



ORIGINAL ARTICLE

# Magnetization reversal in a site-dependent anisotropic Heisenberg ferromagnet under electromagnetic wave propagation



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**Abstract** Information density and switching of magnetization offers an interesting physical phenomenon which invoke magneto-optical techniques employed on the magnetic medium. In this paper, we explore the soliton assisted magnetization reversal in the nanosecond regime in the theoretical framework of the Landau–Lifshitz–Maxwell (LLM) model. Starting from the Landau–Lifshitz equation, we employ the reductive perturbation method to derive an inhomogeneous nonlinear Schrödinger equation, governing the nonlinear spin excitations of a site-dependent anisotropic ferromagnetic medium under the influence of electromagnetic (EM) field in the classical continuum limit. From the results, it is found that the soliton undergoes a flipping thereby indicating the occurrence of magnetization reversal behavior in the nanoscale regime due to the presence of inhomogeneity in the form of a linear function. Besides, the spin components of magnetization are also evolved as soliton spin excitations.

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## 1. Introduction

The continuous effort in minimizing and search for ultimate speed of modern computers and magnetic recording media for data storage give rise to a constant strive to derive and potentially optimize mechanisms in order to manipulate fast reversal of magnetic moment of a material (see Fechner et al., 2012; Kaka and Russek, 2002; Tudosa et al., 2004;

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Adam et al., 2012; Schumacher et al., 2003). The interaction of light with matter spans a wide range of applications in all optical devices and has intensive focus in experimental research, with special attention paid to magnetization reversal that is running down to the limit of subpico/femtosecond regime. Generally, so-called magneto-optical switching combines the merits of magnetic and optical techniques and refers to a qualitative method of reversing magnetization in a ferromagnet simply by circularly polarized light, where the magnetization direction is controlled by the light helicity (see Stanciu et al., 2007a). In particular, the magnetization direction is well controlled by the direction of angular momentum of the photons (see Stanciu et al., 2006, 2007b). On theoretical grounds, it is shown that such effect may even lead the switching process down to femtosecond time scale that would be based on the application of shaped ultrashort laser pulse of certain frequency, duration and polarization (see Gmez-Abal et al., 2004). Experiments generally use pump-probe processes in which a high energy laser pulse is used to heat the sample and a low energy probe pulse is used to monitor the magnetic response using the magneto-optical Kerr effect (MOKE) (see Moradi and Ghanaatshoar, 2010; Chau and Hsieh, 1973; Liu et al., 2011; Montoncello et al., 2008; Georg and Back, 2007).

More qualitative features of the effect of laser on magnetization reversal can be well understood theoretically on the basis of electromagnetic wave equation proposed by Maxwell and Landau–Lifshitz (LL) equation that governs the spin dynamics of magnetic materials. The LL equation constitutes the basic governing equation for the spin–spin exchange interaction in ferro/antiferromagnet which includes crystal anisotropy with some other dominant higher order interactions namely biquadratic, weak interaction such as Dzyaloshinskii–Moriya (DM) interaction, dipole–dipole, and octupole–dipole interactions (see Daniel and Kavitha, 1999, 2002; Ahmad Abliz et al., 2009; Kavitha et al., 2010a,b, 2011a,b, 2012). Crystal field anisotropy being a primary interaction in all magnetic materials elucidates the spin reversal actively and simultaneously controls the characteristic switching time (see Uzdin et al., 2012). Moreover, higher order magnetic interactions also provide significant contribution for magnetization switching and the inhomogeneity present in the medium can too support for magnetization reversal as demonstrated by Kavitha et al., recently (see Kavitha et al., 2010a,b, 2011a,b, 2011). Thus the fundamental and the practical limit of speed of magnetization reversal is a subject of vital importance as well as one of the intriguing questions of modern magnetism.

In the above respect, recently the study of interaction of electromagnetic (EM) field in ordered magnetic media has become an emerging and growing field of research. In this case, the magnetic field component of the electromagnetic field is found to excite the magnetization of the ferromagnetic medium in solitonic form and also the small amplitude plane electromagnetic wave propagates in the form of EM solitons (see Leblond, 2005). Nakata (1991a,b) and Leblond (2008, 2010) also show the soliton excitations of EMW components in a ferromagnetic medium using a reductive perturbation method by neglecting the spin-spin exchange energy. Similarly an extension of the above investigation is made by taking into account the basic magnetic interaction namely the spin-spin exchange interaction in isotropic/anisotropic ferro and antiferromagnetic media (see Veerakumar and Daniel, 1998, 2001). Recently, the present authors made a rigorous study on the

effect of DM interaction in an antiferromagnet (see Kavitha et al., 2011), thereby showing that DM interaction induces breatherlike spin excitations in the medium and in addition it enhances the amplitude of the EM soliton.

