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Removal of the center of mass in nuclei and its effects on ^{4}He

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ABSTRACT

The singular value decomposition of rectangular matrices is shown to provide the recipe for removing the center of mass spurious admixtures from the multiphonon basis generated by an equation of motion method for solving the nuclear eigenvalue problem. It works for any single particle basis without any energy restriction on the selection of the configurations. Its effects on ^{4}He are illustrated.

1. Introduction

Decoupling the center of mass (c.m.) from the intrinsic motion in finite nuclei remains a challenging problem despite the many attempts made in the past [1–7].

A rigorous solution is provided by the no-core shell model (NCSM) [8–10] which adopts the Lawson method [5] to factorize intrinsic and c.m. wavefunctions by adding an external c.m. harmonic oscillator (HO) potential to the nuclear Hamiltonian. This recipe, however, has a restricted validity. The factorization, in fact, is achieved only in a HO single particle (s.p.) basis and if all and only configurations up to $N\hbar\omega$ are included.

The same Lawson method was adopted within shell model to remove the c.m. from level densities [11].

An analogous prescription was used in random-phase approximation (RPA) using a HO potential plus a schematic interaction [12]. For realistic interactions, the only recipe consists in using a huge Hartree-Fock (HF) basis in order to push the c.m. mode to zero energy [13]. In any case, the problem remains unsolved in higher or extended RPA.

Few years ago, we decoupled the c.m. from the Tamm-Dancoff approximation (TDA) states [14] by exploiting the Gram-Schmidt orthogonalization method, a procedure adopted recently in RPA [15].

The TDA states enter as building blocks of the multiphonon states generated within the equation of motion phonon method (EMPM) and used to solve the nuclear eigenvalue problem [16–18].

As we shall see, it is not sufficient to use the c.m. free TDA phonons to avoid the contamination of the multiphonon states.

Here, we obviate this drawback by proposing a method based on the singular value decomposition (SVD) which removes completely the c.m. admixtures from the full basis.

The SVD has been adopted in several contexts of nuclear structure. It was exploited to restore the unitarity of the Bogoliubov transformation in a truncated quasiparticle space [19], to obtain an optimization of the interaction within the energy density functional approaches [20,21] and, also, within Gamow shell model (GSM) [22], to extract the spectral weight function from the propagator within a Monte Carlo approach [23,24], to factorize nuclear structure properties from the reaction dynamics in deuteron electrodisintegration [25].

The method has one important limitation with respect to NCSM. It does not apply to the unperturbed HF ground state whose c.m. admixtures affect the bulk properties of the system. We minimize these impurities by using intrinsic operators as done, for instance, in coupled-cluster (CC) [26–28], where it is shown empirically that the mere use of an intrinsic Hamiltonian yields a factorized ground state to a good approximation.

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On the other hand, it has the advantage of working for any s.p. basis and with no energy constraint on the selection of the HO configurations.

The method can be easily extended to odd systems [29–34] and can be adopted within the quasi-particle version of the EMPM [35] to remove at once the contamination induced by the c.m. and the violation of the particle number in open-shell nuclei.

The EMPM so reformulated is implemented numerically for studying the properties of ^{4}He. This nucleus, especially sensitive to the c.m. motion, was explored mostly in approaches using Jacobi and hyperspherical coordinates (see for references [36,37]). In the last two decades, it was investigated in several *ab-initio* approaches like NCSM [38,39,9], CC [40,41], and in medium similarity renormalization group (IMSRG) [42]. All mentioned studies were focused on its bulk properties. The low-lying levels were studied in early NCSM using effective two-body interactions [43,44] and in a variational approach based on correlated Gaussians using the Argonne v8' potential plus a phenomenological three-body force [45].

We use the NNLO$_{\rm sat}$ potential [40] to generate a HF basis from the lowest six HO major shells. This restricted basis allows us to solve the eigenvalue problem in a space encompassing the full three-phonon basis.

2. The method

The basic ingredients are the HF particle-hole (p-h) vacuum $\mid 0\rangle$ and the TDA states $\mid \lambda\rangle = O^\dagger_\lambda \mid 0\rangle$, where

$$O^\dagger_\lambda = \sum_{ph} c^\lambda_{ph} (a^\dagger_p \times b_h)^\lambda. \tag{1}$$

Here, $a^\dagger_p = a^\dagger_{x_p j_p m_p}$ and $b_h = (-)^{j_h+m_h} a_{x_h j_h -m_h}$ create a particle and a hole of energies ϵ_p and $-\epsilon_h$.

