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The importance of dynamic behaviour of vibrating systems on the response in case of non-Gaussian random excitations

C. Braccesi[a], F. Cianetti[a], M. Palmieri[a,*], G. Zucca[b]

[a]Unversity of Perugia, Department of Engineering, Via G. Duranti 93, 06125 Perugia, Italy

[b]Italian Air Force, Flight Test Center, Technology Materials for Aeronautics and Space Department, Military Airport M. De Bernandi, via Pratica di Mare, 000040 Pomezia (RM), Italy.

Abstract

Dynamic response of vibrating system subjected to non-Gaussian random loads was investigated through a set of numerical simulation on several lumped systems aimed to determine whether and in what form the dynamic behaviour of a vibrating system transfers or masks non-Gaussianity features of the input to the output response. Indeed, in several numerical and experimental activities performed on a Y-shaped specimen it was observed how the system response, both in terms of displacement or stress, changed according to an input variation (stationary and non-stationary Gaussian and non-Gaussian load time histories) and according to a change of the system frequency response function. Moreover, it was observed that even if the system was excited in its frequency range, the response remains unchanged and similar to the input in case of non-stationary and non-Gaussian load, removing preliminarily the possibility to use spectral methods for damage evaluation, going necessarily back to a more "expensive" time-domain analysis. Since the system response characteristics may change significantly according to the input excitation features and to the dynamic system parameters allowing, in some cases, the use of spectral techniques for fatigue damage evaluation also in case of non-Gaussian input loads, the aim of this paper is to understand whether and how the dynamic behaviour of a generic mechanical system transforms the non-Gaussian input excitations into a Gaussian response. To this aim several numerical displacement responses of 1-dof lumped systems characterized by different frequency response functions (resonance frequency position and damping) were analysed and investigated for different stationary and non-stationary Gaussian and non-Gaussian excitations. In such a way, it was possible to a-priori establish under what circumstances the frequency-domain approaches can be adopted to compute the fatigue damage of real mechanical systems.

Keywords: Non-Gaussianity; Non-stationarity; Vibration fatigue.

* Corresponding author
E-mail address: massimiliano.palmieri@studenti.unipg.it

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1. Introduction

The use of mechanical components for critical applications such as aerospace, military or automotive, necessarily requires numerous tests to evaluate the resistance and the behavior of these components when subjected to severe operating conditions (vibration, temperature and humidity). The vibration tests imposed by the reference specifications in the military, aerospace or automotive sectors envisage subjecting the component to specific vibrating loads to be applied through the use of electrodynamic shakers or vibrating tables (MIL-STD-810F (2000)). Generally, all standards provide the designer with a random input in the form of Power Spectral Density (PSD) whose content theoretically represents the same content of an infinte time signal. Although a test phase is necessary before the commissioning of critical mechanical components, during the design phase it is still necessary to evaluate, by replicating through numerical simulation, what happens to the component when subjected to certain operating conditions with the intent of prevent premature fatigue failure as much as possible. To evaluate the damage theoretically accumulated on a real component during a vibration test by numerical simulation, it is preferable to use a frequency domain approach (Bishop (1998)). Frequency domain methods for the evaluation of fatigue damage, such as that of Dirlik (Dirlik (1985)), Tovo-Benasciutti (Benasciutti et al. (2005)) or Braccesi et al. (Braccesi et al. (2015)) allow to obtain accurate results in a very short time (Braccesi et al. (2017)) if compared to classical methods over time such as rainflow counting or range count, especially when dealing with long time histories. The evaluation of the fatigue damage through a frequency domain approach is based on the use of the stress PSD and on the properties of multiaxial stress criteria (Mrš nik et al. (2016), Braccesi et al. (2018)). In general, however, standard frequency methods assume that the time signal associated to the PSD is both Gaussian and stationary (Bendat (2010)). When these conditions are met, the frequency methods provide accurate results in terms of accumulated damage and estimated duration.

Unfortunately, the real operating conditions of mechanical components see these subjected to non-Gaussian and non-stationary loads (Rouillard (2007), Shuang et al. (2018), Gao et al. (2007)). If the system response turns out to be non-Gaussian, the frequency domain methods for fatigue damage estimation provide unreliable results since they are based on the Gaussian hypothesis.

For this reason, non-Gaussianity is subject of many researches where the purpose is to assess how non-Gaussianity has an influence on fatigue behavior on one hand and to understand how much the role of the dynamic behavior of the systems is so influential in the responses that the non-Gaussianity of the inputs can be directly ignored (Benasciutti et al. (2016), Braccesi et al. (2009), Niesłony (2016)). Kihm et al. (2013) and Rizzi et al. (2011), have numerically demonstrated that in case of non-Gaussian stationary inputs, the system response is always Gaussian, justifying the use of frequency methods in any load condition. Furthermore, Wang et al. (2013) conducted a comparative study on the effect of numerous Gaussian and non-Gaussian inputs on the fatigue behavior of multi-degree systems certifying that the damage induced by non-Gaussian processes is higher than that induced by Gaussian inputs.

