The Astrophysical Journal, 587:L1–L4, 2003 April 10

THE LACK OF OBSERVATIONAL EVIDENCE FOR THE QUANTUM STRUCTURE OF SPACETIME AT PLANCK SCALES[1]

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Received 2003 January 26; accepted 2003 March 3; published 2003 March 11

ABSTRACT

It has been noted (Lieu & Hillmann) that the cumulative effect of Planck-scale phenomenology, or the structure of spacetime at extremely small scales, can be lead to the loss of the phase of radiation emitted at large distances from the observer. We elaborate on such an approach and demonstrate that such an effect would lead to an apparent blurring of distant point sources. Evidence of the diffraction pattern from the *Hubble Space Telescope* observations of SN 1994D and the unresolved appearance of a Hubble Deep Field galaxy at $z = 5.34$ lead us to put stringent limits on the effects of Planck-scale phenomenology.

Subject headings: gravitation — time

1. INTRODUCTION

It is generally believed that a description of gravity consistent with quantum theory (quantum gravity) should imply properties of the spacetime much different from the conventional ones, when the latter is being observed at the so-called *Planck scale*. Such a Planck scale is obtained as a combination of fundamental constants and corresponds to a characteristic length $l_P \approx 1.6 \times 10^{-35}$ m and time interval $t_P \approx 5.4 \times 10^{-44}$ s given by

$$l_P = ct_P = \sqrt{\frac{G\hbar}{c^3}}. \tag{1}$$

Space and time, when observed at such scales, are expected to exhibit a grainy, fuzzy, or foamlike structure, as depicted by several authors (see, for instance, Rovelli 1998, Garay 1998, and Kempf 1999). The operational definition of the measurement of a length or of a time interval should be affected by such a property of spacetime (Wigner 1957; Salecker & Wigner 1958; Adler & Santiago 1999), and one can conceive several gedankenexperiments that should be affected by the so-called Planck-scale phenomenology (PSP; see Amelino-Camelia 2001a).

In spite of the extremely small size of l_P, several authors recently pointed out that the *systematic accumulation* of such an effect during the long journey of a photon propagating through a spacetime affected by the PSP could lead to observable consequences. Several possible measurements have been proposed so far (Amelino-Camelia et al. 1997, 1998; Ellis, Mavromatos, & Nanopoulos 2000; Ng & van Dam 1999) and were eventually later criticized (Adler et al. 2000).

Most recently, Lieu & Hillman (2003, hereafter LH03) suggested that differential phase measurements of light propagated over a long distance, as implicitly made by the interferometry of an extragalactic source, can place much tighter constraints on the PSP. They derived the effect on the random phase variation by depending on the ratio of the photon wavelength λ to the Planck length l_P.

It is important to point out that the effects described by LH03 refer to a model of the PSP leading to random variations of the light phase, while other effects exhibit a definite modification of radiation behavior for a given wavelength (Jacobson, Liberati, & Mattingly 2002; Amelino-Camelia 2002). In other words, any spacetime structure model that yields a definite modification at a given wavelength is unconstrained by the random phase approach.

Furthermore, we are aware that LH03 has been the subject of even more recent criticism (Ng, van Dam, & Christiansen 2003), essentially based on the idea that such a random perturbation of spacetime should add incoherently along the propagation path, leading to a square-root dependency on the distance between the source and the observer. Here we note that such an assumption is the basis of other PSPs (Amelino-Camelia 1994) already ruled out by experimental verifications (Ng & van Dam 1999) and that other theories also do not incorporate such dependencies (Karolyhazy 1966; Ng & van Dam 1994). In this Letter, we argue that the use of diffraction, as an interferometry effect by a telescope dish, can put stringent limits on the PSP with random phase variations.

