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INTERACTIONS OF MAGNETIC MONOPOLES WITH NUCLEI AND ATOMS: FORMATION OF BOUND STATES AND PHENOMENOLOGICAL CONSEQUENCES

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We discuss the binding of magnetic monopoles to nuclear and atomic systems. The energy spectrum is calculated, by assuming an interaction with a hard core. The formation process of monopole-nucleus bound states is analysed and it is shown that monopoles reaching Earth are most likely bound to a proton. We also discuss phenomenological implications of the existence of bound states in connection with the monopole catalysis of proton decay.

1. Introduction

The theoretical discovery of magnetic monopole-like solutions in gauge theory [1] and the report of the possible observation of such a particle [2] have rekindled interest in the search of magnetic monopoles.

Monopoles are expected to be very massive, $m_M \sim 10^{16}$ GeV, and to reach Earth with very low velocity, $v_M \lesssim 10^{-3}c$. As a consequence of these unusual properties, negative results of previous accelerator and cosmic ray searches are not to be regarded as discouraging [3, 4]. Indeed the full picture of slow monopoles interacting with matter has to be developed, in order to plan for decisive searches of these particles. With this spirit we discuss here the possibility that slow magnetic monopoles bind to extended particles, such as nuclear and atomic systems. The idea is that the interaction of the monopole with the magnetic moments of these systems can be strong enough to produce bound states. Indeed we find that nucleons, as well as some nuclei and atoms, can bind to monopoles and provide estimates of the binding energies. Other authors have previously considered this idea [5, 6]. With respect to their work, our treatment for the search of bound states is considerably simpler, thanks to the recent formalism of ref. [7]. Also, our determination of the energy spectrum of the monopole-nucleus system is more accurate than in ref. [6], where the alleged values in several cases are incorrect.

We also consider the reactions which lead to the formation of the bound states. From a discussion of this process, grounded in the similarity with the formation of mesic atoms [8, 9], it emerges that formation of the bound states is quite likely at the low velocities expected for magnetic monopoles. Also we find that if a monopole-atom bound state is formed, soon it squeezes into a system where the atomic nucleus is directly bound to the monopole provided that the nuclear magnetic moment is sufficiently large.

Next we consider the phenomenological consequences of the occurrence of nucleus monopole bound states*. We find that monopoles reaching Earth are most likely accompanied by a proton, a circumstance which has direct implications on the detection of magnetic monopoles. Bound states of monopoles and nuclei are also interesting in connection with Rubakov's effect [11], i.e. the monopole catalysis of proton decay:

$$\mathrm{M} + \mathrm{p} \rightarrow \mathrm{M} + \mathrm{e}^{+} + \mathrm{X} . \tag{1.1}$$

The Rubakov effect can be strongly enhanced if bound states are formed, as a consequence of the large overlap of the proton-monopole wave function. By using this consideration we derive bounds on the cross section for reaction (1.1) and on the monopole flux from data on proton decay experiments.

Finally, we discuss the possibility that monopoles catalyse nuclear fusion reactions of light nuclei.

We would like to remark on the exploratory spirit of this paper, the principal aim being to outline some consequences of the existence of monopole-nucleus bound states.

It is worth warning the reader about some crude approximations used in the paper.

(i) We assume the monopole-nucleus interaction to have a hard core for distances less than the nuclear radius. This is clearly unphysical, in that the monopole has a colour gauge field surrounding it up to a distance of order 1 fm and yielding a highly non-trivial monopole-nucleus interaction at nuclear distances. Our approximation, however, is sufficient to establish the existence of monopole-nucleus bound states. The true potential, whatever it is, is more attractive than our approximate form. Thus, if we find that a nucleus binds to a monopole within our approximation, a fortiori it will bind when the correct interaction is taken into account.

(ii) In the discussion of the Rubakov effect in the bound monopole-nucleus system, one needs to know the overlap between the nucleus and the monopole:

$$\rho_{\mathrm{eff}} = v_{\mathrm{N}}^{-1} \int_{v_{\mathrm{N}}} \mathrm{d}v \, |\psi(\boldsymbol{r})|^{2} , \tag{1.2}$$

* It is worth observing that we are concerned with electrically neutral magnetic monopoles. Binding of dyons to nuclei was discussed by us in a recent paper [10].

where v_N is the nuclear volume. Clearly the determination of ρ_{eff} is beyond the hard core approximation, which obviously yields $\rho_{eff} = 0$. As an educated guess we will use $\rho_{eff} = L_N^{-3}$, where L_N is a typical dimension of the bound system. One has to keep in mind, however, that the actual value of ρ_{eff} could only be calculated by resorting to the exact potential.

Our work can be improved in many respects. At the end of the paper we briefly discuss some aspects which, in our opinion, deserve further investigation.

2. Interaction of monopoles with extended particles

2.1. INTERACTION ENERGY OF A MONOPOLE AND A MAGNETIC DIPOLE

According to the Dirac quantization condition [12] the magnetic charge q_M of monopoles can have values*:

$$q_M = n/2e, \tag{2.1}$$

where $-e$ is the electron charge and n is a relative integer. We will restrict ourselves to monopoles with $n = \pm 1$, as we expect that only these can be stable on the scale of the universe's age:

$$q_M = \varepsilon/2e, \qquad \varepsilon = \pm 1. \tag{2.2}$$

Consider a particle (say a nucleon, a nucleus or an atom) with electric charge $q_E = Ze$, mass m, spin $\boldsymbol{S}$ and magnetic moment

$$\boldsymbol{\mu} = \frac{e\kappa}{m}\boldsymbol{S}. \tag{2.3}$$

For simplicity we will restrict to spin-$\frac{1}{2}$ particles. The interaction energy of the magnetic moment with the field $\boldsymbol{B}$ of the monopole as a function of the relative distance R is given by:

$$H_{dip} = -\boldsymbol{\mu}\cdot\boldsymbol{B} = -\frac{\kappa\varepsilon}{2m}(\boldsymbol{S}\cdot\boldsymbol{R})R^{-3}. \tag{2.4}$$

Clearly this description is adequate only for distances R larger than the size of the particle, a. When $R \lesssim a$, one has to take into account the internal structure of the particle and to replace (2.4) with an equation that takes into account the interaction between the monopole and the constituents. Generally this results in an interaction which is weaker than that which is given in eq. (2.4) and which is regular at the origin. Consider, for example, that the particle is a nucleus and that its magnetic

* We use units such that $\hbar = c = 1$. m_p and m_e denote the proton and electron mass.

moment μ_N arises from an unpaired nucleon. Eq. (2.4) has then to be replaced by the interaction with the nucleon magnetic moment μ_n ($\mu_n = (e\kappa_n/m_n)S_n = \mu_N$). This interaction has then to be averaged over the nucleon wave function, i.e. over the volume of the nucleus. In this way one gets, for R smaller than the radius a of the nucleus:

$$H_{\mathrm{dip}} \approx -(\boldsymbol{S}\cdot\boldsymbol{R})\frac{\kappa\varepsilon}{2ma^3}\,. \tag{2.5}$$

One sees that the singularity of eq. (2.5) at $R=0$ is avoided and a smooth behaviour of the interaction is obtained for $R \leqslant a$. Although a realistic calculation of the interaction at $R \lesssim a$ has to take into account the internal structure of the system, the above consideration suffices to get a prescription for the approximate evaluation of the energy levels and wave functions. In order to prove the existence of bound states we can replace eq. (2.4) by an infinite repulsive potential for $R \leqslant a$:

$$V(R) = +\infty\,, \qquad \text{for } R \leqslant a\,. \tag{2.6}$$

If we find bound states for the hard core potential, a fortiori they will exist for the real potential. Indeed this latter is more attractive than the hard core potential. Also the fact that the true potential is regular at the origin guarantees that the hamiltonian is a well-behaved operator (i.e. there is no fall onto the center).