In this scenario, this work initially presents an experimental test campaign concerning non-Gaussianity and non-stationarity in random fatigue applications with the aim of investigating how non-Gaussianity and non-stationarity affect the fatigue behavior of a real system. Several Y-shaped specimens were excited with different random signals obtained by combining different levels of non-Gaussianity and non-stationarity (Cianetti et al. (2017), Palmieri et al. (2017)). The obtained results showed that the system response, expressed in terms of stress, is always Gaussian if the input signal is stationary non-Gaussian. On the contrary, in case the system is subjected to non-stationary non-Gaussian excitations, the system response turns out to be non-Gaussian and non-stationary with a level of non-Gaussianity close to that of the input (Palmieri et al. (2017)). In order to extend this result to all possible mechanical systems and to any input excitation, the second part of this work is aimed to study how different dynamic 1-dof systems (characterized by different damping) respond to different inputs with different characteristics of non-Gaussianity and frequency content.

2. Theoretical background

This section will report the theoretical aspects that will be used later to evaluate the influence of vibrating system dynamics on responses when subjected to non-Gaussian and non-stationary inputs. First of all, some aspects about the properties of random processes is given. Subsequently the techniques used for the generation of stationary and non-stationary non-Gaussian signals are described. Lastly, the general equations used for the reconstruction of the responses of vibrating systems,in terms of both displacement and stress, are presented.

2.1. Random Signals Properties

Due to their nature, random processes are generally treated through a statistical approach and are therefore described through a probability distribution (Bendat (2010)). In daily practice, it is usual to use a Gaussian probability distribution and therefore represented by Eq. 1.

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{1}$$

where μ represents the mean value and σ is the standard deviation.The mean value μ and the variance σ^2 are the first and the second central moment, M_1 and M_2 respectively of the distribution $p(x)$ (Bendat (2010)). The central moments, associated to the pdf describe the properties of the distribution itself, and they can be evaluated according to Eq. 2.

$$M_j = \frac{1}{n} \sum_{i=1}^{n} [x_j - \mu]^j \tag{2}$$

j represents the order of the central moment and n is the number of points of the recorded process. Even if for Gaussian process, only two moments, the mean value μ and variance σ^2 suffice for a complete definition of the shape of the pdf, in case of non-Gaussian process only the mean value μ and the variance σ^2 are not enough to completely characterized the process. The principal metrics describing non-Gaussian features of the pdf are the kurtosis k_u and the skewness s_k that are expressed in terms of the central moments as:

$$k_u = \frac{M_4}{M_2^2} = \frac{M_4}{\sigma^4} \qquad\qquad s_k = \frac{M_3}{M_2^{3/2}} = \frac{M_3}{\sigma^3} \tag{3}$$

The kurtosis k_u characterizes the sharpness of the pdf peak and the width of the pdf tails while the skewness s_k is a measure of the asymmetry of the pdf. For a Gaussian distribution $k_u = 3$ and $s_k = 0$. A process is regarded as leptokurtic if its kurtosis is higher than 3, and platykurtic if it is smaller than 3.

2.2. Generation of non-Gaussian process

In this section, the techniques used for the generation of stationary and non-stationary non-Gaussian process are given. In this work, in order to generate stationary non-Gaussian signals, the indirect method (Braccesi et al. (2017)),which first provides the generation of a Gaussian signal was used. From a given PSD, a Gaussian process can be generated through the use of Eq. 4, where $C_x(k)$ are the coefficients of the Fourier series.

$$x(t) = \frac{1}{N} \sum_{k=2}^{N-1} C_x(k) e^{\frac{j2\pi k}{N}} \qquad\qquad k = 1, 2, \ldots, N-1 \tag{4}$$

By re-arranging the Parseval theorem (Bendat (2010)), it is possible to obtain the following equation:

$$2 \sum_{k=1}^{N/2} |C_x(k)|^2 = 2 \sum_{k=1}^{N/2} G(k\Delta\omega)\Delta\omega \tag{5}$$

from which it is possible to calculate the absolute value of $C_x(k)$. To generate a random signal, it is important moreover to choose the phase ϕ_k , mutually independent and uniformly distributed in the interval $[0, 2\pi]$ so that the coefficients $C_x(k)$ are given by:

$$C_x(k) = |C_x(k)| e^{j\phi_k} \qquad\qquad k = 1, 2, \ldots, N-1 \tag{6}$$

Once the coefficients are known, by Eq. 4 it is possible to generate a stationary Gaussian random signal. The generation of stationary non-Gaussian processes is based on the assumption that a generic Gaussian process $x(t)$ is related to a non-Gaussian process $y(t)$ by:

$$x(t) = g(y(t)) \tag{7}$$

where g represents a transformation function. In this activity, the transformation function is that proposed by Winterstein (Winterstein (1988)) which is modelled as a monotonic cubic Hermite polynomial function. This method allows to estimate the transformation from the first fourth central moments:

$$g = \frac{x-\mu}{\sigma} - \frac{s_k}{6}\left(\left(\frac{x-\mu}{\sigma}\right)^2 - 1\right) - \frac{k_u-3}{24}\left(\left(\frac{x-\mu}{\sigma}\right)^3 - 3\left(\frac{x-\mu}{\sigma}\right)\right) \tag{8}$$

Once the transformation is known, by computing the inverse of the transformation (9) it is possible to generate the stationary non-Gaussian signal:

$$y(t) = G(x(t)) \qquad G = g^{-1} \tag{9}$$

The generation of non-stationary non-Gaussian processes, on the other hand, involves the use of an amplitude modulation of a stationary Gaussian process. This modulation is obtained through a ”carrier wave”, that is a low frequency function independent from the Gaussian signal. This carrier wave is a random variable characterized by a β distribution (Bendat (2010)). The β distribution in fact allows only positive numbers to be generated between 0 and 1 and is extremely flexible. The mean value of the distribution is fixed while the variance allows to control the kurtosis. Furthermore, the parameters a and b are chosen in such a way that the kurtosis of the carrier wave is one third of the desired kurtosis. In relation to this, the Gaussian process can easily be generated through Eq. 10.

$$z(t) = x(t) \cdot y(t) \tag{10}$$

Where $z(t)$ rappresents the generated non-stationary non-Gaussian process, $y(t)$ is the carrier wave and $x(t)$ is the stationary Gaussian process.