Starting from $\mid 0\rangle$ and the TDA one-phonon states $\mid \lambda\rangle$, we intend to generate iteratively an orthonormal basis of n-phonon ($n = 2, 3...$) correlated states $\mid \beta\rangle = \mid \alpha_n\rangle$ assuming known the $(n-1)$-phonon basis states $\mid \alpha\rangle = \mid \alpha_{n-1}\rangle$. To this purpose we construct the set of redundant states

$$\mid (\lambda \times \alpha)^\beta\rangle = \left\{ O^\dagger_\lambda \times \mid \alpha\rangle \right\}^\beta \tag{2}$$

and extract from them a basis of linearly independent states by resorting to the Cholesky decomposition method. We are then allowed to write the n-phonon states we search for in the expanded form

$$\mid \beta\rangle = \sum_{\lambda\alpha} C^\beta_{\lambda\alpha} \mid (\lambda \times \alpha)^\beta\rangle. \tag{3}$$

They can be determined by solving the generalized eigenvalue equation within the n-phonon subspace

$$\langle (\lambda \times \alpha)^\beta \mid H \mid \beta\rangle = E_\beta \langle (\lambda \times \alpha)^\beta \mid \beta\rangle. \tag{4}$$

For our purposes, however, it is more useful to exploit the structure (2) of the states $\mid (\lambda \times \alpha)^\beta\rangle$ and use the equivalent equation of motion in the reduced form

$$\langle \beta \parallel [H, O^\dagger_\lambda] \parallel \alpha\rangle = (E_\beta - E_\alpha) \langle \alpha \parallel O^\dagger_\alpha \parallel \beta\rangle. \tag{5}$$

Once expanded, the commutator contributes through terms like $\langle \beta \parallel [(a^\dagger_p \times b_h)^{\lambda'} \times (a^\dagger_r \times b_s)^\sigma]^\lambda \parallel \alpha\rangle$, where $(rs) = (pp')$ and $(rs) = (hh')$.

We then need just to act on these matrix elements by using the closure $I_{n-1} = \sum_\alpha \mid \alpha\rangle\langle\alpha \mid$ and expressing the $(a^\dagger_p \times b_h)^{\lambda'}$ in terms of $O^\dagger_{\lambda'}$ upon inversion of Eq. (1).

These operations lead to the generalized eigenvalue equation within the n-phonon subspace

$$\mathcal{H}C = (\mathcal{A}\mathcal{D})C = E\mathcal{D}C. \tag{6}$$

The explicit form is

$$\sum_{\lambda\alpha\lambda'\alpha'} \mathcal{H}^\beta_{\lambda\alpha\lambda'\alpha'} C^\beta_{\lambda'\alpha'} = E_\beta \sum_{\lambda\alpha\lambda'\alpha'} \mathcal{D}^\beta_{\lambda\alpha\lambda'\alpha'} C^\beta_{\lambda'\alpha'}. \tag{7}$$

Here

$$\mathcal{D}^\beta_{\lambda\alpha\lambda'\alpha'} = \langle (\lambda \times \alpha)^\beta \mid (\lambda' \times \alpha')^\beta\rangle \tag{8}$$

is the overlap or metric matrix which preserves the Pauli principle and

$$\mathcal{H}^\beta_{\lambda\alpha\lambda'\alpha'} = \sum_{\lambda_1\alpha_1} \mathcal{A}^\beta_{\lambda\alpha\lambda_1\alpha_1} \mathcal{D}^\beta_{\lambda_1\alpha_1\lambda'\alpha'}, \tag{9}$$

where

$$\mathcal{A}^\beta_{\lambda\alpha\lambda_1\alpha_1} = (E_\lambda + E_\alpha)\delta_{\lambda\lambda_1}\delta_{\alpha\alpha_1} + \mathcal{V}^\beta_{\lambda\alpha\lambda_1\alpha_1}. \tag{10}$$

The expressions of the overlap matrix $\mathcal{D}$ and of the phonon-phonon interaction $\mathcal{V}^\beta_{\lambda\alpha\lambda_1\alpha_1}$ can be found, for instance, in Ref. [46].

