

Art Collections 2020, Safety Issue (ARCO 2020, SAFETY)

The prediction of collapse mechanisms for masonry structures affected by ground movements using Rigid Block Limit Analysis

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Abstract

Masonry structures belonged to the Cultural Heritage suffered severe damages in the last decades due to the action of the settlement-induced ground movements. The researchers have been developing numerical tools for the vulnerability analysis and assessment of masonry structures subjected to settlements. Continuous, discrete and rigid block models were proposed in literature. The analysis of both local or global failure modes due to settlement is a still debated topic, involving several questions related to the modelling techniques and to the investigation of the parameters which affect the masonry behaviour against foundation movements. In this framework, the paper focuses on a numerical approach for the settlement analysis based on the rigid block limit analysis. The Italian Code (NTC 2018) also suggests linear kinematic approach for the seismic-induced collapse mechanisms analysis. In such a formulation, the structure is modelled as a collection of polyhedral rigid blocks assuming frictional contact interfaces with infinite compressive strength and zero tensile strength and neglecting the mortar contribution. Originally formulated for the in-plane and out-of-plane mechanisms analysis, the numerical formulation was recently improved in order to analyze blocky-structures subjected to uniform settlement.

Numerical case study of a monumental masonry church façade subjected to uniform settlement at the base was presented in this paper aiming at testing the numerical procedure. The results were discussed to evaluate the software capability and accuracy in the settlement-induced collapse mechanisms prediction.

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Peer-review under responsibility of Marco Tanganelli and Stefania Viti

Keywords: Masonry monumental structures, settlement vulnerability, rigid block limit analysis, collapse mechanism analysis, mathematical programming.

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1. Introduction

The Architectural Cultural Heritage must be protected against natural risks, not only seismic but also hydrogeological one. The issue of the vulnerability of masonry structures subjected to foundation movements is a still open topic in literature (Como (2015); Ochsendorf (2006); DeJong (2016)). A number of challenges exists in this research field. Among them, numerical and modelling approaches for the performance assessment of structures affected by settlement plays an important role. Various modelling approaches were already proposed in literature, some of them considering masonry as assemblage of discrete blocks (Baggio and Trovalusci (1998); Orduña and Lourenço (2005-1); Orduña and Lourenço (2005-2); Bui and Limam (2012); Bui et al. (2017); McInerney and DeJong (2015); Cascini et al. (2020); Portioli and Cascini (2016); Portioli and Cascini (2017); Portioli (2020); Galassi et al. (2018-1); Galassi et al. (2018-2); D’Altri et al. (2019); Angelillo et al. (2018); Di Carlo et al. (2018)) others like a continuum medium (de Felice and Malena (2019); Spada (2019); Milani et al. (2016); Rossi et al. (2016); Calìo et al. (2012); Calìo et al. (2016); Giardina et al. (2013); Giardina et al. (2015); Giardina et al. (2019); Reccia et al. (2014); Alessandri et al. (2015); Amorosi et al. (2012); Amorosi et al. (2014); Amorosi et al. (2016); Lasciarrea et al. (2019); Torres et al. (2019); Drougkas et al. (2019); Tubaldi et al. (2018); Casalegno et al. (2013)). Applied element models were also introduced in literature for the performance assessment of masonry structure (Malomo et al. (2019-1); Malomo et al. (2019-2)). The issue of the analytical model for the collapse mechanism assessment of masonry types subjected to spreading support is also a high debated topic (Zampieri et al. (2018); Zampieri et al. (2019)). Comparisons between different numerical formulations devoted to the assessment of masonry structures subjected to settlements or spreading support were presented in Pepe et al. (2020-1), Pepe et al. (2020-2) and Landolfo et al. (2020).

In this framework, this conference paper aims at assessing the structural response of masonry monumental church façades under settlement using a modelling strategy based on a discrete Rigid Block Limit Analysis (RBLA). The procedure analyzes structure modelled as a collection of polyhedral rigid blocks assuming frictional contact interfaces with infinite compressive strength and zero tensile strength and neglecting the mortar contribution. The blocks interaction takes place at the interfaces, adopting a concave contact point formulation, where the internal forces are located at the vertexes of the blocks and are essentially the normal force and the shear forces. The formulation involves both sliding and opening at contact surfaces for the failure conditions, assuming a cohesionless Coulomb failure criterion.