2.3. Structural dynamics

The equation of motion of $n-$ degree of freedom system, subjected to a vector force $\{f\}$ can be written in the physical space as:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{f\} \tag{11}$$

were $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrix respectively. Eq. 11 represents a system of $n-$ coupled second order differential equations. Applying the substitution:

$$\{u\} = [\Phi]\{q\} \tag{12}$$

it is possible to obtain the vector $\{u\}$ as image of vector $\{q\}$, determined in the modal space, where the equation of motion are decoupled (Maia (1998)). Substituing Eq.12 into Eq. 11, Eq. 11 can be written as:

$$[M][\Phi]\{\ddot{q}\} + [C][\Phi]\{\dot{q}\} + [K][\Phi]\{q\} = \{f\} \tag{13}$$

By multiplying Eq. 13 for $[\Phi]^T$ and exploiting the orthogonality properties of the eigenvector of Eq. 14:

$$[\Phi]^T[M][\Phi] = [I] \qquad [\Phi]^T[C][\Phi] = [2\xi\omega_0] \qquad [\Phi]^T[K][\Phi] = [\omega_0^2] \tag{14}$$

Eq. 13 can be written as:

$$[I]\{\ddot{q}\} + [2\xi\omega_0]\{\dot{q}\} + [\omega_0^2]\{q\} = [\Phi]^T\{f\} \tag{15}$$

Eq. 15 represents a system of n decoupled second order differential equations that can be solved independently of each other, thus constituting a system of n decoupled differential equations. It is important to highlight that it is possible to obtain a system of decoupled equations only if the damping matrix is defined as a linear combination of the mass and stiffness matrix. Under this hypothesis, also the damping matrix $[C]$ is diagonal and therefore the system results to be decoupled.

Each of Eq. 15 is of the form:

$$\ddot{q}_j + 2\xi\omega_{0,j}\dot{q}_j + \omega_{0,j}^2 q_j = [\Phi]_j^T\{f\} \tag{16}$$

Eq. 16 represents the equation of motion of an elementary oscillator, and therefore can be solved independently from the others as it depends solely on the modal parameters related to the $j-th$ mode. Once the n solutions of the n differential equations (Eq. 15) have been determined, it is possible to obtain, through Eq. 12, to the nodal displacement vector $\{u\}$.

Therefore, the vector $\{u\}$, whose elements describe in time domain the displacements and rotations of the n degrees of freedom of the structure, can be evaluated by means of a linear combination between the variables $\{q\}$, called modal or lagrangian coordinates, and the modal shapes $[\Phi]$. Having hypothesized that the system is linear, then the same approach can be used not only to evaluate the displacements and rotations of the n degrees of freedom of the structure but also the stress. In fact, in linear conditions, the stress is directly related to the deformation and therefore to the displacement. For this reason, once the lagrangian coordinates $\{q\}$ are known, the stress can be obtained simply by exploiting the Eq. 12 , where the modal shapes $[\Phi]$ must be expressed in terms of stress (Braccesi et al. (2016)). In this activity, having considered only single degree of freedom systems, all the evaluations are referred to the modal coordinate q. For a single degree of freedom system in fact, the modal form $[\Phi]$ that allows to pass from modal space to physical space is only a multiplicative factor and therefore does not make any changes to the statistics of the process.

3. Experimental test with stationary and non-stationary non-Gaussian excitations

The first part of this work is focused on the experimental evaluation of how non-Gaussianity and non-stationarity affect the fatigue behavior of a real component. In particular, the Y-shaped specimen shown in Fig. 1 was used. The specimen has a $10x10mm$ section and the "arms" are arranged at 120° with respect to the vertical axis. The specimen is made of $A-S8U3$ aluminium alloy. To reduce the natural frequencies, two masses of $52.5g$ were added to the extremities of the arms. The tests were conducted by fixing the specimen to an electrodynamic shaker and exciting it with a constant flat PSD (Fig. 2), defined between 600 and $850Hz$, so that the $4th$ natural frequency of the sample, equal to $775Hz$, fell inside of the frequency band of the input PSD. To evaluate how non-Gaussianity and non-stationarity affect the fatigue behavior of the considered specimen, the following procedure was used: initially, a series of specimens were excited with a stationary Gaussian signal with a constant flat PSD (Fig. 2) with different levels of amplitude with the intent of evaluating the coefficients of the fatigue strength curve. Subsequently, a series of random signals were generated to be applied to the specimen with the same PSD but with different kurtosis and different levels of stationarity (Tab. 1).

Table 1. Excitation signal types.