The solution of Eq. (7) yields the n-phonon basis states (3). The iteration of the procedure up to an arbitrary n produces a set of states which, added to the HF ($\mid 0\rangle$) and TDA ($\{\mid \alpha_1\rangle\} = \{\mid \lambda\rangle\}$) states, form an orthonormal basis $\mid \alpha_n\rangle$ ($n = 0, 1, 2, 3, ...$).

Such a basis is then adopted to diagonalize the residual Hamiltonian in the full space. The matrix elements between different subspaces $\langle \alpha_n \mid H \mid \alpha_{n'}\rangle$ ($n' \neq n$) can be found, for instance, in Ref. [47]. For $n' = n+2$ ($n \neq 0$) we write here the equivalent more transparent formula

$$\langle \alpha_n \mid H \mid \alpha_{n+2}\rangle = \sum_{\alpha_2} \langle 0 \mid H \mid \alpha_2\rangle \langle (\alpha_2 \times \alpha_n)^{\alpha_n} \mid \alpha_{n+2}\rangle. \tag{11}$$

The coupling of the vacuum to the two-phonon states is

$$\langle 0 \mid H \mid \alpha_2\rangle = \sum_{\lambda\lambda'} \langle (\lambda \times \lambda')^0 \mid \alpha_2\rangle \langle \lambda \mid V \mid \lambda'\rangle, \tag{12}$$

where V is the two-body interaction in normal order form. The diagonalization of the Hamiltonian yields the final eigenvalues $\mathcal{E}_\nu$ and the corresponding eigenstates

$$\mid \Psi_\nu\rangle = \sum_{\alpha_n}^{n=0,1,2..} \mathcal{C}^\nu_{\alpha_n} \mid \alpha_n\rangle. \tag{13}$$

They can be used to evaluate the transition amplitudes of the multipole operators

$$\langle \Psi_{\nu'} \parallel \mathcal{M}(\lambda) \parallel \Psi_\nu\rangle = \sum_{\alpha_n\beta_{n'}} \mathcal{C}^\nu_{\alpha_n} \mathcal{C}^{\nu'}_{\beta_{n'}} \langle \beta_{n'} \parallel \mathcal{M}(\lambda) \parallel \alpha_n\rangle. \tag{14}$$

3. Removal of the c.m. motion

Following Ref. [14], we adopt Gram-Schmidt to extract from the n$_{ph}$ 1^- p-h configurations a set of $n_{ph} - 1$ states orthogonal to

$$\mid \lambda^{(cm)}_\mu\rangle = \frac{1}{N} R_\mu \mid 0\rangle, \tag{15}$$

where R_μ ($\mu = -1, 0, 1$) are the spherical c.m. coordinates and N is a normalization constant. Such a set yields a basis of $n_{ph} - 1$ c.m. free TDA phonons.

These TDA states, however, do not guarantee the purity of the multiphonon basis. Let us consider the two-phonon subspace and

separate the n states $\{|\, i\rangle\} = \{|\, (\lambda \times \lambda')^{\alpha})\}$, composed of the c.m. free phonons $|\, \lambda\rangle$, from the m ones $\{|\, s\rangle\} = \{|\, (\lambda \times \lambda_{cm})^{\alpha}\rangle, |\, (\lambda_{cm} \times \lambda_{cm})^{\alpha}\rangle\}$ containing at least one c.m. phonon $|\, \lambda_{cm}\rangle$. The overlap between the two set of states is non zero ($\mathcal{D}^{(cm)}_{si} = \langle s \,|\, i\rangle \neq 0$) because of the exchange terms between the constituent phonons.

