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ABSTRACT

The form factors of the $N_{33}^{*}N$ weak-axial vectors are expressed in terms of the vector and axial vector form factors of the nucleon. A relation between the $N_{33}^{*}N\pi$ and $NN\pi$ coupling constants and an estimate of the mass of a hypothetical 1^{+} meson are also obtained.

EVALUATION OF THE $N^*_{33}N$ WEAK COUPLING CONSTANTS BY MEANS OF CURRENT ALGEBRA

Recently many outstanding results have been obtained from the use of the equal time commutation relations which generate the algebra of a symmetry group, but without assuming invariance under it[1]. In particular the $SU(2) \otimes SU(2)$ algebra has shown itself very promising, when supplemented by the PCAC assumption, which represents the bridge between strong interactions and weak phenomena.

In this note we investigate the form factors $H_i(\Delta^2)$ of the $N^*_{33}N$ weak axial vertex, which we define through

$$\langle n | A^{(-)}_\mu | N^{*+}_{33} \rangle = i \bar{u} \left[-H_1 \delta_{\mu\nu} - \frac{i}{m_\pi} H_2 p_\nu \gamma_\mu + \frac{1}{m_\pi^2} H_3 p_\nu (p^* + p)_\mu + \frac{1}{m_\pi^2} H_4 p_\nu (p - p^*)_\mu \right] u_\nu \qquad (1)$$

The form factors of the corresponding vector vertex $\langle n | V^{(-)}_\mu | N^{*+}_{33} \rangle$ can be derived by CVC from the electromagnetic ones occurring in electroproduction. From the knowledge of the two vertices it is possible to calculate the amplitude for the process

$$\nu + N \rightarrow \mu + N^* \quad (N^* \rightarrow N + \pi)$$

Our aim is to express the $H_i(\Delta^2)$ in terms of the vector and axial form factors of the nucleon. As byproducts we obtain a relation between the $N^*_{33}N\pi$ and $NN\pi$ coupling constants, which is well satisfied experimentally, and an estimate of the mass of an hypothetical 1^+ meson, provided the weak axial form factors are dominated by its pole.

We start by considering the following commutators of the SU(2) x SU(2) algebra

$$[\bar{I}^{(3)}, A_\mu^{(3)}] = 0 \qquad (2)$$

$$[\bar{I}^{(+)}, A_\mu^{(-)}] = 2V_\mu^3 \qquad (3)$$

where V_μ, A_μ are the vector and axial vector currents, $\bar{I}^i = \int A_0^i \, d\bar{x}$, $A_\mu^- = A_\mu^1 - i\, A_\mu^2$ etc, and we use isotopic spin to relate, for instance, A_μ^3 to A_μ^- .

We take the matrix elements of Eqs. 2),3) between nucleon states $|N_1\rangle$, $|N_2\rangle$, of momenta p_1 , p_2 . According to the covariant method of ref. 2), we introduce

$$B_\mu^{(i,j)} = \int d^4x \, \theta(-x_0) e^{-iqx} \langle N_2 | [\bar{D}^{(i)}(x), A_\mu^{(j)}(0)] | N_1 \rangle , \quad \bar{D} = \partial_\nu A_\nu$$

so that from Eq.s 2), 3) we get the "low energy theorems"

$$\lim_{q \to 0} B_\mu^{(3,3)} = 0 \qquad (4)$$

$$\lim_{q \to 0} B_\mu^{(+,-)} = 2 \langle N_2 | V_\mu^3 | N_1 \rangle \qquad (5)$$

For a further analysis of the sum rules 4), 5) we treat any $B_\mu^{(i,j)}$ according to dispersion relation techniques. To this purpose we introduce the scalar variables

$$\Delta^2 = (p_2 - p_1)^2 \geq 0 , \quad p_1 \cdot q = p_2 \cdot q = -m\nu$$

where we impose $q^2 = 0$; $q \cdot \Delta = 0$.