2. SINGLE-APERTURE OBSERVATIONS

Following LH03, we assume that the error in the phase of a wave front is just a different way of expressing the impossibility of measuring, by means of light at wavelength λ, a distance L with a precision ΔL such that

$$\frac{\Delta L}{L} < a_0 \left(\frac{l_P}{\lambda}\right)^{\alpha}, \tag{2}$$

where the parameters a_0 and α characterize the theory being tested. For instance, $\alpha = \frac{1}{2}$ corresponds to the random walk approach (Amelino-Camelia 2000), $\alpha = \frac{2}{3}$ corresponds to the holography principle (Wheeler 1982; Hawking 1975), and $\alpha = 1$ is the natural choice in a linearized theory. The coefficient a_0 can reasonably be expected to be of the order of the unity, but according to Amelino-Camelia (2001b), it can be *a few orders of magnitude* below unity. This gives us an idea of the region of the parameter space described by a_0 and α, where a meaningful search should be done.

Let us assume, in fact, that the measurement of *any* distance

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[1] Based on observations made with the NASA/ESA *Hubble Space Telescope* (*HST*), obtained from the data archive at the Space Telescope Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under the NASA contract NAS 5-26555.

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between a light source and a telescope of diameter D is affected by the PSP described above and translates into an independent modification of the wave-front phase as determined from two distinct positions (this assumption, in the LH03 framework, corresponds to the obviously verified requirement of $D \gg l_P$). Let us consider the distances L_1 and L_2 as measured from a point source placed at a distance $L \approx L_1 \approx L_2$ from the two sides of a telescope of aperture D (see Fig. 1). Any intrinsic variation ΔL in the wave front along the two lines of sight will translate into an apparent angular shift $\Delta\theta$ given by

$$\Delta\theta \approx \frac{\Delta L}{D}, \tag{3}$$

where we did not co-add the (independent) uncertainties over the possible set of sight lines starting from any point of the telescope pupil (for instance, considering only L_1 and L_2, a $\sqrt{2}$ factor should be inserted into eq. [3]) as this would not change the order of magnitude of the result.

It is important to emphasize that this result does not presume our knowledge of either L_1 or L_2 with the accuracy stated in equation (2). Actually, the distance of any astronomical object is known with a much poorer accuracy than that required to test equation (2). The key point here is the *independence* of the accuracies on the measurements. Specifically, the difference in the optical paths joining various points of the telescope pupil and the (unresolved) source are randomly modified by the PSP.

If the PSP effects of equation (2) are present, will this lead to a deterioration of any interference pattern (e.g., the Airy rings of a filled aperture) seen at the diffraction limit? Such a consequence is inevitable—to avoid it, one must invoke the highly unlikely scenario of *correlated* fluctuations in the optical paths over all points along the entire span of the light footprint, which in general has a size $\gg l_P$ [excepting only an initial segment of paths, $\sim l_P(L/D)$ in length, which for the purpose of this work is an irrelevantly short (i.e., $\ll\lambda$) segment].

Thus, it is reasonable to deduce that the PSP leads to an apparent angular broadening of a light source placed at a distance L, as seen from a telescope of diameter D, given by

$$\Delta\theta = a_0 \frac{L}{D}\left(\frac{l_P}{\lambda}\right)^{\alpha}. \tag{4}$$

We compare such an angular broadening with the diffraction limit imposed by the telescope aperture by introducing a ratio η defined as

$$\eta = \frac{\Delta\theta}{\lambda/D} = a_0 \frac{L}{\lambda}\left(\frac{l_P}{\lambda}\right)^{\alpha}. \tag{5}$$

The meaning of η is that it directly influences fringe visibility in the case of an interferometer or the Strehl ratio S of the deterioration in the point-spread function in the case of a telescope. This is because one can write, following Sandler et al. (1994) and assuming that the broadening is equivalent to a blurring effect due to, e.g., atmospheric disturbance, the following equation for S:

$$S = \exp(-\eta^2). \tag{6}$$

It is reasonable to adopt $\eta = 1$ as a rough criterion for any experimental setup of this kind to secure a reliable test of PSP effects. At $\lambda = 1\ \mu$m, a representative wavelength for diffraction-limited optical telescopes (including the $D = 2.4$ m aperture of the *HST* and the $D = 8, \ldots, 10$ m class ground-based telescopes equipped with adaptive optics facilities), this criterion requires the observation of sources at a minimum distance $L_{\rm min} \approx 6.2 \times 10^{22}$ m ≈ 2.1 Mpc, as already noted in LH03, to detect or to rule out the case for $\alpha = 1$ and $a_0 = 1$.

