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Editorial

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A festschrift in honor of Professor Patrizia Pucci's 65th birthday

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This issue is dedicated to the 65th birthday of Professor Patrizia Pucci, honoring her wide and great mathematical achievements in nonlinear analysis, partial differential equations, and mathematical physics.

Professor Patrizia Pucci was born on May 11, 1952, in Perugia. She obtained her degree in Mathematics in 1975 at the University of Perugia with the grade 110/110 and laudem. After some scholarships of the *Consiglio Nazionale delle Ricerche*, Patrizia Pucci started her academic career at the University of Perugia. Since 1990 until 1991, she was *straordinarious professor* and *ordinarious professor* in Mathematical Analysis at the University of Modena. Starting with November 1991, Patrizia Pucci has been *ordinarious professor* at the University of Perugia. In this role, Professor Patrizia Pucci worked with a lot of devotion for her students and has been involved in numerous varied activities, including coordination of doctoral students, coordinator or participant at scientific projects, invited scientific activities, organization of international conferences, or editorial appointments. The multiple professional responsibilities of Professor Patrizia Pucci include: member of the *Collegio dei Docenti del Dottorato di Ricerca in matematica*, administration at the University of Florence (since 1997); member of the *Giunta del Dipartimento di Matematica e Informatica* of the University of Perugia (1998–2001); president of the CCL in Matematica of the University of Perugia (since 2011), etc.

Since January 2012, Professor Patrizia Pucci is member of the editorial board of Advances in Nonlinear Analysis since the birth of this journal in 2012. Since August 2016, she is co-Editor-in-Chief of Advances in Differential Equations as well as member of the editorial board of several journals like Nonlinear Analysis, Advanced Nonlinear Studies, Complex Variables and Elliptic Equations, Bollettino dell'Unione Matematica Italiana, etc.

In 2004, Professor Patrizia Pucci received Prize Professor Luigi Tartufari for Mathematics, awarded by the *Accademia Nazionale dei Lincei* of Rome. In 2016 and 2017, she received the *Doctor Honoris Causa* honorary degrees awarded by the University of Craiova and the Babeş-Bolyai University of Cluj Napoca.

Since February 2017, Professor Patrizia Pucci is a member of the prestigious *Accademia delle Scienze dell'Umbria*.

The whole scientific work of Professor Patrizia Pucci combines the passion for developing applied mathematics with tireless intellectual curiosity. A particular place in the scientific career of Patrizia Pucci is her long time collaboration with James Serrin in various very relevant research fields such as the maximum principle, the critical point theory and singular phenomena. At the same time, Professor Pucci has constantly been interested in a high-level scientific collaboration with young researchers, students, or colleagues. We briefly describe in what follows some of Professor Pucci's outstanding contributions to four different fields in nonlinear analysis.

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(i) The *maximum principle* is a powerful and widely used analytic tool in the study of second-order linear and nonlinear elliptic and parabolic equations. The absence of such principles for other types of equations and systems leads to enormous extra-difficulties in constructing theories. Professors Pucci and Serrin have studied the strong maximum principle and a compact support principle for a wide class of elliptic inequalities. Consider a domain ω of $\mathbb{R}^N$, $N \geqslant 2$, and the differential inequality

$$\nabla \cdot (A(|Du|)Du) - f(u) \leqslant 0, \quad u \geqslant 0, \tag{0.1}$$

where A is a continuous function in $(0, \infty)$ such that $\Omega(t) \equiv tA(t)$ is strictly increasing and $\Omega(t) \to 0$ as $t \to 0$; f is a continuous function in $[0, \infty)$, $f(0) = 0$, and f is non-decreasing on some interval $[0, \delta)$. Define

$$H(t) = \int_0^{\Omega(t)} \Omega^{-1}(s)\, ds \quad \text{for } t \geqslant 0, \qquad F(u) = \int_0^u f(s)\, ds,$$

