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TOWARD A UNITARIZATION OF THE VENEZIANO MODEL*

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The *N/D* method is used to constrain the parameters of the Veneziano model. A more refined procedure, preserving exact crossing, is noted.

Veneziano [1] has suggested an Ansatz for a two-particle scattering amplitude that is crossing-symmetric, has Regge asymptotic behaviour, and satisfies finite-energy sum-rules (FESR) in the Dolen-Horn-Schmid [2] sense. Its main shortcoming is that it is not unitary. Indeed, it is not even possible for the trajectory function $\alpha(s)$, to have a right-hand cut, if "ancestor" poles are to be avoided#. If one could modify the Veneziano amplitude in such a way to make it unitary, while still maintaining crossing and Regge behaviour, then one would have a very interesting model of two-particle scattering.

We have found that the Veneziano amplitude can be unitarized, at low energies, without too much alteration, only if the parameters of the model satisfy certain contraints. It is the purpose of this paper to exhibit these constraints, and to report a preliminary numerical calculation of a

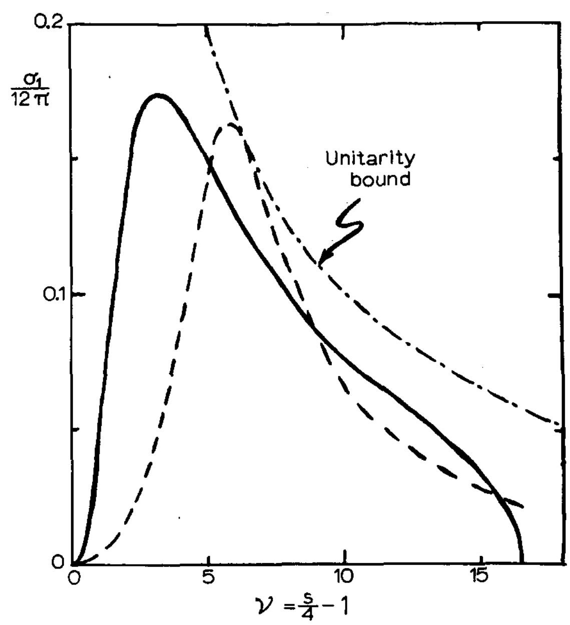

Fig. 1. Plot of the partial-wave cross-section, σ_1, as calculated from the *N/D* equations, with $B_1(s) = B_1(4)$, $\alpha' = \frac{1}{80}$, $\bar{\beta} = 1.53$ and $\alpha(30) = 1$ (solid curve). The dotted curve resonance at $s = 30$ and a width corresponding to the same $\bar{\beta}$.

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It may be possible to remove these by adding an infinite number of Veneziano satellite terms, however.

partially unitarized version of the Veneziano amplitude, when the contraints are satisfied.

The idea is to replace the first resonance pole in a partial-wave projection of the Veneziano amplitude by a cut extending from the threshold to a point mid-way between the first two resonances. This is the usual point taken to divide the resonance and Regge regions in applications of FESR [2]. One then demands that the discontinuity across this cut be given by unitarity. This leads to a nonlinear equation, which can be solved by the N/D method. Of course, the original FESR are no longer satisfied, in general; but they can be partially retored by requiring, say, the first two moments over the cut to be equal to those over the original resonance. This procedure is only meaningful if the unitarized absorptive part exhibits a peak, in which case the requirement amounts, roughly speaking, to demanding that the position and width of the original resonance be reproduced. This contraint leads to a determination of two of the three parameters of Veneziano's Ansatz.

To improve this calculation, one could replace the first N resonance poles by a cut extending from the threshold to a point mid-way between the Nth and the $(N+1)$ st. resonances. Inelastic effects could be included by replacing the phase-space factor, $\rho(s)$, by $-\mathrm{Im}[1/A_l^{\mathrm{Regge}}]$, in the inelastic region‡. Naturally, one would not expect a CDD pole-free calculation to give anything resembling N resonances, since the higher resonances, since the higher resonances of the Veneziano model are not likely to be reproduced by a one-channel calulation. It is nevertheless possible to avoid the explicit inclusion of other channels by adding $(N-1)$ CDD poles. This does not give rise to any extra parameters (as one might at first expect) since the first N poles (which have been replaced by the cut) are related to each other by the Veneziano formula, and one can always require the output states to agree with them. This "agreement" can be effected, much as before, by requiring that the first $2N$ moments over the cut be equal to those over the original states‡‡.

Consider $I = 1$ $\pi\pi$ scattering, for simplicity. The general crossing-symmetric Veneziano form for this process is

$$V(s,t) = -\bar{\beta}\left[\frac{\Gamma(p-\alpha(s))\Gamma(p-\alpha(t))}{\Gamma(r-\alpha(s)-\alpha(t))} - (t\to u)\right] \qquad (1)$$

where s, t and u are the usual Mandelstam variables, $\alpha(s)$ is the Regge trajectory function (which is taken to be lenear), and $\bar{\beta}$ is constant. The leading term corresponds to $r = p$, whereas satellite terms correspond to $r > p$. Since the ρ-meson is the lowest lying resonance on $\alpha(s)$, the leading term must correspond to $p = 1 = r$.

