Preprint & this talk at: http://eotvos.dm.unipi.it/documents/generalpapers/EPwithSLR.html

Limitations to testing the Equivalence Principle with Satellite Laser Ranging

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EP & UFF

- The Equivalence Principle (EP) is tested by testing the Universality of Free Fall (UFF) with test masses of different composition freely falling in the gravitational field of a source body

$$\eta \equiv \frac{\Delta a}{a} \equiv \frac{differential\ acceleration\ between\ free\ falling\ test\ masses}{free\ fall\ acceleration\ of\ test\ masses\,(\text{"}driving\text{"}\ acceleration)}$$

$$\eta = 0 \Rightarrow UFF\ holds;\ no\ EP\ violation$$

the closer to zero is the value of η measured with a given experiment, the better this experiment tests the EP:

$\eta_{RTB} \simeq 0.9 \cdot 10^{-13}$ *w.r.t. Sun* "Eot-Wash" group, PRL **83** 3585 (1999)

$\eta_{LLR} \simeq 10^{-13}$ *w.r.t. Sun* J.G. Williams et al. PRL **93** 261101 (2004)

The driving acceleration

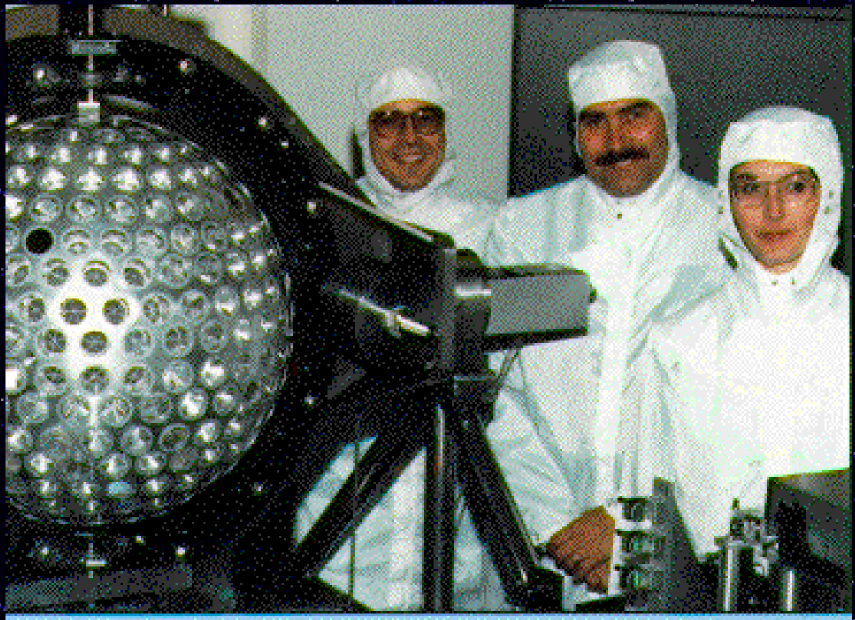
a (m/s^2): driving acceleration		
	Source body	
	Earth	Sun
Test masses suspended on Earth (best tests with RTB w.r.t. Sun)	1.69×10^{-2} max (at 45° latitude)	0.006 (max)
Test masses free falling on Earth (Galileo mass dropping experiments…not competitive with TB…)	9.8	0.006 (max)
Test masses free falling around the Earth (Galileo mass dropping from a tower of "infinit" height…)	8.4 (at low altitude: $h \cong 500$ km)	0.006 (max)

$$\frac{8.4}{0.006} \simeq 1400$$

numerical factor to gain (over torsion balance ground experiments) by doing an EP experiment with test masses in low orbit around the Earth

We do already have test masses orbiting around the Earth….

- LAGEOS/LAGEOS II: passive, compact (low A/M ratio to reduce non gravitational perturbations), cannon-ball satellites covered with corner cube laser reflectors. The closest possible artificial objects to the ideal "test mass" of celestial mechanics!



