

Random coupling between groups of degenerate fiber modes in mode multiplexed transmission

Cristian Antonelli,^{1,*} Antonio Mecozzi,¹ Mark Shtaif,² and Peter J. Winzer³

¹ Dept. of Physical and Chemical Sciences, University of L'Aquila, 67100 L'Aquila, Italy

² School of Electrical Engineering, Tel Aviv University, Tel Aviv, Israel 69978

³ Bell Labs, Alcatel-Lucent, 791 Holmdel-Keyport Rd., Holmdel, New Jersey 07733, USA

[*cristian.antonelli@univaq.it](mailto:cristian.antonelli@univaq.it)

Abstract: We study random coupling induced crosstalk between groups of degenerate modes in spatially multiplexed optical transmission. Our analysis shows that the average crosstalk is primarily determined by the wavenumber mismatch, by the correlation length of the random perturbations, and by the coherence length of the degenerate modes, whereas the effect of a deterministic group velocity difference is negligible. The standard deviation of the crosstalk is shown to be comparable to its average value, implying that crosstalk measurements are inherently noisy.

© 2013 Optical Society of America

OCIS codes: (060.2330) Fiber optics communications; (060.4510) Optical communications; (060.4230) Multiplexing.

References and links

1. P. J. Winzer and G. J. Foschini, "MIMO capacities and outage probabilities in spatially multiplexed optical transport systems," *Opt. Express* **19**, 16680–16696 (2011).
2. K. P. Ho and J. M. Kahn, "Statistics of group delays in multi-mode fibers with strong mode coupling," *J. Lightwave Technol.* **29**, 3119–3128 (2011).
3. R. Ryf, S. Randel, A. H. Gnauck, C. Bolle, A. Sierra, S. Mumtaz, M. Esmaelpour, E. C. Burrows, R. J. Essiambre, P. J. Winzer, D. W. Peckham, A. H. McCurdy, and R. Lingle, "Mode-division multiplexing over 96 km of few-mode fiber using coherent 6×6 MIMO processing," *J. Lightwave Technol.* **30**, 521–531 (2012).
4. C. Koebele, M. Salsi, D. Sperti, P. Tran, P. Brindel, H. Mardoyan, S. Bigo, A. Boutin, F. Verluise, P. Sillard, M. Astruc, L. Provost, F. Cerou, and G. Charlet, "Two mode transmission at 2x100Gb/s, over 40km-long prototype few-mode fiber, using LCOS-based programmable mode multiplexer, and demultiplexer," *Opt. Express* **19**, 16593–16600 (2011).
5. D. Gloge, "Weakly guiding fibers," *Appl. Opt.* **10**, 2252–2258 (1971).
6. R. Ryf, R. J. Essiambre, S. Randel, M. A. Mestre, C. Schmidt, and P. J. Winzer, "Impulse response analysis of coupled-core 3-core fibers," *Proceedings of ECOC 2012, Paper Mo.1.F* (2012).
7. C. Antonelli, A. Mecozzi, M. Shtaif, and P. J. Winzer, "Stokes-space analysis of modal dispersion in fibers with multiple mode transmission," *Opt. Express* **20**, 11718–11733 (2012).
8. The deterministic coupling between members of an LP mode group [9] is not included in **B** as its effect is assumed to be masked by the presence of the random perturbations which are represented by the second term in the square brackets of Eq. (1). This is consistent with the fact that strong random coupling between the constituent pseudo-modes is observed in experiments.
9. H. Kogelnik and P. J. Winzer, "Modal birefringence in weakly guiding fibers," *J. Lightwave Technol.* **30**, 2240–2243 (2012).
10. J. P. Gordon and H. Kogelnik, "PMD fundamentals: polarization mode dispersion in optical fibers," *Proc. Natl. Acad. Sci. USA* **97**, 4541–4550 (2000).
11. Strictly speaking the matrices U_l and U_q are independent up to a scalar phase factor that is negligible as compared to the phase difference due to the deterministic wavenumber mismatch.

12. The quantity that we refer to as the *coherence length* of the field describes the propagation distance along which the field decorrelates due to the fiber perturbations. It is not related to the coherence length of the light source.
13. A. Galtarossa, L. Palmieri, M. Schiano, and T. Tambosso, "Measurement of birefringence correlation length in long, single-mode fibers," *Opt. Lett.* **26**, 962–964 (2001).
14. J. Vuong, P. Ramantanis, A. Seck, D. Bendimerad, Y. Frignac, "Understanding discrete linear mode coupling in few-mode fiber transmission systems," *Proceedings of ECOC 2011*, Paper Tu.5.B.2 (2011).
15. C. W. Gardiner, *Stochastic methods for physics, chemistry and natural sciences* (Springer-Verlag, NY, 1983).