A completely c.m. free basis can be obtained only if we can determine a basis of states $|\, \alpha\rangle = \sum_i C^{(\alpha)}_i \,|\, i\rangle$ fulfilling the orthogonality condition $\langle \alpha \,|\, s\rangle = 0$ for all $|\, s\rangle$ and any $|\, \alpha\rangle$, which amounts to determine the right null space of the rectangular matrix $\mathcal{D}^{(cm)}$

$$\mathcal{D}^{(cm)} C = 0. \tag{16}$$

This goal is achieved here by a procedure exploiting the SVD. According to the SVD, the $m \times n$ rectangular matrix $\mathcal{D}^{(cm)}$ undergoes the following decomposition

$$\mathcal{D}^{(cm)} = U \Sigma V^T = \sum_{i=1,m} \mathbf{u}_i \sigma_i \mathbf{v}_i, \tag{17}$$

where U is a left-singular orthonormal $m \times m$ matrix composed of the row singular vectors $\mathbf{u}_i$ acting on the c.m. states, V^T is the transpose of a right-singular orthonormal $n \times n$ matrix V composed of the column singular vectors $\mathbf{v}_i$ acting on the states composed of c.m. free phonons $|\, \lambda\rangle$, and Σ is an $m \times n$ rectangular diagonal matrix with m non vanishing singular values $\sigma_i \neq 0$.

It is to be noted that the other $n - m$ singular values vanish, $\sigma_i = 0$ for $i = m + 1, n$.

Thus, the right-singular matrix V decomposes into two submatrices. One is composed of the vectors $\mathbf{v}_s$ (s=1,m) and yields the transformed states

$$|\, \nu_s\rangle = \sum_{i=1,n} v_{si} \,|\, i\rangle \qquad (s = 1, m) \tag{18}$$

spanning the c.m. spurious subspace.

The other submatrix, which we denote by $\mathbb{V}$, is composed of the singular vectors $\mathbf{v}_r$ (r=m+1,n) and generates the $n - m$ transformed states

$$|\, \nu_r\rangle = \sum_{i=1,n} v_{ri} \,|\, i\rangle \qquad (r = m + 1, n) \tag{19}$$

orthogonal to the c.m. states $|\, \nu_s\rangle$

$$\langle \nu_r \,|\, \nu_s\rangle = 0. \tag{20}$$

These states form the intrinsic subspace we searched for.

We can then apply the transformation $\mathbb{V}$ to the eigenvalue Eq. (6) obtaining

$$\mathcal{H}' C' = E \mathcal{D}' C', \tag{21}$$

where $C' = \mathbb{V} C$, $\mathcal{D}' = \mathbb{V} \mathcal{D} \mathbb{V}^T$, and $\mathcal{H}' = \mathbb{V} \mathcal{H} \mathbb{V}^T$.

The eigenstates $|\, \alpha_2\rangle = \sum_r (\mathbb{V} C)_r \,|\, \nu_r\rangle$ can be recast in terms of the original basis states $|\, i\rangle = \{|\, (\lambda \times \lambda')^{\alpha}\rangle\}$.

We adopt the same procedure for the three-phonon subspace once we identify the c.m. spurious states. These are $|\, s\rangle = \{|\, (\lambda_{cm} \times \alpha)^{\beta}\rangle, |\, (\lambda \times \alpha_{cm})^{\beta}\rangle, |\, (\lambda_{cm} \times \alpha_{cm})^{\alpha}\rangle\}$, where $|\, \alpha_{cm}\rangle$ are just the transformed two-phonon states $|\, \nu_s\rangle$ $(s = 1, m)$ corresponding to the non vanishing singular values $\sigma_s \neq 0$.

4. Numerical implementation

The Hamiltonian is composed of an intrinsic kinetic term plus the $\mathrm{NNLO_{sat}}$ potential [40]. Its three-body force is fully taken into account in producing the HF basis, while it is truncated at the normal ordered two-body level in the multiphonon calculation.

Table 1
Excitation energies of the lowest computed states and their n-phonon composition.

J^{π}_{ν}	ω_{ν}	$\lvert C_0\rvert^2$	$\lvert C_1\rvert^2$	$\lvert C_2\rvert^2$	$\lvert C_3\rvert^2$
0^+_1	0.000	0.894	0.003	0.102	0.001
0^+_2	23.472	0.002	0.788	0.152	0.059
0^-_1	25.817	0.000	0.776	0.176	0.048
2^-_1	27.005	0.000	0.800	0.163	0.037
2^-_2	27.994	0.000	0.850	0.108	0.042
2^+_1	28.123	0.000	0.781	0.163	0.057
1^-_1	28.691	0.000	0.823	0.123	0.054
0^-_2	29.750	0.000	0.769	0.181	0.050
1^-_2	30.571	0.000	0.679	0.264	0.057
1^-_3	31.279	0.000	0.843	0.096	0.060
1^+_1	31.813	0.000	0.818	0.127	0.055

A HF basis is created from the first six ($\mathrm{N_{max}} = 5$) HO major shells at $\hbar\omega = 20$ MeV. The HF ground state is the only quantity which is outside the SVD domain and, therefore, contains c.m. admixtures. Their effects are minimized by the use of the adopted intrinsic Hamiltonian which yields the energy $E_{\mathrm{HF}} = -14.256$ MeV.