-400 kg, 60 cm diameter; CuBe

-Semimajor axis: $2R_{\oplus}$

-Launched:

-LAGEOS 1976 (NASA)

-LAGEOS II 1992 (NASA & ASI)

-Tracked by Laser ranging stations on the Earth

numerical factor to gain (over torsion balance ground tests) by doing an EP test at the altitude of LAGEOS: $\simeq 1200$

The theoretical background: 2-body problem with EP violation (I)

- ….for a satellite around the Earth of negligible mass compared to the planet (will show that the general case of comparable masses is an easy extension..) whose motion obeys $1/r^2$ law but may show a dependence on composition

$$M_g = M_i \equiv M \qquad m_g = m_i(1+\eta)$$

$$\ddot{\vec{r}} = -\frac{GM(1+\eta)}{r^3}\vec{r} \qquad \eta \neq 0$$

amounts to changing the product (GM) and therefore can only "rescale" the orbit of the satellite….

$$\vec{J} = \vec{r} \times \dot{\vec{r}} \quad , \quad \vec{e} = \frac{1}{GM(1+\eta)}\dot{\vec{r}} \times \vec{J} - \frac{\vec{r}}{r}$$

orbital angular momentum and Lenz vector integrals of motion…

$$r = \frac{J^2/[GM(1+\eta)]}{1+e \cdot \cos f}$$

orbit equation…

true anomaly (angle measured from Lenz vector)

orbit eccentriicity (modulus of Lenz vector integral)

The theoretical background: 2-body problem with EP violation (II)

$$\frac{r}{a} = \frac{1-e^2}{1+e\cdot\cos f}$$

note that this ratio does not depend on η ……

orbit semimajor axis

$$J = \sqrt{GM(1+\eta)a(1-e^2)}$$

$$E = -\frac{GM(1+\eta)}{2a}$$

$$e$$

The orbital angular momentum integral and the energy integral are composition dependent, but the eccentricity is not….

$$n^2a^3 = GM(1+\eta)$$

third Kepler's law…

$n=2\pi/P$ mean motion (average orbital angular velocity)

The theoretical background: 2-body problem with EP violation (III)

..so, to account for EP violation in the 2-body problem we only need to change

$$GM \Rightarrow GM(1+\eta)$$

keeping in mind that whenever the equations are written per unit of the secondary mass, this is the inertial mass of the secondary body, which may now differ from its gravitational mass (by η) resulting in an orbit of different size, depending on the value of η

The relevant physical quantity which needs to be measured in order to test the EP with a satellite in orbit around the Earth is the semimajor axis a of its orbit

Previous literature on the use of free orbiting test masses to test the EP did not provide a clear theoretical beckground and focused instead on the issue of the initial conditions of the test masses…(EP violation simply rescales the orbit and therefore no point along the orbit has any special physical meaning, not even the initial one!!)

(e.g. Blaser, CQG 18, 2509 (2001))

The theoretical background: 2-body problem with EP violation (IV)

If the masses of the 2 bodies are comparable, all the previous results hold provided that the mass of the primary is substituted by

$$M_{tot} \equiv M + m_g$$

and whenever the equations were written per unit of the secondary (inertial) mass, they are now written per unit of reduced (inertial) mass of the system

$$\mu_i \equiv \frac{Mm_i}{M + m_i}$$

Once the reduced problem is solved, it is well know that it provides the solution for the individual orbits of the two bodies…

A “simple” EP test with LAGEOS…. (I)

- Can the orbit of LAGEOS tell us about its η ? And how accurately?

From 3rd Kepler’s law, since the period (mean motion) is the best measured quantity, we take it to be known exactly and compare all relevant physical quantities in the 2 cases :

$\eta = 0$ classical 2-body problem of newtonian celestial mechanics (relativistic corrections are applied to the orbit of real LAGEOS…)

$\eta \neq 0$ violation of UFF (and of EP). We call this case non-galilean…

$$a_{ng} \simeq a_c(1+\frac{1}{3}\eta)$$

$$E_{ng} \simeq E_c(1+\frac{2}{3}\eta)$$

$$J_{ng} \simeq J_c(1+\frac{2}{3}\eta)$$

A “simple” EP test with LAGEOS…. (II)

- We now define $\Delta a_{EP} \equiv a_{ng} - a_c$

  as the difference in the semimajor axis of the satellite orbit in the two cases, with and without, an EP violation to the level. η

