Polarization holograms allow highly efficient generation of complex light beams

U. Ruiz,[1] P. Pagliusi,[1,2] C. Provenzano,[1] K. Volke-Sepúlveda,[3] and Gabriella Cipparrone[1,2,*]

[1] *Dipartimento di Fisica, Università della Calabria, I-87036 Rende (CS), Italy*
[2] *IPCF-CNR, UOS Cosenza, and Excellence Centre CEMIF.CAL, Ponte P. Bucci 33B, I-87036 Rende, Italy*
[3] *Instituto de Física, Universidad Nacional Autonoma de México, Apdo. Postal 20-364, 01000 México D.F., México*
[*] *gabriella.cipparrone@fis.unical.it*

Abstract: We report a viable method to generate complex beams, such as the non-diffracting Bessel and Weber beams, which relies on the encoding of amplitude information, in addition to phase and polarization, using polarization holography. The holograms are recorded in polarization sensitive films by the interference of a reference plane wave with a tailored complex beam, having orthogonal circular polarizations. The high efficiency, the intrinsic achromaticity and the simplicity of use of the polarization holograms make them competitive with respect to existing methods and attractive for several applications. Theoretical analysis, based on the Jones formalism, and experimental results are shown.

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1. Introduction

Polarization holography, based on the vector character of the interference, relies on the interference of orthogonally polarized waves, when a modulation of the polarization state of the light occurs in the superposition region, as a function of the phase difference between the beams [1]. Conventional holographic media, sensitive only to the light intensity, are not suitable to record polarization holograms (PHs), and thus photo-anisotropic materials, sensitive to the light polarization, are required, since they can store the polarization information. In such materials, linear or circular optical anisotropies (either birefringence and/or dichroism) can be photoinduced depending on the polarization state of the incident light, and the resulting PHs exhibit unique properties with respect to the diffraction efficiency and the polarization states of the diffracted beams [1]. PHs recorded with two interfering Gaussian beams of equal intensity and orthogonal circular polarization have been largely used for applications in several fields, such as polarizing elements, optical switching, polarimetry, among others [1–4]. In this geometry, only the zero (0) and first (± 1) diffracted orders appear, in which the $+1$ (-1) order is a left- (right-) circularly polarized wave and is proportional to the right- (left-) hand component of the incident wave, whereas the zero order keeps the same polarization state of the incident probe beam [1]. Moreover, 100% diffraction efficiency can be achieved for proper values of the optical retardance (i.e., thickness and photoinduced birefringence) [1, 2]. Recently, polarization holography has been adopted to build up forked polarization gratings, able to efficiently generate optical vortices, with the main purpose to demonstrate spin to orbital angular momentum conversion [5, 6]. In this way, the possibility of encoding the phase of a beam by means of a PH has also been probed. Moreover, amplitude encoding in PHs has been demonstrated as well for a small finite number of amplitude values, related with the exposure time in a temporally modulated and phase-shaped beam, generated with a phase spatial light modulator (SLM) and imaged onto a photoactive polymer film [7]. In fact, there are fundamental aspects of light and light-matter interaction that have been studied thanks to the use of these holographic techniques for the generation of optical vortices, such as the conversion between spin and orbital angular momentum. For example, the so called q-plate, introduced in 2006 by Marrucci and associates, is a liquid crystal device for converting a variation of the spin angular momentum of light into orbital angular momentum [8], and this is also what has been done with forked PHs [5, 6]. Nowadays, besides the case of the simplest optical vortices generated by a forked grating, there are several other complex fields with continuous variations of phase and amplitude, which have interesting dynamical properties in interaction with matter and a number of applications. Such is the case, for instance, of propagation-invariant optical fields (PIOFs), characterized by a transverse intensity profile ideally independent of the axial propagation coordinate z. In addition to the plane wave, there are several families of PIOFs, each defined by its own geometry in the transverse plane; circular Bessel, elliptic Mathieu, parabolic Weber, and Airy beams [9–11]. All these beams have revealed themselves as interesting and in some cases exclusive tools to perform different specific tasks according to their geometric characteristics. For instance, high order Bessel beams (BBs) are optical vortices as well, possessing orbital angular momentum that can be transferred [12–15]. Mathieu beams have found relevant applications in nonlinear optics, since their topological