The HF basis is, then, used to generate the TDA phonons. The c.m. spuriousness gets concentrated almost totally in the lowest 1^-_1 which results to be quasi-degenerate with the HF vacuum ($\omega_{1^-_1} = E_{1^-_1} - E_{\mathrm{HF}} = 0.053$ MeV) just as in RPA. The close link between TDA and RPA was discussed in Ref. [14].

The Gram-Schmidt orthogonalization removes this level as well as all residual admixtures from the other 1^-. The c.m. free TDA phonons are used to build up the full basis $\{|\, \alpha_n\rangle\}$ up to $n = 3$ and, then, to solve the full eigenvalue problem.

The binding energy $B = 26.526$ MeV differs from the experimental value $B_{exp} = 28.296$ MeV by ~ 1.8 MeV and comes almost entirely from the coupling to the two-phonon subspace. In fact, if we exclude the three phonons, we get $B_2 = 26.345$ MeV.

Though HF and correlation energies are comparable, the HF vacuum remains by far the dominant component of the ground state wavefunction ($\sim 89\%$), while the two-phonon components account only for $\sim 10\%$ (Table 1).

If we use as constituents the contaminated TDA phonons, including the 1^-_1, a calculation up to two phonons yields $B_2 = 26.557$ MeV, only ~ 200 keV away from the value obtained after the elimination of the c.m. motion. The two-phonon components remain unchanged ($\sim 10\%$) and contain only $\sim 0.4\%$ of spurious contaminants.

The above numbers support the CC numerical proof that the effects of the c.m. on the ground state of ^{4}He are negligible if an intrinsic Hamiltonian is used [26,27].

It is worth to point out that the ground state undergoes modest changes if we enlarge the HO space up to $\mathrm{N_{max}} = 12$. We obtain $E_{\mathrm{HF}} = -14.371$ MeV and $B_2 = 26.865$ MeV with a ~ 500 keV gain with respect to $\mathrm{N_{max}} = 5$. The two-phonon content of the wavefunction is unchanged ($\sim 10\%$). The missing contribution to the binding energy should come from the residual NNN interaction not taken into account here. In fact, we have checked that the same calculation reproduces exactly the experimental value if the two-body Daejeon16 potential [49] derived from the N3LO interaction is used.

If the c.m. is not removed, several spurious multiphonon states intrude into the low energy region, because of the very low lying 1^-_1 phonon. A calculation up to two phonons yields a 1^- at ~ 6.281 MeV above the correlated ground state, originated from the 1^-_1 phonon (94%). The 0^+_2 and 2^+_1 states, with a dominant (70%) $1^-_1 \times 1^-_1$, are few MeV above ($\omega_{0^+_2} \simeq 9.185$ MeV and $\omega_{2^+_1} \simeq 9.302$

