A COMPUTATIONAL MODEL OF VISCOELASTIC COMPOSITE MATERIALS FOR LIGAMENT OR TENDON PROSTHESES

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ABSTRACT

A constitutive model and a finite element formulation for viscoelastic anisotropic materials subject to finite strains is expounded in this paper. The composite material is conceived as a matrix reinforced with stiff fibres. The constitutive relations are obtained by defining a strain energy function and a relaxation function for each constituent. By means of this approach, the viscoelastic properties of the material constituents can be taken into account and therefore different time dependent behaviour can be assigned to the matrix and to the reinforcing fibres. The response provided by this kind of constitutive formulation allows for the description of mechanical behaviour for either natural anisotropic tissues (such as tendons and ligaments) and for the composite materials which are currently adopted for tissue reconstruction. The main features of those mechanical properties observed in an ideal uniaxial test are: a non linear stress-strain response and a time dependent response which is observed in relaxation of stresses for a prescribed constant stretch and in a moderate strain rate dependence of the measured response.

1. INTRODUCTION

Ligaments and tendons are biological structures with a relevant load-bearing role during the relative movements of body parts. Even if the roles of these two kinds of structure are distinct, they display a qualitatively comparable mechanical behaviour. Experimental studies conducted on both kinds of tissue show non-linear force-displacement response and a viscoelastic behaviour. Moreover, cyclic loading are characterised by a pre-conditioning stage followed by a stationary response [1]. An important subject of research, in this field, is the clinical treatment of tissue reconstruction in case of rupture. One of the possible technique to recover the functionality is the implantation of a totally new ligament or tendon made of composite material. For a clinically successfully implantation a fully biomechanically compatible prosthesis should be conceived [2]. To this end the mechanical behaviour of composite materials for prostheses and of natural tissues are compared with particular reference to their viscoelastic properties. This paper aims at studying the viscoelastic constitutive laws for composites subjected to finite deformations that simulate the behaviour of ligaments and tendons. A computer implementation will be used for predicting the mechanical response of artificial devices and the influence of the kinematic and mechanical non-linearities on the global behaviour will be addressed. A large number of theoretical and numerical studies have been presented for biological tissues and, in particular, the papers by [3-5] were dedicated to ligaments and tendons. All those models describe, through different approaches, the common behaviour of such tissues: the non-linear stress strain response and the time dependent behaviour. The proposed theories can be classified in three main classes: general continuum theories, phenomenological approaches and structural approaches. The continuum theory is based on the extension of continuum based theory to the viscoelastic case. [6] presented the extension to the finite strains range of the quasi-linear viscoelasticity presented for the first time by [1]. Phenomenological approaches are developed by means of formulations that describe the mechanical response in the simplest possible way and by fitting experimental results. As last, the structural approaches are based on the knowledge of the mechanical response of each material constituent [7-8]. In this paper a continuum approach with a structural interpretation has been used for the description of the viscoelastic behaviour of composite materials, here analytical aspects and the finite element implementation are investigated. A possible time integration scheme of the presented viscoelastic model is presented in [9]. This model is the extension of the constitutive laws, formulated in [2], to the case of the viscoelastic response. The structural approach to the viscoelastic constitutive laws presented in this paper relies on the assumption that fibres and matrix can exhibit different time dependent properties in terms of relaxation spectrum and total relaxation ratios. As such, this model can be regarded as a quasi-linear model, where linear viscoelasticity is associated to a non-linear mechanical response.

2. THEORETICAL DEVELOPMENTS AND NUMERICAL IMPLEMENTATION

Theoretical developments of the viscoelastic material description are presented on the basis of the constitutive equations suited for time independent mechanical behaviour of anisotropic materials, subject to finite deformations, which was reported by the Author in [2]. Such constitutive equations were based on the assumption of an elastic potential for each composite constituents, namely for the matrix and the fibres. The interaction between the two components was taken into consideration when assuming a perfect bond between those two materials and therefore by setting up a kinematic relationship between fibre stretch and matrix deformation. In a three-dimensional Cartesian space, let the current and the reference body configuration is denoted by $\mathbf{x}$ and $\mathbf{X}$ respectively. In a Lagrangean description, where the reference configuration is the initial undeformed state of the body, the deformation tensors are the right and left Cauchy-Green stretch tensors defined as:

$$\mathbf{C} = \mathbf{F}^T\mathbf{F};\ \mathbf{B} = \mathbf{F}\mathbf{F}^T \qquad (1)$$

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \qquad (2)$$

where the superscript T denotes the transpose. A set of reinforcing fibres, parallel to the vector $\mathbf{N}$ in the reference configuration, are considered. The fibre stretch λ_f can be obtained if the perfect bond between fibre and matrix is assumed:

$$\lambda_f^2 = \mathbf{N}^T\mathbf{C}\mathbf{N} \qquad (3)$$

The elastic potentials adopted for the matrix W_m and the K^{th} family of fibres W_{fk} are:

$$W_m = [\alpha(I_c - 3) + \beta(II_C - 3)]; \qquad (4)$$

$$W_{fk} = \frac{1}{2}E_f\left(\lambda_{fk}^m - 1\right)^q \qquad (5)$$

where I_C and II_C are the first and second invariant of tensor $\mathbf{C}$, while, α, β, E_f and q are material parameters.

