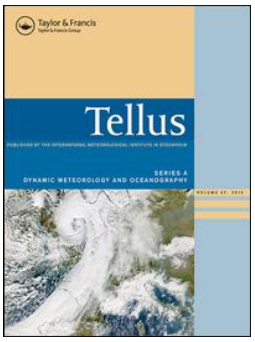

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Formula for a baroclinic adjustment theory of climate

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ABSTRACT

Recently, a theory relating baroclinic neutrality and midlatitudes tropopause height has been proposed. However, GCM results have shown that the dependence of the theory on external parameters is not consistent with that displayed by these numerical experiments. In the present paper we suggest an analytic formula for baroclinic adjustment to the neutrality of Eady waves through tropopause modification. This formula extends considerably the abovementioned theory by taking into account both a simple representation of the stratosphere and the topography. These modifications alter the tropopause condition for a baroclinically neutral state and its sensitivity to the external parameters. In particular, the topography introduces a dependence on the tropospheric vertical wind shear of the neutrality condition. This feature is not present in other models that assume a background state with a zero potential vorticity gradient in the troposphere. We show, furthermore, that the modified neutrality condition has sensitivities that may resemble those displayed by GCM simulations, with respect to the parameters defining the background flow.

1. Introduction

Climate physics is the study of physical laws that govern the behaviour of the climatic system and, in particular, the atmosphere, where radiative, thermodynamic, chemical and mechanical processes occur at all the spatial and temporal scales. The parameterisation of these processes may lead to a useful simplification of the overall model.

A typical example of such a parameterisation is the theory of the 'convective adjustment' proposed by Manabe and Strickler (1964), who assume that the climate rapidly adjusts to a radiative–convective equilibrium. Therefore, the effects of the convective small-scale motions can be parameterised in terms of macroscopic variables such as the tropospheric lapse rate and the thickness of the convective layer.

Green (1970) and Stone (1973), among others, proposed that an adjustment mechanism might also occur for baroclinic eddies. In particular, Stone (1978) noted that disturbances, growing because the baroclinic instability of the zonal mean flow, may leave the atmosphere close to a state of 'baroclinic neutrality'.

Many authors have proposed other baroclinic adjustment theories. Welch and Tung (1998) provide for an account of a recent approach and a review of other efforts (see also Caballero and Sutera, 2000). These theories take advantage of the constraint implied by the Charney and Stern (1962) theorem, which relates a sign change in the meridional gradient of the potential vorticity (hereafter PV) to a necessary condition for baroclinic instability. Then, a parameterisation of the baroclinic instability process can be obtained by

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removing the sign change in the background PV, so that an adjusted neutral state can be found. As discussed extensively by Mak (2000) in the context of a β-plane quasi-geostrophic model, by Caballero and Sutera (2000) in a simple two-level quasi-geostrophic model on the sphere and by Caballero et al. (2001) in a primitive-equation framework; unfortunately there is no unique way of modifying the background PV. However, these studies show that the modification of the meridional structure of the zonal mean wind appears to be the dominant mechanism in reducing the baroclinic growth of the non-zonal disturbances. As pointed out by Caballero et al. (2001), this mechanism, often referred to as 'the barotropic governor', appears to be operating at any value of the external forcing, regardless of the nature of the axisymmetric basic state that is in a dynamical balance with the radiative forcing. Moreover, the 'barotropic governor' appears to be particularly strong when a primitive-equation model in spherical geometry is considered.

Neverthless, seasonally averaged zonal mean tropospheric PV seems to be well mixed. Sharp gradients of PV are mostly found at ground level (where the temperature field is hardly dependent on the atmospheric dynamics) and at the tropopause level (where a sharp gradient is probably maintained by the stratospheric dynamics). Insofar as a seasonal average is a good description of a climate state, we may assume that the zonal mean PV is close to that considered in an 'Eady model'. Sun and Lindzen (1994) show some observational results supporting this point of view. On the other hand, an Eady basic state may be baroclinically unstable; thus it may be suggested that there are other mechanisms of stabilisation that differ from the 'barotropic governor'.

In fact, Lindzen (1993), hereafter L93, noted that, when an Eady basic state is considered, the condition for a marginally unstable baroclinic flow can be achieved by raising the tropopause height. Undoubtedly, deep baroclinic cyclones are known to modify the tropopause structure; therefore, such an approach may claim some observational support.

The L93 theory leaves the adjusted atmosphere with an unchanged surface meridional gradient of temperature, while the tropopause height is raised. A classical view of climate changes induced by external factors (Manabe and Wetherald, 1987) relates the planet surface temperature modifications to the tropopause height. As a consequence, it appears that the L93 method, besides the contribution given to the baroclinic adjustment theory, has great pertinence for attempts to evaluate the effect induced by an external stress on the climatic equilibrium.

Recently, the predictions of the L93 theory have been tested against the outcome of some numerical experiments performed with more complex models (Egger, 1995; Thuburn and Craig, 1997). When the values of the external parameters were varied, a weak agreement (if any) between the L93 theory and the numerical experiments was found. In these GCM simulations, in fact, the tropopause seems to be strongly related to the stratospheric radiative constraint rather than to a mechanism such as that suggested in L93. Therefore, it could be argued that such a baroclinic adjustment process does not appear to be effective in maintaining the model atmosphere close to neutrality through a re-definition of the tropopause height (Thuburn and Craig, 2000; Barry et al., 2000).

Nevertheless, we feel that, before dismissing this theory, a better understanding of the dependence of the tropopause height on dynamical factors should be required. In fact, the mentioned lack of consistency might be a consequence of the simple formulation of the boundary conditions considered in L93.

For this reason, in the present paper we follow L93 by adding simple improvements to the treatment of the classical Eady problem, namely we remove the rigid lid condition and examine the effect of topography. The results to be presented show new features in the condition on the tropopause height for baroclinic stability. In particular, we find a more complex dependence on the external parameters than that predicted by L93. Moreover, we will show that our results partially restore the consistency between this theory and GCM experiments.

We notice that, while in a 'tropopause adjustment theory' the study of the effect of topography is a new concept, the rigid lid removal has already been considered by Harnik and Lindzen (1998). However, in the latter work the dispersion relationship is numerically computed, while in the present paper, at the marginal cost of a simplification of the background flow, an analytic expression for the neutrality condition is obtained. This formula is easy to use if a sensitivity study is performed or if we wish to compare the theory with GCM outcomes. Therefore, we also thought to present the case where the effect of the stratosphere is considered.