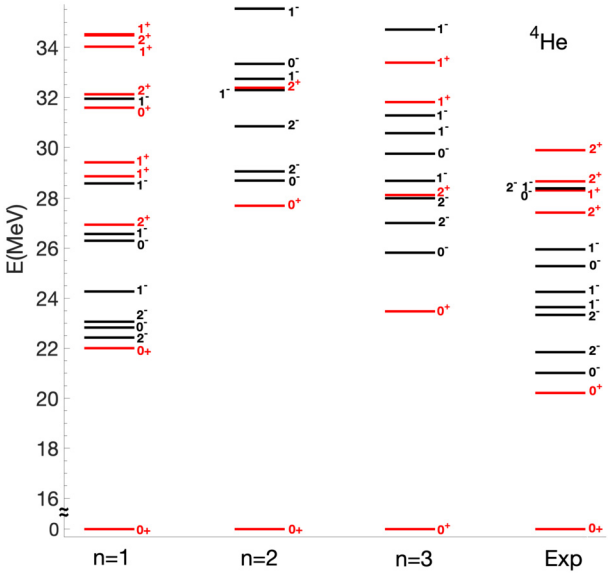

Fig. 1. Level schemes computed in different multiphonon spaces (n=1,2,3) versus the experimental spectrum [48]. For $n=1$ (TDA) $\omega_\lambda = E_\lambda - E_{\rm HF}$, for $n>1$ $\omega_\nu = E_\nu - E_0$, where $E_0 = E_{\rm HF} + E_{corr}$.

MeV). The other states are at ~ 23 MeV or more, but the two-phonon components of some of them are strongly contaminated.

It should be clear from this example that this anomalous two-phonon spectrum is due to the peculiar nature of our method which magnifies maximally the role of the c.m. motion by concentrating the c.m. spuriousness into a single low lying constituent TDA phonon.

We could avoid these pathological levels by simply removing the 1^-_{cm}. This approximate procedure does not avoid the c.m. admixtures induced by the exchange terms of $\mathcal{D}$ which affect especially the states with two-phonon dominance and, therefore, enhance the contamination of the three-phonon states.

Only by combining Gram-Schmidt for TDA with SVD for the other n-phonon subspaces we can decouple rigorously the intrinsic from the c.m. subspaces and, thereby, obtain a completely c.m. free basis in each n-phonon subspace for any n. The resulting spectra are shown in Fig. 1.

The appreciable overlap between TDA and experimental spectra is illusive since the TDA levels are referred to the HF vacuum. The spectrum resulting from the calculation up to two phonons is shifted up in energy because of the depression of the correlated ground state induced by the strong coupling of the HF vacuum to the two-phonon subspace (Eq. (12)). The closely related one-phonon to three-phonon coupling (11) pushes appreciably, though not completely, the excited levels down toward the experimental region. For a further improvement we should enlarge the HO space. For $N_{max} = 12$ up to two phonons, the TDA as well as the two-phonon levels are ~ 2 MeV lower in energy than the corresponding ones for $N_{max} = 5$.

All low-lying theoretical levels have a one-phonon character (Table 1) and, with the exception of the 2^+_1 which does not have a counterpart, are in one to one correspondence with the experimental ones. In fact, the p-h proton neutron content of each state is compatible with the decay of the corresponding level (Table 2). Only the exclusive proton decay of the 0^+_1 remains unexplained.

The experimental 2^+_ν $(\nu = 1,2,3)$ levels, as well as the 0^-_1, 1^-_4, and 2^-_3, undergo a D-decay and, presumably, are to be associated to two-phonon states. These, however, fall at too high energy, above ~ 40 MeV, and can be pushed down only by their coupling to four phonons. We have used Eq. (11) to compute such a coupling under the assumption that the four-phonon states are products of non interacting two-phonon constituents ($|\,\alpha_4\rangle \sim |\,\alpha_2 \times \alpha'_2\rangle$) and that the exchange terms between α_2 and α'_2 in the overlap matrices can be neglected. We found that all two-phonon states get shifted downward considerably. For instance, the first two-phonon 2^+ occurs now at 30.285 MeV.

Table 2
Proton ($\tau = p$) and neutron ($\tau = n$) p-h weights $W_\tau = \sum_{ph} |c^\tau_{ph}|^2$ of the one-phonon (TDA) components of the lowest excited states compared with the proton (%p) and neutron (%n) decay modes of the experimental levels [48].

J^π_ν	W_p	W_n	%p	%n
0^+_2	51	49	100	0
0^-_1	56	44	76	24
2^-_1	79	21	63	37
2^-_2	21	79	53	47
2^+_1	53	47	3	3
1^-_1	53	47	55	45
0^-_2	44	56	52	48
1^-_2	54	46	50	47
1^-_3	44	56	52	48
1^+_1	72	18	48	47

These preliminary estimates suggest that the four phonons are needed for a satisfactory complete description of the spectrum.