The Cauchy stress components for a time independent behaviour are:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_m + \sigma_f\,\mathbf{n}\mathbf{n}^T + p\mathbf{I} \qquad (6)$$

$$\sigma_{ij} = \frac{2}{J}\left[(\alpha + I_C\beta)B_{ij} - \beta B_{ik}B_{kj}\right] + p\delta_{ij} + E_f\frac{1}{2}mq\left(\lambda_f^m - 1\right)^{q-1}q\lambda_f^m n_i n_j \qquad (7)$$

The structural approach to viscoelasticity for composites is based on the knowledge of the viscous properties of each constituent. The behaviour of the composite will be obtained by summing up the contribution of all the constituents. In a small strains setting and for problems with incompressible materials, a linear viscoelastic response can be modelled by means of time integration as follows:

$$\boldsymbol{\sigma}(t) = \int_0^t \frac{G(t-s)}{G(0)}\frac{d\boldsymbol{\sigma}_e^d(s)}{ds}ds + \mathbf{I}p(t) \qquad (8)$$

This equation states that the deviatoric stress components at a time instant t is obtained by integrating over the time interval $s = 0,t$ all the previous elastic deviatoric stress components weighted by the function G(t). It is implicitly assumed that the deformation process starts at time instant $t = 0$, and that at previous time instants the body was in the undeformed and unstressed state. This function G(t) is a relaxation function which provides a fading memory to the material. By means of this function, recent events have more influence on the state at the current time than older ones. The constant p in equation (8) represents the hydrostatic stress components, which are related to the volumetric deformation, which is assumed not subject to viscous effect. Equation (8) is the starting point for the description of the viscous effects in the matrix of the composite material. A generalisation to the finite strains is however required. This generalisation should take into consideration that rigid rotations of a body particle do not contribute to viscous behaviour [9,10]. Equation (8) can be generalised for use in the finite strain range, as in [6], in the following way:

$$\sigma(t) = \mathbf{R}(t)\left[\int_0^t \frac{G(t-s)}{G(0)} \frac{d}{ds}\left[\mathbf{R}^T(s)\sigma_{em}^d(s)\mathbf{R}(s)\right]\right] \mathbf{R}^T(t) + \mathbf{I}p \tag{9}$$

During the deformation process the body particle is subjected to a finite rotation determined by the tensor $\mathbf{R}(t)$, which satisfies the relation $\mathbf{R}^T\mathbf{R} = \mathbf{I}$. The rotation tensor $\mathbf{R}$ is obtained by applying the multiplicative decomposition of the deformation tensor $\mathbf{C}$ as follows:

$$\mathbf{C} = \mathbf{R}^T\mathbf{U}^2\mathbf{R} \tag{10}$$

The equation (10) states that the a deformation tensor can be decomposed in a pure stretching represented by the positive definite tensor **U** and a rigid rotation represented by the orthogonal matrix **R**. The effects of those rigid rotations, which do not influence the viscous behaviour of the material, can be eliminated by rotating the Cauchy stress tensor to a fixed reference system. It is obtained by means of the double product of the elastic deviatoric stress components $\sigma_{em}^d(s)$ by the matrices $\mathbf{R}^T$ and **R**, (see the integral argument of the right-hand side of the equation (9). The stress components modified by the viscous effect can be reoriented to the current reference system by means of a further double product by **R** and $\mathbf{R}^T$. Equation (9) represents the time dependent effects given by the viscous behaviour of the matrix which depends on the relaxation function G(t)/G(0). This is a monotonic decreasing function of the time which starts from $G(t)/G(0)|_{t=0} = 1$ and has an horizontal asymptote attaining the value denoted as $G(\infty)/G(0)$, that is the ratio between initial and long term stress of an ideal relaxation test. The viscous effect of the reinforcing fibre is now taken into consideration. It must be considered that rigid rotation of fibres does not have any viscous effect on the fibre itself. If a fibre is subject to rotation only, without getting stretched, the stress components in fibre direction do not change and therefore do not give any contribution in the time integral which can be written in the following form:

$$\sigma_f = \int_0^t \frac{\Phi(t-s)}{\Phi(0)} \frac{d\sigma_f^e(s)}{ds} ds = \int_0^t \frac{\Phi(t-s)}{\Phi(0)} \frac{d\sigma_f^e(s)}{d\lambda_f} \frac{d\lambda_f}{ds} ds \quad . \tag{11}$$

In equation (11), σ_f^e is the Cauchy stress component in the current fibre direction. The viscous effect given by the fibres will be described by the relaxation function $\Phi(t)/\Phi(0)$, which is a monotonic decreasing function. It starts from a value given by $\Phi(t)/\Phi(0)|_{t=0} = 1$ and has an horizontal asymptote at a value denoted as $\Phi(\infty)/\Phi(0)$. The total stress tensor for composite viscoelastic solids is:

$$\boldsymbol{\sigma}(t)=\mathbf{R}(t)\left[\int_0^t \frac{G(t-s)}{G(0)}\frac{d}{ds}\mathbf{R}^T(s)\boldsymbol{\sigma}^d_{em}(s)\mathbf{R}(s)ds\right]\mathbf{R}^T(t)+$$
$$+\sum_{j=1}^{N}\left[K_j\int_0^t \frac{\Phi(t-s)}{\Phi(0)}\frac{d\sigma^e_f(s)}{d\lambda_f}\frac{d\lambda_f}{ds}ds\,\mathbf{n}(t)\mathbf{n}^T(t)\right]+p(t)\mathbf{I} \tag{12}$$

The hydrostatic component $p(t)$, included in the formulation in order to account for the incompressibility condition, is not known beforehand and it will be determined by means of equilibrium considerations. The elastic stress components of matrix and fibres are obtained by using equation (7). The proposed model has, therefore, the double advantage to put together, in a unified constitutive theory, the mechanical non-linearity and time dependent properties of materials. The formulation can easily be adopted for three-dimensional problems by implementation into a finite element code. In the following sections, the finite element formulations, a comparison between analytical and numerical results and a boundary value problem for a time dependent behaviour will be presented.