The paper is organised as follows: in Section 2 results for the Eady model with a rigid lid assumption and for a vertically unbounded atmosphere are briefly presented. In Section 3 we study the effect of topography on the condition for stability and in Section 4 some sensitivity studies are performed. In Section 5 a summary and some speculations on the nature of the equilibration process are offered.

2. Conditions for baroclinic neutrality of the Eady basic state

Let us consider different formulations of the Eady problem.

2.1. *The rigid lid*

In the standard formulation of the Eady problem in a quasi-geostrophic flow (Pedlosky, 1979), the basic state consists of a Boussinesq stratified quasi-geostrophic atmosphere characterised by a constant PV and vertically confined between an upper rigid lid and a lower flat surface. The linearised form of the quasi-geostrophic PV conservation equation can be expressed in terms of the geopotential perturbation φ:

$$(\partial_t + U\partial_x)q + \partial_y \Pi \partial_x \varphi = 0 \tag{1}$$

where $U = U(y, z)$ is the zonal mean basic state wind,

$$q = \nabla^2\varphi + \partial_z(\varepsilon\partial_z\varphi),$$

$$\Pi = f_0 + \beta y + \nabla^2\Phi + \partial_z(\varepsilon\partial_z\Phi)$$

and $\varepsilon = f_0^2/N^2$. Here, f_0 is the Coriolis parameter at a given latitude ϕ_0, β is the meridional gradient of the planetary vorticity, N is the Brunt–Väisälä frequency and Φ the background geopotential. For the Eady problem,

$$U(y, z) = u_0 + \Lambda z$$

(where u_0 is the wind at the surface and Λ is the vertical zonal wind shear) and, moreover, $\partial_y \Pi = 0$.

The boundary conditions are:

$$(\partial_t + U\partial_x)\partial_z\varphi - \partial_z U \partial_x \varphi = 0 \quad \text{at } z = 0,\ z = z_T \tag{2}$$

where z_T is the height of the atmosphere here assumed to be the tropopause height.

Considering the solution in the form

$$\varphi(x, y, z, t) = \Psi(z)\sin(ly)\,e^{ik(x-ct)} + c.c., \tag{3}$$

with

$$\Psi(z) = a\sinh(\alpha z) + b\cosh(\alpha z)$$

and

$$\alpha^2 = (k^2 + l^2)/\varepsilon,$$

where k and l are the zonal and meridional wavenumber respectively. The condition for stability is

$$\Delta = -4x\tanh(x) + x^2\tanh^2(x) + 4\tanh^2(x) \geqslant 0, \tag{4a}$$

where

$$x = \alpha z_T. \tag{4b}$$

The solutions of eq. (4a) are

$$z_T = 0$$

$$z_T \cong 2.4\frac{f_0}{N\sqrt{k^2 + l^2}} \tag{4c}$$

The short-wave cutoff satisfies

$$\sqrt{k^2 + l^2} \cong (2.4 f_0)/N z_T. \tag{4d}$$

As a consequence, by raising the tropopause height, the short-wave cutoff moves to a lower total wavenumber. On the other hand, the smallest value of the total wavenumber is assumed to be given since the jet-width (which constrains the meridional scale of the wave), is held unchanged (Ioannou and Lindzen, 1986). We remark that the above assumption may be hard to justify, unless we do not explicitly assume the maintenance of a zero background PV gradient during the wave growth. This assumption amounts to keep unchanged each term contributing to the PV balance, i.e. the wind and the temperature meridional structure, while, as we know, in a quasi-geostrophic theory the static stability cannot change.

Let the free parameters be

$$\Lambda = 2.6 \times 10^{-3}\,\mathrm{s}^{-1}, \quad k = 2\pi s/L_x, \quad l = \pi/L_y,$$

$$\phi_0 = 30^\circ, \quad \Delta\phi = 27^\circ, \quad L_x = 2\pi r_a\cos(\phi_0) \quad \text{and}$$

$$L_y = r_a\Delta\phi,$$

where r_a is the Earth's radius, $\Delta\phi$ is the jet width and s is the non-dimensional zonal wavenumber.

Let us define z_s the height of the tropopause for which the atmosphere is baroclinically stable with respect to the perturbations of zonal wavenumber s. Taking $N = 1.05 \times 10^{-2}\,\mathrm{s}^{-1}$, we reproduce the L93 result, i.e. z_1 is about 16 km.

2.2. *The effect of the stratosphere*

The rigid lid assumption may be removed by imposing, for example, that the geopotential and the vertical velocity are continuous at the tropopause, while the perturbation fields vanish in the limit for z going toward the outer space. In this case the equations of motion are (Rivest et al., 1992):

$$\begin{aligned}(\partial_t + U_1\partial_x)q_1 + \partial_y\Pi_1\partial_x\varphi_1 = 0\\(\partial_t + U_2\partial_x)q_2 + \partial_y\Pi_2\partial_x\varphi_2 = 0\end{aligned}\qquad(5)$$

with

$$q_i = \nabla^2\varphi_i + \partial_z(\varepsilon_i\partial_z\varphi_i)$$

and

$$\Pi_i = f_0 + \beta y + \nabla^2\Phi_i + \partial_z(\varepsilon_i\partial_z\Phi_i)\quad\text{for } i = 1, 2.$$

The subscripts 1, 2 refer to the troposphere and the stratosphere, respectively. Similarly to the previous case, let us consider $U_1 = u_0 + \Lambda_1 z$ for $z \leqslant z_T$, $U_2 = U_1(z_T) + a_0\Lambda_1(z - z_T)$ for $z > z_T$. The parameter a_0 accounts for the direction and the strength of the zonal wind vertical shear in the upper atmosphere.

The boundary conditions for this case, hereafter referred to as the 'two-layer' model, are:

$$\begin{aligned}&(\partial_t + U_1\partial_x)\partial_z\varphi_1 - \partial_z U_1\partial_x\varphi_1 = 0\quad\text{at } z = 0\\&\frac{1}{N_1^2}\{(\partial_t + U_1\partial_x)\partial_z\varphi_1 - \partial_z U_1\partial_x\varphi_1\}\\&\quad= \frac{1}{N_2^2}\{(\partial_t + U_2\partial_x)\partial_z\varphi_2 - \partial_z U_2\partial_x\varphi_2\}\quad\text{at } z = z_T\\&\varphi_1 = \varphi_2\quad\text{at } z = z_T\\&\varphi_2 \to 0\quad\text{for } z \to \infty.\end{aligned}\qquad(6)$$

As in the previous case, the PV meridional gradient of the basic state is set to zero everywhere, i.e. $\partial_y\Pi_i = 0$ (an assumption that will be removed later on).

