous conductivity of semiconductor films illuminated by laser pulses, which strongly depends on the distance from the surface (due to the exponential attenuation of the laser intensity inside the film). But it was shown in [17] that formulas for the resonance frequency shift, given by Eq. (26), remain valid for inhomogeneous plasma films as well, if one replaces the product $\varepsilon_2 G$ in the definition of parameter A (24) with the integral $\int \varepsilon_2(x)\,dx$ across the semiconductor film. Another difference is that measurements performed here have been done for conductivities much larger than the expected conductivity of the semiconductor plasma films. For example, considering a thick film with $G = 1$ μm, corresponding to the absorption length of 800 nm light in GaAs at 5 K, one obtains for the impinging laser energy/pulse of $W = 100$ μJ and the surface area $S = 10$ cm^2 the following estimation of the concentration of carriers created inside this film: $n \sim W/(E_g SG) \sim 5 \times 10^{17}$ cm^{-3} (where $E_g \sim 1.4$ eV is the energy gap of GaAs). Taking for a free carriers mobility the value $b = 10^4$ cm^2/V s (which is realistic, according to preliminary measurements), we can evaluate the average resistivity inside the conducting film as $\rho = 1/(nbe) \sim 10^{-3}$ Ω cm. Then formula (28) gives for the total thickness of semiconductor slab $D = 2$ mm (which will be used in the experiment) the value $A = 4$, which is still bigger than unity, so that the frequency shift will be more than 90% of the value corresponding to the ideal metallic boundary, according to Eq. (26). The account of losses made in [17] also shows that the final generation rate of "Casimir photons" will be positive and big enough.

Acknowledgements

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