2a Finite Element Formulation

A commercial finite element code has been adopted for the implementation of the previous expounded constitutive relations. It has been accomplished by adding a user's subroutine to the computer code (ABAQUS). The subroutine provides to the finite element package the Cauchy stress tensor at each given gradient tensor **F** and the tangent operator $\mathbf{C}^S$. This latter is defined as the corotational variation of the Cauchy stress tensor components with respect to the strain components:

$$\mathbf{C}^S=\frac{\partial\boldsymbol{\sigma}}{\partial\mathbf{e}}\quad C_{ijkl}=\frac{\partial\sigma_{ij}}{\partial e_{kl}} \tag{13}$$

where **e** is the deviatoric strain rate defined as:

$$\mathbf{e}=\mathbf{D}-\frac{1}{3}\varepsilon_{vol}\mathbf{I} \tag{14}$$

$$D_{ij}=\frac{1}{2}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]\quad \varepsilon_{vol}=D_{ii} \tag{15}$$

Note that the spatial derivative of the displacement field is taken with respect to the current spatial co-ordinate system (x_1, x_2, x_3) because an updated Lagrangian formulation is adopted. Therefore the reference configuration is the last converged equilibrium configuration state. In the case of incompressible materials subject to finite strains, the Cauchy stress variation is obtained by summing up the contribution of the corotational variation and the contribution given by the rigid rotation of the body particle :

$$d\left(J\sigma^d\right)=J\left(\mathbf{C}^s d\mathbf{e}+d\boldsymbol{\Omega}\boldsymbol{\sigma}^d-\boldsymbol{\sigma}^d d\boldsymbol{\Omega}\right) \tag{16}$$

$$\Omega=\frac{1}{2}\left[\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}\right]\quad . \tag{17}$$

In the previous formula, $\mathbf{C}^s$ accounts for the corotational components of stress variation and the skew tensor Ω takes into account of the rigid rotation of the elementary volume. For incompressible homogeneous materials modelled through the Mooney Rivlin constitutive equations the tangent operator $\mathbf{C}^{sm}$ takes the expression:

$$\mathrm{C}^{sm}_{ijhl}=\frac{2}{\mathrm{J}}\left[(\alpha+\mathrm{I}_C\beta)\mathbf{H}^1_{ijhl}-\beta\mathbf{H}^2_{ijhl}+2\beta\mathbf{B}_{ij}\mathbf{B}_{hl}\right] \tag{18}$$

where the apex *m* denotes matrix and the matrixes **H** are:

$$\mathbf{H}^1_{ijhl}=\delta_{ih}\mathbf{B}_{jl}+\delta_{jl}\mathbf{B}_{ih} \tag{19}$$

$$\mathbf{H}^2_{ijhl}=\delta_{hi}\mathbf{B}_{jp}\mathbf{B}_{pl}+\delta_{jl}\mathbf{B}_{ip}\mathbf{B}_{ph}+2\mathbf{B}_{ih}\mathbf{B}_{jl} \tag{20}$$

The Second Piola Kirchhoff stresses induced in the medium by the fibres is obtained by differentiating the fibre elastic potential with respect to the kinematic variables, i.e.:

$$\mathbf{T}^f=2\frac{\partial W_f}{\partial\mathbf{C}}=2\frac{\partial W_f}{\partial\lambda_f}\frac{\partial\lambda_f}{\partial\mathbf{C}}\mathbf{N}\mathbf{N}^T \tag{21}$$

The Cauchy stresses are obtained by the usual transformation:

$$\boldsymbol{\sigma}^f=\frac{1}{J}\mathbf{F}\mathbf{T}^f\mathbf{F}^T=\frac{\lambda_f}{J}\frac{\partial W}{\partial\lambda_f}\mathbf{n}\mathbf{n}^T\quad . \tag{22}$$

The tangent operator or elasticity tensor can be obtained by making the second derivative of second Piola Kirchhoff tensor stresses with respect to the strain components. Let us denote by $\mathbf{C}^K$ the following partial derivatives:

$$C^k_{ijkl} = 2\frac{\partial T_{ij}}{\partial C_{kl}} = 4\frac{\partial^2 W}{\partial C_{ij}\partial C_{kl}} \tag{23}$$

The tangent operator that relates the corotational increments of the Cauchy stress σ^f to the kinematic quantities can be obtained as follows:

$$C^{sf}_{ijkl} = \frac{1}{J} F_{im} F_{jn} F_{kp} F_{lq} C^k_{mnpq} \tag{24}$$

$$\mathbf{C}^{sf} = \frac{\partial \sigma^f}{\partial \mathbf{e}} = 2\frac{\partial \sigma^f}{\partial \lambda_f}\frac{\partial \lambda_f}{\partial \mathbf{C}}\frac{\partial \mathbf{C}}{\partial \mathbf{e}} \tag{25}$$

and in the index form it becomes:

$$C^{sf}_{ijhk} = \frac{\lambda_f^2}{J}\left[-\frac{1}{\lambda_f}\frac{\partial W_f}{\partial \lambda_f} + \frac{\partial^2 W_f}{\partial \lambda_f^{\,2}}\right] n_i n_j n_h n_k \tag{26}$$

Where apex f denotes fibres. In the case of $m = 1$ and $q = 2$ of formula (3), the tangent operator for the fibres is:

$$C^{sf}_{ijhk} = \lambda_f E_f n_i n_j n_h n_k \tag{27}$$

The total tangent operator that takes into account of both the matrix and the fibres is obtained by summing up their contributions as follows:

$$\mathbf{C}^s = \mathbf{C}^{sm} + \sum_k \mathbf{C}^{sf_k} \tag{28}$$

The discretisation in the time domain allows for a step by step calculations of stresses and tangent operator [9].