2.2. SCHRÖDINGER EQUATION

If one does not consider the magnetic moment interaction, the non-relativistic hamiltonian of a charged particle interacting with a heavy magnetic monopole is simply the kinetic energy operator*:

$$H_{\mathrm{charge}} = \tfrac{1}{2}mv^2 = \frac{1}{2m}(\boldsymbol{P} - Ze\boldsymbol{A})^2\,.$$

This operator can be written as:

$$H_{\mathrm{charge}} = \frac{1}{2mR^2}\left\{-\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + (\boldsymbol{L}^2 - q^2)\right\}, \tag{2.7}$$

with

$$q = q_{\mathrm{M}}Ze = \tfrac{1}{2}Z\varepsilon\,. \tag{2.8}$$

The conserved vector $\boldsymbol{L}$ is the sum of the kinetic and field angular momenta:

$$\boldsymbol{L} = \boldsymbol{R}\times(\boldsymbol{P} - Ze\boldsymbol{A}) - q\hat{\boldsymbol{R}}\,. \tag{2.9}$$

* We refer to ref. [7] for a detailed discussion of the Schrödinger equation following from eq. (2.7). We will follow the treatment of that paper.

As clearly implied by this equation, the eigenvalues of L^2, $l(l+1)$, are restricted to be:

$$l \geqslant |q|\,, \qquad l - |q| = \text{integer}. \tag{2.10}$$

The eigenfunctions of (L^2, L_Z), the so called monopole harmonics, are described in ref. [7]. The hamiltonian (2.7) is always positive, as it represents the kinetic energy. Consequently the system described by (2.7) cannot have bound states. The magnetic moment interaction is taken into account by adding it to H_{charge}. By using eqs. (2.4) and (2.7) we get the following hamiltonian, valid as long as $R \geqslant a$:

$$H = \frac{1}{2mR^2}\left\{-\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \left(\boldsymbol{L}^2 - q^2\right) - \boldsymbol{S}\cdot\hat{\boldsymbol{R}}\kappa\varepsilon\right\}. \tag{2.11}$$

The prescription on the potential at $R \leqslant a$ (eq. (2.6)) can be translated into a boundary condition on the wave function ψ:

$$\psi(R = a) = 0. \tag{2.12}$$

The total angular momentum of the system,

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}, \tag{2.13}$$

being a constant of the motion, eigenstates of J^2 and J_Z can be considered for the study of the eigenvalues and eigenvectors of (2.11). As the hamiltonian explicitly involves the $\boldsymbol{L}$ operator, (which is not conserved) it is convenient to analyze the l content of the (J, J_Z) states. Two different possibilities arise, which we will discuss separately in the following.

(i) $J = |q| - \frac{1}{2}$. Note that this implies $l = |q|$. We can separate the radial part of the wave function from the spin and angular part in the following way:

$$\psi_{J=|q|-\frac{1}{2}}(\boldsymbol{R}) = \frac{1}{R}\chi(R)|q, l = |q|, J = |q| - \tfrac{1}{2}\rangle. \tag{2.14}$$

A straightforward calculation of the $\boldsymbol{S}\cdot\hat{\boldsymbol{R}}$ operator on the $J = |q| - \frac{1}{2}$ states gives [13]*:

$$\boldsymbol{S}\cdot\hat{\boldsymbol{R}}|q, l = |q|, J = |q| - \tfrac{1}{2}\rangle = \tfrac{1}{2}\varepsilon|q, l = |q|, J = |q| - \tfrac{1}{2}\rangle. \tag{2.15}$$

All other operators acting trivially on the non-radial part of the wave function, one

* It is worth observing that, different from spherical harmonics, monopole harmonics do not have definite parity. This corresponds to the fact that under the $\boldsymbol{r} \to -\boldsymbol{r}$ transformation the operator $\boldsymbol{L}$ has not a definite parity.

gets the following one-dimensional Schrödinger equation for the function χ:

$$\frac{1}{2m}\left(-\frac{\mathrm{d}^2}{\mathrm{d}R^2}+\left(|q|-\tfrac{1}{2}\kappa\right)\frac{1}{R^2}\right)\chi=E\chi, \tag{2.16}$$

with the boundary condition:

$$\chi(a)=0. \tag{2.17}$$

Thus the monopole-particle interaction has been reduced to a one-dimensional problem in an effective potential:

$$V=\lambda_{\mathrm{I}}(2mR^2)^{-1}, \tag{2.18}$$

$$\lambda_{\mathrm{I}}=|q|-\tfrac{1}{2}\kappa. \tag{2.19}$$

If λ_{I} is negative the magnetic moment monopole interaction is attractive. As shown in the appendix, the motion in the potential (2.18), subject to the boundary condition (2.17) has infinite (zero) bound states for $\lambda_{\mathrm{I}}<-\frac{1}{4}$ ($\geqslant -\frac{1}{4}$). Thus the condition for bound states is:

$$\kappa>2|q|+\tfrac{1}{2}.$$

(ii) $J>|q|-\frac{1}{2}$. States with definite (J, J_z) are built up with two different values of l:

$$l_{\pm}=J\pm\tfrac{1}{2}. \tag{2.20}$$

The state vector can then be written as:

$$R^{-1}\chi_{+}(R)|q,l_{+},J,J_z\rangle+R^{-1}\chi_{-}(R)|q,l_{-},J,J_z\rangle. \tag{2.21}$$

The eigenvalue problem of eq. (2.11) will consist of two coupled equations for the wave functions $\chi_{\pm}(R)$. In order to write down these equations one needs to know the matrix elements of the $\boldsymbol{S}\cdot\hat{\boldsymbol{R}}$ operator in the $|q,l,J,J_z\rangle$ basis. A straightforward calculation gives ($j=J+\frac{1}{2}$) [13]:

$$\boldsymbol{S}\cdot\hat{\boldsymbol{R}}|q,l_{\pm},J,J_z\rangle=\pm\frac{q}{2j}|q,l_{\pm},J,J_z\rangle-\frac{\left(j^2-q^2\right)^{1/2}}{2j}|q,l_{\mp},J,J_z\rangle. \tag{2.22}$$

From eq. (2.11) one can now derive a two-component Schrödinger equation for the doublet $\tilde{\chi} = \begin{pmatrix} \chi_- \\ \chi_+ \end{pmatrix}$:

$$-\frac{d^2}{dR^2}\tilde{\chi}^2 + \frac{A}{R^2}\tilde{\chi} = 2mE\tilde{\chi}. \tag{2.23}$$

The 2×2 matrix A is given by:

$$A = \begin{bmatrix} j(j-1) - q^2 + q\kappa\varepsilon/2j & (j^2 - q^2)^{1/2}\kappa\varepsilon/2j \\ (j^2 - q^2)^{1/2}\kappa\varepsilon/2j & j(j+1) - q^2 - q\kappa\varepsilon/2j \end{bmatrix}. \tag{2.24}$$

From the system (2.23) one can get two decoupled equations by transforming to the basis where the matrix A is diagonal. The radial wave functions $\chi_{1,2}$ in this basis are then given by:

$$-\frac{d^2}{dR^2}\chi_i(R) + \lambda_i\frac{1}{R^2}\chi_i(R) = 2mE\chi_i(R), \tag{2.25}$$

where λ_i are the eigenvalues of the A matrix. The boundary condition (2.12) in the new basis is again:

$$\chi_i(a) = 0. \tag{2.26}$$

In this way the interaction with the magnetic monopole is reduced to a one-dimensional problem in a R^{-2} potential. If the lower of the eigenvalues of A, say λ_{II}, is negative, the potential in the corresponding channel is attractive*. In particular, if $\lambda_{II} < -\frac{1}{4}$ we have again an infinite family of bound states. Explicit evaluation of the eigenvalue gives:

$$\lambda_{II} = (j^2 - q^2) - (j^2 - |q|\kappa + \tfrac{1}{4}\kappa^2)^{1/2}. \tag{2.27}$$

By taking into account also eq. (2.10) we find that bound states with total angular momentum $J = j - \frac{1}{2}$ can be formed if the following two conditions are simultaneously satisfied:

$$j^2 \geqslant (|q| + 1)^2, \tag{2.28a}$$

$$j^2 < q^2 + \tfrac{1}{4} + \left||q| - \tfrac{1}{2}\kappa\right|. \tag{2.28b}$$

2.3. ESTIMATE OF THE ENERGY LEVELS

As shown in the foregoing subsections, the search of bound states for the monopole-particle interaction is reduced to the study of an equation of the form:

$$\left(-\frac{d^2}{dR^2} + \frac{\lambda}{R^2}\right)\chi = -k^2\chi \tag{2.29}$$

* The trace of the matrix A being positive it is impossible that both the eigenvalues are negative.