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properties depend on the parameter defining the interfocal distance of the ellipses [16, 17], and they are characterized by a new dynamical quantity, analogous to the angular momentum, but associated with elliptic geometry [18]. Weber beams (WBs) have been recently proposed as atomic beam splitters [19]. These are just examples to illustrate the relevance of PIOFs in several areas of physics.

Here, we present a method for the generation of PIOFs, directly from an incident plane wave, by means of PHs. This is the first demonstration of non-quantized amplitude information storage in a PH. The high efficiency predicted for PHs recorded with orthogonal circularly polarized plane waves is exploited here to efficiently generate complex structured beams. To illustrate the method, we report on the generation of BBs and WBs by PHs, characterized by comparable accuracy and significantly higher efficiency with respect to the synthetic phase hologram methods [20].

The generation of complex beams by means of PHs is relevant not only as an alternative method, but it may also pave the way to study unexplored aspects about the dynamical properties of light and how they change in interaction with a medium. In addition, the possibility of encoding polarization, phase and amplitude in a single optical element may be important also in quantum information.

2. Theory

We adopt the Jones formalism to describe the generation and the diffraction of the PHs, which are assumed to be recorded in a material with only photoinduced linear optical anisotropy. The aim of this work is to generate complex light fields $E_c = A(q_1, q_2)e^{i\vartheta(q_1,q_2)}$, where $A(q_1, q_2)$ and $\vartheta(q_1, q_2)$ are the amplitude and the phase variation of the complex beam, and (q_1, q_2) denote either the circular (r,φ) or the parabolic (ν,ζ) coordinates in the transverse plane. We consider the interference of a plane wave $\vec{E}_p$ and a tailored complex field $\vec{E}_m$, having opposite circular polarizations, whose propagation axes form an angle α:

$$\vec{E}_p = \frac{E_0}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}e^{i\delta}, \quad \vec{E}_m = \frac{E_0}{\sqrt{2}}f(A)\begin{pmatrix}1\\ -i\end{pmatrix}e^{i\vartheta}e^{-i\delta}, \tag{1}$$

where $\delta \equiv \pi x/\Lambda$, $\Lambda \equiv \lambda_R / 2\sin(\alpha/2)$ is the spatial periodicity of the polarization pattern, λ_R is the recording wavelength and $f(A)$ is an amplitude function that will be determined so as to produce the desired complex field in the read out process.

The resulting interference field in the superposition region is:

$$\vec{E}_{tot} = \vec{E}_p + \vec{E}_m = \frac{E_0}{\sqrt{2}}\begin{pmatrix}e^{i\delta} + f(A)e^{i\vartheta}e^{-i\delta}\\ ie^{i\delta} - if(A)e^{i\vartheta}e^{-i\delta}\end{pmatrix}. \tag{2}$$

The transmission matrix T of the PH, evaluated according to Ref [1], can be written as the sum of the three matrices associated with the 0 and ± 1 orders

$$T = T_0 + T_{+1} + T_{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\cos(M) + \frac{1}{2}\begin{bmatrix}i & -1\\ -1 & -i\end{bmatrix}\sin(M)e^{i(2\delta-\vartheta)} + \frac{1}{2}\begin{bmatrix}i & 1\\ 1 & -i\end{bmatrix}\sin(M)e^{-i(2\delta-\vartheta)}, \tag{3}$$

where $M = 2kd\beta_{lin}E_0^2 f(A)$, k is the wave number, d is the thickness of the recording material, and β_{lin} is a coefficient related to the photoinduced linear birefringence of the material, so that the three diffraction orders are $\vec{E}_j = \vec{T}_j\vec{E}_{in}$, with $j = 0, \pm 1$. As can be seen in Eq. (3), the amplitude and phase information of the field E_m are on the ± 1 diffracted orders, hence, we will only consider them in the next discussion.