Then we decompose $B_\mu^{(i,j)}$ into invariant functions and we assume for each of them an unsubtracted dispersion relation in ν , at fixed Δ^2 , i. e.

$$B_\mu = \Sigma_s M_\mu^s B^s(\nu, \Delta^2)$$

$$B^s(\nu, \Delta^2) = \frac{1}{\pi}\int \frac{A_I^{(s)}(\nu', \Delta^2)}{\nu' - \nu} d\nu' - \frac{1}{\pi}\int \frac{A_{II}^{(s)}(\nu', \Delta^2)}{\nu' - \nu} d\nu'$$

where the $A_{I,II}^{(s)}$ can be deduced from the general quantities

$$A_I = \frac{i}{2}\Sigma_\alpha (2\pi)^4 \delta(p_2 + q - p_\alpha) \langle N_2 | \bar{D} | \alpha \rangle \langle \alpha | A_\mu | N_1 \rangle$$

$$A_{II} = \frac{i}{2}\Sigma_\alpha (2\pi)^4 \delta(p_1 - q - p_\alpha) \langle N_2 | A_\mu | \alpha \rangle \langle \alpha | \bar{D} | N_1 \rangle$$

Owing to the limit $q \to 0$ ($\nu \to 0$) involved in our Eq.s 4), 5) only some M_μ^s survive and they are chosen to be $(p_1+p_2)_\mu$, $(p_2-p_1)_\mu$, γ_μ.

For a practical evaluation we keep as intermediate states the nucleon and the $N_{33}^*(1236)$ resonance. Such an approximation has shown itself rather satisfactory in previous works[3,4], to which we refer a discussion of its validity. To express the matrix elements of $\bar{D}$ in terms of the physical vertices we use the relation

$$\langle \alpha | \bar{D}^{(-)} | \beta \rangle = - \frac{\sqrt{2}\, m\, m_\pi^2}{(p_\alpha - p_\beta)^2 + m_\pi^2} \frac{g_A}{g} \langle \alpha | j_\pi^{(-)} | \beta \rangle \qquad (6)$$

derived assuming the dominance of the pion pole in $(p_\alpha - p_\beta)^2$.

The vertices we need are

$$\langle p_2 | j_\pi^{(3)} | p_1 \rangle = i g \bar{u}_2 \gamma_5 u_1 \ ,$$

$$\langle p_2 | j_\pi^{(3)} | N_{33}^{*+} \rangle = \sqrt{\frac{2}{3}} \frac{\lambda}{m_\pi} \bar{u}_2 u_\nu (p^* - p_2)_\nu \ ,$$

$$\langle p_2 | V_\mu^3 | p_1 \rangle = \frac{1}{2} \bar{u}_2 \left[F_1^V \gamma_\mu - \frac{i}{4m} F_2^V (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)(p_2 - p_1)_\mu \right] u_1$$

$$\langle n_2 | A_\mu^- | p_1 \rangle = \bar{u}_2 \left[i r_A G \gamma_5 \gamma_\mu + \beta \gamma_5 (p_2 - p_1)_\mu \right] u_1$$

where $F_1^V(0) = 1$, $F_2^V(0) = 3.71$, $G(0) = 1$, $r_A \simeq -1.18$, $\beta(0) \simeq -2mr_A/m_\pi^2$.

With these definitions, we obtain one relation from the sum rule 4)

$$\frac{m\lambda}{\sqrt{6}\, m_\pi g} \frac{M+m}{M} \left[\frac{1}{3} H_1 - \frac{M}{3m_\pi} H_2 + \frac{M^2}{m_\pi^2} \rho_+ H_4 \right] = \frac{1}{4} M \beta \qquad (7)$$

and two relations from the sum rule 5), by comparing the terms in γ_μ and $(p_1+p_2)_\mu$ of its l.h.s. and r.h.s.

$$r_A \frac{m\lambda}{\sqrt{6}\, m_\pi g} \frac{M+m}{M} \left[\frac{2}{3} H_1 - \frac{2M}{m_\pi} \rho_- H_2 \right] = r_A^2 G - F_1^V - F_2^V \qquad (8)$$

$$r_A \frac{m\lambda}{\sqrt{6}\, m_\pi g} \frac{M+m}{M} \left[-\frac{1}{3} H_1 - \frac{M}{3m_\pi} H_2 + \frac{M^2}{m_\pi^2} \rho_+ H_3 \right] = \frac{M}{4m} F_2^V \qquad (9)$$

where $\rho_\pm = 1 \pm \frac{m}{3M} - \frac{M^2+m^2}{3M^2} - \frac{\Delta^2}{3M^2}$.