and suppose that either $f(s) = 0$ for $s \in [0, \tau)$, $\tau > 0$ or

$$\int_0^\delta ds/H^{-1}(F(s)) = +\infty.$$

Under these conditions, the strong maximum principle is satisfied in the following sense: whenever u is a nonnegative solution of (0.1) with $u(x_0) = 0$ for some $x_0 \in \omega$, then $u \equiv 0$ in ω. If the strong maximum principle is not valid, then there are nonnegative solutions which vanish at some points but not identically. These results have been extended by P. Pucci and J. Serrin to the case of fully quasilinear inequalities of the form

$$\nabla \cdot (a^{ij}(x)A(|Du|)D_j u) - B(x, u, Du) \leqslant 0 \quad (\text{resp.} \geqslant 0).$$

We refer to [4, 22–24, 26] as basic contributions to the understanding of the maximum principle in a general abstract setting; we also refer to the marvelous monograph [25].

(ii) One of the most important mini-max properties is the so-called *mountain pass* theorem, which marks the beginning of a new approach in nonlinear analysis. The mountain pass theorem was established by A. Ambrosetti and P. Rabinowitz [1]. Their original proof relies on some deep deformation techniques developed by R. Palais and S. Smale, who put the main ideas of the Morse theory into the framework of differential topology on infinite-dimensional manifolds. P. Pucci and J. Serrin have been concerned with the *limiting case* of the mountain pass theorem, which corresponds to *mountains of zero altitude*. Their main abstract result is stated in the following theorem.

Theorem. *Let X be a real Banach space and let $I : X \to \mathbb{R}$ be a C^1-functional satisfying the following conditions:*
(i) *there exist a, r, R with $0 < r < R$ and $I(u) \geqslant a$ for all $u \in A := \{u \in X : r < \|u\| < R\}$,*
(ii) *$I(0) \leqslant a$ and $I(e) \leqslant a$ for some $e \in X$ with $\|e\| \geqslant R$.*
Then I has a critical point x_0 in X, different from 0 and e, with critical value $b \geqslant a$; in addition, $x_0 \in A$ when $b = a$.

We refer to the papers [13, 14, 16] for several important contributions to the critical point theory. A survey on the impact of the mountain pass theory in nonlinear analysis can be found in [11].

(iii) An important contribution of Professor Pucci concerns the qualitative and asymptotic analysis of various types of nonlinear differential systems that describe the behavior of some phenomena in mathematical physics. We mainly refer to precise damping conditions for global asymptotic stability, asymptotic stability for nonautonomous dissipative wave systems, or stability for nonlinear systems with time-dependent restoring potentials; see [18–20].

(iv) A particular interest in the research papers of Professor Pucci has been given to the study of some models arising in mathematical physics, such as Kirchhoff equations and non-homogeneous problems driven by differential operators with variable exponents. Some of the most relevant to these fields may be found in the papers [2, 3, 5, 6, 9, 12, 29, 30].

The most cited work of Professor Patrizia Pucci is the reference book [25] with 236 citations (cf. MathSciNet). Other papers very well received in the mathematical community are [7, 8, 15, 17, 21, 27, 28].

An exceptional tribute to Professor Patrizia Pucci may be found in the paper [10] by Professor Jean Mawhin. In this work, written by one of the best nonlinear analysts of our times, it is clearly pointed out that Patrizia Pucci is "*a leading mathematical force in Perugia*".

A (temporary) conclusion raised by Jean Mawhin in [10] is the following: "*Patrizia's achievements cannot be disjoined from her rich personality. A frail appearance hides a strong character and an incredible energy, but reveals an extreme sensibility. Friendship and fairness mean a lot for Patrizia, and the human side in scientific collaboration and contacts always plays an essential role. Patrizia cherishes and cultivates her Perugian roots and her erudite taste for art and architecture, in the great tradition of Italian Renaissance*."

This issue of Advances in Nonlinear Analysis is to recognize major work in mathematics that is impressive both for its variety than its depth, and that completely transformed our approach and insight into several major problems in nonlinear analysis. It also shows our gratitude and friendship for a colleague who has always shared with us her passion for research and mathematics.

Happy 65th Birthday, Patrizia!

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