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HOW WELL CAN WE INFER THE PROPERTIES OF THE SOLAR ACOUSTIC SOURCES?

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ABSTRACT

Measurements of the *p*-mode line asymmetry in the solar oscillation velocity power spectrum have been used on several occasions to infer the properties of the acoustic sources. These inferences are based on the assumption that, unlike the observed intensity signal, the velocity signal does not contain a nonresonant (background) component that is correlated with the *p*-mode signal. Line asymmetry measurements have also been used to draw inferences on the nature of the correlated background signal that is present in intensity observations. By simultaneously modeling the observed velocity and intensity power spectra and the intensity-velocity cross spectrum, we enforce strict observational constraints on the properties of the fitting model. We find that in order to accurately describe the observed data, we have to include a correlated background component in *both* our models for the *V* and *I* signals at low frequencies. Our results also show that we cannot uniquely determine the acoustic source depth for low-frequency waves or the detailed properties of the correlated background signals. It appears that further physical and/or observational constraints are needed before we can obtain this information.

Subject heading: line: profiles — Sun: helioseismology — Sun: oscillations

1. INTRODUCTION

The resonant line profiles in the velocity (*V*) and intensity (*I*) power spectra of the solar acoustic modes are asymmetric at low frequencies. The amount of asymmetry is believed to be related to the type and location of the acoustic sources (Duvall et al. 1993; Gabriel 1995; Abrams & Kumar 1996) and to the level of the interaction of the waves emitted by these sources with convection (Roxburgh & Vorontsov 1997; Nigam et al. 1998; Kumar & Basu 1999). The latter phenomenon, which manifests itself as a coherent signal that is correlated with the oscillation signal (hereafter referred to as a "background" signal as it is a nonresonant phenomenon), is responsible for the differing sense of asymmetry observed in the *V* and *I* power spectra for the same spectral region. Although the details of the correlated background signal are not yet fully understood, the consensus seems to be that this signal most likely is convective in origin and only affects *I* observations. As a consequence, there have been several determinations of the properties of the low-frequency acoustic sources that are based on measurements of the line asymmetry in the *V* power spectrum (Chaplin & Appourchaux 1999; Nigam & Kosovichev 1999; Kumar & Basu 2000). However, recent studies of the complex *I*-*V* cross spectrum, in conjunction with the *V* and *I* power spectra, suggest that the *V* signal does contain a significant correlated background component (Skartlien & Rast 2000; Severino et al. 2001). Obviously, this casts some doubt on the validity of any inference about the depth and type of the low-frequency acoustic sources that is based on measurements of the line asymmetry in the *V* power spectrum and that does not account for the effects of a correlated background signal.

2. SPECTRAL MODEL

We use the model of Severino et al. (2001) to describe the different components in the solar signal that are necessary to describe the total *V* and *I* power spectra and the complex *I*-*V* cross spectrum. Following their terminology, we use "correlated" to mean that there is a certain time lag between the appearance of the nonresonant signal and the oscillation signal, and we use "coherent" to mean that the fluctuations of the *V* and *I* signals are linearly related to a well-defined phase difference. We can then decompose the observed signal into four components: (1) a coherent resonant *p*-mode signal (*p*); a coherent background that comprises (2) a correlated (cc) and (3) an uncorrelated component (cu); and, finally, (4) the incoherent noise (in). Using this model, we can then describe the velocity (P_V) and intensity (P_I) power spectra at a given spherical harmonic degree (ℓ) using

$$P_V(\nu) = |V_{\rm cc}|^2 + |V_{\rm cu}|^2 + |V_{\rm in}|^2 \tag{1}$$

and

$$P_I(\nu) = |I_{\rm cc}|^2 + |I_{\rm cu}|^2 + |I_{\rm in}|^2, \tag{2}$$

where $V_{\rm cc}$ and $I_{\rm cc}$ are the total correlated signals, i.e., the sum of the *p*-mode signal and the correlated background,