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Assuming a normalized linearly or circularly polarized plane wave ($\vec{E}_{in}^{lin} = \begin{pmatrix}1 & 0\end{pmatrix}^T$ or $\vec{E}_{in}^{circ} = 1/\sqrt{2}\begin{pmatrix}1 & \pm i\end{pmatrix}^T$) impinging on the PH, we obtain

$$\vec{E}_{\pm1}^{lin} = \frac{i}{2}\sin(2kd\beta_{lin}E_0^2 f(A))e^{\mp i\vartheta}\begin{pmatrix}1\\ \pm i\end{pmatrix}, \quad \vec{E}_{\pm1}^{circ} = \frac{i}{\sqrt{2}}\sin(2kd\beta_{lin}E_0^2 f(A))\Psi e^{\mp i\vartheta}\begin{pmatrix}1\\ \mp i\end{pmatrix}, \quad (4)$$

where $\Psi = 1$ ($\Psi = 0$) for the + 1 order and $\Psi = 0$ ($\Psi = 1$) for the −1 order when the incident beam is left-handed (right-handed) circularly polarized.

According to Eq. (4) the first-order (± 1) diffracted beams have conjugated phase ϑ and opposite circular polarizations, while exchange of energy between the diffracted beams occurs by varying the ellipticity of the incident plane wave, in which the −1 order reproduces the polarization and phase of the complex field $\vec{E}_m$.

In order to codify also the amplitude A of the complex beam $\vec{E}_c$, we determine $f(A)$ in the recording wave $\vec{E}_m$, such that $\sin(2kd\beta_{lin}E_0^2 f(A))$ is proportional to A. According to Eq. (4), we assume

$$\sin(2kd\beta_{lin}E_0^2 f(A)) = \xi A, \quad (5)$$

where $0 < \xi \le 1$ ξ is a uniform parameter, so that $f(A) = \sin^{-1}(\xi A)/2kd\beta_{lin}E_0^2$.

The parameter ξ is evaluated by considering the PH created by the interference between two plane waves (i.e.,$f(A) = A = 1$). In this case, in fact, Eq. (5) becomes $\xi = \sin(2kd\beta_{lin}E_0^2) = \sqrt{\eta_p}$, where $\eta_p = \left(\left|\vec{E}_{+1}\right|^2 + \left|\vec{E}_{-1}\right|^2\right)/\left|\vec{E}_{in}\right|^2$ is the total diffraction efficiency, as reported in [1].

In the general case, the total diffraction efficiency can be calculated from Eq. (4)

$$\eta \equiv \frac{\int_\Omega\left(\left|\vec{E}_{+1}\right|^2 + \left|\vec{E}_{-1}\right|^2\right)d\Omega}{\int_\Omega\left|\vec{E}_{in}\right|^2 d\Omega} = \frac{\eta_p}{\Omega}\int_\Omega |A|^2 d\Omega, \quad (6)$$

where Ω represents the total area in which the complex beam is defined. In the case of the BB $A(r,\varphi)e^{i\vartheta(r,\varphi)} = J_1(2\pi k_r r)e^{i\varphi}$, where J_1 is the first order Bessel function, k_r is the spatial frequency, and the diffraction efficiency calculated by Eq. (6) is $\eta_{Bessel} \cong 0.15\times\eta_p$. In the case of the third order odd WB $A(\nu,\zeta)e^{i\vartheta(\nu,\zeta)} = P_o(\nu,\zeta;3)$ [11], the diffraction efficiency is $\eta_{Weber} \cong 0.10\times\eta_p$.