2b numerical tests

Some numerical experiments have been performed in order to validate the implementation of the stress-strain relation and the tangent operator within the finite element code by comparing numerical and analytical solutions. Only analytical solutions for time independent problems were available. Numerical tests carried out on a single element model have shown a fairly good agreement between theoretical and numerical results. A more complex geometrical model has been used for comparing analytical and numerical solutions in the case of non-uniform deformation field. The model consists of an hollow cylinder of composite material obtained by reinforcing a matrix with a double helicoidal fibre pattern. The fibres form an angle with the longitudinal axis, which is variable with the distance from the centre according to the following formula:

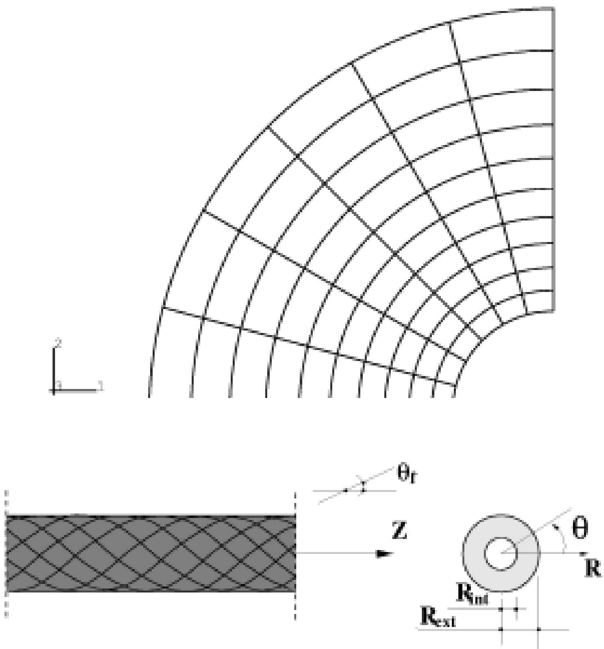

Fig. 1(a) :top view of the three-dimensional finite element model, (**b**) :simplified geometrical model of a composite tendon reinforced by using helicoidal fibres

$$\tan\theta_f = AR^n + B \tag{29}$$

where A, B and n are constants and R is the radial distance from the axis. The geometric model of the composite tendon is shown in Fig.1 [b,c]. The inner and outer surfaces of the model are free of stresses and the two end transversal sections are subject to a uniform field of displacements in longitudinal direction (z direction) in order to simulate a uniaxial tensile test. The analytical solution of this boundary value problem is available in [2]. The cylinder is subject to a uniform state of stress and strain in the

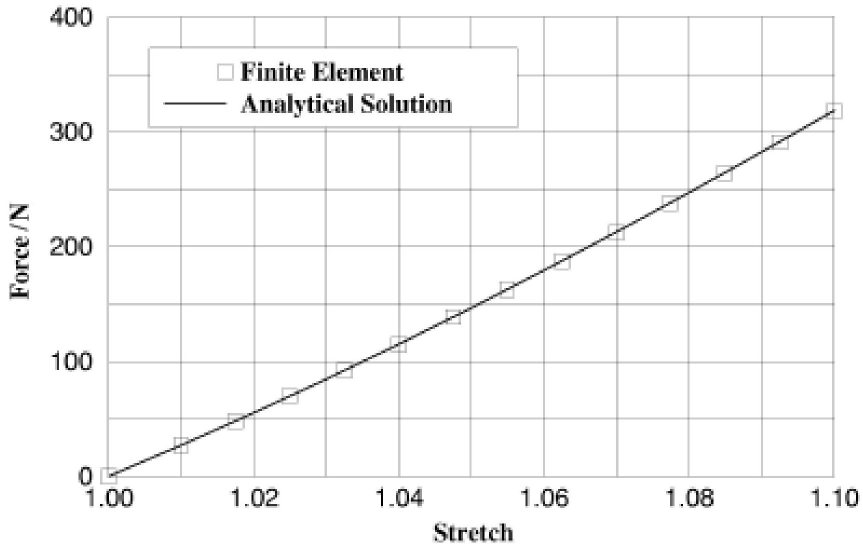

Fig. 2 :Longitudinal force in the extension of the tube model: analytical and finite element results.

longitudinal direction, whereas in the radial direction gradients of stress and strain arise. Symmetry considerations allow modeling only one quarter of the whole solid. The top view of the finite element model is shown in Fig.1 [a]. A 20-node brick element with quadratic displacement interpolation functions has been used. Hybrid element formulation with linear hydrostatic pressure approximation can take into consideration the incompressibility condition imposed on the matrix material. A finite element simulation has been performed by using the parameter values in table 1.

Table 1: Mechanical properties of materials used in the tube model.