$$V_{\rm cu}(\nu) = |V_{\rm cu}|e^{i\phi_{V,\,\rm cu}} \tag{3}$$

and

$$I_{\rm cu}(\nu) = |I_{\rm cu}|e^{i\phi_{I,\,\rm cu}} \tag{4}$$

are the coherent, uncorrelated components, and $|V_{\rm in}|$ and $|I_{\rm in}|$ are the incoherent noise components. The *I*-*V* coherence (ρ) and phase difference (Φ) spectra are modeled as

$$\rho(\nu) = \frac{|X|}{\sqrt{P_V P_I}} \tag{5}$$

and

$$\Phi(\nu) = \tan^{-1}\left[\frac{\Im(X)}{\Re(X)}\right], \tag{6}$$

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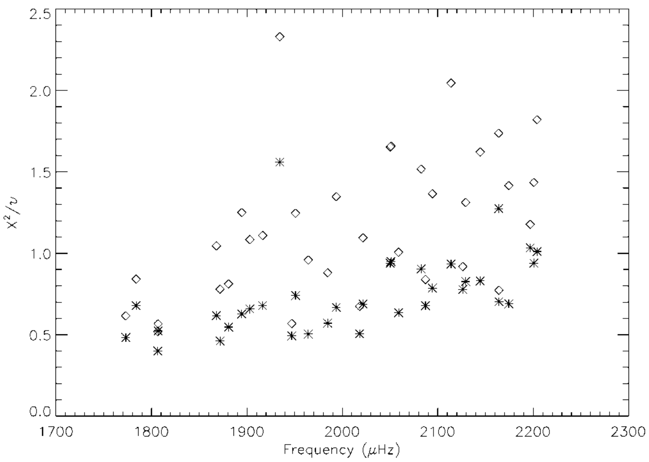

FIG. 1.—Individual reduced χ^2 values for fits with $B_V = 0$ (*diamonds*) and B_V as a free parameter (*asterisks*). The decrease in reduced χ^2 with decreasing frequency is most likely due to our overestimation of the observed dispersion in the measured data in the vicinity of the peaks after the frequency-averaging process. However, this type of systematic error, which would be the largest for the narrowest peaks (i.e., at low frequencies), will affect both types of fit equally. The F_χ ratio, which can be used to determine the validity of adding extra parameters to the fitting model (Bevington & Robinson 1992), shows that the fits made with $B_V = 0$ are significantly worse than the fits where B_V is allowed to vary.

where

$$X(\nu) = I_{cc}V_{cc}^* + I_{cu}V_{cu}^* \tag{7}$$

is the complex cross spectrum (the asterisk denotes complex conjugation). For the resonant *p*-mode signal, we use a two-wave interference model in which the source lies outside the resonant cavity (Duvall et al. 1993; Meunier & Jefferies 2000). The total coherent, correlated signals V_{cc} and I_{cc} are then given by

$$V_{cc}(\nu) = A_V\left(1 + \frac{e^{-i2(\theta-\delta\theta)}}{1 - Re^{-i2\theta}}\right) + B_V e^{i\phi_{V,\,cc}} \tag{8}$$

and

$$I_{cc}(\nu) = A_I\left(1 + \frac{e^{-i2(\theta-\delta\theta)}}{1 - Re^{-i2\theta}}\right) e^{i\phi_p} + B_I e^{i\phi_{I,\,cc}}. \tag{9}$$