1. Introduction

Spatially multiplexed transmission is considered to be a promising solution for increasing the information throughput of fiber-optic systems [1], and significant efforts are being invested into exploring the properties of fiber structures supporting several propagation modes. A typical situation occurring in multimode structures is that groups of modes are characterized by similar propagation constants, implying strong coupling between them [2]. We refer to such modes as degenerate. On the other hand, modes that belong to different groups characterized by notably different propagation constants are weakly coupled, as has been observed in a series of experiments [3, 4]. This reality is nicely exemplified in the case of single-core fibers operating in the weakly guiding regime, where the linearly polarized (LP) mode approximation is valid [5]. A multi-core fiber forms another example as, depending on the overall core number and geometry, multiple degeneracies between the resultant super-modes may exist [6].

Our goal in this paper is to model the coupling between non-degenerate groups of modes and to understand its dependence on various fiber parameters. We obtain an expression for the crosstalk and find that the degree of coupling is mainly determined by the mismatch between the wavenumbers of the two groups, whereas the effect of the mismatch on group velocities is negligible. In addition, we show that the standard deviation of the crosstalk is comparable to its average value, a property that has to be taken in proper consideration when characterizing crosstalk in multimode fibers.

2. Analysis

As elaborated in [7], linear propagation in a fiber supporting N orthogonal spatial modes is described by the equation

$$\frac{d\vec{\mathcal{E}}}{dz} = i \left[\mathbf{B} + \frac{1}{2N} \tilde{\vec{b}}(\omega, z) \cdot \vec{\Lambda}^{(2N)} \right] \vec{\mathcal{E}}, \quad (1)$$

where $2N$ is the total number of propagation modes (with the factor of 2 accounting for polarizations), and $\vec{\mathcal{E}}$ is a $2N$ -dimensional column vector whose components are the complex envelopes of the electric field in the individual modes. The overall number of modes is $2N = \sum_{j=1}^M 2g_j$, where g_j is the degeneracy of the j -th group of modes and M is the number of groups. For example, in a weakly-guiding step index fiber supporting the first two LP mode groups, we have $N = 3$, $M = 2$, $g_1 = 1$, and $g_2 = 2$. The $2N \times 2N$ matrix \mathbf{B} describes propagation in the absence of perturbations, where the modes are uncoupled. Its only nonzero elements are on the main diagonal and they equal the wavenumbers of the various modes [8]. The vector $\tilde{\vec{b}}(\omega, z)$ has $4N^2 - 1$ real-valued components and it generalizes the familiar birefringence vector used in the modeling of polarization effects in single mode fibers [10]. The tilde above the vector sign serves to distinguish between generalized birefringence vectors [7] and electric field vectors such as $\vec{\mathcal{E}}$. The term $\vec{\Lambda}^{(2N)}$ is the generalized Pauli matrix vector [7] and its elements are $2N \times 2N$ traceless Hermitian matrices Λ_i , which reduce to the standard Pauli matrices [10] in the case of the single mode fiber ($N = 1$). These matrices constitute a basis for the space

of $2N \times 2N$ traceless Hermitian matrices. The scalar product $\tilde{\mathbf{b}} \cdot \tilde{\Lambda}^{(2N)}$ is to be interpreted as $\sum_{j=1}^{4N^2-1} b_j \Lambda_j$, and (when $\tilde{\mathbf{b}}$ is assigned the appropriate statistics) accounts for the effect of mode coupling resulting from random perturbations in the fiber structure. For ease of representation, it is convenient to decompose the electric field vector $\vec{\mathcal{E}}$ into vectors \vec{E}_j of smaller dimension $2g_j$, whose components are the field envelopes of the degenerate members of the j -th group. In this representation, the first $2g_1$ terms of the main diagonal of the matrix \mathbf{B} are equal to β_1 (the wavenumber of the modes in the first group), the following $2g_2$ terms are equal to β_2 (the wavenumber of the second group) and so on.