The condition for stability is easily calculated from eqs. (5) and (6) when the following perturbations are considered:

$$\varphi_i(x, y, z, t) = \Psi_i(z)\sin(ly)\,\mathrm{e}^{\mathrm{i}k(x-ct)} + c.c.\qquad(7)$$

Letting $\Psi_1(z) = a\sinh(\alpha_1 z) + b\cosh(\alpha_1 z)$ and $\Psi_2(z) = d\,\mathrm{e}^{-\alpha_2 z}$, the resulting solvability condition leads to the following algebraic equation for the phase speed c:

$$Ac^2 + Bc + D = 0$$

where

$$\begin{aligned}A &= \alpha_1^2[\tanh(\alpha_1 z_T) + \gamma],\\B &= -\alpha_1\Lambda_1[\tanh(\alpha_1 z_T)(\alpha_1 z_T + \gamma) + \alpha_1 z_T\gamma + a_0\gamma^2],\\D &= \Lambda_1^2[\alpha_1 z_T + \tanh(\alpha_1 z_T)(-1 + \alpha_1 z_T\gamma + a_0\gamma^2)\end{aligned}$$

and

$$\gamma = N_1/N_2.$$

Solutions are

$$c_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AD}}{2A}.$$

Stability requires that

$$\begin{aligned}\Delta = {}&-4x\tanh(x) + x^2\tanh^2(x) + 4\tanh^2(x)\\&+\gamma[-4x + 2x^2\tanh(x) + 4\tanh(x)\\&\quad-2x\tanh^2(x)]\\&+\gamma^2[x^2 - 2x\tanh(x) + \tanh^2(x)\\&\quad+a_0(2x\tanh(x) - 4\tanh^2(x))]\\&+\gamma^3[a_0(2x - 2\tanh(x))] + \gamma^4 a_0^2 \geqslant 0\end{aligned}\qquad(8)$$

where x is defined as in eq. (4b).

In order to illustrate some new (with respect to L93) features implied by eq. (8), let us consider the following parameter settings: $\Lambda_1 = 2.6 \times 10^{-3}\,\mathrm{s}^{-1}$, $\gamma = 0.5$, $u_0 = 0$ (the latter without loosing generality).

In Fig. 1 we show z_s as a function of the zonal wavenumber s where $a_0 = 0$ (as in Rivest et al., 1992) and $a_0 = -1$ (dashed line and thin line, respectively). The rigid lid case corresponds to the thick line. The height of the tropopause has been normalised with respect to z_1 corresponding to the rigid lid case. We remark a lowering of z_s when a simple representation of the stratosphere is introduced. For example, if $a_0 = 0$ the value of z_1 is about 13.5 km (rather than 16 km obtained for the rigid lid case), which is sensibly closer to the observed midlatitude tropopause height. This

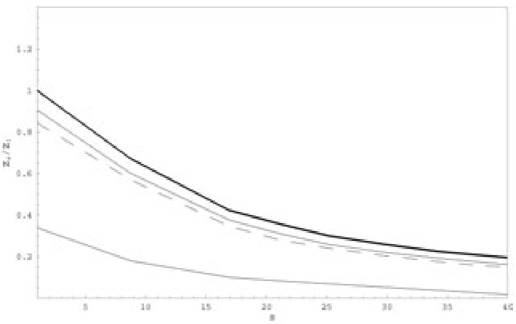

Fig. 1. z_s as a function of s for the 'rigid lid' model (thick line), the 'two-layer' model with $a_0 = 0$ (dashed line), and the 'two-layer' model with $a_0 = -1$ (thin lines). The height of the tropopause has been normalised with respect to $z_1 = 16$ km corresponding to the rigid lid case. The values of the parameters are $N_1 = 1.05 \times 10^{-2}\,\mathrm{s}^{-1}$, $\phi_0 = 30°$, $\Delta\phi = 27°$, $\Lambda_1 = 2.6 \times 10^{-3}\,\mathrm{s}^{-1}$ and $\gamma = 0.5$. Instability regions are below the thick and dashed lines, and between the two thin lines. The lowest thin line pertains to the long-wave cutoff.

shows that when we compare GCM experiments that account for the stratospheric dynamics, formula (8) should be considered in place of eq. (4c). Moreover, if $a_0 = -1$ (or, more generally, a value different from zero) a branch of lower, stabilising, tropopause is introduced (the lowest thin line in Fig. 1). This behaviour of z_s pertains to a long-wave cutoff, which is induced by the presence of the stratospheric vertical wind shear. This is more clearly shown in Figs. 2a and b, where the dimensionless phase speed c and the growth rate are plotted as a function of s. For illustrative purpose, we set $z_T = 4.5$ km. As it can be seen, there is a long-wave cutoff for s of about 4, and the corresponding neutral waves are both westerly propagating.

Formula (8) shows immediately three main differences with respect to the rigid lid case:

(1) The removal of the rigid lid assumption introduces a lowering of z_s. It means that the perturbation wave can remain neutral, despite of a lower tropopause, by leaking the upper edge wave energy towards the stratosphere; in other words, the two boundaries are too many Rossby heights apart to have instability (we recall that the Rossby height is defined in the rigid lid case as α^{-1}). This implies an overall damping on the instability process.

(2) The presence of a stratospheric vertical wind shear ($a_0 \neq 0$) removes the $x = 0$ solution of the stability condition (8) (and thereby a twofold value of the critical Rossby height is introduced). This leads to a planetary wave cutoff, a property that does not occur in the rigid lid case. It follows that the presence of a stratosphere, even if it is described in a simplistic form, seems to modify also the behaviour of the planetary-scale waves.

(3) A dependence of z_s on the static stability ratio γ is introduced; hence, the stratospheric radiative equilibrium plays an important role in determining the tropopause height condition for neutrality.