Nr.	Signal type	k_u	s_k
1	Gaussian stationary	2.96	0.00212
2	Non-Gaussian stationary	5.43	0.00031
3	Non-Gaussian stationary	7.36	0.00345
4	Non-Gaussian non-stationary	7.08	0.00081

The influence of non-Gaussianity and non-stationarity has been studied by comparing the experimental life and the estimated life obtained by exciting the specimen with the generated signals.

The $S-N$ curve of the specimens experimentally obtained (Palmieri et al. (2017)) is shown in Eq. 17.

$$\sigma = 987.5\dot{N}^{-0.169} \tag{17}$$

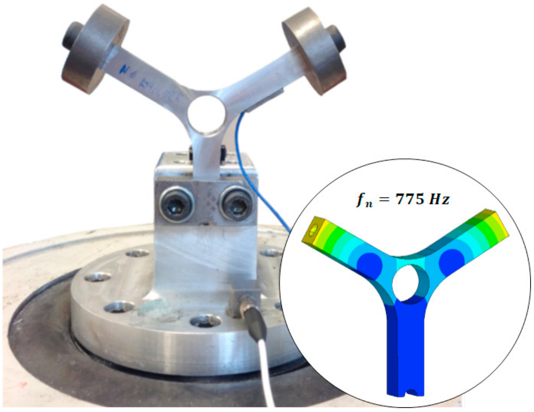

Fig. 1. Y-shaped specimen

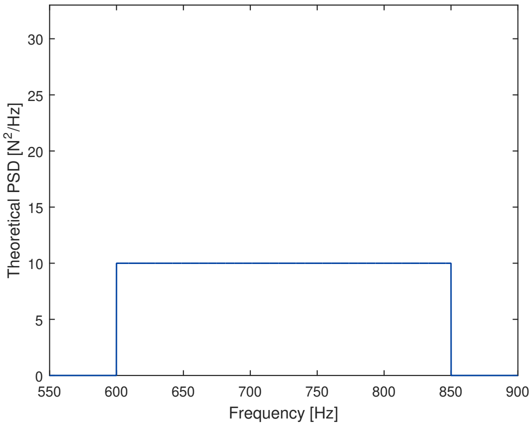

Fig. 2. Input constant flat PSD

A numerical model was created with the state-space approach and then validated by comparing the acceleration PSD as shown in Fig. 3. Note the $S - N$ curve, a series of specimens were excited with random signals, where the level of non-Gaussianity and non-stationarity is summarized in Tab. 1. Signals 3 and 4 of Tab. 1 are shown in Fig. 4 and in Fig. 5. To study the influence of non-Gaussianity on the fatigue behavior of a real component, and then to evaluate when it is justifiable the use of classical frequency domain methods for damage evaluation even in case of non-Gaussian excitations, the Tovo-Benasciutti method (Benasciutti et al. (2005)) is used also in non-Gaussian conditions. A comparison between the experimentally measured life and that obtained by simulation is shown in Fig. 6. From Fig. 6 it is easily visible how in case of stationary non-Gaussian inputs, the estimated life differs slightly if compared to the experimentally measured one. On the contrary, in case of non-Gaussian and non-stationary inputs, it is easy to see how there is a clear difference between the measured and the estimated life.

The great difference obtained between the experimental and numerical life is due precisely to the non-Gaussianity of the output stress. For this reason, a strain gauge applied on one ”arm” of the specimen (most stressed area) (Fig. 1) has allowed to certify how in case of stationary non-Gaussian inputs the kurtosis of the output stress is always around 3 while in case of non-Gaussian and non-stationary inputs, the numerical and experimental response, in terms of stress, shows very high kurtosis and similar to those of the input. This result is also summarized in Tab. 2.

From this test campaign it was concluded that in case a system is excited around its resonance frequency, with a non-Gaussian stationary input, the response is Gaussian distributed, thus justifying the use of frequency methods for the damage evaluation. On the contrary, in case the system is excited with a non-Gaussian and non-stationary input,

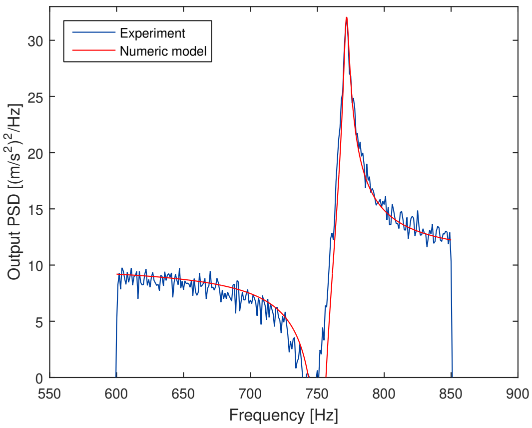

Fig. 3. Comparison between numerical and experimental acceleration PSD

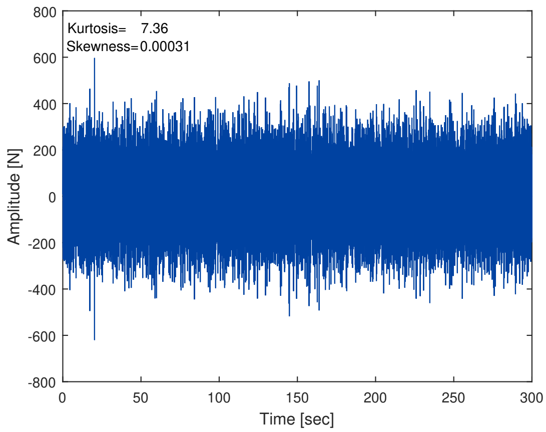

Fig. 4. Input stationary non-Gaussian signal with k_u = 7.36

Table 2. Comparison between input and output kurtosis.