The partial-wave projection of (1), divided by the threshold factor, $(s-4)^l$, say $V_l(s)$, has a left-hand cut, and also a sequence of poles on the right, arising from the function, $\Gamma(1-\alpha(s))$, in the numerator. Consider the $l = 1$ projection, for simplicity. This is the lowest physical partial-wave in the $I = 1$ state, and has the ρ-meson as its lowest resonance. Of course $V_1(s)$ does not satisfy unitarity on the right. It is to be modified by subtracting out the ρ-pole contribution, $V_1^{\rho}(s)$, and replacing it by a cut. Thus

$$A_1(s) = \frac{1}{\pi}\int_4^{s_1} \frac{\mathrm{d}s'\rho(s')}{s'-s}|A_1(s')|^2 + B_1(s), \qquad (2)$$

where $\alpha(s_1) = \frac{3}{2}$ and $B_1(s) = V_1(s) - V^{\rho}(s)$. This nonlinear integral equation can be solved by the Uretsky form of the N/D method [3].

The above procedure is similar to that of the strip approximation [4, 5]. Its main advantage over first calculating the left-hand cut discontinuity from (1), and then constructing the entire right-hand cut by the N/D method, is that elastic unitary is imposed only in the low-energy region, $4 \leq s \leq s_1$, where it is most likely to be valid. Moreover, the present treatment is considerably simpler in practice. The high-energy region, $s > s_1$, continues to be given by the original Veneziano formula.

The lowest moment sum-rules are now restored by imposing

$$\int_4^{s_1} \mathrm{d}s\,(s-4)^n\,\mathrm{Im}\,A_1(s) = \int_4^{s_1} \mathrm{d}s\,(s-4)^n\,\mathrm{Im}\,V_1(s), \qquad (3)$$

for $n = 0$, 1. If $\mathrm{Im}A_1(s)$ is dominated by a narrow peak, these conditions simply amount to requiring the position and with of the resonance to match those of the original resonance in $V_1(s)$. Eq. (3) is equivalent to requiring that the coefficients of s^{-1} and s^{-2}, in the large-s expansion of A $A_1(s) - B_1(s)$, be the same as those of $V_1^{\rho}(s)$.

A preliminary calculation (good to about 20%) was made by approximating $B_1(s)$ by its threshold value, $B_1(4)$, which is particularly easy to evaluate. It is now trivial to solve the N/D equations

‡ In addition to the Regge poles in the Veneziano model, the Pomeranchuk contribution should probably be included.

‡‡ Alternatively, it may be better to consider sum-rules with a continuous moment index, n, and then to impose the corresponding conditions on the value and derivatives, with respect to n, taken at n=0.

[5] and to impose conditions (3) for $n = 0, 1$. This implies two constraints upon the three parameters of the Veneziano model (these parameters are $\bar{\beta}$, and the slope and intercept of $\alpha(s)$, which is assumed linear). A third contraint could perhaps be obtained by extending the calulation to all isospin states, and by imposing the Veneziano requirement of degeneracy between the $I = 0$ and $I = 1$ trajectories, at least at $\alpha(s_\rho) = 1$. In the present more limited calculation, $\alpha(s_\rho) = 1$ is forced to correspond to the experimental ρ mass ($s_\rho = 30$). Then the two moment conditions serve determine α', the slope, and $\bar{\beta}$. The results of the calculations are $\alpha' = \frac{1}{80}$ and $\bar{\beta} = 1.5$. The corresponding experimental values are $\alpha' = \frac{1}{50}$ (if α is linear and passes through the ρ and f_0 experimental points), and $\bar{\beta} = 0.6$ (corresponding to a ρ-width of 125 MeV). These results are confirmed by more accurate computer calculations, which will be reported elsewhere [6].

A plot of the partial-wave cross-section, $\sigma_1 = 12\pi[(s-4)/s]^{\frac{1}{2}} \operatorname{Im} A_1(s)$ (see fig. 1), shows that the maximum occurs at $s = 18$, instead of $s = 30$. Presumably, this is a reflection of the crudity of the calulation. Perhaps an even more serious difficulty is that, although σ_1 does exhibit a peak, the phase-shift never passes through 90°. Unless this is rectified when the calculation is improved by increasing s_1 and adding CDD poles (as described above), this would indicate that higher channels are important, and must be included explicitly.

If the N/D calculations were extended to more partial waves (an if other channels were included), one might well find the system to be overdetermined . However, in such a case, one could always add suplementary terms of the form (1), but with different r and p. Any extra constraints could then be used to determine the coefficients $\bar{\beta}$ associated with these supplementary terms.

Finally, it should be remarked that the unitarization procedure destroys strict crossing symmetry. However, it is straightforward to set up a scheme in which above calculation is but the first approximation. One would first calculate all partial waves up to, say, $l = L$, by the above method, thus generating unitarized cuts in the direct channel. The next stage would be replace the corresponding poles in the crossed channel by the unitarized cuts from the first stage, and to calculate a new Born term, $B_l(s)$, from which new N/D calculations could be made for $l \leq L$. The whole process could be cycled any number of times. If this were to converge, one would have, in the limit, a crossing-symmetric amplitude that is unitary for $4 \leq s \leq s_1$, $l \leq L$, in both channels.

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