with $k^2 = -2mE > 0$. Eq. (2.29) holds for $R > a$ and χ is subject to the boundary condition

$$\chi(a) = 0. \tag{2.30}$$

As previously mentioned, and as discussed in the appendix, an infinite number of bound states exist provided that

$$\lambda = -\sigma^2 < -\tfrac{1}{4}. \tag{2.31}$$

For the sake of having some analytical estimate of the eigenvalues one can resort to semiclassical approximations. By requiring the semiclassical wave function to vanish at $R = a$ one gets a Bohr-Sommerfeld quantization condition:

$$\int_a^{R_S} P_R \, dR = (n - \tfrac{1}{4})\pi, \qquad n \geqslant 1. \tag{2.32}$$

R_S is the classical turning point:

$$R_S = \sigma/k \tag{2.33}$$

and P_R is the radial momentum:

$$P_R^2 = -k^2 + \sigma^2/R^2. \tag{2.34}$$

The action integral is then:

$$\int_a^{R_S} P_R \, dR = -\sigma\left\{(1 - a^2\chi^2/\sigma^2)^{1/2} - \ln\frac{1 + (1 - a^2k^2/\sigma^2)^{1/2}}{ak/\sigma}\right\}. \tag{2.35}$$

Eqs. (2.32) and (2.35) provide an expression for the energy levels in the form:

$$E_n = -\frac{\sigma^2}{2ma^2} f^2[(n - \tfrac{1}{4})\pi/\sigma]. \tag{2.36}$$

The function $f(y)$ is shown in fig. 1.

As is clear from eq. (2.35), at large y, f can be approximated by:

$$f(y) \simeq 2\exp[-1 - y]. \tag{2.37}$$

For any $y > 0$, f can be well-approximated by:

$$f(y) \simeq 0.74\exp[-y] + 0.26\exp[-6y]. \tag{2.38}$$

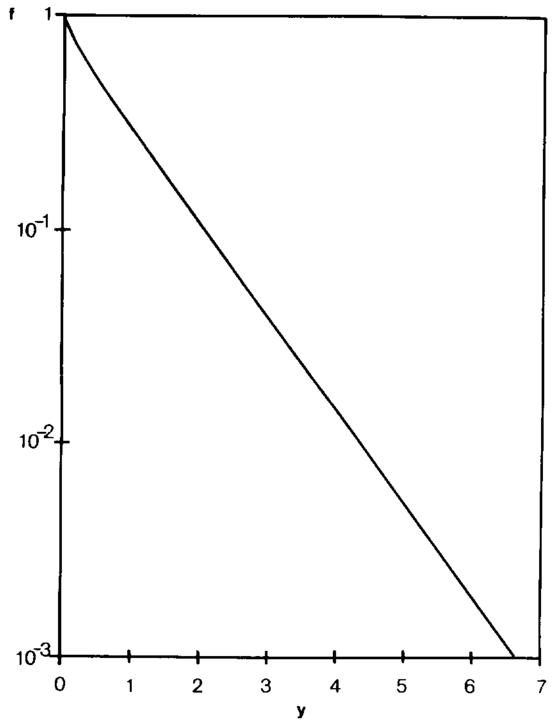

Fig. 1. The function $f(y)$ giving the semiclassical energy levels (eq. (2.36)).

Thus one has an infinite family of energy levels. The typical energy scale is:

$$E_0 = \sigma^2/(2ma^2), \tag{2.39}$$

and the energy levels tend exponentially to zero energy*:

$$E_n \approx E_{n-1}\exp[-2\pi/\sigma]. \tag{2.40}$$

So far we have used a non-relativistic approximation. To judge about its validity one needs to compare E_0 with the mass of the particle. We will see that in any case of interest one has:

$$E_0/m \ll 1.$$

So the non-relativistic approximation is well justified. On the other hand it is worth remarking that the condition of validity of the semiclassical approximation,

* It is also interesting to have an estimate of the linear size L of the bound states. By computing $L^{-1} = \langle r^{-1} \rangle$ over the semiclassical wave function with energy $E = -k^2/2m$ one gets $L \approx \sigma/k$.

$|(\mathrm{d}/\mathrm{d}R)1/P_{\mathrm{R}}| \ll 1$, when applied to the point $R = a$ gives:

$$\sigma\left(1 - k^2a^2/\sigma^2\right)^{3/2} \gg 1. \tag{2.41}$$

This implies small binding energies, $(k^2a^2/\sigma^2 \ll 1)$ as well as $\sigma \gg 1$. If these conditions are not satisfied one has to resort to the exact solution of eq. (2.29) with boundary condition (2.30). While deferring to the appendix for this discussion, we summarize here the relevant results.

By writing the binding energy of the nth level as

$$E_n = -\frac{1}{2ma^2}x_n^2 \tag{2.42}$$

the values of x_n as a function of

$$\nu = \left(\sigma^2 - \tfrac{1}{4}\right)^{1/2}, \tag{2.43}$$

are shown in fig. 2 for the first few n's. For the higher n's one can use the following recursion formula:

$$x_{n+1} \simeq x_n \exp\{-2\pi/\nu\}. \tag{2.44}$$

The above equation, which can be proved for high n, actually works also for $n = 2$.

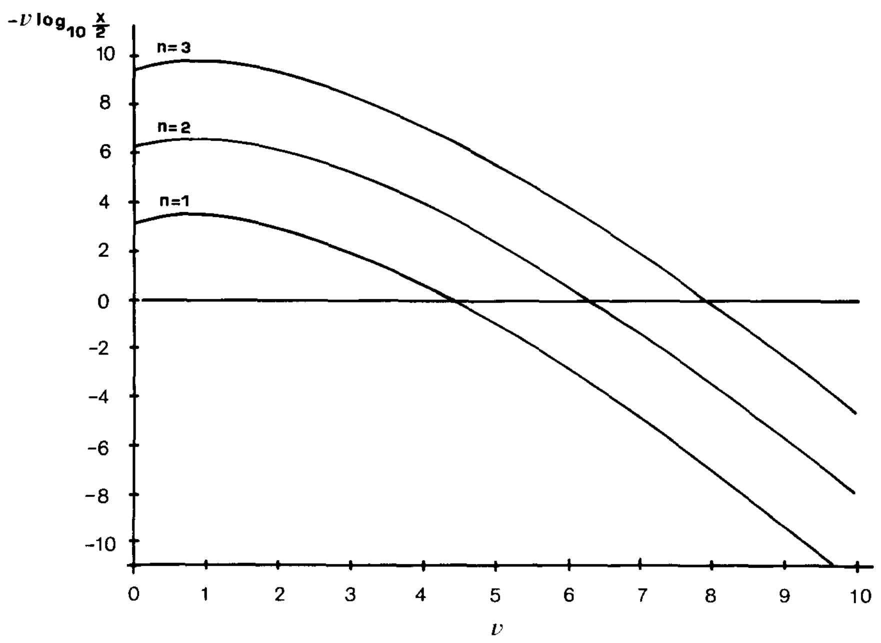

Fig. 2. Location of the first three zeroes of the function $K_{i\nu}(x)$, as a function of ν. The approximate parallelism of the three curves exhibits the property written in eq. (2.44).

The eigenfunction corresponding to energy $E = -k^2/2m$ is

$$\chi(R) = \begin{cases} \sqrt{R}\,K_{i\nu}(kR), & R > a \\ 0 & R < a, \end{cases} \tag{2.45}$$

where $K_{i\nu}$ is the modified Bessel function of order $i\nu$.

As already mentioned, the introduction of a hard core at $r = a$ is somewhat an artifice. Particularly, the choice of the core radius is to some extent arbitrary. However, one sees from eq. (2.42) that the qualitative features of the energy spectrum are independent of the value of a. The condition for the existence of bound states involves the giromagnetic ratio and not at all a (see eqs. (2.43), (2.31), (2.27) and (2.19)). On the other hand, the energy scale is fixed by a (see again eq. (2.42)).

3. Energy levels of nuclear and atomic systems bound to monopoles

We make use of the results of sect. 2 (particularly eqs. (2.19), (2.27) and (2.31)) to study the binding of monopoles to simple nuclear and atomic systems.