The case study of the monumental façade of the church of the Natività della Beata Vergine Maria in Bondeno (Italy) is investigated in this paper. A uniform foundation displacement is applied at half of the model base aiming at the assessment of the crack pattern and structural behavior at collapse. The value of the loss of the base reaction is associated to the activation of the failure mechanism corresponding to the imposed settlement.

The computational ability of the numerical procedure is discussed in terms of CPU time to solve the mathematical problem and convergence features.

The paper is organized as follows: the rigid block model is described in section 2. Then, the case-study of the monumental church façade is presented and analyzed in section 3. Finally, the outputs are illustrated and compared.

Nomenclature

n_k	normal force component at contact point k
t_{1k}	shear force component at contact point k along local coordinate axis 1
t_{2k}	shear force component at contact point k along local coordinate axis 2
c	vector of the contact forces
α	load factor
A	equilibrium matrix of the rigid block model
f_D	vector of dead loads
f_S	vector of live loads
μ	friction coefficient
ρ	weight for unit volume

2. Modelling approach and numerical formulation

The numerical model aims at analyzing settled structures assembled as a collection of polyhedral elements using rigid block limit analysis. The settlement is simulated by an additional block located in the settled area moving downward, whose action produces a gradual loss of the base reaction until the collapse.

The geometrical model is drawn using Computer Aided Design (CAD) tools, such as AutoCAD® or Rhinoceros®, where the overall model is discretized into several block types attached by some attributes. The attributes are essentially the cartesian coordinates of the vertices and of the centroid of the blocks, the labels of the quadrangular contact surfaces and the amount of the block volume. The GUI was designed to define the masonry mechanical properties (friction angle and weight per unit volume), the boundary and loading conditions (both live and constant loads) and the settlement direction.

A point contact model is adopted to describe the blocks interaction at the interface. In such a model, the internal forces are located at the corner of the blocks k . They are essentially the normal force n_k and shear forces t_{1k} and t_{2k} , collected in the vector c . There are at least 4 contact points per contact interfaces, i.e. 12 vectors (Figure 1b). It is worth noting that the number of vectors per each element is not fixed, depending on the contact interfaces.

The numerical model assumes two contact failure conditions: opening and sliding at interfaces. The procedure solves an optimization mathematical programming problem based on the lower bound theorem of the limit analysis calculating the maximum value of the collapse load factor (or base reaction) subjected to equilibrium and failure conditions constraints. The output is represented by the collapse mechanism and the value of the load factor (or base reaction) at collapse. The problem is described by the following formulation:

$$\begin{aligned} \max \quad & \alpha \\ \text{s.t.} \quad & \mathbf{A}c = \mathbf{f}_D + \alpha \mathbf{f}_L \\ & c \in \left\{ c_k \in \mathbb{R}^3 : \mu n_k \geq \sqrt{t_{1k}^2 + t_{2k}^2}, n_k \geq 0 \right\} \end{aligned} \quad (1)$$

The equation (1) describes the optimization problem. The calculation of the maximum admissible load factor is subjected to two constraints. The first constraint is the equilibrium condition between the contact forces c and external loads, where \mathbf{A} is the equilibrium matrix, \mathbf{f}_D is the vector of the dead loads and \mathbf{f}_L is the vector of live loads, which is coincident with the varying component of the base reaction at the moving support. Then the model assumes a Coulomb friction failure criterion, where the collapse condition is represented by a convex cone as a function of the contact forces and of the friction coefficient μ .

The numerical procedure allows to assume both associative and non-associative friction model. The associative solution represents an upper bound value for the load factor. An iterative procedure is implemented to calculate the non-associative solution in order to obtain a zero-dilatancy sliding behavior.