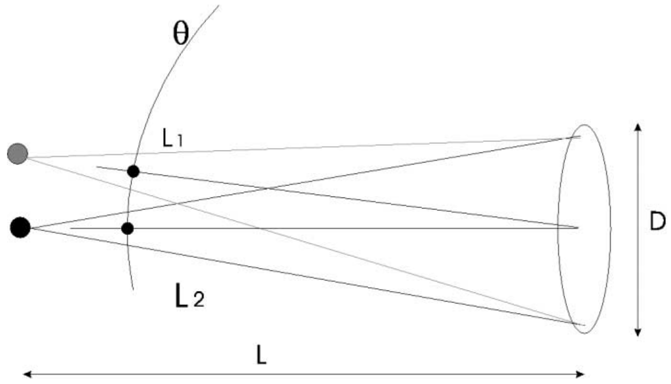

FIG. 1.—Observation of a light source at a distance L from the center of the telescope aperture. The distances between the source and two extremity positions on the aperture are denoted by L_1 and L_2. A variation in L_1 and L_2 will result in an apparent displacement $\Delta\theta$ in the location of the source .

3. ASTRONOMICAL BENCHMARKS

A celestial object that appears extended can be so either because it is in fact genuinely extended or because the PSP causes a blurring of the image in the manner described above. To avoid confusion between the two possibilities, the best target choice is a supernova (SN). This is because for a distant SN, its angular size must remain considerably smaller than the telescope diffraction limit, even if one assumes that the SN shell has been expanding steadily at the speed of light since the initial explosion. Evidently then, our purpose of scrutinizing the PSP will be fulfilled by an investigation of an *HST* archival image of SN 1994D, located at $L \approx 13.7$ Mpc (Patat et al. 1996).

By comparing the *HST*-collected frame of SN 1994D with that of a foreground Galactic star in the same field (see Fig. 2), we see that both objects exhibit no deterioration of their Airy interference patterns. This constrains the Strehl deterioration parameter to $S > 0.2$ (or else the first Airy rings will become invisible) and hence (by eq. [6]) places a lower limit on η at $\eta > 1.3$.

A separate investigation concerns the Hubble Deep Field high-z images. Spectroscopic follow-ups have shown that objects as distant as $z = 5.34$, corresponding to $L \approx 7.7$ Gpc, are as small as $0''.12$ (Spinrad et al. 1998). The distance adopted here is the comoving radial distance, as it is the summation over the journey of the photon of the length experienced by a comoving observer, where we assume the PSP exhibits in the same way. The exact value for L depends on the cosmological model used; here we have chosen $H_0 = 72$ km s^{-1} Mpc^{-1} and $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$ as given in Krauss (2003).

Since $\eta = 1.7$ is the ratio of this observed size to the diffraction limit capability of a $D = 2.4$ m telescope at the relevant wavelength of $\lambda = 814$ nm, one can clinch the PSP even further than before. We note in passing that, in reality, such a ratio for η understates the case against PSP because while propagating from the source to us, a photon initially had a shorter wavelength, so that the quantity ΔL of equation (2), hence $\Delta\theta$ of equation (3), was larger in the past.

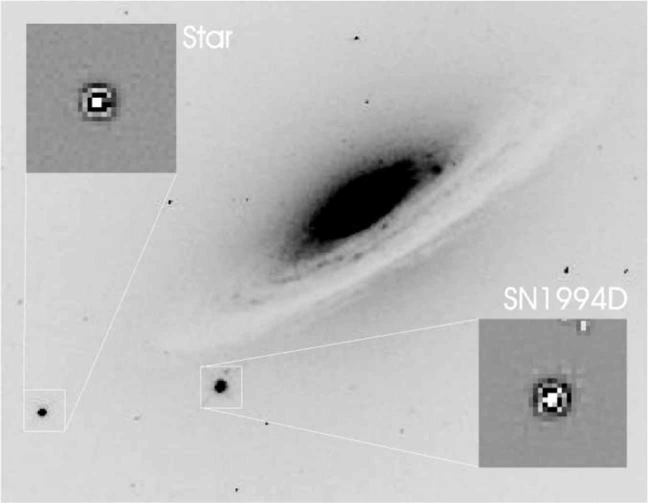

FIG. 2.—SN 1994D as taken with *HST*. In the boxes, a Galactic star and the SN are shown with the high spatial frequency content of both original images enhanced in the same way by subtracting a smoothed version (with a 3 × 3 boxcar) of the same area. The Airy disks are clearly seen in both images, in spite of a small pixel size (equivalent to 0″.046) giving a poor sampling of the diffraction limit.