Nr.	Signal type	Theoretical Input k_u	Experimental Output stress k_u	Numerical Output stress k_u
1.	Gaussian stationary	2.96	2.79	3.21
2.	Non-Gaussian stationary	5.43	2.78	3.05
3.	Non-Gaussian stationary	7.36	2.85	3.26
4.	Non-Gaussian non-stationary	7.08	6.18	6.55

the system response is non-Gaussian with a kurtosis close to that of the excitation. It is therefore evident that, under these excitation conditions, the use of a frequency-domain approach for the fatigue damage estimation would lead to inaccurate results.

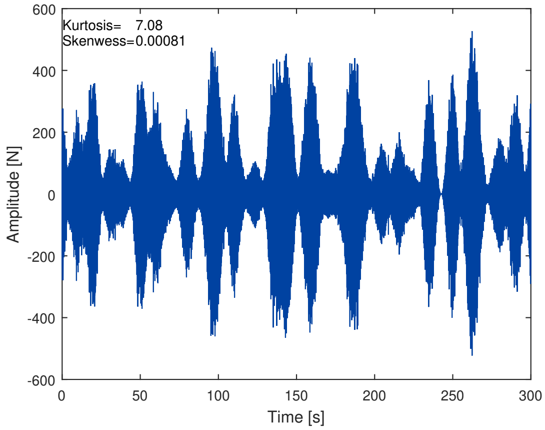

Fig. 5. Input non-stationary non-Gaussian signal with $k_u = 7.08$

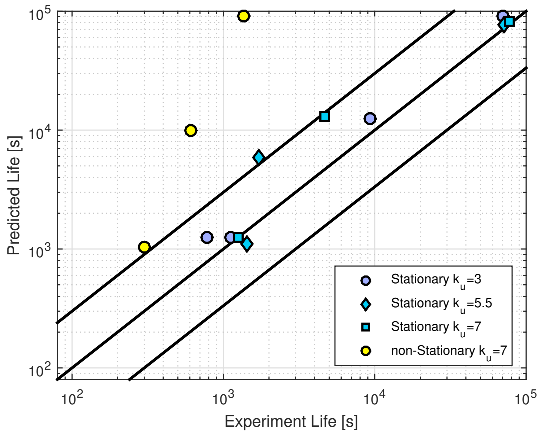

Fig. 6. Comparison between experimental and numerical fatigue life

4. Influence of the dynamic behavior of 1-dof system on the response in case of non-Gaussian excitation

The results of the experimental test campaign showed how the analyzed component responds Gaussianally in case it is excited with non-Gaussian and stationary inputs thus justifying the use of classical spectral methods for the estimation of the fatigue damage even in non- Gaussian loading circumnstances. The component's response in terms of stress is instead non-Gaussian if it is subjected to a non-Gaussian and non-stationary input, with an output kurtosis close to that of the input. In this case, therefore, it is not possible to use the frequency methods to estimate the fatigue damage. In order to investigate the generality of this result, a systematic virtual testing campaigns has been performed on a lumped mass system, with the aim to assess whether any 1-dof system, excited around its resonance frequency, responds Gaussianly if subjected to a stationary non-Gaussian input and if it responds non-Gaussianly if it is subjected to a non-Gaussian non-stationary excitation.

With the aim of having as general a point of view as possible, firstly the influence of the damping on the response of a single degree of freedom systems was study. It was therefore considered a system with a single degree of freedom characterized by a resonance frequency equal to $5Hz$ and by a percentage damping in the range from 1% to 100%. The system has been created with the modal approach described in Sec. 2.3. Fig. 7 shows an example of the system frequency response function for 4 different damping values (1%, 10%, 50%, 100%).

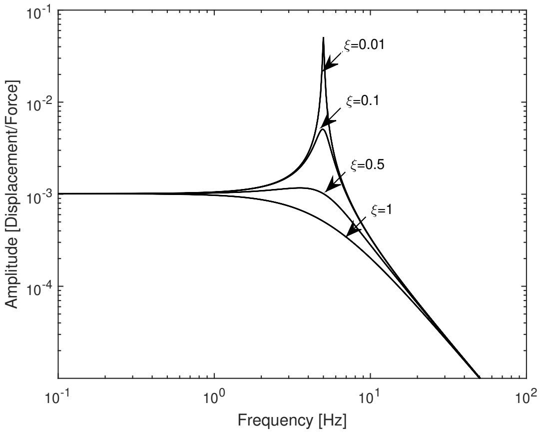

Fig. 7. Example of the 1-dof system frequency response function between the input force and the output displacement

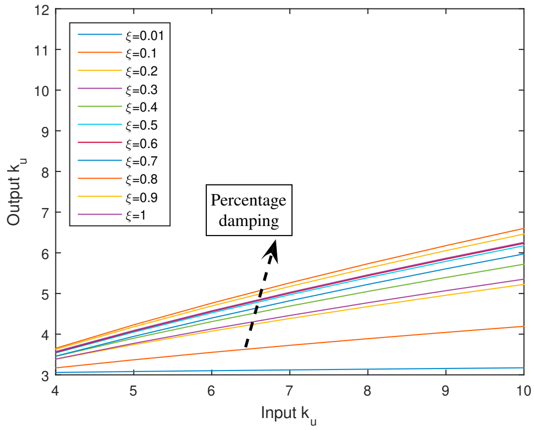

Fig. 8. Output kurtosis trend for different input kurtosis and different percentage damping in case of stationary excitations

From a constant PSD defined between $0.1Hz$ and $10Hz$ with $RMS = 100N$, a large set of stationary non-Gaussian signals were generated with kurtosis in the range 4 to 10 linearly spaced by 1, and zero skewness.