This paper communicates this issue and is constructed as follows. In Section 2, the coupled Landau–Lifshitz–Maxwell (LLM) equation is reduced to perturbed nonlinear Schrödinger (NLS) equation through the reductive perturbation method. In Section 3, we employ the multiple scale perturbation analysis on the perturbed NLS equation and obtain the soliton parameter evolution equations and magnetic spin soliton components are constructed. The occurrence of magnetization reversal is discussed in Section 4. Section 5 concludes the results.

## 2. Model and spin dynamics

The system under consideration is a site-dependent anisotropic ferromagnetic medium exposed to an external magnetic field  $\mathbf{H}$  through the propagation of electromagnetic wave governed by the Landau–Lifshitz equation for the evolution of spin density in the classical continuum limit. The dynamical equation is written as follows

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \times \{J(f\mathbf{S}_{xx} + f_x\mathbf{S}_x) - 2AS^x\hat{n} + 2\beta\mathbf{H}\}, \quad (1)$$

where  $\hat{n} = (1, 0, 0)$  and suffix  $x$  represents partial differentiation,  $J$  is the exchange integral,  $f$  is the site dependent co-efficient which varies appropriately from site to site,  $A$  is the anisotropic parameter that tends the magnetization to favor along the  $x$ -direction and  $\beta = \gamma\mu_B$ , where,  $\gamma$  is the gyromagnetic ratio and  $\mu_B$  represents Bohr magneton. In general the inhomogeneity occurs in the magnetic system if (a) the distance between neighboring atoms varies along the lattice, thereby altering the overlap of electronic wave functions, e.g. charge transfer complexes TCNQ or organometallic insulators TTF bisdithiolenes, and (b) the wave function itself varies from site to site although the atoms themselves may be equally spaced, e.g. a one-dimensional magnetic insulator placed in a weak, static, inhomogeneous electric field with the deliberate introduction of imperfections in the vicinity of a bond so as to alter the electronic wave functions without causing appreciable lattice distortion. Since the overlapping of wavefunction varies from site to site, the associated interaction is termed as ‘site-dependent interaction’ as designated by  $f(x)$  in Fig. 1.

We consider the propagation of electromagnetic waves in a magnetic material medium in the presence of an external magnetic field. The governing Maxwell’s equations are the following (see Jackson, 1993):

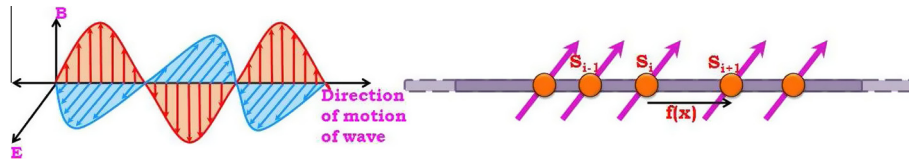
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (2)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}. \quad (3)$$

In Eqs. (2) and (3),  $\mathbf{E}$ ,  $\mathbf{D}$ ,  $\mathbf{H}$  and  $\mathbf{B}$  are respectively the electric field, the electric induction, the magnetic field and the magnetic induction. The constitutive equation for  $\mathbf{E}$  and  $\mathbf{D}$  is

$$\mathbf{D} = \epsilon\mathbf{E}, \quad (4)$$

where we shall assume that  $\epsilon$  is the scalar permeability of the magnetic medium, whereas the constitutive equation for  $\mathbf{H}$  and  $\mathbf{B}$  is



**Figure 1** Schematic diagram representing the inhomogeneous ferromagnetic system excited by the electromagnetic wave (EMW).

$$\mathbf{B} = \mu(\mathbf{H} + \mathbf{S}), \quad (5)$$

where,  $\mu$  is the magnetic permeability of the medium and  $\mathbf{S}$  is the magnetization density in the magnetic medium of propagation. Eliminating  $\mathbf{E}$ ,  $\mathbf{D}$  and  $\mathbf{H}$ , from Eqs. (2)–(5), we have

$$\mathbf{B}_{xx} - \mu\epsilon\mathbf{B}_{tt} = \mu[\mathbf{S}_{xx} - \hat{n}S_{xx}^x]. \quad (6)$$