Choosing in Eq. 6) $|\alpha\rangle$ and $|\beta\rangle$ to be a nucleon and an N_{33}^* state, we can add another relation

$$-H_1 + \frac{M-m}{m_\pi} H_2 - \frac{M^2-m^2}{m_\pi^2} H_3 - \frac{\Delta^2}{m_\pi^2} H_4 = -r_A \frac{\sqrt{2}\, \lambda m}{\sqrt{3}\, g\, m_\pi} \frac{m_\pi^2}{\Delta^2 + m_\pi^2} \qquad (10)$$

Thus we have at our disposal four equations from which, in principle, we can evaluate the four form factors $H_i(\Delta^2)$ in terms of F_1^V, F_2^V, G and β .

It is known that the pion pole is present in $\beta(\Delta^2)$ and $H_4(\Delta^2)$ only. Our equations should in principle hold for spacelike Δ^2, but we can extrapolate them until the pion pole $\Delta^2 = -\omega_\pi^2$, owing to the smallness of ω_π. By comparing the residua of $H_4(\Delta^2)$ as given by Eq.s 7), 10) we get

$$\frac{\lambda^2}{g^2} = \frac{9\omega_\pi^2 M^2}{2\omega (M+\omega)^2 (2M-\omega)} \tag{11}$$

This gives $\lambda = 1.90$, while the experimental value[5] turns out to be 2.12. The agreement can be considered satisfactory, and the corrections due to the higher states are seen to give a 10% contribution, as expected on the basis of the discussion given in ref. 3), 4). To get a better appreciation of this result, we recall that in a treatment of π N scattering where the N_{33} is considered a true particle, unitarity considerations give, in the static limit[6],

$$\frac{\lambda^2}{g^2} = \frac{9}{8} \frac{\omega_\pi^2 (\omega^* + \omega)}{\omega^3} \tag{12}$$

with $\omega^* = M - m$. We note that, in the "static limit" $\frac{M}{(M+\omega)^2 (2M-\omega)} \rightarrow \frac{1}{4\omega^2}$, the Eq.s 11) and 12) coincide.

Using Eq. 11), the Eq.s 7) - 10) give, at $\Delta^2 = 0$

$$\begin{aligned} H_1(0) &= -0.41 \\ H_2(0) &= -1.13 \\ H_3(0) &= -0.088 \\ H_4(0) &= 0.86 \end{aligned} \tag{13}$$

We can remark that our $H_1(0)$ is much smaller than the one estimated by Berman and Veltman[7] by means of Eq. 10) by neglecting terms in $H_2(0)$ and $H_3(0)$.

Standard dispersive treatment in the Δ^2 channel suggests that a 1^+ meson (if it exists) might dominate the weak form factors $G(\Delta^2)$, $H_i(\Delta^2)$ i=1,2,3 , so that

$$G(\Delta^2) = \frac{H_i(\Delta^2)}{H_i(0)} = \frac{m_A^2}{\Delta^2+m_A^2} \qquad i = 1,2,3,$$

where m_A is the meson mass. We can try to estimate it by looking at the form factors slope at $\Delta^2 = 0$. In Eq.s 7),10) the terms with the pion pole are enhanced in taking derivatives, masking the smoother behaviour of the other terms. From Eq.s 8),9) assuming $F_1^V(\Delta^2) = F_2^V(\Delta^2)/3.71 = m_\rho^2/(\Delta^2 + m_\rho^2)$, we obtain respectively $m_A = 1.07$ Gev and $m_A = 1.18$ Gev.

In a previous work[4], from the commutators

$$[\overline{I}^{(3)}, V_\mu^{(3)}] = 0 \qquad [\overline{I}^{(-)}, V_\mu^{(3)}] = A_\mu^{(-)}$$

and in the same approximation as above, we derived

$$G(\Delta^2) = F_1^V(\Delta^2) + \frac{\Delta^2}{4m^2} F_2^V(\Delta^2)$$

From the slope at $\Delta^2 = 0$, we obtained $m_A = 1.24$ Gev (in a more refined calculation of $G(\Delta^2)$ the result was 1.1 Gev). The agreement among the various estimates seems to be rather encouraging.

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