3. Experimental investigation

We have experimentally investigated the generation of BBs and WBs, by means of PHs recorded in an amorphous azo-polymer [21]. The latter is confined between two glass plates in a 23µm thick film, prepared by heating the polymer above the glass transition temperature T_g and cooling it down to room temperature. The polymer film is exposed to the polarization patterns obtained by the interference of the plane wave $\vec{E}_p$ and the complex beam $\vec{E}_m$ in Eq. (1), having orthogonal circular polarization. Both the beams are generated from an Ar^+ laser (wavelength λ_R = 488nm) over a circular area $\Omega \cong 0.05\,\text{cm}^2$ using a phase spatial light modulator (SLM) (PLUTO, HOLOEYE GmbH), addressed by means of a synthetic phase hologram [20]. The recording waves are superposed on the recording medium at a small angle $\alpha \cong 0.4°$, which corresponds to a spatial period Λ = 72µm for the PH. The recording intensity

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of the plane wave $\vec{E}_p$ (25 mW/cm^2) and the exposure time (60s) have been chosen by maximizing the total diffraction efficiency η_p of the PH recorded by two plane waves, in the range of linear photoresponse of the polymer film ($\Delta n_{lin} = \beta_{lin} E_0^2$). Indeed, this latter condition is necessary to fit the theoretical approach (see Eqs. (3) and (5)). Depending on the maximum photoinduced birefringence of the polymer far from saturation and on its absorption at the recording wavelength λ_R, we have experimentally found a value for η_p = 0.70, corresponding to the cited intensity and exposure time. A circularly polarized He–Ne laser probe beam (λ = 633nm), whose wavelength is far from the absorption band of the polymer, has been used to reconstruct the BBs and WBs from the PHs. A CCD camera has been used to capture the intensity distribution of the complex beams $\vec{E}_{-1}^{circ}$ diffracted by the PHs and compare them with the corresponding BBs and WBs generated by the SLM.

In Fig. 1 we analyse the case of the first order BB, i.e $Ae^{i\vartheta} = J_1(2\pi k_r r)e^{i\varphi}$, with $k_r \cong 17 cm^{-1}$. According to the selected parameters of the BB and the optical components employed to reconstruct the BB, the maximum non-diffracting distance is $z_{\max} \cong 1.2m$. The intensity profiles of the BBs generated by the SLM and reconstructed by the PH are reported in Fig. 1(a) and 1(b), respectively. A comparison between the two beams is reported in Fig. 1(d), which shows the 1D normalized intensity profiles of the SLM-generated (black) and the PH-reconstructed (red) BBs evaluated along the white line in Fig. 1(a) and 1(b). In Fig. 1(c), we show the propagated PH-reconstructed BB at $z = 30cm$. In order to evaluate quantitatively the agreement between the intensity modulation of the BBs produced by the SLM and the PH, we use the root-mean squared error, $RMSE \equiv \sqrt{\frac{1}{\Omega}\iint_{\Omega}\left(I_{SLM} - I_{PH}\right)^2 d\Omega}$.

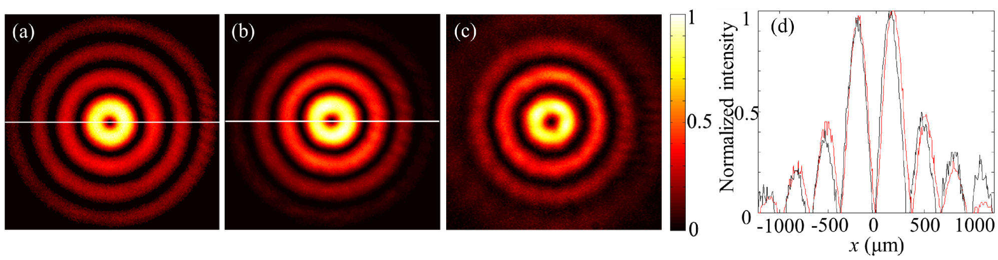

Fig. 1. First order BB. (a) Intensity distribution of the beam generated by the SLM, (b) intensity distribution of the beam reconstructed by the PH, (c) the propagated PH-reconstructed BB at the plane z = 30cm, and (d) 1D intensity profiles of the SLM-generated (black line) and PH-reconstructed (red line) BBs.