α /MPa	β /MPa	E_f /MPa	A /mm^{-1}	B
10.1	3.25	750	0.31	0.282

The longitudinal force obtained by integrating, along the current transversal area the σ_{zz} stress component, provided by the finite element model, are in good agreement with the values given by the analytical solution (Fig. 2). The stress components σ_{rr}, σ_{zz} and $\sigma_{\theta\theta}$ also exhibit a rather good agreement with the analytical results (Figures 3-5). In Fig. (3) the stress components along the radial co-ordinate of the cylinder for two different values of the stretch ratio (λ=1.04 and λ =1.1), are shown. The model and the boundary conditions adopted for results validation have already been considered as a simple model of an artificial ligament in Vena et al. 1998. The above composite has been supposed to be used to replace damaged ligaments. In fact, preliminary numerical studies for evaluating the mechanical behaviour of such prosthetic devices, suggest that a suitable choice of constitutive and geometrical parameters could lead to the required mechanical response.

3 APPLICATIONS: NUMERICAL RESULTS OF THE TIME DEPENDENT CONSTITUTIVE MODEL

Biological tissues are complex structures made of collagen fibres organised in a more or less parallel pattern. Those fibres are initially crimped and, as the deformation process develops, they tend to straighten and to exhibit increasing stiffness. Those complex microstructural arrangements and complex interaction phenomena between fibres and the surrounding matrix make ligaments and tendons biological structures exhibiting a complex mechanical behaviour. If an ideal uniaxial pull test is performed on a tendon sample, which can be

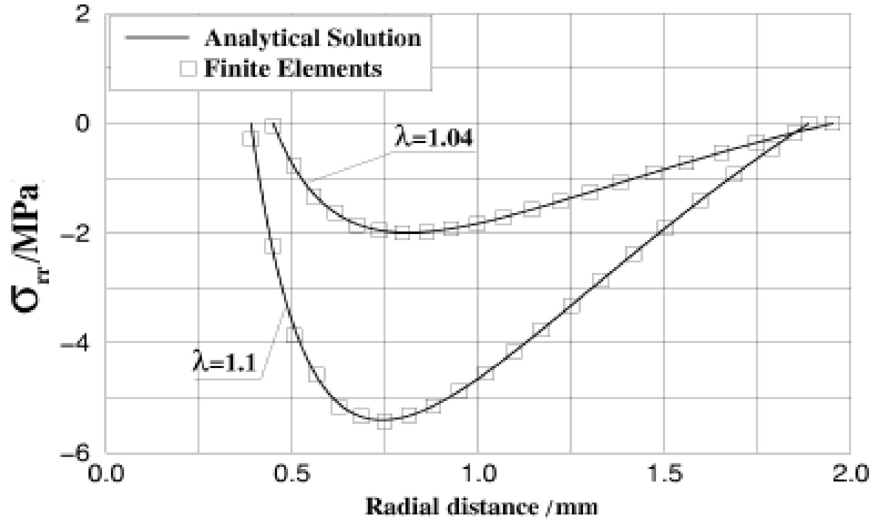

Fig. 3 :Radial Cauchy stress in the tube model, analytical and finite element results.

assumed to as a uniaxial structure, a typical stress-strain curve is observed. This curve is characterised by two distinct regions: an initial region displaying low stiffness and non-linearity is followed by a rather linear region, which exhibits higher stiffness. Those two regions are separated by a transition zone denoted as toe region. The stiffening effect is given by the progressive straightening and recruitment of crimped fibres. Moreover, experimental results taken from the literature [11] show a time dependent behaviour such as relaxation of stresses, creep and strain rate dependence. Relaxation experiments carried out on natural ligaments have shown that the total stress relaxation can reach the 40% of the initial stress in hundreds of seconds and that a fraction of the total relaxation is obtained in the very first period of the test. Obviously this is only a qualitative description of the viscous behaviour of ligaments. From a quantitative point of view, the results show some scatter due to the fact that samples taken from natural tissues are always different if observed at a microscopic level. The theoretical description of the constitutive behaviour of the previous section can in principle be used both

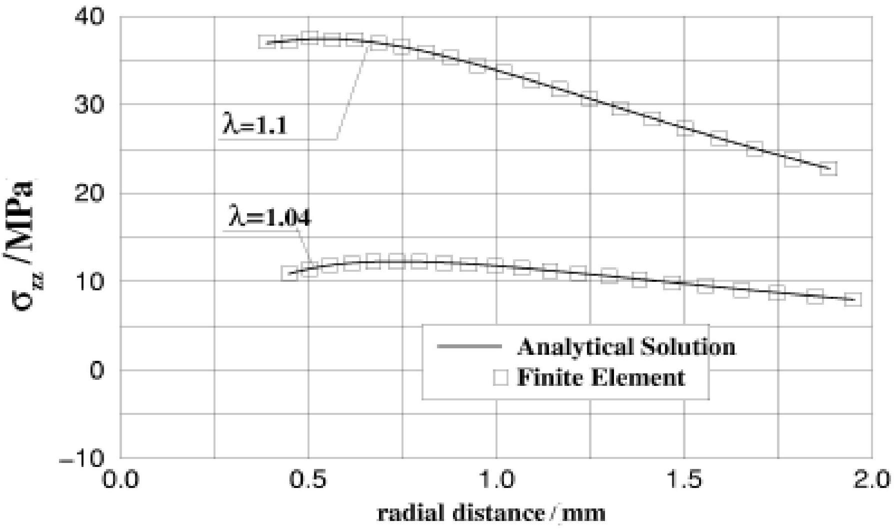

Fig. 4 :Cauchy stress in the tube model, analytical and finite element results.