  With

$$a_{ng} \simeq a_c (1 + \frac{1}{3}\eta) \quad \Rightarrow \quad \eta \simeq 3 \frac{\Delta a_{EP}}{a_c}$$

  hence

$$r_{ng} \simeq r_c (1 + \frac{\Delta a_{EP}}{a_c})$$

$$E_{ng} \simeq E_c (1 + 2 \frac{\Delta a_{EP}}{a_c})$$

$$J_{ng} \simeq J_c (1 + 2 \frac{\Delta a_{EP}}{a_c})$$

A “simple” EP test with LAGEOS…. (III)

- But it is clear that the same value Δa_{EP} might simply be due to measurement error in the semimajor axis of the satellite, with no EP violation at all. Then, we would call it Δa_{meas} and get an equivalent “classical”

$$\eta_{class} \simeq 3\frac{\Delta a_{meas}}{a_c}$$

which sets the limit to which a LAGEOS-like laser tracked satellite can test the EP.

Note that, from 3rd Kepler’s law in the classical case, it is well known that the measurement error in semimajor axis of the satellite also limits the knowledge of (GM) of the planet:

$$\frac{\Delta(GM)}{GM} \simeq 3\frac{\Delta a_{meas}}{a_c}$$

A "simple" EP test with LAGEOS…. (IV)

- The numbers for LAGEOS are…

International Earth Rotation and Reference System Service (IERS), 2003

$$\frac{\Delta(GM)}{GM} = \frac{3.986004418 \cdot 10^{14} \, m^3 s^{-2}}{8 \cdot 10^5 \, m^3 s^{-2}} \simeq 2 \cdot 10^{-9}$$

and (with 15d orbital arcs, from normal points to about 1-3 mm):

$$\Delta a_{meas} \simeq 1 \, cm$$

from which:

$$\eta_{classSLR} \simeq 3 \frac{\Delta a_{meas}}{a_c} \simeq 3 \frac{10^{-2} \, m}{1.23 \cdot 10^7 \, m} \simeq 2.4 \cdot 10^{-9}$$

This results will improve with the square root of the integration time, but still it would need about 120000 yr of LAGEOS data to get this limit down to the 10^{-12} level of EP test already reached by torsion balances!!!

Note that this result holds without even considering the perturbing effects of non gravitational forces on 2 LAGEOS of different composition ….

Would it help to use the pericenter of LAGEOS? NO…

- The pericenter of the orbit of LAGEOS has a secular variation (unlike the semimajor axis...) due to the flattening of the Earth, and so does the effect of an EP violation on it, so the effect grows linearly with time

$$\dot{\omega} \simeq -\frac{3}{4}\frac{(GM(1+\eta))^{1/2}}{a^{7/2}} J_{2\oplus} R^2_{eq\oplus} \frac{1-5\cos^2 I}{(1-e^2)^2}$$

however:

$$\frac{\dot{\omega}_{ng}}{\dot{\omega}_c} \simeq 1-\frac{2}{3}\eta$$

and in order to make this ratio small (in order to measure η) it is required that the classical change of the argument of pericenter be very accurately determined. But it depends on the semimajor axis, so we are back to the previous case…

In point of fact, long arc data sets are not used for LAGEOS, not even for detremining the argument of epricemter, because of the accumulation of non gravitational perturbation.

Why then is LLR so successful in EP testing?

…clearly it cannot be because non gravitational forces are negligible on celestial bodies while they are relevant on artificial satellites (though this is true..), since we have not considered them at all in the previous analysis for LAGEOS

The reason is that the distance of the test masses (Earth & Moon) from the source body (Sun) is much larger than it is for LAGEOS around the Earth:

we take a 1 cm measurement error as for LAGEOS…

$$\eta_{classLLR} \simeq 3\frac{\Delta a_{meas}}{d_{\oplus\odot}} \simeq 3\frac{10^{-2}\,m}{1.5\cdot 10^{11}\,m} \simeq 2\cdot 10^{-13}$$

So, the success of LLR in EP testing to about this level is no mistery. Though, there is not much room for considerable further improvement….

Why not use LAGEOS in the field of the Sun, similarly to the Moon? (I)

- One could consider LAGEOS and the Earth for EP violation in the field of the Sun, or the orbits of two LAGEOS-like satellites of different composition also in the field of the Sun… There would be no gain in the driving signal (bad news), but may be one could make a test close to that of LLR (same driving signal form the Sun)…. Satellite ranging data would be the same, simply analysed for EP violation w.r.t. the Sun..