It is interesting to note that if the magnetization  $\mathbf{S}$  is zero, then the above Eq. (6) would be the linear wave equation, satisfied by the isotropic, dispersionless transverse waves, propagating at speed  $c = (\mu\epsilon)^{-1/2}$ . The set of coupled Eqs. (1) and (6) completely describe the propagation of EM wave in an anisotropic site-dependent ferromagnetic medium which we wish to solve in the consecutive section using a perturbation theory.

### 3. Perturbation scheme and nonlinear Schrödinger equation

Having derived the equations of motion, the task now lies in solving them to understand the underlying spin excitations. We find Eqs. (1) and (6) as a set of highly nontrivial nonlinear coupled partial differential equations which are not amenable to exact analysis in general. In this section, we attempt to solve the coupled Landau–Lifshitz equations for magnetization and the Maxwell’s equations for electromagnetic field within the frame work of reductive perturbation method along the lines of Taniuti, Yajima and others (see Taniuti and Yajima, 1969; Leblond, 2008). This technique adopts the nonlinear modulation of the slowly varying envelopes of EM plane waves of small but finite amplitude in the antiferromagnetic medium. In order to carry out this perturbation, the magnetization of the medium and the magnetic induction of the EM field have to be expanded nonuniformly in the anisotropic weak antiferromagnetic medium. Since, the easy axis of magnetization of the anisotropic medium lies parallel to the direction of propagation ( $x$ -direction), we assume that at the lowest order of expansion, the magnetization of the medium and the magnetic induction lie parallel to the propagation axis and turn around to the ( $y-z$ ) plane at higher orders. Therefore writing  $\mathbf{S} = (S^x, S^y, S^z)$  and  $\mathbf{B} = (B^x, B^y, B^z)$  and expressing the Fourier components of  $\mathbf{S}$  and  $\mathbf{B}$  in powers of a small parameter  $\epsilon$ , we write

$$\begin{aligned} S^x &= S_0 + \sum_{n=1}^{\infty} \epsilon^n S_n^x(\zeta, \tau) + \dots, \\ S^z &= \epsilon^{1/2} \sum_{n=1}^{\infty} \epsilon^n S_{n+1}^z(\zeta, \tau), \end{aligned} \quad (7)$$

and

$$\begin{aligned} B^x &= B_0 + \sum_{n=1}^{\infty} \epsilon^n B_n^x(\zeta, \tau) + \dots, \\ B^z &= \epsilon^{1/2} \sum_{n=1}^{\infty} \epsilon^n B_{n+1}^z(\zeta, \tau). \end{aligned} \quad (8)$$

In the above perturbative expansions,  $S_0$  and  $B_0$  characterize the unperturbed state of the system, the small parameter  $\epsilon$  measures the perturbation of the applied magnetic field or the amplitude of the perturbed field and  $\alpha = y, z$ . Also, we consider, the components are functions of the slow variables  $\zeta$  and  $\tau$  introduced through the stretching  $\zeta = \epsilon(x - vt)$  and  $\tau = \epsilon t$  with  $v$  being the velocity of the pulse and time variable accounts for the evolution of the propagating pulse. As the medium is ferromagnetic in character, we assume that the value of the dielectric constant  $\epsilon$  of the medium is small and hence we rescale  $\epsilon$  as  $\epsilon^2\epsilon$ . Also, we assume that the bilinear exchange interaction is stronger than Zeeman energy, hence rescaling  $J$  as  $\epsilon^{-1}J$  and  $\beta$  as  $\epsilon\beta$ . We now substitute the expansions for  $\mathbf{S}$  and  $\mathbf{B}$  as given in Eqs. (7) and (8) in the component form of Eqs. (1) and (6) with the subsequent scaling defined above and then collecting and solving the coefficients at different orders of  $\epsilon$ , we get