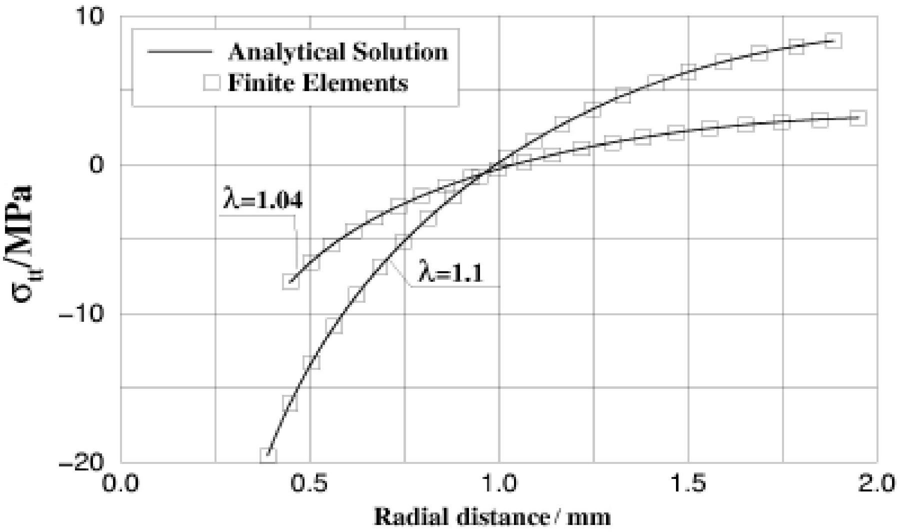

Fig. 5 :Circumferential Cauchy stress in the tube model, analytical and finite element results.

for natural ligaments and artificial composites conceived for ligaments replacements. These latter must be manufactured with materials such that their mechanical behaviour is comparable to the one exhibited by natural tissues.

Let us consider a strip of composite material reinforced by fibres initially oriented with angle $+\theta$ and $-\theta$ with respect to the X_1 axis as in fig. 6. Such composite is subject to stretching in the X_1 direction while the displacements in the remaining two directions are free. Equilibrium conditions impose that faces with normal directed in the X_3 and X_2 directions are stress free. The mechanical response of such boundary value problem, in the case of time independent constitutive equations (6), is already described in [2]. In this problem the Cauchy stress components related to the matrix are always referred to a Cartesian reference system which remains parallel to the main reference system drawn in fig. 6. Therefore the rotation matrix **R**(t) of equation (12) is the identity matrix **I**. On the other hand the fibres are subject to stretching and rigid rotation. The fibre stretch λ_f is obtained as follows:

$$\lambda_f = \sqrt{\lambda_1(\sin\theta)^2 + \lambda_3(\cos\theta)^2} \tag{30}$$

where λ_1 and λ_3 are stretch ratio in the X_1 and X_3 directions obtained by equilibrium considerations. In the present application the relaxation functions where assumed as series of exponential terms for both the matrix and the fibres as follows:

$$\frac{G(t)}{G(0)} = G_\infty + \sum_{i=1}^{N_g} G_i e^{-\frac{1}{\tau_g^i}} \tag{31}$$

$$\frac{\Phi(t)}{\Phi(0)} = \Phi_\infty + \sum_{i=1}^{N_\Phi} \Phi_i e^{-\frac{1}{\tau_\Phi^i}} \tag{32}$$

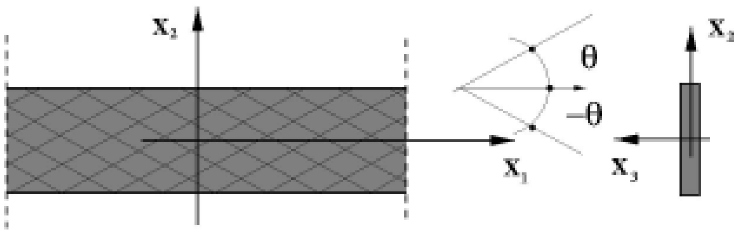

Fig. 6 :Geometric model of the composite strip.

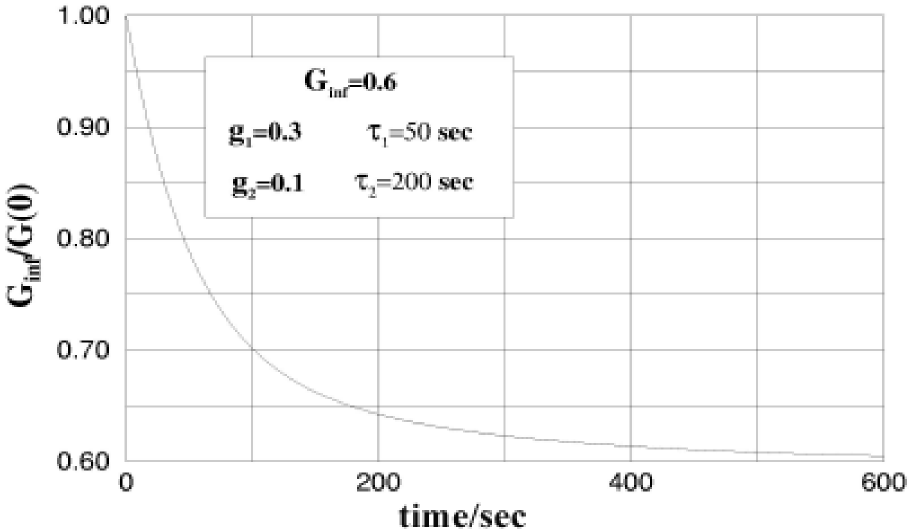

Fig. 7 :Relaxation function with two elements of the exponential series.