We now proceed by dividing the matrix $\tilde{\mathbf{b}} \cdot \tilde{\Lambda}^{(2N)}$ into rectangular blocks denoted by \mathbf{b}_{lj} whose dimension is $2g_j \times 2g_l$, which can be demonstrated to have the property $\mathbf{b}_{lj}^\dagger = \mathbf{b}_{jl}$. With this notation, and substituting for convenience $\vec{e}_l(z) = \exp(-i\beta_l z) \vec{E}_l(z)$, Eq. (1) can be rewritten as a set of coupled equations

$$\frac{d\vec{e}_l}{dz} = \frac{i}{2N} \mathbf{b}_{ll} \vec{e}_l + \frac{i}{2N} \sum_{j \neq l}^M e^{i\beta_{jl}z} \mathbf{b}_{lj} \vec{e}_j, \quad (2)$$

where we defined $\beta_{jl} = \beta_j - \beta_l$. A formal solution of Eq. (2) is given by

$$\vec{e}_l(z) = \mathbf{U}_l(z, 0) \vec{e}_l(0) + \frac{i}{2N} \sum_{j \neq l}^M \int_0^z \mathbf{U}_l(z, z') \mathbf{b}_{lj}(z') \vec{e}_j(z') e^{i\beta_{jl}z'} dz', \quad (3)$$

where $\mathbf{U}_l(z, 0)$ is a unitary matrix satisfying $d\mathbf{U}_l/dz = i\mathbf{b}_{ll}\mathbf{U}_l/2g_l$, and where $\mathbf{U}_l(z, z') = \mathbf{U}_l(z, 0)\mathbf{U}_l^\dagger(z', 0)$ describes propagation in the l -th group of modes from z' to z in the absence of random coupling with other groups.

2.1. Coupling between groups of modes

We now study the signal leaking from one group of degenerate modes to another. To that end, we evaluate $\vec{E}_l(z)$ when the input signal is injected only into group q , with $q \neq l$. Since we expect the coupling between different groups to be small due to the large wavenumber difference, we extract $\vec{E}_l(z)$ via a first-order perturbation analysis, which yields

$$\vec{e}_l(z) = \frac{i}{2N} \int_0^z e^{i\beta_{ql}z'} \mathbf{U}_l(z, z') \mathbf{b}_{lq}(z') \mathbf{U}_q(z', 0) \vec{e}_q(0) dz'. \quad (4)$$

The interpretation of Eq. (4) is rather intuitive. The electric field generated in group l at position z' is proportional to the product of the random coupling matrix $\mathbf{b}_{lq}(z')$ by the field $\vec{e}_q(z') = \mathbf{U}_q(z', 0) \vec{e}_q(0)$. The matrix $\mathbf{U}_l(z, z')$ accounts for the propagation of the field in group l from z' to z and the term $\exp(i\beta_{ql}z')$ accounts for the mode-independent phase that it accumulates. In what follows we will assume that the input vector is normalized, so that $|\vec{e}_q(0)| = 1$.

It is of interest to note that the inverse Fourier transform of Eq. (4) is the impulse response of the system describing coupling between the q -th and the l -th groups of modes. When the delay spread in the fiber is dominated by the deterministic walk-off between the groups, the impulse response measured after a distance z will be limited to a time interval whose duration is $|z\beta_{ql,1}|$, where $\beta_{ql,1}$ is the frequency derivative of β_{ql} at the carrier frequency, namely at $\omega = 0$. This picture is in agreement, in terms of the general shape of the impulse response, with the experimental results reported in [3] and it can be deduced from Eq. (4) by approximating $\beta_{ql} \simeq \beta_{ql,0} + \beta_{ql,1}\omega$ and by neglecting the frequency dependence of the perturbation vector $\tilde{\mathbf{b}}(z)$, which is responsible for random modal dispersion [7].

In order to quantify the extent of coupling between different groups of modes we consider the total energy received in the l -th group when a signal is transmitted in the q -th group. This quantity is given by

$$u_{lq} = \int_{-\infty}^{+\infty} \tilde{P}(\omega) |\vec{e}_l(z, \omega)|^2 \frac{d\omega}{2\pi}, \quad (5)$$

where $\tilde{P}(\omega)$ is the input signal spectrum, normalized so that $\int_{-\infty}^{+\infty} \tilde{P}(\omega) d\omega / 2\pi = 1$. Since this problem is stochastic in nature, we now examine the statistics of u_{lq} over fiber realizations.