At the order ( $\epsilon^0$ ):

$$B_0 = 0, \quad (9)$$

$$B_1^y = \mu S_1^y, \quad (10)$$

$$B_1^z = \mu S_1^z, \quad (11)$$

and

At the order ( $\epsilon^1$ ):

$$B_1^x = 0, \quad (12)$$

$$B_2^y = \mu S_2^y, \quad (13)$$

$$B_2^z = \mu S_2^z, \quad (14)$$

$$\frac{\partial S_1^x}{\partial \tau} - v \frac{\partial S_1^x}{\partial \zeta} = Jf \left( S_1^y \frac{\partial^2 S_1^z}{\partial \zeta^2} - S_1^z \frac{\partial^2 S_1^y}{\partial \zeta^2} \right) + Jf_z \left( S_1^y \frac{\partial S_1^z}{\partial \zeta} - S_1^z \frac{\partial S_1^y}{\partial \zeta} \right), \quad (15)$$

$$v \frac{\partial S_1^y}{\partial \zeta} - \frac{\partial S_1^y}{\partial \tau} = S_0 \left( Jf \frac{\partial^2 S_1^z}{\partial \zeta^2} + Jf_z \frac{\partial S_1^z}{\partial \zeta} \right) + 2AS_1^y S_1^z + \frac{2\beta}{\mu} S_0 B_1^z, \quad (16)$$

$$\frac{\partial S_1^z}{\partial \tau} - v \frac{\partial S_1^z}{\partial \zeta} = S_0 \left( Jf \frac{\partial^2 S_1^y}{\partial \zeta^2} + Jf_z \frac{\partial S_1^y}{\partial \zeta} \right) + 2AS_1^y S_1^z + \frac{2\beta}{\mu} S_0 B_1^y. \quad (17)$$

Now, we define a new complex variable  $u = S_1^y - iS_1^z$ ,  $|u|^2 = \frac{A}{J\beta S_0} S_1^y$ , such that Eqs. (16) and (17) can be reduced to the following NLS equation with the inhomogeneity function  $f = ax + b$  and an appropriate rescaling yields

$$iu_t + u_{xx} + 2|u|^2 u = \lambda[xu_{xx} + u_x], \quad (18)$$

where,  $\lambda = -\frac{a}{b}$ ,  $a$  and  $b$  are arbitrary constants. When  $\lambda = 0$ , Eq. (18) is a well known completely integrable cubic nonlinear Schrödinger (NLS) equation with associated soliton solution (see Zakharov and Shabat, 1973; Ablowitz and Segur, 1981) and the term on the right side of (18) represents a small perturbation added to the simple cubic NLS equation. It is the aim of the present paper to look for magnetization switching in the ferromagnetic medium for which we wish to solve Eq. (18) for the soliton parameter evolution equations in the next section. Some insights into the mechanism of controlling

soliton were gained in Ref. (see [Kavitha et al., 2010a,b](#); [Kavitha and Daniel, 2003](#)) where the magnetization reversal in ferromagnetic material was established through soliton. The electromagnetic wave dynamics exploiting localized excitations experimentally have already been reported in materials such as yttrium iron garnet (see [Chen et al., 1993](#)) and Cu benzoate AFHC ([Dender et al., 1997](#)).

#### 4. Multiple scale perturbation analysis

It is not easy to obtain exact solutions of nonlinear systems in all cases. In order to obtain information about the system under consideration, we are forced to attempt approximation methods such as perturbation analysis. There are a number of perturbation schemes available in the literature which exploit the proximity of the perturbed NLS Eq. (18) to the completely integrable case of the pure NLS equation, i.e., Eq. (18) with  $\lambda = 0$ . We employ one such method namely multiple scale perturbation analysis suggested by [Kodama and Ablowitz \(1981\)](#) to witness the presence of soliton in the ferromagnetic medium through which magnetization reversal has been established. The envelope one soliton solution for the cubic NLS can be written as  $u = \eta \text{sech} \eta (\theta - \theta_0) \exp[i\xi(\theta - \theta_0) + i(\sigma - \sigma_0)]$ , where  $\theta_x = -2\xi$ ,  $\theta_x = 0$ ,  $\sigma_x = \eta^2 + \xi^2$ ,  $\sigma_x = 0$ . The parameter  $\eta$  and  $\xi$  are related to the scattering parameter of the inverse scattering transform analysis. Now we write  $\eta$ ,  $\xi$ ,  $\theta$ ,  $\theta_0$ ,  $\sigma$  and  $\sigma_0$  as functions of a new time scale  $T$ . Considering  $\hat{u}_0$  as the exact one soliton solution corresponding to the unperturbed part of Eq. (18), we introduce certain fast variable  $\theta$  and a slow variable  $T = \lambda t$ . Therefore Eq. (18) has the solution of the form