Here A is the amplitude of the upward- and downward-emitted waves from the source (the subscripts V and I denote the quantity as measured in velocity and intensity, respectively), R is the wave reflection coefficient at the solar surface, θ is the phase delay of a wave (incurred when traveling from the top of the acoustic cavity to the bottom), $\delta\theta$ is the phase delay between the acoustic source and the top of the acoustic cavity, ϕ_p is the phase difference between an evanescent wave measured in intensity and velocity, B_V and B_I are the amplitudes of the correlated components of the background signals, and $\phi_{V,\,cc}$ and $\phi_{I,\,cc}$ are the phases of these signals with respect to the oscillation mode. The parameter θ is related to the mode frequency, ν_0, through the expression

$$\theta(\nu) = \pi(\nu_0 - \nu)\left(\frac{d\nu}{dn}\right)_\ell^{-1}, \tag{10}$$

where $(d\nu/dn)_\ell$ is the separation between radial orders (n) at a constant degree ℓ. We note that the difference between this source-outside-the-cavity (SOC) model and a source-inside-the-cavity (SIC) model is that in the latter, the first term in the parentheses in equations (8) and (9) is divided by $(1 - Re^{-i2\theta})$. In this Letter, we concentrate on the SOC model as it represents the expected scenario for the low-frequency waves (Kumar & Basu 1999).

3. MODELING THE OBSERVATIONS

We exercised our model on 9 months of V and I observations taken by the Global Oscillation Network Group's instruments (Oliviero, Severino, & Straus 2001) during the period from 1996 October 28 to 1997 September 16. The rotation-corrected, *m*-averaged, V and I power spectra and the I-V coherence and phase difference spectra were generated from the raw V and I coefficients as described in Severino et al. (2001), and then averaged in frequency to a final resolution of 0.4 μHz. This frequency smoothing of the data significantly reduces the noise level and provides us with a cleaner (albeit lower resolution) estimate of the underlying "limit spectrum" (Gardner 1988).

We simultaneously model all four observed spectra by finding the parameters[4] that minimize the χ^2 cost function. We note that for the coherent, uncorrelated components of the background signal, we can only measure the intensity-velocity phase difference $\Delta\phi = \phi_{I,\,cu} - \phi_{V,\,cu}$ and not the individual phases $\phi_{I,\,cu}$ and $\phi_{V,\,cu}$. All background components are assumed to be frequency-independent over the range of frequencies in the fitting region. To minimize contamination from spatial leaks, we restrict the fitting region to $\pm\frac{1}{2}(d\nu/d\ell)_n$ of the mode's eigenfrequency [i.e., half-way to the adjacent peaks at $(n, \ell - 1)$ and $(n, \ell + 1)$] and only use modes with $10 \le \ell \le 20$ and 1.7 mHz $\le \nu \le$ 2.2 mHz.

Fits were made with and without a correlated background signal in V using a two-step procedure. The first step consists of using a genetic algorithm (Charbonneau 1995) to locate the region in parameter hyperspace that contains the global minimum. The second step consists of using a variable metric (quasi-Newton) method (Press et al. 1986) to "home in" on the global minimum and to also provide error estimates for the fit parameters (via the Hessian matrix). Figure 1 shows that the fits with B_V fixed at zero (13 free parameters) are systematically worse than those obtained with B_V as a free parameter (14 free parameters). (In both cases, $\phi_{V,\,cc}$ was fixed to zero.) Figure 2 shows both types of fit to the $n = 19$, $\ell = 9$ mode. The general characteristics of these fits are representative of the fits to all the modes represented in Figure 1. It can be seen that the bulk of the difference in the fits is in the modeling of the line asymmetry in the velocity power spectrum. The difference in the line profiles with and without a correlated background comes through the extra cross terms between the *p*-mode signal and the correlated background in the former. Figure 2 and Figure 3, which shows the fits to the velocity power spectrum for the $n = 10$, $\ell = 10$ mode, show that the difference in the fits is subtle and is visually discernable only with logarithmic scaling. However, although the difference is visually subtle, the F_χ values (Bevington & Robinson 1992, p. 208) for the two fits show that the difference is significant at the 95% confidence level or better. This improvement in the fitting of the velocity

[4] Eqs. (1)–(10) formally require 17 parameters: A_V, R, ν_0, $\delta\theta$, B_V, $\phi_{V,\,cc}$, $|V_{cu}|$, $|V_{in}|$, A_I, ϕ_p, B_I, $\phi_{I,\,cc}$, $|I_{cu}|$, $|I_{in}|$, $\phi_{I,\,cu}$, $\phi_{V,\,cu}$, and $(d\nu/dn)_\ell$. However, we determine $(d\nu/dn)_\ell$ from available tables of mode frequencies.