Such an EP violation would result in a “polarization” of the satellite orbit around the Earth in the Earth-Sun direction: the center (or focus) of the satellite orbit would not coincide with the center of mass of the Earth, but be displaced torwards the Sun, or away form it depending on the sign of η by :

mean motion of the satellite around the Earth (n_{sat})

mean motion of the Earth around the Sun ($n_\oplus$)

$$\left|\delta r_{pol}\right| \simeq \frac{1+\dfrac{2n_{sat}}{n_{sat}-n_\oplus}}{n_{sat}^2-(n_{sat}-n_\oplus)^2}\, n_\oplus^2 d_{\oplus\odot}\eta$$

polarization of the orbit of an Earth satellite in presence of EP violation in the field of the Sun ($\left|\delta r_{pol}\right|$)

Why not use LAGEOS in the field of the Sun, similarly to the Moon? (II)

and, in terms of the dimensionless parameter:

$$m = \frac{n_{\oplus}}{n_{sat} - n_{\oplus}}$$

$$\left|\delta r_{pol}\right| = \frac{3}{2} m \left[1 + \frac{1}{6} m - \frac{1}{12} m^2 + \cdots \right] d_{\oplus\odot} \eta$$

1 for LAGEOS (*m* parameter very small…)

about 1.6 for the Moon

Why not use LAGEOS in the field of the Sun, similarly to the Moon? (III)

…giving the ratio of orbit polarization for LAGEOS and the Moon:

$$\frac{\left|\delta r_{polLAGEOS}\right|}{\left|\delta r_{polMoon}\right|} \simeq \frac{n_{Moon} - n_{\oplus}}{n_{LAGEOS}} \cdot \frac{1}{1.6} \simeq \frac{1}{300}$$

This means that an EP violation for LAGEOS of the same amount as for the Moon would require to detect, in the case of LAGEOS, a polarization of its orbit about 300 times smaller than for the Moon by a factor 1/300,,,

… LAGEOS is closer to Earth than the Moon, by about a factor 30, and therefore its orbit is less affected by the Sun.. which would be the case also for an EP violation effect driven by the Sun…

Still, an EP test with LAGEOS in the field d of the Sun to 10^{-11} might be possible, and would be of interest for both LLR and SLR communities:

LLR community could try its data analysis methods in a different physical system with different systematics; SLR community could check its model with an additional parameter which is already constrained by other experiments…

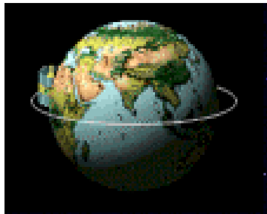
What does really limit laser ranging for EP testing?

An EP test requiers to measure a differential acceleration between the test masses:

$$\eta \equiv \frac{\Delta a}{a} \equiv \frac{\textit{differential acceleration between free falling test masses}}{\textit{free fall acceleration of test masses ("driving" acceleration)}}$$

Laser ranging performs from Earth an absolute measurement of the orbit of the test bodies, which is clearly much more difficult (and contains a lot of physical information… just think to the wealth of science provided by LAGEOS and by LLR -in addition to EP testing…) but was not meant for EP testing…

If the test bodies are coupled and put inside a spacecraft to orbit the Earth at low altitude, their relative motion can be monitored in situ far more accurately, by many orders of magnitude… EP can be tested in the field of the Earth, thus also exploiting the much larger driving acceleration….



GG satellite experiment and GGG lab test

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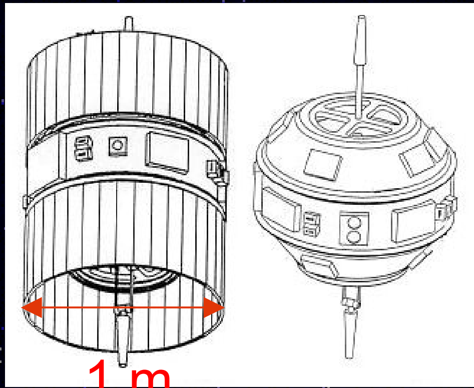

$\eta \simeq 10^{-17}$ *target*