The results of this systematic campaign of tests produced a conspicuous amount of data that observed from afar allowed to affirm that in front of a mechanical system characterized by a vibrating mode whose natural frequency falls within the frequency range of the input and from the application of a single stationary non-Gaussian load the response is not necessarily Gaussian and that the level of this non-Gaussianity depends first of all on the percentage damping of the system. As can be seen from Fig. 8 in fact, as percentage damping increases, the system tends to respond non-Gaussianly, thus not confirming the hypothesis initially made downstream of the experimental tests. In fact, the system tends to respond Gaussianly for any value of input kurtosis only if its percentage damping is lower than 1 %. If the percentage damping results to be higher instead, the response expressed in terms of modal coordinates q turns out to be non-Gaussian although there is an attenuation of the input kurtosis. It should also be emphasized that it was decided to evaluate the non-Gaussianity of the modal coordinates q, since according to what is described in Sec. 2.3, the modal coordinates q are directly connected to the stress time histories through a scale factor, the modal shapes, which do not modify the statistics of the process. On the basis of what was obtained in the case of stationary inputs, it is therefore evident that a frequency approach can be adopted to estimate the fatigue damage only if the analyzed system is characterized by a small percentage damping, otherwise the adoption of a time-domain approach results necessary. In case non-Gaussian response occurs however, an hybrid approach can be used. This

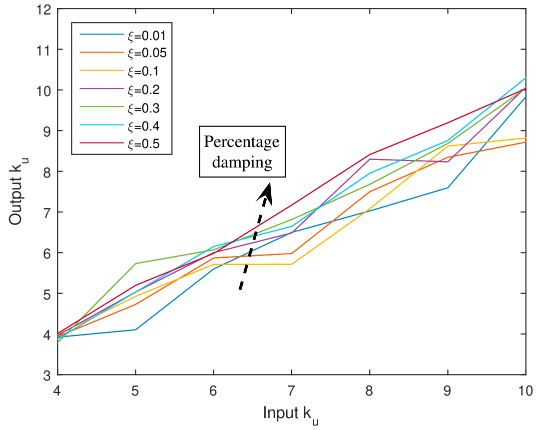

Fig. 9. Output kurtosis trend for different input kurtosis and different percentage damping in case of non-stationary excitations

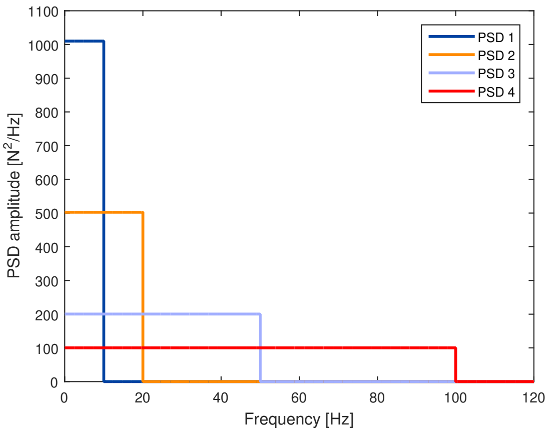

Fig. 10. Input PSDs

technique foresees to use whatever frequency domain method for damage evaluation but accompanied by a damage correction coefficient that takes into account the non-Gaussianity of the response (Braccesi et al. (2009)). In order to further confirm what was obtained during the experimental test campaign, the 1-dof systems previously illustrated were excited with non-Gaussian and non-stationary inputs. Starting from the same constant flat PSD, a large set of non-stationary non-Gaussian signals were generated with kurtosis in the range 4 to 10 linearly spaced by 1 and zero skewness. The percentage damping was vary in the range from 1% to 50%, since no substantial differences arises from a certain damping value onwards.The results of this systematic campaign of tests has produced a conspicuous amount of data that observed from afar have allowed to affirm that in front of a mechanical system characterized by a vibrating mode, whose natural frequency falls within the frequency range of the input, and by the application of a single non-Gaussian and non-stationary load, the response tends to be non-Gaussian too, with a kurtosis value close to that of the input. From the results shown in Fig. 9 it is possible to see that the trend of the kurtosis of the responses is almost linear, and therefore given a certain input, characterized by a certain kurtosis, the system response is characterized by a level of non-Gaussianity (kurtosis) very close to that of the input already starting from a 1% of damping value.