In summary, we have shown that modest, but realistic, modifications of the rigid lid model imply different sensitivities of the adjustment process to the external parameters (see also Section 4). Finally, eq. (8) is an expression for the neutral

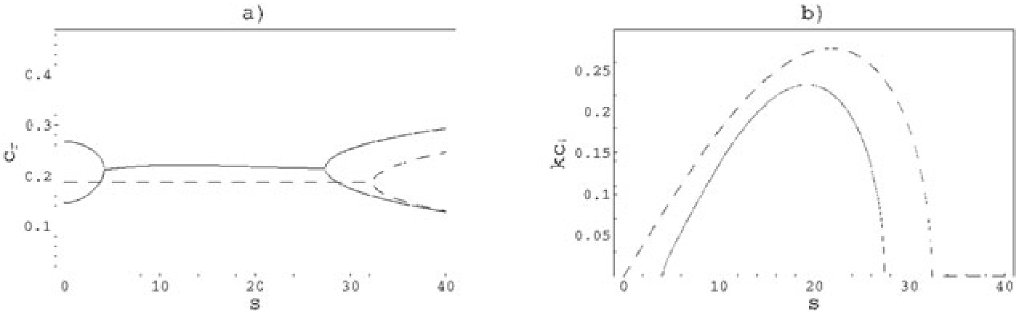

Fig. 2. (a) Dimensionless real part c_r of the phase speed c and (b) growth rate kc_i as a function of the non-dimensional zonal wavenumber s for the 'rigid lid' model (dashed line) and 'two-layer' model with $\gamma = 0.5$, $a_0 = -1$ (solid line). For illustrative purposes, the height of the tropopause is placed at $z_T = 4.5$ km. The other parameter values are as in Fig. 1.

condition that can be compared with the numerical results shown in Harnik and Lindzen (1998).

2.3. *The effect of the stratosphere with a PV gradient*

A property of eq. (8) is that z_s is independent on the tropospheric vertical wind shear, i.e. if we change the imposed meridional temperature gradient there are no changes on Δ, a feature that contrasts with Thuburn and Craig (1997) experiments. This unwished-for occurrence may be due to the assumptions insofar adopted. Perhaps the most unrealistic one consists on setting $\partial_y \Pi_2 = 0$. As already mentioned, while the assumption $\partial_y \Pi_1 = 0$ may be observationally supported, it is well known that the stratospheric PV meridional gradient is surely $O(\beta)$. To account for this effect, we impose $\partial_y \Pi_2 = \beta$ and check whether any dependence on the tropospheric vertical wind shear (Λ_1) is introduced on z_s.

At this purpose, following Lindzen (1994a), we can determine the wind and temperature vertical profiles corresponding to a given constant PV gradient. In fact, by inverting the PV equation, one can consistently adjust the vertical wind and the temperature profiles. As an example, let assume that the surface separating different values of $\partial_y \Pi$ is located at 10 km. Then, keeping the lapse rate unchanged, a consistent temperature and wind field are computed. They are shown together with the associated PV vertical profile in Fig. 3.

In this framework, let us consider a normal mode. The interior equation is

$$\partial_z \frac{f_0^2}{N^2} \partial_z \Psi + \left(\frac{\partial_y \Pi}{U - c} - (k^2 + l^2) \right) \Psi = 0. \qquad (9)$$

The lower boundary condition is given by the first equation in (6), while at the top of the atmosphere ($z_{\mathrm{top}} = 50$ km) we assume that the following radiation condition holds:

$$\partial_z \Psi + \mathrm{i} \sqrt{ - \frac{(k^2 + l^2) N^2}{f_0^2} + \frac{\partial_y \Pi}{U - c} } \, \Psi = 0 \text{ at } z = z_{\mathrm{top}} \qquad (10)$$

where i is the imaginary unit.

The latter equation can be obtained following, for example, the approach presented in Harnik and Lindzen (1998). When eq. (9) is vertically discretised, we must solve an eigenvalue problem for the phase speed c for each wavenumber. For the cases presented here, the vertical grid spacing, Δz, is set to 500 m.

In Fig. 4 we show the dimensionless growth rates as a function of s for different values of Λ_1 (and the corresponding basic fields as in Fig. 3). An inspection of Fig. 4 shows no long-wave cutoff as should be expected, since weakly unstable planetary waves are present (Harnik and Lindzen, 1998). Moreover, we also notice no change in the short-wave cutoff. Therefore, z_s is virtually independent of Λ_1, as it is in eq. (8). A posteriori, this outcome appears to be obvious, since the instability threshold depends on the Dirac-delta at z_T. This Dirac-delta is not a function of Λ_1, and therefore no change must be expected.

We may conclude that the inclusion of a stratosphere (with or without background PV meridional gradient) does not modify the outcome that the neutrality condition is independent of the tropospheric vertical wind shear. Hence, while some improvements on the nature of the stability condition in terms of the tropopause height (and its dependence on some external parameters) has been introduced, the inconsistency remains strong with respect to the experiments of Thuburn and Craig (1997). In fact, when they changed the imposed meridional temperature gradient in their model, the tropopause changed accordingly. However, in other approaches to baroclinic neutrality (Stone, 1978; Held, 1982) the dependence on the vertical wind shear is achieved by including a β effect on the background PV in the troposphere. In this way, the scale of the unstable wave is fixed by β rather than z_s. Therefore, a relationship between tropospheric vertical wind shear and the neutrality condition can be found via the theorem of Charney and Stern (1962). In the Eady model $\partial_y \Pi_1 = 0$, so that such a procedure does not lead to a similar outcome. Therefore, the present approach may appear faulty.

In the next section we will show that the condition of stability is further modified when topography is taken into account, an effect that in the literature has been only sparsely considered. As we shall see, this analysis will be rewarding. It leads, in fact, to an external parameter sensitivity of z_s that depends on Λ_1.