$$u = \hat{u}(\theta, T; \lambda) \exp[i\xi(\theta - \theta_0) + i(\sigma - \sigma_0)], \quad (19)$$

and it may be noted that in the case of one soliton solution, we need only one fast variable. Therefore, we expand  $\hat{u}$  in the form

$$\hat{u}(\theta, T; \lambda) = \hat{u}_0(\theta, T) + \lambda \hat{u}_1(\theta, T) + \dots, \quad (20)$$

and neglecting the higher order terms. In the above Eq. (20),  $\hat{u}_0$  is the unperturbed soliton for the unperturbed schrodinger equation and  $\hat{u}_1$  is the first order perturbed soliton solution. Using Eqs. (19) and (20) in Eq. (18), we obtain

$$-\eta^2 \hat{u}_0 + \hat{u}_{00\theta} + 2|\hat{u}_0|^2 \hat{u}_0 = 0, \quad (21)$$

$$-\eta^2 \hat{u}_1 + \hat{u}_{10\theta} + 4|\hat{u}_0|^2 \hat{u}_1 + 2\hat{u}_0^* \hat{u}_1 = F(\hat{u}_0), \quad (22)$$

where,

$$F(\hat{u}_0) = u_{0\theta} + (\theta - \theta_0) \hat{u}_{0\theta\theta} - \xi^2 (\theta - \theta_0) \hat{u} - \hat{u} [\xi_T (\theta - \theta_0) - \xi \theta_{0T} - \sigma_{0T}] + i[\hat{u}_T + \hat{u} \xi + 2\xi (\theta - \theta_0) \hat{u}_\theta]. \quad (23)$$

Eq. (21) corresponds to the unperturbed NLS equation for the variable  $\hat{u}_0$  which admits the leading order solution in the form of  $\hat{u}_0 = \eta \text{sech} \eta (\theta - \theta_0)$  and Eq. (22) represents the first order perturbed equation. Separating the complex Eq. (22) into a set of real equations by substituting  $\hat{u}_1 = \hat{\phi}_1 + i\hat{\psi}_1$ , where  $\hat{\phi}_1$  and  $\hat{\psi}_1$  are real functions, we obtain

$$L_1 \hat{\phi}_1 = -\eta^2 \hat{\phi}_1 + \hat{\phi}_{10\theta} + 6\hat{u}_0^2 \hat{\phi}_1 = \Re F(\hat{u}_0), \quad (24)$$

$$L_2 \hat{\psi}_1 = -\eta^2 \hat{\psi}_1 + \hat{\psi}_{10\theta} + 2\hat{u}_0^2 \hat{\psi}_1 = \Im F(\hat{u}_0), \quad (25)$$

where,  $L_1$  and  $L_2$  are self-adjoint operators. The real  $\Re F(\hat{u}_0)$  and imaginary  $\Im F(\hat{u}_0)$  part of  $F(\hat{u}_0)$  are given by

$$\Re F(\hat{u}_0) = \hat{u}_{0\theta} + (\theta - \theta_0) \hat{u}_{0\theta\theta} - \xi^2 (\theta - \theta_0) \hat{u}_0 + \hat{u}_0 (\xi_T (\theta - \theta_0) - \xi \theta_{0T} - \sigma_{0T}), \quad (26)$$

$$\Im F(\hat{u}_0) = -\hat{u}_{0T} + \xi \hat{u}_0 + 2\xi (\theta - \theta_0) \hat{u}_{0\theta}. \quad (27)$$

As  $\hat{u}_{0\theta}$  and  $\hat{u}_0$  are solutions to the homogeneous part of Eqs. (24) and (25) respectively the secularity conditions yield

$$\int_{-\infty}^{\infty} \hat{u}_{0\theta} \Re F d\theta = 0, \quad (28)$$

$$\int_{-\infty}^{\infty} \hat{u}_0 \Im F d\theta = 0. \quad (29)$$

On explicitly evaluating the integrals (28) and (29), respectively, we obtain the time evolution of the soliton parameters namely amplitude and velocity.