Although damping plays a fundamental role in the path that goes from inputs to outputs, especially in terms of non-Gaussianity, it is not the only parameter that has a determining role in the kurtosis value assumed by the responses. To get a vision of the phenomenon as more general as possible, the 1-dof systems were excited with a series of input

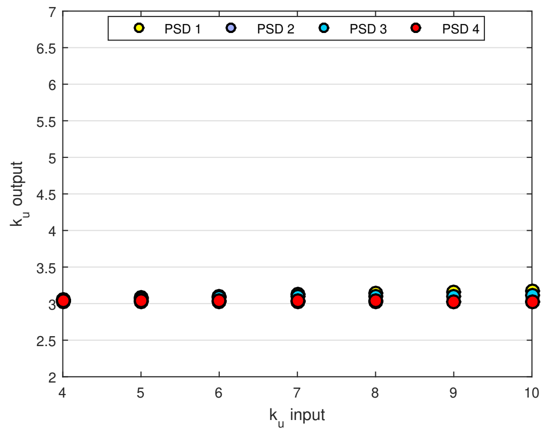

Fig. 11. Output kurtosis trend for different stationary input signal bandwidth and kurtosis for 1% of damping

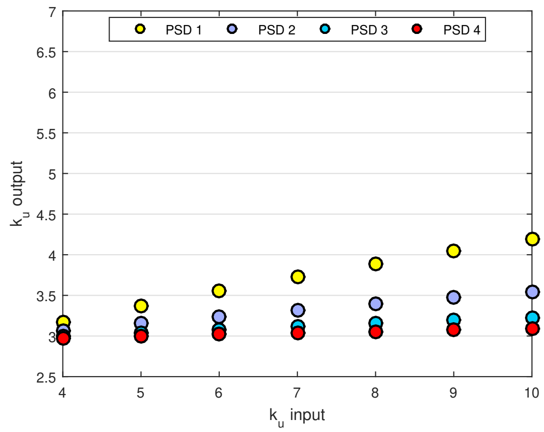

Fig. 12. Output kurtosis trend for different stationary input signal bandwidth and kurtosis for 10% of damping

loads characterized by a PSD with a gradually increasing bandwidth (Fig. 10). In this way much importance is given to the end of the system frequency response function (Fig. 7), and it was possible to evaluate its contribution on the response. The bandwidth of the input PSDs are summarized in Tab. 3.

Table 3. Bandwidth of the input PSDs.

PSD Nr.	Bandwidth Lower limit	Bandwidth Upper limit
1.	0.1 Hz	10 Hz
2.	0.1 Hz	20 Hz
3.	0.1 Hz	50 Hz
4.	0.1 Hz	100 Hz

To evaluate the effect of the bandwidth of the input signal on the kurtosis of the response, three lumped mass systems with resonance frequency fixed at $5Hz$ and three percent damping $(1\%, 10\%, 50\%)$ were considered. These percentage damping values have been chosen since they summarize what has been obtained in the virtual test campaign previously illustrated, where it was observed that the system response is Gaussian only if its damping value is smaller

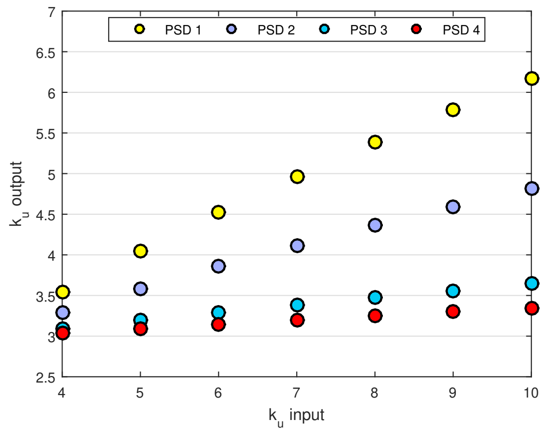

Fig. 13. Output kurtosis trend for different stationary input signal bandwidth and kurtosis for 50% of damping

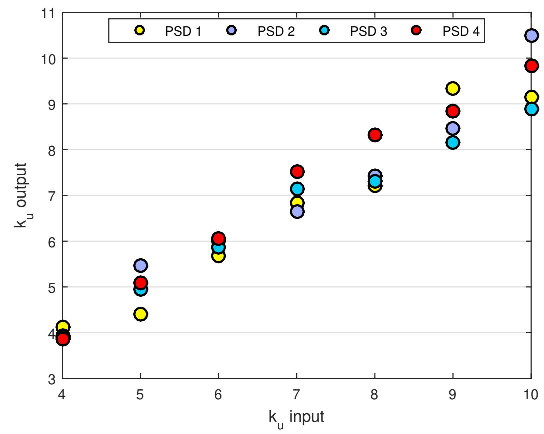

Fig. 14. Output kurtosis trend for different non-stationary input signal bandwidth and kurtosis for 1% of damping

than the 1% and if the excitation is stationary non-Gaussian (Fig. 8). Fig. 11, 12, 13 shows the trend of the kurtosis of the responses to a variation of the bandwidth of the input PSD for the considered percentage damping. Considering the case in which the system is characterized by a percentage damping of 1% (Fig. 11) it is noted that also with an increased bandwidth of the input the kurtosis of the response is very close to 3 certifying as a 1-dof system characterized by a very low percentage damping responds Gaussianally if excited with a stationary input with any level of non-Gaussianity. For cases in which the damping is higher instead (Fig. 12 and (Fig. 13), it is clear that the bandwidth of the input PSD plays an important role in the kurtosis of the response. From Fig. 12 and Fig. 13 it is easy to note that as the bandwidth increases, the response tends to be a Gaussian one. The same evaluations were made in case of non-stationary non-Gaussian signals where the three lumped mass system with natural frequency fixed to $5Hz$ and damping $1\%, 10\%, 50\%$ were excited with non-Gaussian and non-stationary inputs generated from the PSDs shown in Fig. 10. The kurtosis of responses for varying PSD bandwidth for all the considered damping values are shown in Fig. 14,15,16. Analyzing the results shown in Fig. 14, 15,16 it is possible to confirm the same behavior shown in Fig. 9. Indeed, the response of a vibrating system, excited around its resonant frequency with a non-Gaussian and non-stationary input is non-Gaussian, with a kurtosis close to that of the input independently from the bandwidth of the input signal.