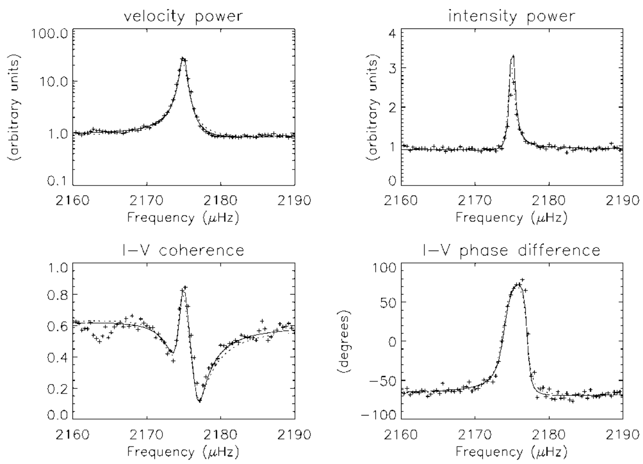

FIG. 2.—Fits to the $n = 9$, $\ell = 19$ mode with $B_V = 0$ (*dotted line*) and B_V as a free parameter (*solid line*). The V and I power spectra are normalized such that the first point in the fitting interval has a value of 1. The frequency region 2161 μHz $\le \nu \le$ 2165 μHz was omitted from the fit as it is contaminated by a small leak from the $\ell = 14$, $n = 10$ mode.

power spectrum when using a model that has a correlated background signal has been noticed before (Meunier & Jefferies 2000). Figure 4 compares the fitted values of B_V and B_I. It can be seen that the level of correlated background, with respect to the total background signal, is the same for the velocity and intensity signals. These results confirm that the observed velocity signal is consistent with the presence of a correlated background signal.

The presence of a background signal that is correlated with the *p*-mode signal suggests that we should try to determine the phase relationship between the two (i.e., measure $\phi_{V,\,\mathrm{cc}}$). Unfortunately, making $\phi_{V,\,\mathrm{cc}}$ a free parameter (15 free parameters in total) does not result in any improvement in the fit quality (i.e., significant reduction in χ^2); it just allows degenerate solutions. This now raises the following question: "Can we uniquely determine the amplitudes and phases of the individual components of the complex constants?" Some insight into this matter is gained by looking at equations (8) and (9) in more detail with respect to the model of Severino et al. (2001). The first terms in the parentheses of these equations, which represent the upward-propagating wave from the source, can obviously be considered as an additional correlated background component. This becomes more evident by rewriting the equations to highlight the dependence on source depth, i.e.,

$$V_{\mathrm{cc}}(\nu) = e^{i2\delta\theta}\left(C_V e^{i\psi_V} + \frac{A_V e^{-i2\theta}}{1 - Re^{-i2\theta}}\right) \qquad (11)$$

and

$$I_{\mathrm{cc}}(\nu) = e^{i2\delta\theta}\left(C_I e^{i\psi_I} + \frac{A_I e^{-i2\theta}}{1 - Re^{-i2\theta}}\right) e^{i\phi_p}, \qquad (12)$$

where

$$C_V e^{i\psi_V} = A_V e^{-i2\delta\theta} + B_V e^{-i(2\delta\theta - \phi_{V,\,\mathrm{cc}})}, \qquad (13)$$

$$C_I e^{i\psi_I} = A_I e^{-i2\delta\theta} + B_I e^{-i(2\delta\theta - \phi_{I,\,\mathrm{cc}} + \phi_p)}. \qquad (14)$$

The upward-emitted wave and the "convective" correlated background term show up in the equations in the same manner, i.e., as a (complex) additive constant (C) to a now symmetric line profile.