#### 4.1. Magnetization reversal

Theoretical investigations on magnetization reversal in magnetic materials have been a growing interest in recent times (see [Sabareesan and Daniel, 2011](#); [Daniel and Sabareesan, 2009](#)). [Sabareesan and Daniel \(2011\)](#), [Daniel et al.](#) show analytically that the switching time in permolloy thin film has been reduced considerably to subpico second level when the external field applied goes beyond some threshold limit also the switching time reduces when the magnetic surface anisotropy increases. Recently, [Rahman et al.](#), elucidate the reversal mechanism in nanocrystalline magnetic films by adopting two reliable techniques namely Magneto-Optical Kerr Effect (MOKE) and nanosecond pulsed field magnetometer (see [Rahman et al., 2007](#)). These authors show that the magnetization reversal in the film occurs at the order of nanosecond. In this direction, we establish the magnetization switching mechanism in the ferromagnetic medium through EM soliton at nanosecond regime on the theoretical grounds. The presence of inhomogeneity in the medium certainly raises question about the role of soliton and its participation in magnetization reversal. It is well known that the inhomogeneity in the form of a linear function completely supports soliton spin excitations that admit Lax pair (see [Porsezian, 1997](#)) and show integrability. However, when the inhomogeneity is in the form of a nonlinear function, the velocity and amplitude of the soliton increase as time passes and reach a maximum and suddenly flip leading to the magnetization reversal and move in the opposite direction (see [Daniel and Kavitha, 2002](#)). Since the velocity of the soliton is inversely proportional to the inhomogeneity the soliton damps very quickly in case of highly inhomogeneous medium. In the case of a localized inhomogeneity (see [Kavitha et al., 2010a,b](#); [Kavitha et al., 2010](#)) the amplitude of the soliton infact oscillates with double periodicity and shows a marginal reversal in the amplitude. The velocity of the soliton shows dramatic turns at the points when it reverses or switches in the nanosecond regime. Thus the presence of inhomogeneity makes a sustainable oscillation for soliton through which the magnetization reversal occurs. In order to witness the presence of magnetization reversal through soliton, we invoke the secularity conditions obtained in the previous section for the evolution of soliton parameters namely velocity  $\xi$  and amplitude  $\eta$ . Now making use of  $\hat{u}_{0\theta}$  and  $\hat{u}_0$  in the above secularity conditions and integrating them leads to

$$\xi_T = \xi^2 + \frac{1}{3} \eta^2 \quad \text{and} \quad \eta_T = 0. \quad (30)$$

Upon solving these equations one can obtain,

$$\xi(T) = \frac{\eta}{\sqrt{3}} \tan\left(\frac{\eta}{\sqrt{3}}(T + c_0)\right), \quad (31)$$

where,  $c_0$  is the constant of integration and the amplitude of the soliton  $\eta$  which is time independent. We demonstrate the magnetization reversal through flipping of EM soliton by plotting Eq. (31) for the evolution of velocity of the soliton  $\xi(T)$  by choosing the parameter  $c_0 = 0.5$ . From the plots shown in Fig. 2, we observe that the presence of inhomogeneity in the spin chain induces the magnetization reversal through flipping of EM soliton. The exact balance between nonlinearity and dispersion of the EM wave leading to the formation of EM soliton in the medium. However, the amplitude evolution of the soliton  $\eta$  remains constant as time passes whereas the velocity evolution of the soliton periodically changes. That is, the soliton moves with varying speed along the spin lattice so that it reaches a maximum velocity. Then, there is an imbalance generated by inhomogeneity between the dispersion and nonlinearity, instigating the soliton to switch over to negative direction in order to balance the influence of dispersion and nonlinearity. Further, the velocity of the soliton reaches a maximum value in a short period in the negative direction and hence again an imbalance is created between the dispersion and nonlinearity and thereby switching over to positive direction very slowly. In this way, a sequential balance and imbalance between the dispersion and nonlinearity leads to the reversal of magnetization through the coherent solitonic evolution periodically. Thus the velocity of the soliton continuously switches over to positive to negative and negative to positive leading to the magnetization reversal as time passes. From the Fig. 2, the switching time reduces considerably from 2.5 to 1.2 nano time units (ntu) for increasing the value of amplitude  $\eta$  as  $\eta = 2.0$ ,  $\eta = 3.0$  and  $\eta = 4.0$  as evidenced from the plots. When the amplitude of the soliton is high, the frequency of flipping soliton shoots up in Giga Hz and the switching time reduces further and which may have potential application in optimizing the speed of data storage and retrieval in magnetic systems.