3.1. NUCLEONS

For the proton one has $\kappa = 2.8$, $q = \frac{1}{2}\varepsilon$ and the total angular momentum of the proton-monopole system is integer. From eq. (2.18a) one finds:

$$\lambda_{\mathrm{I}} = -0.9\,. \tag{3.1}$$

Thus protons can bind to magnetic poles in $J = 0$ states. As $|\lambda_{\mathrm{I}}| \approx 1$, the energy levels have to be calculated by solving the full Schrödinger equation (2.29). The outermost zero of the modified Bessel function $K_{i\nu}(x)$ lies at $x = 2.7 \cdot 10^{-2}$ (see fig. 3). For $a = 1$ fm one finds that the ground state has energy:

$$E_1^{(\mathrm{Mp})} = -15\ \mathrm{keV}\,. \tag{3.2}$$

The excited levels are at:

$$E_n^{(\mathrm{Mp})} = E_{n-1}^{(\mathrm{Hp})} \exp\{-7.8\}\,. \tag{3.3}$$

The linear size of the ground state is:

$$L^{(\mathrm{Mp})} = \langle r \rangle = 32\ \mathrm{fm}\,. \tag{3.4}$$

We do not find bound states with $J > 0$ as eqs. (2.28) cannot be fulfilled simultaneously.

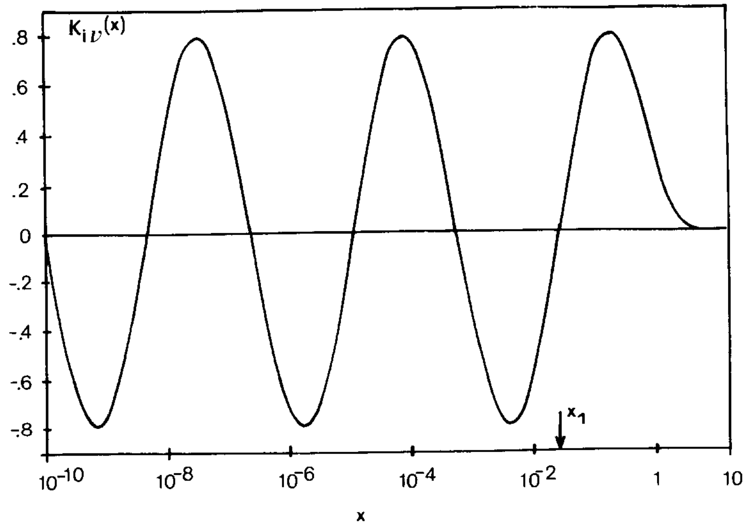

Fig. 3. The function $K_{i\nu}(x)$ corresponding to the (Mp) bound state ($\nu = 0.804$). The outermost zero lies at $x = 2.7 \cdot 10^{-2}$.

For the neutron one has $\kappa = 1.91$, $q = 0$ and the total angular momentum is half integer. From eq. (2.28) one finds that bound states can be formed for $J = \frac{1}{2}$ only. From eq. (2.27) the eigenvalue of the A matrix is:

$$\lambda_{II} = -\sigma^2 = -0.39. \tag{3.5}$$

By taking $a = 1$ fm we get:

$$E_1^{(Mn)} = -1.2\ \text{eV}, \tag{3.6}$$

$$E_n^{(Mn)} = E_{n-1}^{(Mn)} \exp\{-17\}. \tag{3.7}$$

The binding energy is considerably smaller than for the proton, as a consequence of the fact that σ^2 is close to the critical value $\frac{1}{4}$.

In ref. [6] it was found that the binding energy for the ground state of both (Mp) and (Mn) are about 300 KeV. We believe that this result is wrong for the following reasons.

(a) The location of the zeroes of the modified Bessel function was determined by an asymptotic expansion, (eq. (4.14) of ref. [6], $x = \nu$), which does not hold in the region where the outermost zero lies. This is clearly seen from fig. 3.

(b) In ref. [6] the cutoff was set at $a = 6$ fm, which is an unphysical value.

(c) For calculating the binding energy of the neutron the author of ref. [6] used eq. (3.15) (the analog of eq. (2.19) of this paper) which only holds for $q \neq 0$.

3.2. NUCLEI

For a spin-$\frac{1}{2}$ nucleus ${}_Z\mathrm{N}^A$, with magnetic moment $\tilde{\mu}_\mathrm{N}$ in units of the nuclear magneton, $(e\hbar/2m_\mathrm{p}c)$ one has $|q| = \frac{1}{2}Z$, and $\kappa = \tilde{\mu}_\mathrm{N}A$. One must consider separately the cases $J = |q| - \frac{1}{2}$ and that of higher J.

For the first case, the condition for the existence of bound states is (from eq. 2.18a):

$$\lambda_\mathrm{I} = \tfrac{1}{2}(Z - \tilde{\mu}_\mathrm{N}A) < -\tfrac{1}{4}, \tag{3.8}$$

which requires nuclei with unpaired protons ($\tilde{\mu}_\mathrm{N} > 0$). In the second case, for the smallest value of J, $J_m = (|q| + \frac{1}{2})$, the eigenvalue of the A matrix is, from eq. (2.27):

$$\lambda_\mathrm{II} = Z + 1 - \left(Z + 1 + \tfrac{1}{4}(Z - \tilde{\mu}_\mathrm{N}A)^2\right)^{1/2}. \tag{3.9}$$

The condition for the existence of bound states is, again:

$$\lambda_\mathrm{II} < -\tfrac{1}{4}. \tag{3.10}$$

There are several stable spin-$\frac{1}{2}$ nuclei, which satisfy the above conditions. They are listed in table 1, where the relevant results for the energy levels are presented. The binding energy of the ground state is in the range 1–100 keV, the most stable system being ($\mathrm{M}\,{}^{19}_{9}\mathrm{F}$) with $E_1 = 380$ keV. For a few nuclei (${}^3_1\mathrm{H}$, ${}^{19}_{9}\mathrm{F}$, ${}^{203}_{81}\mathrm{Tl}$, ${}^{205}_{81}\mathrm{Tl}$) two families of bound states exist, corresponding to λ_I and λ_II. In this case the first family is always more tightly bound, as can be inferred by the expression (3.8) and (3.9) of λ_I and λ_II. These imply $|\lambda_\mathrm{I}| > |\lambda_\mathrm{II}|$ and consequently the potential is more attractive for family (I). It is worth remarking that semiclassical expressions (eqs. (2.36) and (2.38)) yield a fair approximation also for the ground state, as long as $|\lambda| \gtrsim 10$. The values of the binding energy of ${}^{13}\mathrm{C}$, ${}^{19}\mathrm{F}$ and ${}^{31}\mathrm{P}$ quoted in ref. [6] are in substantial disagreement with our results. The discrepancy is due to the following reasons.

(a) In ref. [6] a finite value of the monopole mass, $M_\mathrm{M} \approx 100 m_\mathrm{p}$ was used. This affects the calculation of the λ's, since, for a finite monopole mass:

$$\lambda_\mathrm{I} = \tfrac{1}{2}\left(Z - \tilde{\mu}\frac{AM_\mathrm{M}}{M_\mathrm{M} + Am_\mathrm{p}}\right),$$

(b) As previously noted, the location of the zeroes of the modified Bessel function was determined too roughly.

(c) The estimate of the nuclear radius is unphysical.

3.3. ATOMIC SYSTEMS

As an illustrative example of monopoles interacting with an atomic system let us consider $^4He^+$. The mass of the system is $m_{He} = 4m_p$, the total charge is $Z = +e$, its typical radius is $a_{He} = 2.5 \times 10^{-9}$ cm and the magnetic moment in the 1 s state fully arises from the electron spin:

$$\mu_{^4He^+} = \mu_e . \tag{3.11}$$

The giromagnetic factor κ is enormous:

$$\kappa = -\frac{m_{He}}{m_e} \approx -7400 . \tag{3.12}$$

Since κ is negative, only family (II) of bound states exist (see eq. (2.19)). Within this family one expects that states with very high values of J can be found. Indeed for eq. (2.28) one finds that the largest value of J is $J_{max} \sim \sqrt{\frac{1}{2}\kappa} \sim 60$. For $J \ll J_{max}$ one finds, from eq. (2.27):

$$\sigma^2 \simeq \tfrac{1}{2}\kappa . \tag{3.13}$$

In the semiclassical approximation the energy levels are then:

$$E_n \approx -\frac{0.55}{4m_e a_{He}^2} \exp\left\{ -\left(n - \tfrac{1}{4}\right) \frac{\pi 2^{3/2}}{\sqrt{\kappa}} \right\} . \tag{3.14}$$

For not too high n, this is a value comparable to the binding energy of $^4He^+$ ($E_{He} = 1/2m_e a_{He}^2$). This shows that generally the coupling to the electron magnetic moment can be quite efficient to bind monopoles with atomic systems. On the other hand, the interaction with the electron being so strong its wave function will be drastically perturbed. Consequently this approach, which in essence relies on the discussion of the electron monopole interaction as a perturbation of the Coulomb interaction between e and α, can just be considered as indicative of the monopole atom interaction*.