2.2. Statistics of mode coupling

We first calculate the average crosstalk $\langle u_{lq} \rangle$. This involves calculating first $\langle |\vec{e}_l(z, \omega)|^2 \rangle$, which in turn involves calculating $\langle \mathbf{U}_q^\dagger(z'', 0) \mathbf{b}_{lq}^\dagger(z'') \mathbf{U}_l^\dagger(z, z'') \mathbf{U}_l(z, z') \mathbf{b}_{lq}(z') \mathbf{U}_q(z', 0) \rangle$, as a consequence of the double integral appearing when evaluating $|\vec{e}_l(z, \omega)|^2$ from Eq. (4). Since the statistics of mode coupling should be stationary with respect to frequency within the telecom bandwidth, we may set the offset from the carrier frequency to zero, namely $\omega = 0$, in $\tilde{\vec{b}}(\omega, z)$ when performing the average. Since the matrices \mathbf{b}_{ll} and \mathbf{b}_{qq} (that determine \mathbf{U}_l and \mathbf{U}_q) are statistically independent of each other [11] and of the coupling blocks \mathbf{b}_{lm} , the averaging of the inner term $\langle \mathbf{U}_l^\dagger(z, z'') \mathbf{U}_l(z, z') \rangle = \langle \mathbf{U}_l(z'' - z', 0) \rangle$ can be performed independently of the outer terms. By isotropy the desired average should be proportional to the identity matrix \mathbf{I}_l , and we may write $\langle \mathbf{U}_l(z, 0) \rangle = \exp(-z/L_l) \mathbf{I}_l$, where we assumed exponential decorrelation of the electric field vector. The physical meaning of the correlation length L_l is seen when expressing the longitudinal autocorrelation function of the field vector as $\langle \vec{e}_l(z'')^\dagger \vec{e}_l(z') \rangle = \vec{e}_l(0)^\dagger \langle \mathbf{U}_l(z'' - z', 0) \rangle \vec{e}_l(0) = |\vec{e}_l(0)|^2 \exp(-|z'' - z'|/L_l)$. We thus refer to L_l as the coherence length of the electric field [12].

We are then left with the calculation of $\langle \mathbf{U}_q^\dagger(z'', 0) \mathbf{b}_{lq}^\dagger(z'') \mathbf{b}_{lq}(z') \mathbf{U}_q(z', 0) \rangle$, where the inner product $\mathbf{b}_{lq}^\dagger(z'') \mathbf{b}_{lq}(z')$ can be averaged separately as it is independent of \mathbf{U}_q . To accomplish this task, we refer the reader to the detailed construction of the $\vec{\Lambda}^{(2N)}$ matrix vector, which is presented in the appendix of [7]. Moreover, since the matrices \mathbf{b}_{lq} represent off-diagonal blocks of the matrix $\vec{\tilde{b}} \cdot \vec{\Lambda}^{(2N)}$, the real and imaginary parts of each of the elements of \mathbf{b}_{lq} are proportional to different components of the vector $\vec{\tilde{b}}$, with the proportionality coefficient equal to \sqrt{N} . Hence, since we model the generalized birefringence vector $\vec{\tilde{b}}$ as consisting of statistically independent components, there is statistical independence between the elements of \mathbf{b}_{lq} and between the real and imaginary parts of each element. Denoting by $f(z)$ the autocorrelation function of the components of $\vec{\tilde{b}}(z)$, namely $\langle b_j(z') b_k(z'') \rangle = f(z' - z'') \delta_{jk}$, we obtain $\langle \mathbf{b}_{lq}^\dagger(z'') \mathbf{b}_{lq}(z') \rangle = 4N g_l f(z' - z'') \mathbf{I}_q$. This leaves us with the calculation of $\langle \mathbf{U}_q^\dagger(z'', 0) \mathbf{U}_q(z', 0) \rangle$, which follows the same lines as in the calculation performed for \mathbf{U}_l , with the result:

$$\langle |\vec{e}_l(z)|^2 \rangle = \frac{g_l}{N} \text{Re} \left\{ \int_0^z e^{-i\beta_{ql}\xi} e^{-(1/L_l + 1/L_q)\xi} (z - \xi) f(\xi) d\xi \right\}. \quad (6)$$

To gain further insight we assume a particular functional form for the autocorrelation function of the mode coupling perturbations, $f(z) = n_0 \exp(-z/L_c)$, where n_0 quantifies the strength of the perturbations and L_c is their correlation length. This is a plausible choice, which is customary in PMD studies [13]. The average coupled energy can thus be written as

$$\langle u_{lq} \rangle = \frac{2n_0 g_l}{N} \text{Re} \left\{ \int_{-\infty}^{+\infty} \tilde{P}(\omega) \frac{e^{-Kz} + Kz - 1}{K^2} \right\} \frac{d\omega}{2\pi}, \quad (7)$$

where $K = 1/L_{\text{eff}} + i\beta_{lq}$, with $L_{\text{eff}} = (1/L_c + 1/L_l + 1/L_q)^{-1}$. Note that for large propagation distances, the fractional expression inside the integral in Eq. (7) can be approximated by z/K ,