It is clear that equations (13) and (14) correspond to four real equations. These equations can thus be used to uniquely determine the values of the two amplitudes and phases of the cc backgrounds once the amplitudes and phases of the C constants and the phase of the source $\delta\theta$ are fixed. Moreover, the four spectra will remain unchanged if $\delta\theta$ and if the amplitudes and phases of the cc backgrounds are varied in such a way as to keep the amplitudes and phases of the C constants fixed. In other words, because the individual amplitudes and phases in the complex constants are free parameters, then even if there is the constraint that the source phase be common to both constants, the amplitudes and phases of the correlated background components are free to adjust to give the appropriate complex constant for the fit. The $\delta\theta$-cc background degeneracy is removed when the cc background in V (or I) is set to zero since it is then not possible to vary $\delta\theta$ and keep C_V (or C_I) constant at the same time (see eq. [13] or eq. [14]).

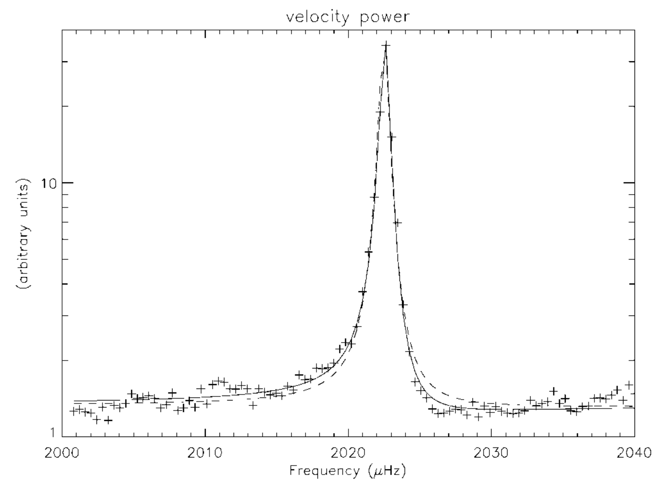

FIG. 3.—Fits to the velocity power spectrum for the $n = 10$, $\ell = 10$ mode with $B_V = 0$ (*dashed line*) and B_V as a free parameter (*solid line*). All four observed spectra were included in the fitting, and the V and I power spectra were normalized such that the first point in the fitting interval had a value of 1.

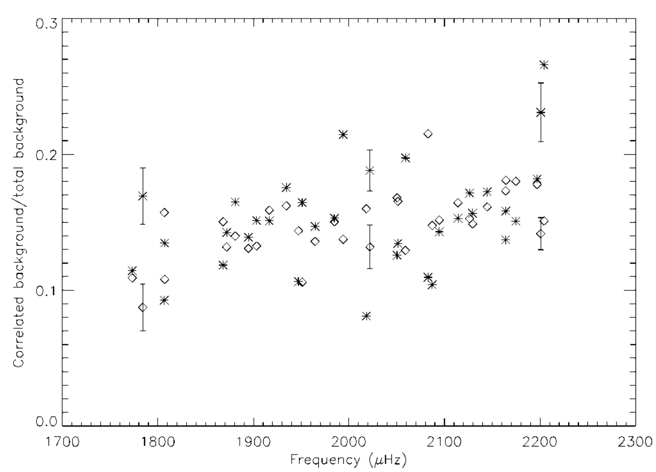

FIG. 4.—Correlated background components in velocity (*asterisks*) and intensity (*diamonds*) as a fraction of the total background signal, i.e., $B_V/(B_V + |V_{\mathrm{cu}}| + |V_{\mathrm{in}}|)$ and $B_I/(B_I + |I_{\mathrm{cu}}| + |I_{\mathrm{in}}|)$. For clarity, we only show $\pm 1\ \sigma$ error estimates for a few points.