4. Formation and deexcitation of bound states

We will consider in some detail the formation of the monopole-nucleus, $(M_Z N^A)$, system. Several reactions which are important for this process will be discussed in the following.

* In ref. [7] the interaction of magnetic monopoles with atoms was treated on a completely different footing. There, the author considers a repulsive interaction, and evaluates the electronic energies assuming that a monopole is located in the nucleus.

4.1. RADIATIVE COMBINATION

The cross section for the reaction:

$$\mathrm{M} + \mathrm{N} \to (\mathrm{MN}) + \gamma \tag{4.1}$$

can be easily estimated by using semiclassical considerations. The amplitude of the radiation field $\boldsymbol{A}$ of M and N being proportional to the acceleration and to the (electric or magnetic) charge of the particle,

$$A_{\mathrm{M}}/A_{\mathrm{N}} = \frac{m_{\mathrm{N}}}{m_{\mathrm{M}}} \frac{1}{Z\alpha} \ll 1\,, \tag{4.2}$$

one can limit to consider the coupling of the radiation field to the nucleus. Consider a monopole-nucleus collision at relative velocity $\boldsymbol{v}$, impact parameter b, resulting in a velocity variation $\Delta\boldsymbol{v}$.

The probability that the nucleus emits a photon in the frequency interval $\omega, \omega + \mathrm{d}\omega$ is [14]:

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} = \begin{cases} \dfrac{2Z^2\alpha}{3\pi\omega}|\Delta\boldsymbol{v}|^2, & \text{for } \omega < v/b \\ 0, & \text{for } \omega > v/b\,. \end{cases} \tag{4.3}$$

A bound state can be formed when the energy ω of the emitted photon is larger than the kinetic energy of the impinging nucleus:

$$\omega > \omega_{\mathrm{I}} = \tfrac{1}{2} m_{\mathrm{N}} v^2\,. \tag{4.4}$$

The probability $w(b)$ of forming an (MN) system is thus obtained by integrating eq. (4.4) in the useful frequency range:

$$\tfrac{1}{2} m_{\mathrm{N}} v^2 < \omega < v/b\,. \tag{4.5}$$

In this way one finds:

$$w(b) = \begin{cases} \dfrac{2\alpha Z^2}{3\pi} \ln\left\{\dfrac{2}{m_{\mathrm{N}} v b}\right\} |\Delta\boldsymbol{v}|^2, & \text{for } m_{\mathrm{N}} v b < 2 \\ 0, & \text{for } m_{\mathrm{N}} v b > 2\,. \end{cases} \tag{4.6}$$

In order to calculate $\Delta\boldsymbol{v}$, we approximate the monopole-nucleus interaction by a central potential:

$$U(R) \approx \mu_{\mathrm{N}} B \approx \mu_{\mathrm{N}}/(2eR^2)\,. \tag{4.7}$$

In this way one finds:

$$|\Delta v|^2 \approx \frac{\pi^2}{m_N^2 v^2 b^4}\left(\frac{\mu_N}{e}\right)^2. \tag{4.8}$$

It is clear from eqs. (4.6) and (4.7) that only the s-wave ($L = mvb = \frac{1}{2}$) contributes appreciably to the cross section. By observing that the cross section in the s-wave is

$$\sigma = \frac{\pi}{m_N^2 v^2} w_0, \qquad w_0 = w\left(b = (2m_N v)^{-1}\right),$$

we get for the radiative combination cross section:

$$\sigma_\gamma^{(MN)} \approx Z^2 \mu_N^2 \frac{2^5}{3} \pi \ln 4 \simeq Z^2 \mu_N^2 50. \tag{4.9}$$

Or equivalently:

$$\sigma_\gamma^{(MN)} = Z^2 \tilde{\mu} \times 0.3 \times 10^{-28}\,\mathrm{cm}^2, \tag{4.10}$$

where $\tilde{\mu}_N$ is the nuclear magnetic moment in units of the nuclear magneton and Z the nuclear charge. In order to judge about the reliability of this estimate let us observe that for $(p\bar{p})$ atom formation through the reaction

$$p + \bar{p} \to (p\bar{p}) + \gamma, \tag{4.11}$$

a similar approach yields:

$$\sigma_\gamma^{(p\bar{p})} \simeq 40\alpha^3 m_p^{-2} v^{-2}, \tag{4.12}$$

whereas the result of the exact quantum mechanical calculation can be fitted as [9]:

$$\sigma \simeq 40\alpha^3 m_p^{-2} v^{-2} \ln(1 + \alpha^2 v^{-2}), \tag{4.13}$$

i.e. the same expression as (4.12) but for the logarithmic correction.

4.2. AUGER AND MOLECULAR PROCESSES

In a hot plasma reaction (4.1) is the principal mechanism of the formation of (MN) states. In cold matter other processes can lead to (MN) formation at a substantially higher rate. Indeed for the case of antiprotonic atoms the Auger process:

$$\bar{p} + H \to (p\bar{p}) + e^- \tag{4.14}$$

at velocities $v \leqslant v^* = \alpha(2m_e/m_p)^{1/2}$ has a cross section:

$$\sigma_{\rm Auger}^{(p\bar{p})} \approx 20\alpha^{-1} m_e^{-3/2} m_p^{-1/2} v^{-1} . \tag{4.15}$$

for $v \sim v^*$ one gets a cross section of a few Å^2, thus overwhelming the radiative recombination process (4.14) by a factor about 10^8. A similar process can occur for slow magnetic monopoles:

$$M + H \rightarrow (Mp) + e . \tag{4.16}$$

As in antiprotonic atoms, in this process the (Mp) is formed in excited states, with dimensions of the order of the Bohr radius, $a_0 = \alpha^{-1} m_e^{-1}$, and then decays to the low-lying levels by emitting radiation. We estimate the cascade time to be very short, about 10^{-16} s. Also, if H is bound in some molecular complex (HX), the monopole could first bind to the H atom through the coupling of the electron magnetic moment and by transferring the binding energy to the molecular excitations:

$$M + (HX) \rightarrow \{(MH), X\}^* . \tag{4.17}$$

In the (MH) system the proton and the monopole are again at a distance $\sim a_0$. The proton then can radiate and reach the deep (Mp) levels,

$$(Mp)^* \rightarrow (Mp) + n\gamma . \tag{4.18}$$

Clearly similar reactions can occur also for interactions of monopoles with atomic species other than hydrogen.

With experience of what occurs for antiprotonic atoms we expect that reactions of the type (4.16) and (4.17) are dominant over the radiative combination process when monopoles interact with cold matter, at monopole velocities $v_M \lesssim v^*$. Thus in cold matter the radiative combination cross section, eq. (4.10), has to be considered as a lower limit to the total (MN) formation cross section.

5. Consequences of the existence of bound states

In this section we discuss several phenomenological consequences of the existence of bound states of monopoles with nuclei, in connection with the possibility of monopole detection.