For the case of the BBs the $RMSE \cong 0.08$, which demonstrates a good agreement between the BBs generated with the two methods. On the other hand, the measured value of the diffraction efficiency is $\eta_{BB}^{\exp} \cong 0.10$, which is very close to the theoretical value calculated from Eq. (6) $\eta_{BB} \cong 0.15 \times 0.70 = 0.105$, and about one order of magnitude higher than the efficiency for the SLM-generated BBs.

In Fig. 2 we analyse the case of the third order odd WBs $A(\nu,\zeta)e^{i\vartheta(\nu,\zeta)} = P_o(\nu,\zeta;3)$ [11]. In this case, the maximum non-diffracting distance is $z_{\max} \cong 0.55m$. The intensity profiles of the SLM-generated and PH-reconstructed beams are reported in Fig. 2(a) and 2(b), respectively. In Fig. 2(c), we show the propagated PH-reconstructed WB at z = 30 cm. The good agreement between the intensity modulation of the WBs produced with the two methods is demonstrated by $RMSE \cong 0.06$. The measured value of the diffraction efficiency is $\eta_{WBs}^{\exp} \cong 0.07$, which is very close to the theoretical value calculated from Eq. (6)

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$\eta_{WBs} \cong 0.10 \times 0.70 = 0.07$, and about one order of magnitude higher than the efficiency for the SLM-generated WBs.

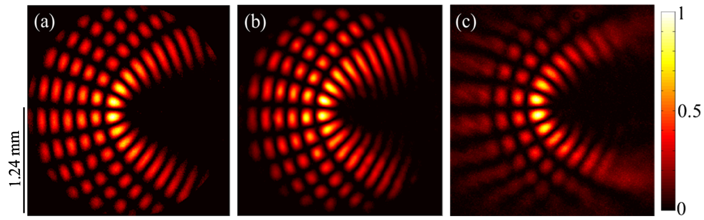

Fig. 2. Third order odd parabolic beam. (a) Intensity distribution of the beam generated by the SLM, (b) intensity distribution of the beam reconstructed by the PH, (c) the propagated PH-reconstructed WB at the plane $z = 30cm$.

4. Conclusions

In conclusion, we report a method for generation of arbitrary complex beams employing polarization holograms, which results from the recording of the interference pattern between a plane wave and a tailored complex beam, with orthogonal circular polarizations, on a polarization sensitive photoanisotropic material. Exploiting an amorphous azopolymer characterized by linear photoinduced birefringence as recording medium, we demonstrate the generation of circularly polarized Bessel and Weber non-diffracting beams from a plane wave with efficiencies 0.10 and 0.07, respectively, which are 70% of the maximum theoretical diffraction efficiencies and about one order of magnitude higher than the efficiency for equivalent SLM-generated beams. The good quality of the generated versus the SLM-generated beams is confirmed by a root mean squared deviation of 0.08 and 0.06 in the normalized intensity modulation of the Bessel and Weber beams, respectively, which were probed to be non-diffracting over an extended region of several centimeters. The transparency of the device in the visible range and the well known achromaticity of the polarization holograms [1] make these elements an excellent alternative to generate complex beams, such as propagation invariant optical fields. The polarization properties of the diffracted beams make them suitable to obtain some kinds of vector beams by direct recombination [22, 23]. Moreover, the PHs we have presented here, are the only optical elements that can store polarization, phase and amplitude information simultaneously. This characteristic makes them attractive for applications in quantum information.

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