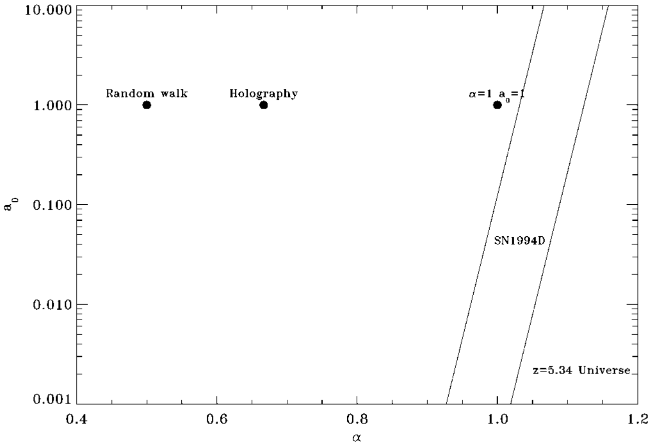

FIG. 3.—Portion of the parameter space a_0-α. The constraints imposed by the observations discussed in the text allow only the regions on the right side of the two oblique lines in order to be consistent with the observations discussed in the text. Three points representing different PSP parameter choices are also shown, where we assumed the coupling coefficient $a_0 = 1$.

4. DISCUSSION

In Figure 3, the implications of the two observations being analyzed thus far are plotted in a_0-α space. We can see that a linear, first-order PSP characterized by $\alpha = 1$ and $a_0 = 1$ is consistently excluded, as are the other cited phenomenologies with smaller α, for all reasonable values of a_0. In particular, when $\alpha = 1$, the upper limit on the angular size of high-*z* objects requires that $a_0 < 3 \times 10^{-4}$.

The two benchmarks presented in the previous section were established with the most powerful instruments currently available (*HST* for measurement of the angular size and Keck for determination of the cosmological distance). It should be realized that, in general, the existence of the PSP with $\alpha \sim a_0 \sim 1$ would render the universe unobservable at any appreciable redshift because of the significant blurring of the images of point sources. This may be regarded as a form of Olbers's paradox. In the case of the far universe, where observations require a special technique, additional benchmarks could be envisaged.

Quantitatively, the limits given above for the exclusion of a first-order PSP understate the case because (1) the errors were estimated conservatively—they would have assumed larger values had we propagated them at every step—and (2) the wavelength of radiation from a distant source is shorter toward the source, meaning that our upper limit on a_0 should in reality be even smaller. Thus, in the same context as that of LH03, the possibility of $\alpha = 1$ may be ruled out with confidence.

Our conclusions may be compared and contrasted with other recent works, notably those of Jacobson et al. (2002) and Amelino-Camelia (2002). The former used X-ray observations of the Crab Nebula to argue against the PSP, and its validity depends on the assumptions made about the physical processes in the Crab Nebula. The latter, however, proposed the *existence* of PSP effects as the reason why gamma rays from a distant quasar survive their journey through the intergalactic medium to reach us. We note here also that alternative interpretations are entirely possible.

In the framework of the assumptions made in LH03, PSP effects are excluded by the observations described in this Letter. Perhaps there exist some ad hoc explanations as to why a first-order PSP cannot be manifested as perturbation of a light pencil. As regards whether the present findings imply that the notion of structural spacetime at Planck scales (a sort of *aether* embedded in the continuum where familiar physics holds) is untenable or whether a subtle mechanism is at play to render such structures evasive, these questions are outside the scope of our Letter.

Thanks are due to Richard Lieu and Lauro Moscardini for their useful suggestions and discussions. We also thank the unknown referee for his/her helpful comments.

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