5.1. ARE MONOPOLES BOUND TO NUCLEI WHEN REACHING EARTH?

In order to study this problem, one has to examine the fate of monopoles since the early stage of the universe, starting from a hot era when protons have been formed, say from a temperature $T \sim 100$ MeV [15]. The rate of (Mp) formation is:

$$\Lambda_{\text{for}} = \sigma_{\gamma}^{(\text{Mp})} \rho_{\text{p}} v_{\text{p}}, \tag{5.1}$$

v_{p} and ρ_{p} are the proton velocity and density. These depend on temperature as:

$$\rho_{\text{p}} \approx \eta T^3, \qquad \eta \sim 10^{-8}\text{–}10^{-10},$$

$$v_{\text{p}} \approx (2T/m_{\text{p}})^{1/2}. \tag{5.2}$$

By using eq. (4.10) one gets:

$$\Lambda_{\text{for}} \approx \eta m_{\text{p}}^{-5/2} T^{5/2}. \tag{5.3}$$

Formation of the (Mp) system takes place if Λ_{for} is larger than the expansion rate of the universe, τ_{U}^{-1}, which is connected to the universe's temperature:

$$\tau_{\text{v}} \approx M_{\text{P}} T^{-2},$$

where M_{P} is the Planck mass. The condition $\Lambda_{\text{for}} \tau_{\text{U}} \geqslant 1$ yields:

$$T \geqslant T_{\text{for}} = (10^{10}\eta)^{-2/3} \text{ keV}, \tag{5.4}$$

i.e. the (Mp) can be formed through the radiative combination process as long as the temperature of the universe is larger than a few keV, the proton density being too low when the temperature drops below T_{for}. On the other hand, in the hot universe the (Mp) system can dissociate by colliding with photons:

$$(\text{Mp}) + \gamma \to \text{M} + \text{p}. \tag{5.5}$$

The photodissociation cross section σ_{dis} can be estimated by approximating the (Mp) system with a hydrogenic atom with an effective charge Z^*. We take $Z^* = 0.8$ since this fairly corresponds to the energy and size of the (Mp) system for a reduced mass equal to the nucleus mass.

By using the well-known result for the photo-electric effect in hydrogenic atoms [16] we find:

$$\sigma_{\text{dis}} \approx 2 \times 10^{-12} m_{\text{p}}^{-2} (m_{\text{p}}/E_{\gamma})^{7/2}, \tag{5.6}$$

E_{γ} being the photon energy, which has to exceed the binding energy of the (Mp) system,

$$E_{\text{b}} \simeq 15 \text{ keV}.$$

If n_γ is the photon density,

$$n_\gamma \approx \tfrac{1}{4}T^3, \tag{5.7}$$

the density of photons which are effective for dissociation is, approximately

$$n_{\text{eff}} \approx n_\gamma \exp(-E_b/T). \tag{5.8}$$

The photo dissociation rate,

$$\Lambda_{\text{dis}} \approx 2\cdot 10^{-12} n_\gamma \exp(-E_b/T) m_p^{-2} (m_p/E_b)^{7/2} \tag{5.9}$$

dominates over the formation rate for

$$T \geqslant T_{\text{dis}} \approx \tfrac{1}{40}E_b. \tag{5.10}$$

As $T_{\text{dis}} < T_{\text{for}}$, in the hot stage of the universe the (Mp) systems are quickly dissociated by collisions with photons. Next, when the universe's temperature drops below a few eV, atoms are stable and the Auger process (reaction (4.16)) becomes important. From the known values of the hydrogen density in the intragalactic and interstellar space [17], we find that (Mp) systems are formed provided that:

$$\sigma_{\text{Auger}} \gtrsim 10^{-20}\ \text{cm}^2. \tag{5.11}$$

It is also clear that in the cold era the photodissociation process is not effective. Thus if (Mp) systems are formed, they are stable.

Although we do not have an estimate of σ_{Auger}, it is likely that the above condition (5.11) is satisfied on grounds of the experience with mesic atom formation [9], where the Auger cross section typically overwhelmes the radiative formation cross section by a factor of order 10^8. Thus we expect monopoles reaching the solar system to be accompanied by a proton.

Let us consider, however, the case of a bare monopole reaching Earth's atmosphere. It is easy to see that it cannot bind to the nuclei of the most abundant atomic species (N, O). Binding to the rare species (for example ${}^{13}_{6}\text{C}$, H, Xe) requires a formation cross section larger than 10^{-20} cm^2. Clearly for so large an Auger cross section (Mp) formation would have occurred before reaching the atmosphere. In conclusion, the Earth's atmosphere is not important for the formation of nucleus-monopole bound states*.

A detector close to the Earth's surface is attained by monopoles which come from above as well as from below. These latter most likely passed through the water of the oceans, where, if they were still free, they surely captured a proton.

* It is worth observing that transfer reactions of the form: (Mp) + N → (MN) + p are unlikely, owing to the Coulomb repulsion between the two nuclei.

In conclusion, a monopole reaching Earth is most likely to be accompanied by a proton and behaves, in many respects, as a dyon. This has to be taken into account when planning an experimental search for magnetic monopoles. In particular, when discussing the energy loss of a slow monopole, the energy loss of a unit electric charge can be used as a lower limit*.

5.2. RUBAKOV EFFECT

The existence of bound (MN) system is also interesting in connection with the Rubakov effect [11], i.e. the reaction:

$$\mathrm{M} + \mathrm{p} \to \mathrm{M} + \mathrm{e}^+ + \mathrm{X}. \tag{5.12}$$

Indeed in the bound (MN) state the Rubakov effect is enhanced as a consequence of the large overlap of the proton and monopole wave function. Let us parametrize the cross section for the Rubakov effect as:

$$\sigma_{\mathrm{Rub}} = w m_{\mathrm{p}}^{-2} v_{\mathrm{rel}}^{-1} \tag{5.13}$$

where v_{rel} is the nucleus monopole relative velocity and w is an unknown parameter. We want to derive some experimental information on w.

A monopole passing through a medium with proton density ρ_{p} can directly induce reaction (5.12) with a rate

$$\lambda_{\mathrm{F}} = \rho_{\mathrm{p}} w m_{\mathrm{p}}^{-2},$$

or form a (MN) bound state, with a rate

$$\lambda_{\mathrm{for}} = \rho_{\mathrm{N}} \sigma_{\mathrm{for}} v,$$

where ρ_{N} is the density of nuclei, and v is the relative nucleus monopole velocity. After formation of the (MN) system proton decay occurs at a rate:

$$\lambda_{\mathrm{B}} = \tfrac{1}{2} A \rho_{\mathrm{eff}} w m_{\mathrm{p}}^{-2}. \tag{5.14}$$

ρ_{eff} is defined in eq. (1.2). The wave function at short distances being unknown, tentatively we take:

$$\rho_{\mathrm{eff}} = L_{\mathrm{N}}^{-3}, \tag{5.15}$$

* It is worth observing that a substantial and so far almost unnoticed mechanism for the energy loss of a slow monopole, in the velocity region $(m_{\mathrm{e}}/m_{\mathrm{p}})^{1/2}\alpha c < v < \alpha c$ arises from the interaction with the electron magnetic moment. By means of this interaction atoms can be adiabatically ionised. We expect the cross section for this process to be an appreciable fraction of atomic dimensions.

where L_N is a typical dimension of the monopole-nucleus system. We use:

$$L_N = 20 \text{ fm}. \tag{5.16}$$

The effective rate of catalysed proton decay is then:

$$\lambda_{eff} = \lambda_F + \frac{\lambda_{for}\lambda_B}{\lambda_{for} + \lambda_B}. \tag{5.17}$$

A flux Φ of monopoles with velocity v_M will catalyse proton decay with a rate:

$$\tau_p^{-1} = \frac{\Phi}{v_M \rho_p} \lambda_{eff}. \tag{5.18}$$

We can use the present upper limit from proton decay experiment, [8] $\tau_p > \tau_0 = 3 \cdot 10^{30}$ years, to derive a bound on Φ and w. From (5.18) one gets:

$$v_M/\Phi > \tau_0 w m_p^{-2}\left\{1 + \left(\rho_N L_N^3 + 0.5 A w m_p^{-2} \sigma_{for}^{-1} v^{-1}\right)^{-1}\right\}. \tag{5.19}$$

We take $A = 50$, $Z = A/2$ as representative values for the nuclei in the proton detector, and assume a density $\rho_N = 10^{23}$ cm^{-3}. For v we take the value corresponding to the thermal velocity of nuclei, $v = v_T = 10^5$ cm/s.