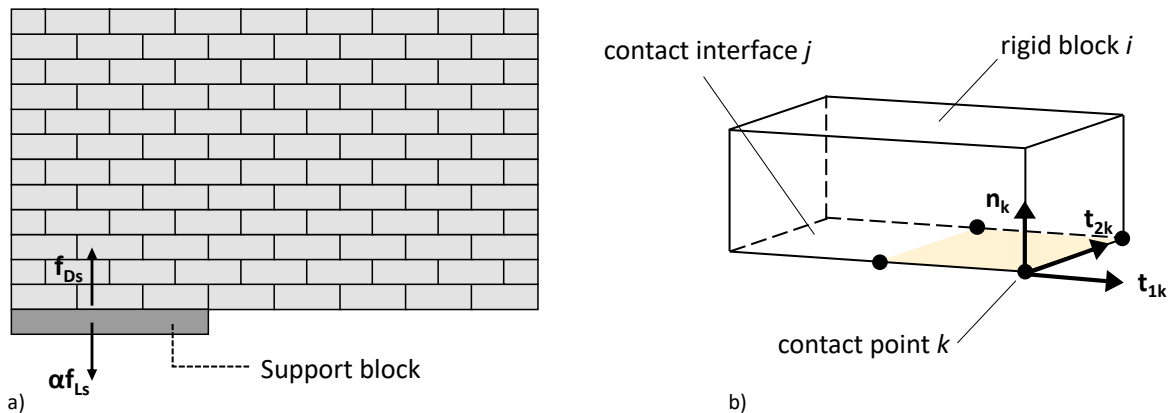


Fig. 1. (a) Rigid block assemblage; (b) internal forces at contact point k .

3. Application to monumental church façade

3.1. The Church of the Natività della Beata Vergine Maria in Bondeno

The case study is represented by the façade of the church of the Natività della Beata Vergine Maria in Bondeno, Italy (Fig. 2a), already investigated in Chiozzi et al. (2018) and Tralli et al. (2020).

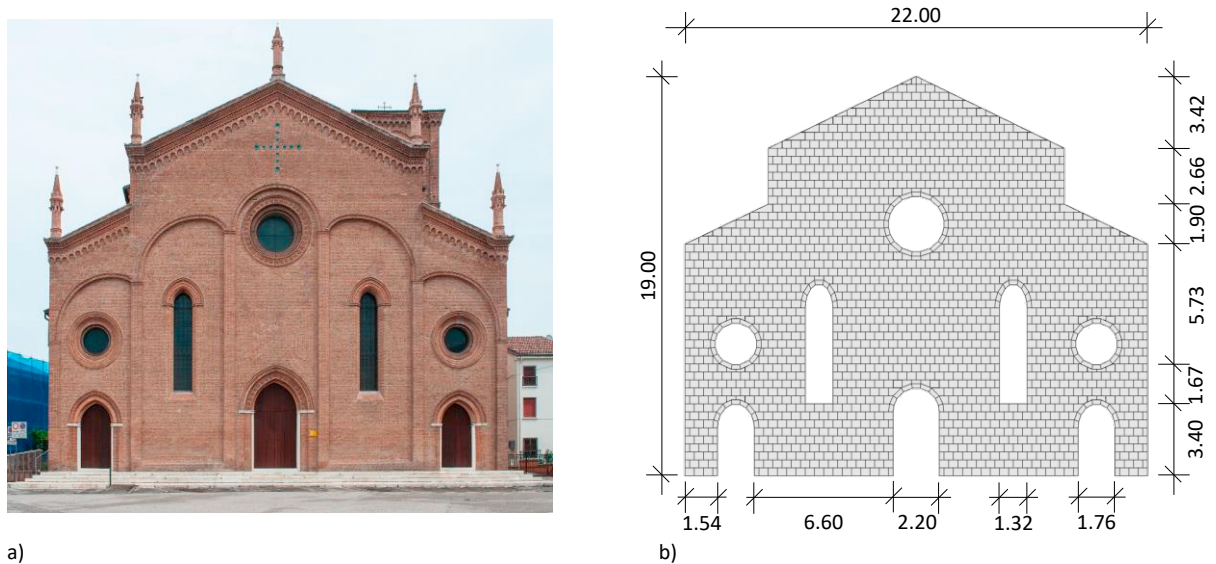


Fig. 2. (a) Façade of the church of the Natività della Beata Vergine Maria in Bondeno; (b) numerical model and geometry.

The façade is part of a single nave church with lateral chapels. The architectural style is typical of the Romanesque church with rose windows and pointed arch doors and windows. The present church was built in the second half of the XV century in place of the original medieval church which dates back to 1114. The floor was completely rebuilt during the XVI century. The roof was restored two times during the XVII century and a curved steel truss structure was finally installed in the XX century to carry the light wooden roof. The present neogothic look was due to the expansion work of the church in 1855-56. The façade assumed the present look after the design by Achille Bonora in 1939. The church was temporarily locked because of damage suffered by the earthquake in 2012.

The façade is 22.00 meters long and 19.00 meters high. The numerical model consists of 1873 polyhedral rigid blocks and 20176 contact points (Fig. 2b). The geometry of the pointed arch doors and windows is simplified in the numerical model by using circular arches. The average block size is equal to 44x50x38 cm. The weight for unit volume ρ and friction coefficient μ were set equal to 18.00 kN/m³ and 0.60 respectively.