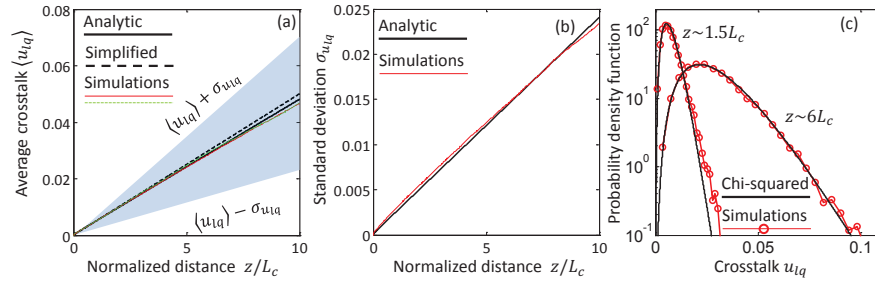


Fig. 1. (a) Average energy coupled from non-degenerate group q to a two-fold degenerate group l versus propagation distance normalized to L_c . The solid black line represents the complete expression given by Eq. (7), whereas the thin red and the dotted green lines were obtained in simulation with different parameters (see text). The dashed black line shows the simplified crosstalk expression given by Eq. (8). The shaded area marks one standard deviation from the mean as given by Eq. (9). (b) Standard deviation of the crosstalk $\sigma_{u_{lq}}$ as obtained from Eq. (9) (thick-black curve) and from numerical simulations (thin red). (c) Crosstalk probability density function (circles) and chi-squared distribution fit (solid line).

which shows that the degree of coupling can be reduced by increasing the wavenumber mismatch. Expanding Eq. (7) to fourth order in β_{lq}^{-1} , and approximating $\beta_{lq} \simeq \beta_{lq,0} + \omega\beta_{lq,1}$, yields

$$\langle u_{lq} \rangle = \frac{n_0}{\beta_{lq,0}^2} \frac{2g_l}{N} \frac{z}{L_{\text{eff}}} (1 + \varepsilon_{lq}), \quad (8)$$

where $\varepsilon_{lq} = 3\omega_{\text{rms}}^2\beta_{lq,1}^2/\beta_{lq,0}^2$, with $\omega_{\text{rms}}^2 = \int_{-\infty}^{+\infty} \omega^2 \tilde{P}(\omega) d\omega / 2\pi$ being the mean square bandwidth of the input signal. The quantity ε_{lq} is a correction term which turns out to be negligibly small for realistic fiber parameters. For example, assuming $\omega_{\text{rms}}/2\pi = 20$ GHz, we obtain $\varepsilon_{lq} \simeq 1.9 \times 10^{-5}/\beta_{lq,0}^2$ for the differential mode group delay value $\beta_{lq,1} = 20$ ps/km [3], and $\varepsilon_{lq} \simeq 0.9/\beta_{lq,0}^2$ for the much greater value $\beta_{lq,1} = 4.35$ ns/km [4], which gives a very small number since $\beta_{lq,0}$ is by orders of magnitude larger than 1 m^{-1} . This means that with a large enough wavenumber mismatch the group velocity difference is expected to have a negligible effect on system performance [14].

We now estimate the standard deviation of the crosstalk $\sigma_{u_{lq}}$. Inspection of Eq. (4) suggests that, if the propagation distance is much longer than the correlation length of \mathbf{b}_{lq} , then the output field is the sum of many independent, random contributions, and hence by the central-limit theorem $\vec{e}_l(z)$ becomes a Gaussian vector with $2g_l$ independent *complex* components. The crosstalk u_{lq} is proportional to the frequency integral of $|\vec{e}_l(z)|^2$ and its statistical properties are affected by the frequency-dependence of the coupling. In the special case where fiber characterization is performed with a continuous-wave (CW) signal, u_{lq} can be approximated by a chi-squared distributed variable with $4g_l$ degrees of freedom and standard deviation given by

$$\sigma_{u_{lq}} = \frac{\langle u_{lq} \rangle}{\sqrt{2g_l}}. \quad (9)$$

With the characteristic values of g_l in the majority of relevant fiber structures (in the case of few mode fibers in the weakly guiding approximation, the largest value of g_l is 2), this implies that measurements of crosstalk are necessarily noisy, a fact that needs to be properly taken into account in practical considerations.