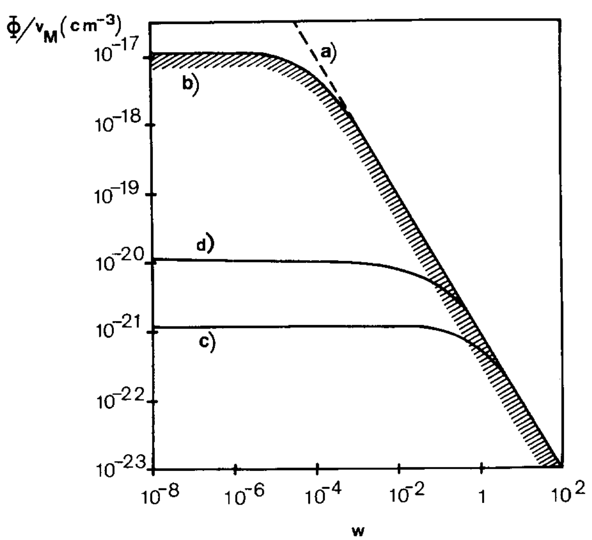

Fig. 4. Bounds on the flux ϕ of monopoles with velocity v_M and on the parameter w of the Rubakov effect (eq. (5.13)), from data on proton stability. The allowed regions lie to the left of the curves, which are calculated according to the following hypotheses: (a) for the Rubakov effect in flight; (b) taking into account (MN) formation, with the formation cross section as in eq. (5.20); (c) the same as in (b), but for a formation cross section 10^4 times larger; (d) the same as in (b), with $v_M = 10^8$ cm/s and a formation cross section as in eq. (5.20).

Fig. 4 shows the allowed regions in the $(w, \Phi/v_M)$ plane for some values of σ_{for}. The straight line (a) corresponds to $\sigma_{for} = 0$, i.e. Rubakov effect in flight eq. (5.13). This case was first discussed in ref. [10].

Curve (b) corresponds to (MN) system formation through the radiative process:

$$\sigma_{for} = \sigma_\gamma^{(MN)} \approx Z^2 m_p^{-2}. \tag{5.20}$$

One sees that the formation of the monopole-nucleus bound states yields more stringent bounds.

As already mentioned, Auger effect should give a value of σ_{for} some orders of magnitude larger than in eq. (5.20). Curve (c) corresponds to $\sigma_{for} = 10^4 \cdot \sigma_\gamma^{(MN)}$ and yields even more stringent bounds. One concludes that it is interesting to have more accurate determination of σ_{for}.

It is worth observing that we have been very conservative when assuming $v = v_T$. Generally one has $v = (v_M^2 + v_T^2)^{1/2}$, and correspondingly (MN) formation occurs at a higher rate. Curve (d) corresponds to $v_M = 10^8$ cm/s and a formation cross section as in eq. (5.20). One concludes that for relatively fast monopoles the radiative formation of (MN) systems enables us to set rather strict bounds on monopole fluxes.

A questionable point in the derivation of the above bounds is the use of eq. (5.15). Clearly the determination of ρ_{eff} is beyond the hard core approximation, which obviously yields $\rho_{eff} = 0$.

To get some estimate of ρ_{eff} we replaced the hard core with a finite potential, $V = -(\lambda/2ma^2)y$, for $R \leqslant a$. By fixing $\lambda = 10$ as a typical value for several nuclei (see table 1), we found $\rho_{eff} a^3 = 0.01$–0.1 when y is varied in the range -1–1. For intermediate nuclei this means $\rho_{eff} \sim 10^{-3}$–10^{-4} fm^{-3}, to be compared with $\rho_{eff} \sim 10^{-3}$ fm^{-3} from eqs. (5.15) and (5.16). Thus the uncertainty on ρ_{eff} does not spoil the above discussion.

5.3. CATALYSIS OF NUCLEAR FUSION REACTIONS

It is interesting to consider the interaction of a (Mp) system with a light nucleus. For a suitable spin orientation, the attraction provided by the monopole magnetic field:

$$U_M \approx -\mu_N/eR^2, \tag{5.21}$$

can compensate the Coulomb repulsion:

$$U_C = Ze^2/R, \tag{5.22}$$

at nuclear distances. Thus one expects an enhancement of the probability of nuclear fusion reactions. In principle this effect could provide a signature of the existence of

TABLE 1

The table presents the binding energy E_1 for the stable nuclei with $S=\frac{1}{2}$ and the first excited level, E_2

Nucleus	$\tilde{\mu}_N$	λ_I	x_1^I	$-E_1^I$	$-E_2^I$	$-E_{1\,s.c.}^I$	λ_{II}	x_1^{II}	$-E_1^{II}$	$-E_2^{II}$	$-E_{1\,s.c.}^{II}$
$_1H^1$	2.79	0.90	0.027	15.1	0.006						
$_1H^3$	2.98	2.97	0.28	112.0	2.5		2.21	0.184	48.4	0.54	
$_2He^3$	−2.13						1.54	0.097	13.4	0.05	
$_6C^{13}$	0.70	1.57	0.102	1.8	0.008	3.42					
$_9F^{19}$	2.63	20.47	2.05	383.0	88.0	378.0	10.72	1.18	126.0	17.2	129.0
$_{15}P^{31}$	1.13	10.04	1.11	49.0	6.3	50.7					
$_{48}Cd^{113}$	−0.62						10.48	1.16	6.3	0.84	6.3
$_{50}Sn^{115}$	−0.92						27.1	2.56	30.0	8.1	29.0
$_{50}Sn^{117}$	−1.0						32.8	2.96	39.0	11.7	38.0
$_{50}Sn^{119}$	−1.05						36.54	3.21	44.0	14.0	43.0
$_{52}Te^{123}$	−0.74						18.62	1.9	14.6	3.1	14.5
$_{52}Te^{125}$	−0.89						28.77	2.88	28.3	8.0	27.8
$_{54}Xe^{129}$	−0.78						22.46	2.21	18.2	4.4	18.0
$_{70}Yb^{171}$	0.49	7.15	0.80	1.48	0.13	1.58					
$_{78}Pt^{195}$	0.61	20.09	2.02	7.6	1.7	7.6					
$_{80}Hg^{199}$	0.50	10.02	1.1	2.2	0.28	2.3					
$_{81}Tl^{203}$	1.61	123.1	7.4	95.0	47.5	97.0	41.42	3.52	21.7	7.4	21.3
$_{81}Tl^{205}$	1.63	126.3	7.53	99.0	50.0	99.0	44.64	3.71	23.8	8.3	23.4
$_{82}Pb^{207}$	0.59	20.0	2.01	6.9	1.5	6.8					

Bound states of both family (I) and (II) are reported, together with the semiclassical values $E_1^{s.c.}$ (left and right part of the table). The energies are in keV. For each family the value of the coupling constant λ and the location of the outermost zero x_1 are also reported. The nuclear magnetic moments $\tilde{\mu}_N$ are reported in the second column. The nuclear radii were calculated according to the rule $a_N = r_0 A^{1/3}$, $r_0 = 1.3$ fm, except for $_1H^1$, $_1H^3$ and $_2He^3$, whose radii were taken to be 1 fm, 2.2 fm and 2.2 fm respectively.

magnetic monopoles. We have considered two processes where the effect could show up. Even if the answer is negative, we think it is interesting to report our arguments.

We have considered the possibility of having the reaction:

$$(\mathrm{Mp}) + \mathrm{d} \to \mathrm{M} + {}_2\mathrm{He}^3 + \gamma, \qquad (5.23)$$

when a monopole passes though a large heavy water detector. The interaction potential of the deuterium nucleus as a function of its distance from the monopole:

$$U_{\text{eff}} = U_N + U_C \simeq -\mu_N/eR^2 + e^2/R, \qquad (5.24)$$

reaches a maximum at $R \approx 25$ fm, where it takes the value $U_{\max} \simeq 25$ keV. The barrier height being considerably larger than the thermal energy, the probability w_T of tunnelling through the barrier turns out to be very small. We find:

$$w_T \simeq \exp[-2m_d/m_e]. \qquad (5.25)$$

The cross section for (5.23) cannot exceed the unitarity limit,

$$\sigma_0 = \frac{\pi}{(m_\mathrm{d} v)^2} w_\mathrm{T} . \tag{5.26}$$

Clearly this is too small to allow for any observable effect*. On the other hand, inside the sun the temperature ($T_\mathrm{sun} \sim 1$ keV) is so large that the presence of monopoles should enhance the rate of the chain of nuclear reactions, particularly the $\mathrm{p} + \mathrm{p} \to \mathrm{d} + \mathrm{e}^+ + \nu$, which is the slowest step of the chain.