To confirm our analytical argument, we performed a series of fits with different fixed values of $\delta\theta$ to a number of the low-frequency modes while allowing $\phi_{V,\mathrm{cc}}$ to be estimated (14 free parameters). As expected, we found that the resulting goodness-of-fit values for a given mode are all the same; i.e., they are independent of the values of $\delta\theta$. Because the analytical argument for the SOC model does not apply in the SIC formulation of the mode profile, as a result of the frequency dependence of the C terms in this case, we repeated our numerical tests with a SIC model. We found the same behavior as for the SOC model. Therefore, the answer to the above question is "no." That is, although the complex I-V cross spectrum places a strong constraint on the total correlated V and I signals, it does not constrain the amplitudes and phases of the individual correlated components. Having said this, we cannot exclude the possibility that the source depth and the properties of the correlated backgrounds can be uniquely determined by using other information that is in the data (e.g., the absolute phases of the V and I signals that are lost in the power and cross spectra) or by using additional physical/mathematical constraints (e.g., by assuming additional relations between the coherent correlated and uncorrelated background components) to transform an ill-posed problem into a well-posed problem.

Finally, we note that equations (11)–(14) nicely demonstrate why the model used by Severino et al (2001), which is based on a symmetric (Lorentzian) line profile for the *p*-mode signal, is able to produce good fits to all four of the observed spectra without considering any "intrinsic line asymmetry" (i.e., the component of the line asymmetry caused by the presence of the source).

4. SUMMARY

We confirm the presence of a correlated background signal in the velocity data at low frequencies. We show that the presence of this signal means that we cannot uniquely determine either the phase associated with the source location or the amplitudes and phases of the correlated background components at low frequencies. This suggests that additional constraints are required to determine these quantities from the observed data. Moreover, it suggests that source depths that have been inferred from modeling the low-frequency modes without including the effects of a correlated background in the model should be viewed with caution. The same is also true for inferences on the properties of the correlated background signal.

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REFERENCES

Abrams, D., & Kumar, P. 1996, ApJ, 472, 882
Bevington, P. R., & Robinson, D. K. 1992, Data Reduction and Analysis for the Physical Sciences (2nd ed.; New York: McGraw-Hill)
Chaplin, W. J., & Appourchaux, T. 1999, MNRAS, 309, 761
Charbonneau, P. 1995, ApJS, 101, 309
Duvall, T. L., Jr., Jefferies, S. M., Harvey, J. W., Osaki, Y., & Pomerantz, M. A. 1993, ApJ, 410, 829
Gabriel, M. 1995, A&A, 299, 245
Gardner, W. A. 1988, Statistical Spectral Analysis: A Nonprobabilistic Theory (Englewood Cliffs: Prentice Hall)
Kumar, P., & Basu, S. 1999, ApJ, 519, 389
———. 2000, ApJ, 545, L65
Meunier, N., & Jefferies, S. M. 2000, ApJ, 530, 1016
Nigam, R., & Kosovichev, A. G. 1999, ApJ, 514, L53
Nigam, R., Kosovichev, A. G., Scherrer, P. H., & Schou, J. 1998, ApJ, 495, L115
Oliviero, M., Severino, G., & Straus, Th. 2001, in Proc. *SOHO* 10/GONG 2000 Workshop: Helio- and Asteroseismology at the Dawn of the Millennium, ed. A. Wilson (ESA SP-464; Noordwijk: ESA), 669
Press, W. H., Flannery, B. P., Teukolsky, S. A., & Vetterling, W. T. 1986, Numerical Recipes: The Art of Scientific Computing (Cambridge: Cambridge Univ. Press), chap. 10
Roxburgh, I. W., & Vorontsov, S. V. 1997, MNRAS, 292, L33
Severino, G., Magrì, M., Oliviero, M., Straus, Th., & Jefferies, S. M. 2001, ApJ, 561, 444
Skartlien, R., & Rast, M. P. 2000, ApJ, 535, 464