The giant dipole resonance cross section is given by

$$\sigma(E1) = \int_0^\infty \sigma(E1,\omega)d\omega = \frac{16\pi^3}{9\hbar c}\int_0^\infty \omega \mathcal{S}(E1,\omega)d\omega \tag{22}$$

where

$$\mathcal{S}(E1,\omega) = \sum_\nu B(E1;0^+_1 \to 1^-_\nu)\delta(\omega - \omega_\nu) \tag{23}$$

is the strength function, $\omega_\nu = \mathcal{E}_\nu - \mathcal{E}_0$ the excitation energies, and

$$B(E1;0^+_1 \to 1^-_\nu) = |\,\langle \Psi_{1^-_\nu} \parallel \mathcal{M}(E1) \parallel 0^+_1\rangle\,|^2 \tag{24}$$

the ground state reduced strength of the $E1$ operator. The transition amplitudes are computed using the Eq. (14). In actual calculations, the δ function is replaced with a Lorentzian having a width $\Gamma = 10$ MeV.

The above formulas show that all 1^-_ν eigenvalues and eigenfunctions enter in the determination of the cross section. Thus, its position and shape depend critically on the energy level distribution.

Therefore, our calculation is more demanding than the *ab-initio* approaches where the cross section is evaluated through the Lorentz integral transformation (LIT) [55] which does not make explicit use of the excited states [56,57,54].

As pointed out already, the position and shape of the cross section are affected by the energy level distribution. In fact, its evolution follows closely the trend of the spectrum (Fig. 2). The depression of the ground state induced by the coupling to two phonons causes a shift of the whole strength at too high energy. The three phonons succeed only partially in bringing the strength back to the experimental energy region.

5. Conclusions

In summary, the combination of Gram-Schmidt and SVD removes exactly and completely the c.m. admixtures from all multiphonon states for any s.p. basis and does not impose any energy constraint on the selection of the HO configurations. It does not act, however, on the HF ground state whose impurities are minimized through the use of an intrinsic Hamiltonian.

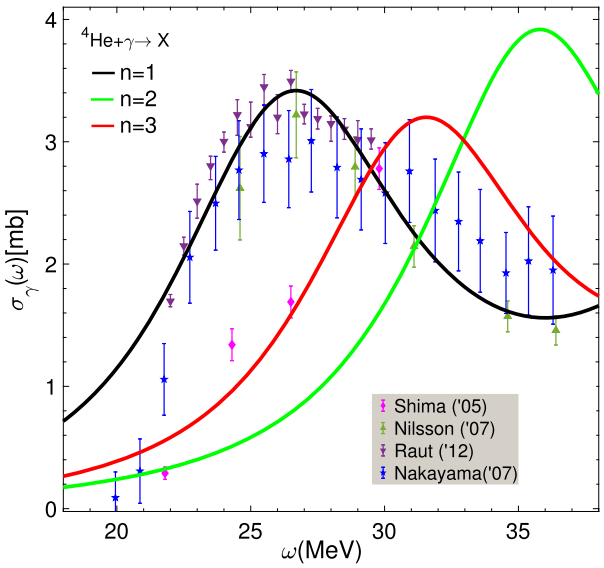

Fig. 2. Theoretical versus experimental [50–53] cross sections. Following Ref. [54], we have assumed $\sigma_\gamma(\omega) \approx 2\sigma_{\gamma,n}(\omega)$ for the data of Ref. [52] and $\sigma_\gamma(\omega) \approx \sigma_{\gamma,p}(\omega) + \sigma_{\gamma,p}(\omega + 0.5$ MeV) for those of Ref. [51].

The numerical implementation of the method for $N_{max} = 5$ illustrates exhaustively its impact on the properties of ^{4}He. It shows that the removal of the c.m. motion through the SVD has a minimal impact on the ground state correlations but affects strongly the spectroscopic properties, at least within our approach.

Only a rough and partial comparison between theory and experiments can be made. A substantial enlargement of the HO space is needed for a better description of the spectroscopic properties. A calculation up to two phonons can be performed easily for any HO space. However, the computational effort increases rapidly with N_{max} and, eventually, becomes prohibitive if we want to include the full three-phonon basis. Fortunately, the method is very versatile and allows us to find out reliable and efficient prescriptions for truncating the basis. Such a truncation is even more necessary for the four phonons which, according to our preliminary estimates, may affect considerably the states of complex structure.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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