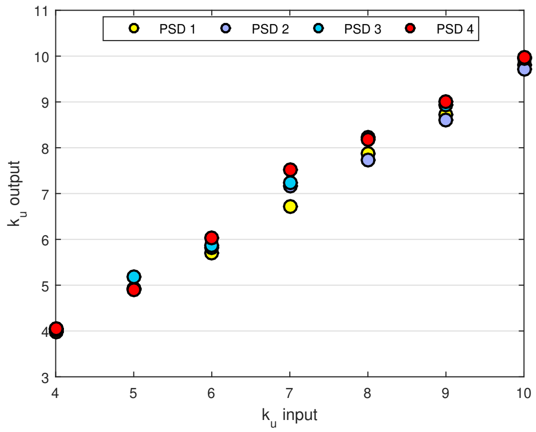

Fig. 15. Output kurtosis trend for different non-stationary input signal bandwidth and kurtosis for 10% of damping

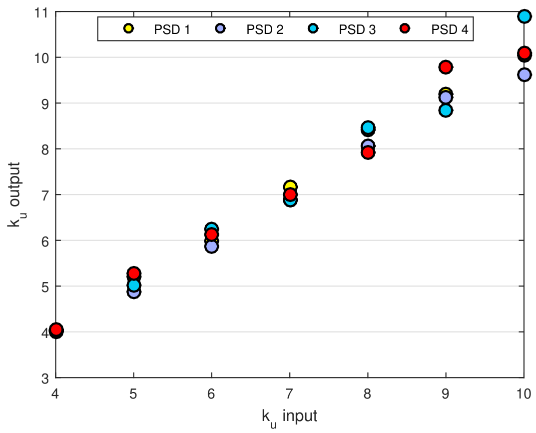

Fig. 16. Output kurtosis trend for different non-stationary input signal bandwidth and kurtosis for 50% of damping

5. Conclusions

In the present research activity, the fatigue behavior of a Y-shaped specimen was studied with the intent of evaluating how non-Gaussianity and stationarity affect the fatigue behaviour of a real component. Initially, several vibration tests were performed with random signals with the same PSD but with different kurtosis values. From the obtained results it was found that the component response is Gaussian when it is excited with a stationary non-Gaussian input. In contrast, in the case of non-Gaussian and non-stationary excitation, the specimen response is non-Gaussian. Moreover, the results obtained show that the fatigue life, in such excitation conditions, is considerably lower due to the strong amplitude modulation of the input, if compared to the duration obtained by exciting the specimen with a stationary input. It has also been verified that if the input is stationary non-Gaussian, the fatigue life estimated with the classical frequency domain methods for fatigue damage evaluation turns out to be identical to that obtained under Gaussian conditions thus justifying the use of frequency methods. In the case where the specimen is excited with non-stationary input instead, the measured life turns out to be considerably lower than that obtained with the classical methods in frequency domain due to the non-Gaussianity of the response. It is therefore evident that the use of frequency methods for calculating the fatigue damage in case the input is non-Gaussian and non-stationary leads to inaccurate results.

The conclusions drawn from experimental tests cannot however be considered as general rules as they have been obtained under certain loading conditions and with a specific specimen. In order to extend the results experimentally obtained, a series of numerical simulations was performed on an elementary oscillator, excited with different stationary and non-stationary non-Gaussian inputs. In such a way it was possible to evaluate how the damping and the bandwidth of the input signal affect the transfer of the kurtosis from the input to the output response. From the outcomes obtained by exciting an 1-dof system with a resonance frequency of $5Hz$ with a stationary non-Gaussian input, results evident that the response is Gaussian only for very small damping values. As damping increases, the response turns out to be non-Gaussian although the output kurtosis is lower than that the input.

This result therefore denies what was obtained from the experimental test campaign. It is possible to state that it is possible to ignore the non-Gaussianity of the inputs and calculate the fatigue damage directly in frequency domain only if the input is stationary and the damping of the system is very small. In the case of non-Gaussian and non-stationary inputs instead, it is possible to extend the results obtained from the experimental tests. In fact, in such loading conditions, even if the damping of the system is very small, the system response shows a kurtosis close to that of the input. This result allows us to state that if the system is excited around its resonance frequency with a non-Gaussian and non-stationary input, the system response is non-Gaussian, with a non-Gaussian level close to that of the input. To conclude this activity, it was assessed how much the bandwidth of the input signal influences the system response. To this, the system was excited with 4 different PSDs with increasing bandwidth. From the results obtained with stationary inputs it is clear how the system response tends to Gaussianity as the bandwidth increases. Conversely, in the case where the system is excited with non-Gaussian and non-stationary inputs, the bandwidth of the input signal does not significantly affect the response.

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