3.2. Numerical outcomes

The rigid block model of the church façade above described was submitted to a uniform foundation settlement of half of the base to investigate the structural performance at collapse in terms of both global failure mode and loss of the reaction at base. The collapse mechanism is showed in Fig. 3a. The façade under settlement exhibits a global failure mode with several cracks, involving both translations and rotations of the rigid blocks. The foundation movement caused a diagonal crack moving from the right end side of the façade at the bottom and involving the first pointed arch door and rose window from the right. Two main cracks appears in the central zone of the façade: a diagonal crack develops from the arch of the main door to the right side of the façade; a less severe vertical crack also moves in the zone between the main door and the monumental rose window, keeping on over the rose window. Two symmetrical diagonal cracks move upward from left and right side of the centered rose window, creating a rotational wedge on the top.

Both associative and non-associative flow rules were considered in the paper. The choice of the flow rule affects the numerical procedure in terms of quality of collapse mechanism and CPU time. In the case of non-associative solution, the failure mode is not affected by block dilatancy, but the CPU time is larger because of the iterative procedure adopted. The value of the base reaction at collapse is equal to 881.75 kN and 898.85 kN in the case of associative and non-associative solutions respectively. The numerical results show the computational ability of the procedure to find the solution of the numerical problem in very few iterations as shown in the convergence plot (Fig. 3b) and the high speed of calculation (CPU Time in Table 1).

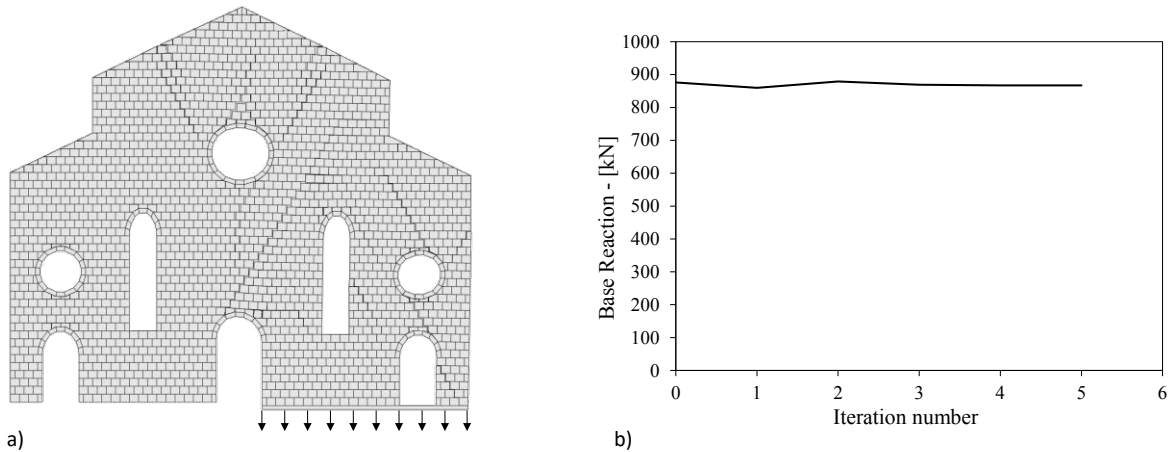


Fig. 3. (a) Collapse mechanism of the façade subjected to settlement (non-associative solution); (b) convergence plot.

Table 1. Numerical results.

Model size (block x contacts)	μ [-]	ρ [kN/m ³]	Associative Solution		Non-Associative Solution	
			Base Reaction [kN]	CPU Time [s]	Base Reaction [kN]	CPU Time [s]
1873 x 20176	0.60	18.00	881.75	2.68	898.85	23.63

4. Conclusions

A numerical procedure for limit analysis of monumental masonry structures subjected to settlement-induced foundation movements was presented in the present conference paper. The model is based on a contact point formulation, assuming infinite compressive strength and zero-tensile strength. The numerical approach was applied to the case study of a Romanesque church façade, with pointed arch doors and rose windows. The façade was involved in a settlement at half of the base length. The outcomes were presented and discussed in terms of loss of the reaction at the base and crack pattern of the façade. The ability of the numerical procedure was discussed considering the time of calculation and the convergence of the iterative procedure for the non-associative solution.

Acknowledgements

The financial support of PRIN 2015 Programme by the Ministry of Education, University and Research (MIUR) is gratefully acknowledged for funding the research project “Protecting the Cultural Heritage from water-soil interaction related threats” (Prot. No. 2015EAM9S5), which is the main framework of the study presented in this article.

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