If $\gamma = n_\mathrm{M}/n_\mathrm{p}$ is the ratio between the number of monopoles and protons inside the sun, the energy produced by the sun per unit time is:

$$P_\mathrm{sun} = P_\mathrm{sun}^{(0)}(1 + \gamma\alpha) , \tag{5.27}$$

where $P_\mathrm{sun}^{(0)}$ is the energy produced in the absence of monopoles and α is an enhancement factor. We find that the enhancement factor is $\alpha \lesssim 10^9$.

Since we believe we understand stellar energy production without taking into account monopoles, i.e. $P_\mathrm{sun} \simeq P_\mathrm{sun}^{(0)}$, the monopole catalysis of nuclear reactions implies a bound on monopole abundance:

$$\gamma \lesssim 10^{-10} . \tag{5.28}$$

This bound, however, is quite poor as compared to bounds which can be derived by using other arguments**.

6. Conclusions

Let us summarise the main points of our discussion.

(a) We find that magnetic monopoles can form bound states with nucleons, nuclei and atoms through the interaction of the magnetic dipole moment with the monopole magnetic field. We calculate the energies of these systems. The values we find (eqs. (3.6), (3.14) and table 1) are to be considered as upper limits, since the real interaction with the monopole is more attractive than the potential used throughout.

* It is interesting to observe that two nuclei could bind to the same monopole, thus forming a sort of nuclear molecule. Our picture of the monopole-nucleus interaction is so rough that we cannot make an estimate of the binding energy of these systems. It is worth observing however that if these states exist the probability that they are formed is very small as a consequence again of the barrier. Eq. (5.26) can be used as an upper limit for the formation cross section.

** For example, the known values of the sun's mass and size imply: $\gamma < m_\mathrm{p}/m_\mathrm{M} \sim 10^{-16}$. Also, it is reasonable to assume that only monopoles with magnetic charge of the same sign survive inside the sun. These monopoles will create a magnetic field which exceeds the value measured on the sun surface unless: $\gamma < 10^{-23}$. A similar bound can be derived by considering that monopoles are repelled by the sun when the magnetic repulsion exceeds the gravitational attraction.

(b) We have shown that the probability of forming these bound states can be quite large. A lower limit $\sigma_{\text{for}} \gtrsim Z^2 m_p^{-2}$ was obtained for the (MN) formation cross section.

(c) We conclude that monopoles reaching Earth are most likely accompanied by a proton, thus behaving in many respects as dyons.

(d) We consider the formation of (MN) systems in connection with the Rubakov effect. From data on proton stability we derive bounds for the monopole flux and the Rubakov cross section. These bounds could be significantly improved with a better knowledge of the (MN) formation process.

(e) We discuss the possibility that monopoles catalyse fusion reactions of light nuclei.

One can envisage several improvements of the present discussion. By taking into account the nucleon magnetic form factor a more quantitative description of (Mp) bound states can be provided*. In this frame the existence of nuclear molecules could also be discussed. It is worth investigating further the atomic and molecular physics processes involved in the formation of (MN) bound states, as this could provide stricter bounds on the parameters of the Rubakov effect.

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Appendix

The square integrable solution of the Schrödinger equation,

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}R^2}+\frac{\lambda}{R^2}\right)\chi=-k^2\chi, \qquad R>a, \tag{A.1}$$

can be written as [6]:

$$\chi(R)\approx R^{1/2}K_{i\nu}(kR), \tag{A.2}$$

where K_i is the modified Bessel function [19].

The boundary condition $\chi(a)=0$ ($K_{i\nu}(ka)=0$) can be satisfied only for real ν [6], i.e. for $\lambda<-\frac{1}{4}$. In this case K_i has an infinite sequence of zeroes, for which $x=0$ is a limiting point, as can be seen from the following representation, valid for small

* After this work was completed we became aware that this point has been recently discussed (ref. [21]). We thank the authors for sending us their preprint.

values of x:

$$K_{i\nu}(x)=\frac{i\pi}{2\sinh(\pi\nu)}\left\{\frac{e^{i\nu\ln x/2}}{\Gamma(1+i\nu)}-\frac{e^{-i\nu\ln x/2}}{\Gamma(1-i\nu)}\right\}. \tag{A.3}$$

(We recall that $\Gamma(1+i\nu)=\overline{\Gamma(1-i\nu)}$.)

In fig. 3 we plot the ground state wave function for the case of a proton bound to a monopole, where the parameter ν is $\nu=0.804$.

One finds that the largest zero of $K_{i\nu}$ is at $x_1=2.7\times10^{-2}$, a value rather different from that quoted in ref. [6].

For a given system, according to the above equations the energy levels are given by:

$$E_n=-x_n^2/(2ma^2), \tag{A.4}$$

where the x_n are the zeroes of $K_{i\nu}$. In fig. 2 we report the first few zeroes of $K_{i\nu}(x)$ as a function of ν.

The ratio between two consecutive x_n is approximately constant. This can be inferred from eq. (A.3). Indeed, by writing $\Gamma(1+i\nu)=\rho_\nu e^{i\delta_\nu}$, the equation $K_{i\nu}(x)=0$ is equivalent to:

$$\sin\left(\nu\ln\tfrac{1}{2}x-\delta_\nu\right)=0. \tag{A.5}$$

This latter yields:

$$x=2\exp\left[-\frac{n\pi+\delta_\nu}{\nu}\right], \tag{A.6}$$

and consequently:

$$x_{n+1}/x_n=\exp[-\pi/\nu]. \tag{A.7}$$

Eq. (A.6), which is expected to hold for small values of x, is in fact a fair approximation of x_n since the first values of n, at least for $\nu\leqslant10$.

References

[1] P. Goddard and D.I. Olive, Reports Prog. Phys. 41 (1978) 1357;
P. Rossi, Phys. Reports 86 (1982) 317

[2] B. Cabrera, Phys. Rev. Lett. 48 (1982) 1378

[3] E. Amaldi, in Old and new problems in elementary particle physics, ed. G. Puppi (Academic Press, NY, 1968)

[4] G. Giacomelli, Bologna preprint IFUB 81/26; invited paper at Conf. on monopoles in quantum field theory, Trieste 1981, to be published

[5] W.V.R. Malkus, Phys. Rev. 83 (1951) 899

[6] D. Sivers, Phys. Rev. D2 (1970) 2048
[7] T.T. Wu and C.N. Yang, Nucl. Phys. B107 (1976) 365
[8] M. Leon, Exotic atoms '79, eds. K. Crowe, J. Duclos, G. Fiorentini and G. Torelli (Plenum, NY, 1980) and reference therein
[9] L. Bracci, G. Fiorentini and O. Pitzurra, Phys. Lett. 85B (1979) 280
[10] L. Bracci and G. Fiorentini, Phys. Lett. 124B (1983) 29
[11] V.A. Rubakov, JEPT Lett. 33 (1981) 644; preprint P0211 (1981);
G.C. Callan, Nucl. Phys. B212 (1983) 391
[12] P.A.M. Dirac, Proc. R. Soc. A133 (1931) 60; Phys. Rev. 74 (1948) 817
[13] I. Kazama, C.N. Yang and A.S. Goldhaber, Phys. Rev. 15 (1977) 2287
[14] J.D. Jackson, Classical electrodynamics (Wiley 1st edition, NY, (1962)) p. 506
[15] S. Weinberg, Gravitation and cosmology (Wiley NY, 1972) chap. 15
[16] H.A. Bethe and E.E. Salpeter, Quantum mechanics of one and two electron atoms (Springer, Berlin, 1957)
[17] C.W. Allen, Astrophysical quantities (The Athlone Press, University of London, 1973)
[18] F. Reines and M.F. Grouch, Phys. Rev. Lett. 32 (1974) 493;
J. Learned, F. Reines and A. Soni, Phys. Rev. Lett. 43 (1979) 907
[19] J. Ellis, D.V. Nanopoulos and K.A. Olive, Cern preprint TH-3323 (1982)
[20] Handbook of mathematical functions, eds. M. Abramowitz and I.A. Stegun, National Bureau of Standards, Washington (1965)
[21] J. Makino, M. Maruyama and O. Miyamura, Osaka preprint 82-11-2