3. Results

In order to validate the above theory, one would like to compare its predictions with the simulation of a realistic fiber structure, supporting multiple groups of modes. Such a procedure would however be prohibitively inefficient because of the multiple length scales involved in the processes of propagation and coupling. On the one hand, the beat-length associated with the wavenumber difference between any two groups of modes can easily be of the order of a millimeter or less, while on the other hand, the correlation lengths characterizing perturbations in the fiber are likely to be on the multiple meters scale (as can be expected based on polarization studies in single-mode fibers). A meaningful statistical study requires simulation of multiple correlation lengths, a prohibitively time-consuming procedure when a step-size of a fraction of a millimeter needs to be used. In order to bypass this difficulty we present in Fig. 1 the results of a computation performed where the correlation length of the perturbations L_c is much shorter than in reality and equal to only fifteen beat lengths ($L_c = 15 \frac{2\pi}{\beta_{l,q,0}}$). In addition, the coherence lengths of the fields in the two groups of modes were assumed to be $L_l = L_q = L_c/10$. We assumed degeneracy factors of $g_q = 1$ and $g_l = 2$, for the two groups of modes and the perturbation vector $\tilde{\vec{b}}(z)$ was assigned Gaussian statistically independent components. Since we do not seek to resolve the details of the formation of strong coupling within each of the two groups of modes, we take the components of $\tilde{\vec{b}}(z)$ that are responsible for such coupling as white noise processes in the longitudinal dimension z . It can be shown that their power spectral density (which is constant) is equal to $8g_{l,q}^2(4g_{l,q}^2 - 1)^{-1}L_{l,q}^{-1}$. The other components of $\tilde{\vec{b}}(z)$, which are responsible for coupling between the two groups of modes, were produced as Ornstein-Uhlenbeck processes [15], with the above specified correlation length L_c and with a variance n_0 which was chosen such that a coupling of $\sim 5 \times 10^{-2}$ is predicted by Eq. (8) at $L = 10L_c$.

In Fig. 1(a) we plot the average crosstalk $\langle u_{lq} \rangle$ as a function of propagation distance, assuming transmission of a CW signal. The thick solid lines (black) represent the full expression Eq. (7) and the thin lines (red) represent the results of Monte Carlo simulations with 50,000 fiber realizations. The dashed lines represent the simplified expression Eq. (8). The excellent agreement between the analytical solutions and the simulations is evident. The small deviation observed when the crosstalk level rises towards 5% represents the saturation of the first order analysis that we used. We note that the accuracy of the analytical results depends only on the overall level of coupling and not on the exact parameter combinations. To demonstrate this, the dotted green curve in Fig. 1(a), which overlaps with the red curve, was calculated with different parameters; $L_c = 30 \frac{2\pi}{\beta_{l,q,0}}$ and $L_l = L_q = L_c/5$. The shaded area surrounding the curves marks one standard deviation as given by Eq. (9). Figure 1(b) shows the standard deviation $\sigma_{u_{lq}}$ as a function of propagation distance. Once again, the thick black and the thin red lines represent the analytical expression Eq. (9) and the numerical results, respectively. In Fig. 1(c) we plot the crosstalk probability density function for the displayed values of the propagation distance. Symbols represent Monte Carlo simulations, solid lines are the plot of a chi-squared distribution with $4g_2 = 8$ degrees of freedom and mean value given by Eq. (7).

4. Conclusions

We studied the crosstalk between groups of degenerate modes induced by random perturbations in multimode fibers. We showed that the crosstalk is determined almost exclusively by the wavenumber difference between the groups, whereas the effect of the group-velocity difference is negligible. In addition, the standard deviation of the crosstalk was found to be comparable in magnitude with the average crosstalk. This result is of major significance for experimental fiber characterization as it questions the reliability of isolated crosstalk measurements.

Acknowledgments

This work was funded by Alcatel-Lucent in the framework of Green Touch. MS also acknowledges financial support from Israel Science Foundation (grant 737/12). CA and AM acknowledge financial support from the Italian Ministry of University and Research through ROAD-NGN project (PRIN 2010-2011). Discussions with Herwig Kogelnik are gratefully acknowledged.