3. The effect of topography

In this section we analyse the role of topography on the baroclinic adjustment process. Here we

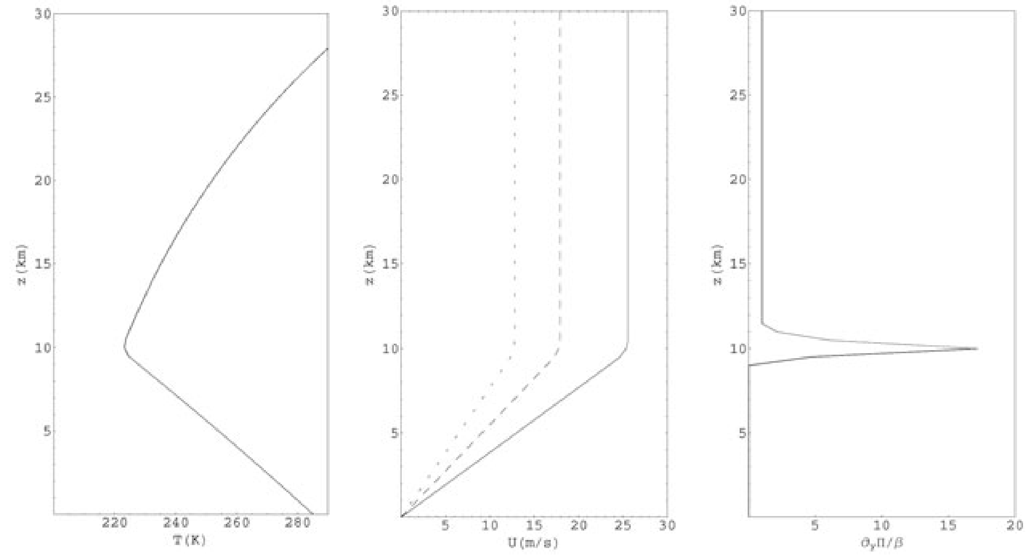

Fig. 3. Basic states with $\Lambda_1 = 2.6 \times 10^{-3}\ s^{-1}$ (solid line), $\Lambda_1 = 1.8 \times 10^{-3}\ s^{-1}$ (dashed line), $\Lambda_1 = 1.3 \times 10^{-3}\ s^{-1}$ (dotted line), while $\partial_y \Pi_2 = \beta$, $a_0 = 0$, $\gamma = 0.5$. The tropopause height is placed at $z_T = 10$ km. The temperature and the normalised PV gradient corresponding to these values of Λ_1 are virtually indistinguishable.

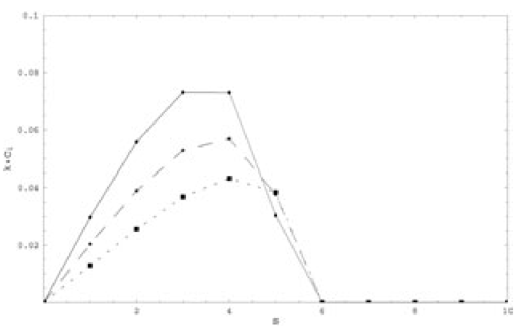

Fig. 4. Dimensionless growth rates as a function of s for the three basic states in Fig. 3: $\Lambda_1 = 2.6 \times 10^{-3}\ s^{-1}$ (solid line), $\Lambda_1 = 1.8 \times 10^{-3}\ s^{-1}$ (dashed line) and $\Lambda_1 = 1.3 \times 10^{-3}\ s^{-1}$ (dotted line).

follow the formulation described in Pedlosky (1980).

The equations of motion and the boundary conditions remain as in eq. (6) except for the lower boundary condition, which becomes

$$(\partial_t + u_0 \partial_x)\partial_z \varphi_1 - \partial_z U_1 \partial_x \varphi_1 + \frac{N_1^2}{f_0} w_1 = 0 \quad \text{at } z = 0 \tag{11}$$

where $w_1 = \vec{u}' \cdot \vec{\nabla} h$, $\vec{u}' = (u'_x, u'_y)$ and the prime stands for the perturbation wind velocity. Here, $h = h_0\, G(y)$, where $G(y)$ is the zonal mean meridional topography profile and h_0 is its height. Notice that here, as in Pedlosky (1980), h is independent on the zonal direction.

This latter assumption is needed only if we wish to keep separable solution both in the vertical and horizontal domains. If, instead, we wish to account for the zonal variation of topography, we must project horizontally the lower boundary condition onto the Eady normal mode (7). In this case [if we interpret $G(y)$ as a normalised zonally averaged topography] the analysis that follows remains unchanged. In fact:

$$\vec{u}' \cdot \vec{\nabla} h = u'_x \partial_x (h(x, y) + u'_y \partial_y h(x, y), \tag{12}$$

then it is easy to show that, while the first term on the right-hand side of eq. (12) is zero because the zonal periodicity, only the second one may have a non-zero projection on the mode.

To avoid possible misunderstanding, we remark that in this paper the three-dimensional domain-averaged zonal wind does not interact with topography and remains constant in time. This assumption excludes that other forms of instability, such as the form-drag instability, can develop. Moreover, no forced, stationary waves are induced in the model. Hence, the zonal wind at the bottom may be set to zero without loosing generality. The removal of this assumption leads to other interesting results that we will report elsewhere.

We choose, as an example, a normalised topographic meridional profile, non-symmetric with respect to the middle of the channel, that increases (or decreases) with latitude:

$$G(y) = \frac{1}{2}\left[1 - \cos\left(\frac{\pi}{L_y} y\right)\right]$$
$$\left\{\text{or } G(y) = \frac{1}{2}\left[1 + \cos\left(\frac{\pi}{L_y} y\right)\right]\right\} \tag{13}$$

Setting $\varphi_i = \Psi_i(z)\, e^{ik(x-ct)}\, g_i(y) + c.c.$ [$g(y)$ may be chosen as the trigonometric function that satisfies the rigid wall boundary condition as in Section 2] and

$$\delta = \frac{N_1^2}{f_0} h_0 \frac{\int g_1^2(y) \partial_y G(y)\, dy}{\int g_1^2(y)\, dy}, \tag{14}$$

the condition for stability becomes:

$$\begin{aligned}\Delta = & -4x \tanh(x) + x^2 \tanh^2(x) + 4 \tanh^2(x) + \frac{\delta}{\Lambda_1}\left(\frac{\delta}{\Lambda_1} + 2x \tanh(x) - 4 \tanh^2(x)\right) \\ & + \gamma \left[\frac{\delta}{\Lambda_1}\left(2x - 6 \tanh(x) + 2x \tanh^2(x) + 2 \frac{\delta}{\Lambda_1} \tanh(x)\right) \right. \\ & \left. \quad - 4x + 2x^2 \tanh(x) + 4 \tanh(x) - 2x \tanh^2(x)\right] \\ & + \gamma^2 \left\{x^2 - 2x \tanh(x) + \tanh^2(x) + \frac{\delta}{\Lambda_1} \tanh(x) \left[2x + \left(\frac{\delta}{\Lambda_1} - 2\right) \tanh(x)\right]\right. \\ & \left. \quad + a_0 \left(-2 \frac{\delta}{\Lambda_1} + 2x \tanh(x) + 4 \frac{\delta}{\Lambda_1} \tanh^2(x) - 4 \tanh^2(x)\right)\right\} \\ & + \gamma^3 \left[a_0 \left(2x - 2 \tanh(x) + 4 \frac{\delta}{\Lambda_1} \tanh(x)\right)\right] + \gamma^4 a_0^2 \geqslant 0. \end{aligned} \tag{15}$$

Notice that the result above can be easily obtained if a perturbative (in h) analysis is performed, as can be found in Speranza et al. (1985) [see also Buzzi and Speranza (1986) and references therein for a non-perturbative approach].