with the conditions:

$$G_{\infty} + \sum_{i=1}^{N_g} G_i = 1 \tag{33}$$

$$\Phi_{\infty} + \sum_{i=1}^{N_{\Phi}} \Phi_i = 1 \tag{34}$$

The choice of exponential series for relaxation function implicitly assumes a discrete relaxation spectrum [12]. In fig. 7 the normalised relaxation curve is shown. It has been obtained by using two terms of the series as indicated in the graph. The physical meaning of those parameters is that a maximum decay of 40 per cent of the total stress is obtained at 200 seconds but the 75 per cent of the total relaxation is already obtained in the first 50 seconds. The higher is the number of terms in the series, the more continuous is the relaxation spectrum. The application of this model to the boundary value problem sketched in fig. 6 brought to results that put in evidence a complex interaction between viscous behaviour of the matrix and of the fibres. Fig. 8 shows the mechanical response in two cases: the viscoelastic behaviour and the elastic behaviour of the material. The elastic response is obtained as a viscous material with an extremely short relaxation time. The two curves clearly exhibit

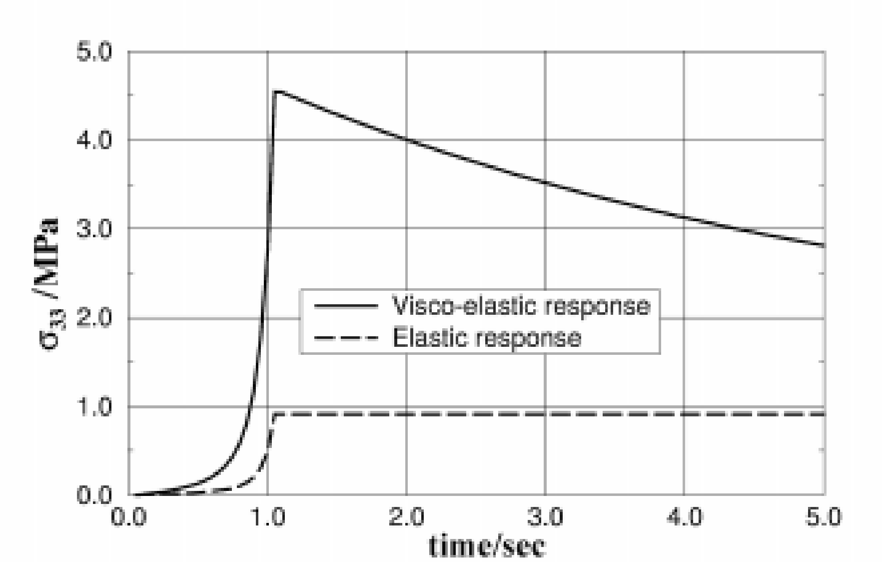

Fig. 8 :Purely elastic and viscoelastic response in a numerical tension test.

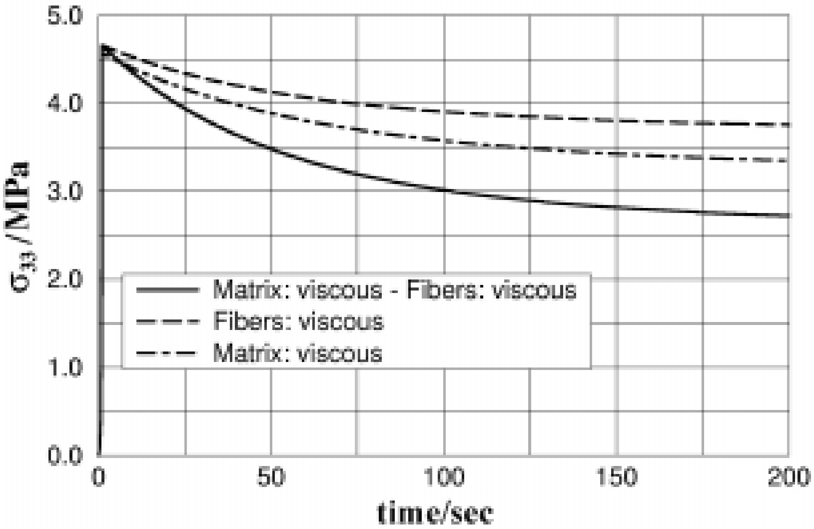

Fig. 9 :Relaxation phase for different viscous properties.

the typical non-linear (toe-region) behaviour of ligaments or tendons, whereas stress relaxation is observed for the viscous case. The results of a stress relaxation test performed for a maximum stretch ratio of λ=1.5 is shown in (Fig. 9). The three curve are related to a test where the viscous properties are given to: a) both the matrix and the fibres, b) to the fibres only and c) to the matrix only. As expected, a larger amount of stress reduction is reached if both matrix and fibres are subjected to viscous effects. The effect of the maximum stretch ratio on a relaxation tests is shown in (Figures 11,12). Fig.11 is related to the case of both matrix and fibres subject to viscous effects, whereas fig. 12 is related to the case in which only the matrix is subject to stress relaxation. The global behaviour of the material is different in the two cases and a complex interaction of the viscous effect can be observed. The behaviour is non-linear since the stress relaxation is not proportional to the maximum imposed stretch. Moreover, a small strain rate sensitivity is observed (Fig. 10). This sensitivity is however largely dependent on the value of the total stress relaxation and the range of rates taken into consideration.