A cursory look at eq. (15) suggests that we can expect modifications of z_s because of the topographic action, both in the rigid lid case $\gamma \to 0$ [notice that this case has been investigated by Pedlosky (1980), though for other purposes] and when this assumption is removed ($\gamma \neq 0$). To the best of our knowledge, the latter has not been considered elsewhere in this or other applications. Therefore, it appears that an analysis of the stability diagram may be a useful digression.

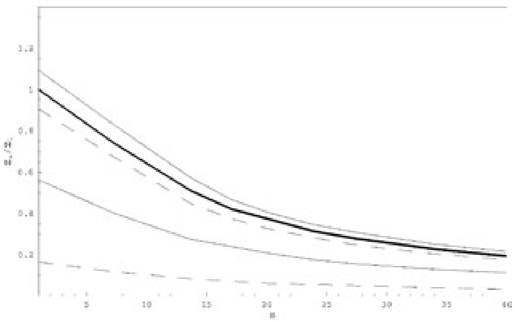

Fig. 5. z_s as a function of s for the 'rigid lid' model when $\delta/\Lambda_1 < 0$ (thin line), $\delta/\Lambda_1 > 0$ (dashed line) and $\delta = 0$ (thick line). Curves are normalised with respect to $z_1 = 16$ km, corresponding to $\delta = 0$ (see Fig. 1). The other parameter values are as in Fig. 1, while $|\delta/\Lambda_1| = 1/3$.

It is worth noticing that eq. (15) depends on topography through the ratio δ/Λ_1. Thus, as an example, we show in Fig. 5 z_s for the rigid lid model as a function of s (normalised with respect to z_1 computed without topography) setting $\delta/\Lambda_1 = -1/3$ (thin line) and $\delta/\Lambda_1 = 1/3$ (dashed line). The other parameters are chosen as in Fig. 1. For comparison we also plot the case $\delta = 0$ (thick line). An inspection of Fig. 5 shows that, even in the case where $\gamma = 0$, topography introduces a new branch of stability, which corresponds again to a long-wave cutoff.

Similar behaviour is also found for the 'two-layer' model as shown in Fig. 6, for the case where $a_0 = 0$, $\gamma = 0.5$ and $|\delta/\Lambda_1| = 1/3$. The effect of topography, for a zonal wind increasing with height, depends on its meridional structure, i.e. the sign of δ. If $\delta < 0$ topography raises z_s, while if $\delta > 0$ topography has a stabilising effect (Blumsack and Gierasch, 1972).

Moreover, in both cases (the 'rigid lid' and the 'two-layer'), topography introduces a tropopause dependence on the meridional temperature gradient of the basic state [see the dependence on Λ_1 in eq. (15)]. This is a new feature with respect to the models with a flat bottom, where z_s has been shown to be independent on Δ_1 and the ultra-long waves are always unstable modes. A dependence on the vertical wind shear was also found in the study by Thuburn and Craig (1997).

The aforementioned behaviour may be tested in a GCM letting, for example, the stratospheric radiative equilibrium approaching the case $\gamma = 0$ and varying the meridional topographic profile. Thus, if predictions of eq. (15) are found closely verified, then this equation could be readily used as a closure method for an otherwise zonally symmetric model.

The effect of topography seems to be qualitatively independent of the PV meridional gradient associated with the basic state. To investigate this point a little further, we have considered the model described in Subsection 2.3. Figure 7 shows the behaviour of z_s as a function of s (normalised with respect the z_1 corresponding to $\partial_y \Pi_2 = \beta$ and no topography) when the basic state is that described in Fig. 3 with $\Delta_1 = 2.6 \times 10^{-3}\,\mathrm{s}^{-1}$. The case $\delta < 0$ corresponds to the upper thin line, while the $\delta > 0$ one is described by the lower dashed line. Finally, $\delta = 0$ case is drawn as a thick line. We remark that the meridional structure of topography, like in the 'two-layer' model, varies the baroclinic neutrality condition of the basic state. Moreover, as shown in Fig. 8 (at variance with Fig. 4), the short-wave cutoff depends on Λ_1.

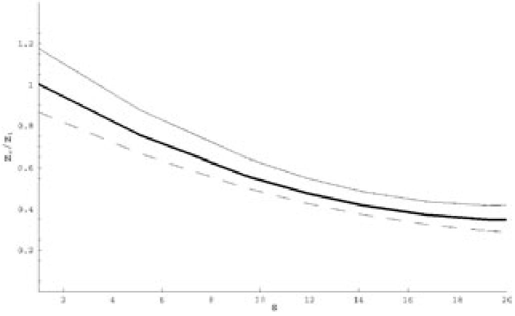

Fig. 7. z_s corresponding to the basic state shown in Fig. 3 as a function of s with $\Lambda_1 = 2.6 \times 10^{-3}\,\mathrm{s}^{-1}$ for $\delta = 0$ (thick line), $\delta = -0.9 \times 10^{-3}\,\mathrm{s}^{-1}$ (thin line), $\delta = 0.9 \times 10^{-3}\,\mathrm{s}^{-1}$ (dashed line). We normalise z_s with respect to $z_1 = 15.5$ km, corresponding to $\delta = 0$.

Further results (not shown here) suggest that this general behaviour is weakly dependent on the structure of the assumed vertical profiles of the basic state fields.

As an aside issue, it could be thought that the

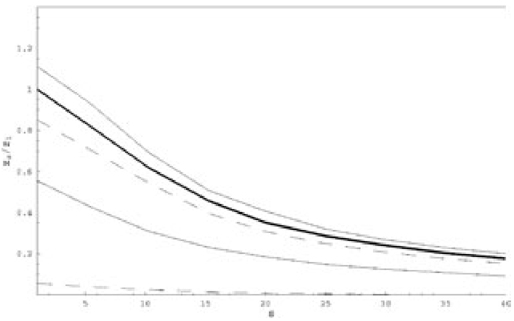

Fig. 6. z_s as a function of the zonal non-dimensional wavenumber s for the 'two-layer' model when $\delta/\Lambda_1 < 0$ (thin line), $\delta/\Lambda_1 > 0$ (dashed line) and $\delta = 0$ (thick line). Curves are normalised with respect to $z_1 = 13.5$ km, corresponding to $\delta = 0$. The other parameter values are $a_0 = 0$, $\gamma = 0.5$, $|\delta/\Lambda_1| = 1/3$.

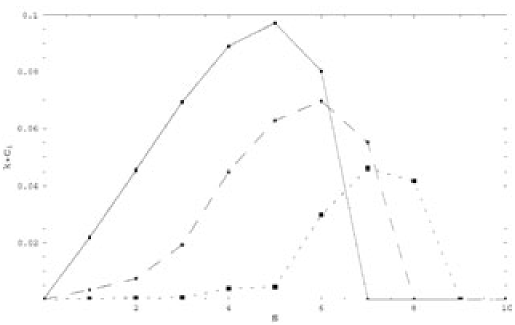

Fig. 8. Dimensionless growth rates as a function of s for the three basic states in Fig. 3 with $\delta = -0.9 \times 10^{-3}\,\mathrm{s}^{-1}$. Here, $\Lambda_1 = 2.6 \times 10^{-3}\,\mathrm{s}^{-1}$ (solid line), $\Lambda_1 = 1.8 \times 10^{-3}\,\mathrm{s}^{-1}$ (dashed line) and $\Lambda_1 = 1.3 \times 10^{-3}\,\mathrm{s}^{-1}$ (dotted line).

effect of topography amounts to adding a β term in the tropospheric background PV while keeping its meridional gradient zero. Preliminary calculations, not presented here, show that this is not the case. In fact, as shown by Lindzen (1994b), a long-wave cutoff can be also obtained in these circumstances. However, we can rule out that the topographic effect falls in this category. This can be shown as follows. If $\delta > 0$, the neutral long waves are two westerly propagating ones, while when $\delta < 0$ they are both shallow easterly propagating. These features are different from those presented in Lindzen (1994b), where the two corresponding neutral long waves are propagating in opposite directions.

The occurrence that the topography adds a gradient (of either sign) of PV just and only at the ground leads to an effect on the lower edge wave that may help (or may not) the interaction with the upper level wave. On the other hand, the upper level edge wave is strongly affected by the stratospheric dynamics of the background flow. Thus, these results show a peculiar feature of Eady modes that appear to have no similarity to other baroclinic disturbances growing on a different background flow.

In summary, in this section, among other things, we have partially restored the confidence on a neutral tropopause theory by means of the topographic action. In the next section, we will show how far these modifications restore the consistency between the neutral tropopause theory and GCM experiments.

4. Sensitivity studies

In order to study the sensitivity of the adjustment mechanism proposed here with respect to physical processes determining the background flow, we will consider the behaviour of z_1, given by eq. (15), as a function of the external parameters.

The parameters that may be easily varied are: (i) the ratio δ/Λ_1; (ii) the Coriolis parameter, f_0; (iii) the stratospheric vertical shear of the zonal wind (namely a_0); and (iv) the static stability ratio γ. Generally, changes in one of these parameters can modify the structure of the assumed basic state. Here, we limit ourselves to consider the response of eq. (15) to variations in (i)–(iv) by keeping unchanged the remaining parameters that characterise the basic state.

The sensitivity of z_1 to δ/Δ_1, for the 'rigid lid' and the 'two-layer' models, is shown in Fig. 9. The height of the tropopause depends strongly on the size and the sign of δ/Λ_1. It must be noted that, for higher positive values of δ/Λ_1, the atmosphere is neutral for any values of z_1, while for $\delta/\Lambda_1 < 0$ there are two possible z_1. However, the most prominent feature shown in this figure consists of the dependence of z_1 on the tropospheric vertical wind shear, an occurrence that is lacking when $\delta = 0$. In particular the tropopause height may rise even if the imposed meridional temperature gradient decreases, as occurs in the experiments of Thuburn and Craig (1997). In L93 theory such a dependence is not achievable.

In L93, z_1 is linearly related to f_0 [eq. (4c)], while in our study such a dependence is highly non-linear; however, it appears that the simple behaviour described by eq. (4c) is not so drastically modified. This can be understood by inspecting Fig. 10, where the behaviour of z_1 as a function of f_0 is shown for the 'two-layer' and the 'rigid lid' models. With respect to the rigid lid (thin line), the stratospheric vertical wind shear seems to modify the z_1 dependence on f_0 for the planetary-scale waves only (lower dashed line). Similar behaviour (not shown) is also found for $\delta \neq 0$, except

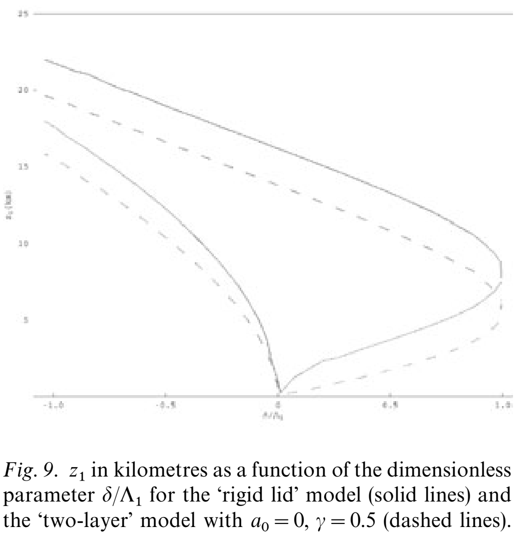

Fig. 9. z_1 in kilometres as a function of the dimensionless parameter δ/Λ_1 for the 'rigid lid' model (solid lines) and the 'two-layer' model with $a_0 = 0$, $\gamma = 0.5$ (dashed lines). The other parameters are as in Fig. 1. Instability regions are between solid and dashed lines, respectively.