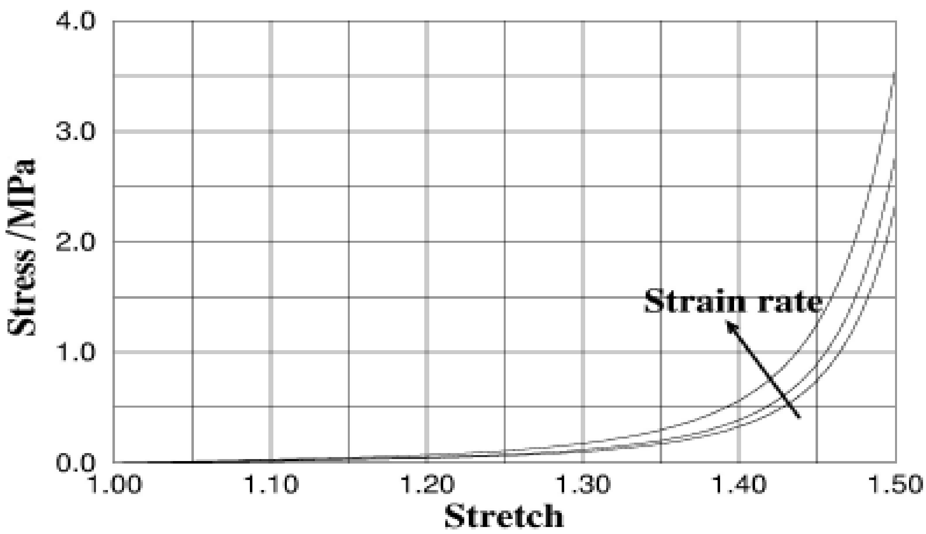

Fig. 10 :Strain rate effect.

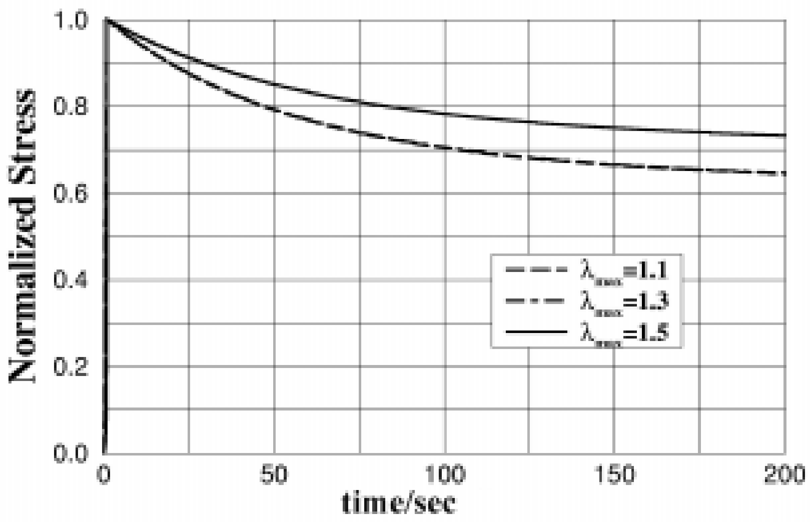

Fig. 11 :Normalised relaxation curves for different maximum stretch (both fibres and matrix have viscosity).

4. CONCLUSIONS

A theoretical and computational description of the non-linear viscoelastic behaviour for fibre reinforced materials was presented in this paper. An original structural approach has been formulated, which is characterised by two independent visco-elastic models for the matrix and the fibres. The effect of the kinematic non-linearities was also taken into account by using suitable stress and strain measures. The model response displays both the mechanical non linearity exhibited by those materials, their anisotropic and visco-elastic behaviour. The relaxation function has been assumed as an exponential series such that a general relaxation spectrum could be described. The model shows a complex interaction between viscous properties of fibres and viscous properties of matrix, thus resulting in a non-linear viscous behaviour. The implementation of such viscoelastic constitutive equations into a finite element code is already ongoing. A coupled numerical-experimental analysis is now required for establishing the suitability of such model for the description of the behaviour of real ligaments and for identification of the model parameters.

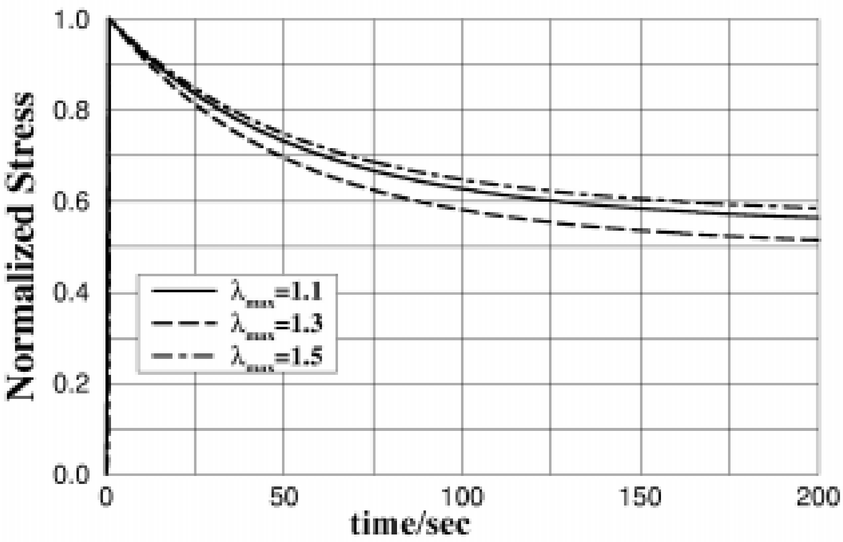

Fig. 12 :Normalised relaxation curves for different maximum stretch (only matrix has viscosity).

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