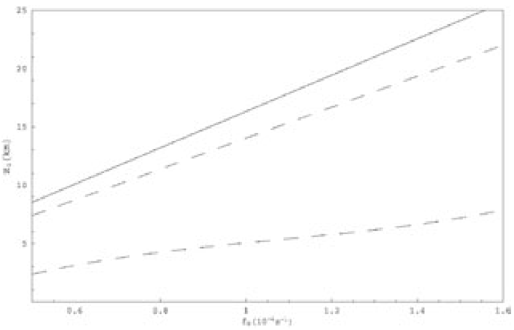

Fig. 10. z_1 in kilometres as a function of f_0 for the 'rigid lid' model (thin line) and the 'two-layer' model with $a_0 = -1$, $\gamma = 0.5$, $\delta = 0$ (dashed lines). The lowest dashed line pertains to the long-wave cutoff.

for the z_1 related to the long-wave cutoff. In the latter case, in fact, z_1 becomes virtually independent on f_0, as is shown in Fig. 11.

Thus, the behaviour of the long-wave cutoff appears to be consistent with Thuburn and Craig (1997), where the neutral tropopause was quite independent on f_0 variations.

We also varied the stratospheric vertical wind shear and the static stability (by changing the parameters a_0 and γ). When $\gamma \to 1$, z_1 is more sensitive to the stratospheric wind shear. This is illustrated in Fig. 12, where contours pertaining to the short-wave cutoff are plotted as a function of a_0 and γ. Now, let us suppose that changes in the stratospheric static stability and vertical wind shear are the responses to a change in the ozone distribution (for instance, because it has an effect on the radiative equilibrium in the stratosphere). Then, our results are compatible with those shown by Thuburn and Craig (1997), who observed a tropopause change as a function of the ozone concentration and its height distribution. Obviously, L93 theory cannot be sensitive to the stratosphere, since it is a rigid lid theory ($\gamma = 0$).

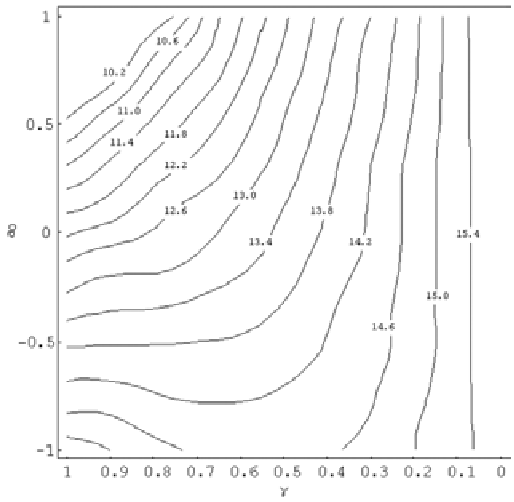

Fig. 12. z_1 in kilometres pertaining to the short-wave cutoff as a function of the stratospheric vertical wind shear a_0 and the static stability ratio γ for the 'two-layer' case. The contour interval is 0.4 km. Here $\delta = 0$ and the other parameter values are as in Fig. 1.

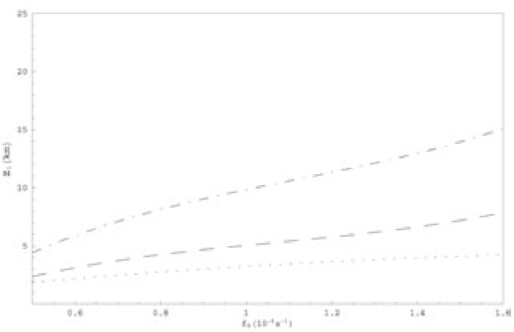

Fig. 11. Curves of z_1 in kilometres pertaining to the long-wave cutoff as a function of f_0 for the 'two-layer' model with $\delta = 0$ (dashed lines), $\delta/\Lambda_1 < 0$ (dot-dashed lines) and $\delta/\Lambda_1 > 0$ (dotted lines). In these cases $a_0 = -1$, $\gamma = 0.5$, $|\delta/\Lambda_1| = 1/3$ and the other parameters as in Fig. 1.

Finally, Thuburn and Craig (1997) showed a change on the tropopause height when they uniformly changed the surface temperature. This effect cannot be associated with a baroclinic adjustment theory, while it can be easily ascribed to a convective adjustment mechanism. The latter process is by far the most rapid to happen, and its interaction with a baroclinic adjustment mechanism may not occur at all.

5. Conclusions

In Sections 2 and 3 we have studied the baroclinic adjustment theory with respect to the tropopause height. We have noted that the neutrality for an Eady baroclinically unstable basic field depends on: (i) rigid lid assumption; and (ii) meridional slope of the lower boundary (i.e. topography). We have derived a general formula that clearly shows the different behaviour of the 'neutral tropopause' when the effect of a stratosphere and the effect of topography are considered.

For the 'two-layer' model, we have found that a reduction of the tropopause height is consistent with a baroclinic neutrality hypothesis. A long-wave cutoff, which does not occur in the rigid lid model, is present when there is a vertical wind shear in the stratosphere.

By including a bottom slope, the planetary-scale waves remain neutral both for the rigid lid and for the 'two-layer' model. Moreover, topography introduces a tropopause dependence on the imposed meridional temperature gradient.

We have studied the sensitivity of the neutral tropopause height to some of the parameters of the basic state, such as the height of topography, its meridional structure, the static stability ratio γ and the stratospheric vertical wind shear. The results obtained show a fair consistency with GCM simulations performed by Thuburn and Craig (1997). Instead, when the Coriolis parameter is varied, we have found remarkable differences with the above-cited experiments, except when the long-wave cutoff is considered. However, it is fair to recall that a change of the rotation rate may have a profound effect on the meridional scale of the background flow. Therefore, our keeping the jet width unchanged, as we have done, may be an irreparable oversimplification of the actual dynamics.

There are many items that here have been left untouched. Among them we like to mention a few as possible future developments: (1) the role of the form drag on the adjustment theory; (2) the effect of the spherical geometry. In fact, as it has been shown by Malguzzi et al. (1996, 1997), the spherical geometry has a profound impact on the relationship between the zonal wind and the stability of the associate stationary wave (even in a meridionally bounded domain); (3) the mechanisms that keep the zero gradient of the background PV, at least in the troposphere. These points and others that have been noted within the body of the paper will be addressed in a forthcoming work.

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