#### 4.2. Perturbed solitons

The perturbed soliton solutions can be constructed by solving Eq. (24) for  $\hat{\phi}_1$  and Eq. (25) for  $\hat{\psi}_1$  using  $\xi_T$  and  $\eta_T$ . The homogeneous part of Eq. (24) admits two particular solutions  $\hat{\phi}_{11}$  and  $\hat{\phi}_{12}$  which are of the form

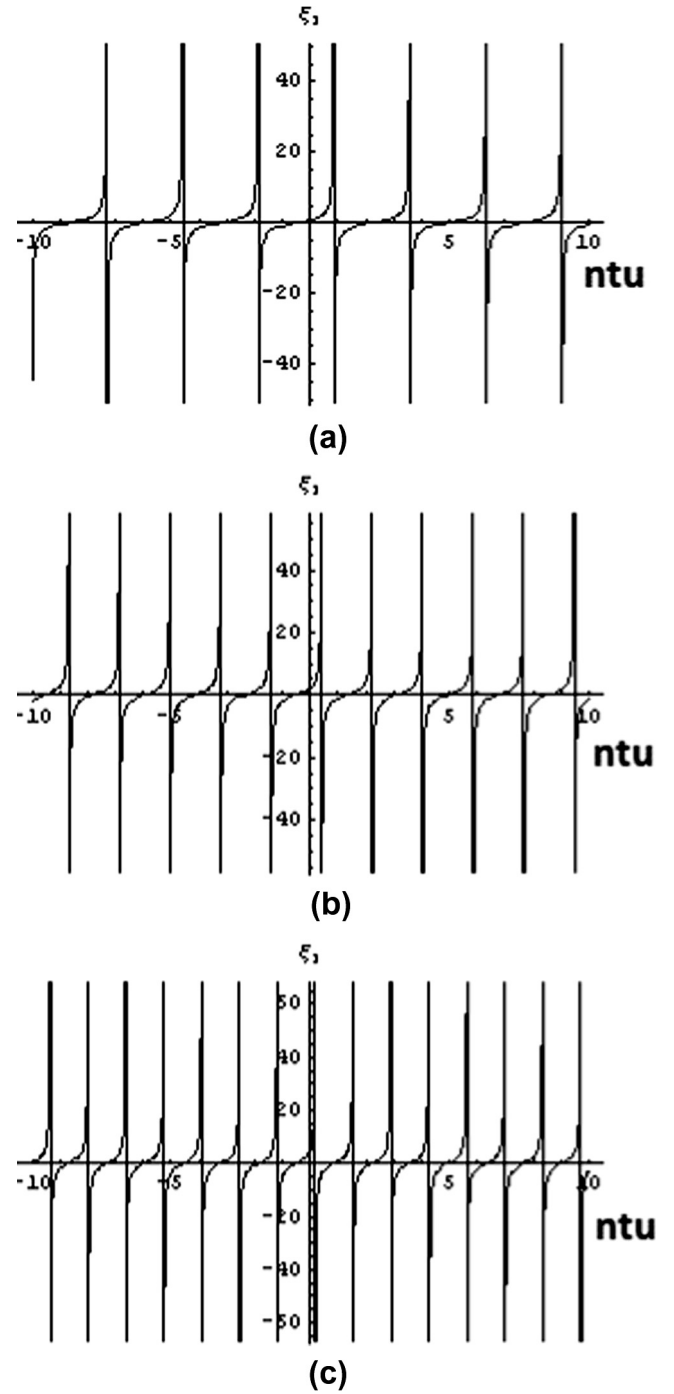
$$\hat{\phi}_{11} = \text{sech}\eta(\theta - \theta_0) \tanh \eta(\theta - \theta_0), \quad (32)$$

$$\hat{\phi}_{12} = -\frac{1}{\eta} \left[ \text{sech}\eta(\theta - \theta_0) - \frac{3}{2}\eta(\theta - \theta_0)\text{sech}\eta(\theta - \theta_0) \tanh \eta(\theta - \theta_0) - \frac{1}{2} \tanh \eta(\theta - \theta_0) \times \sinh \eta(\theta - \theta_0) \right]. \quad (33)$$

Knowing the two particular solutions, the general solution for  $\hat{\phi}_1$  can be obtained through the formula

$$\hat{\phi}_1 = \delta_1 \hat{\phi}_{11} + \delta_2 \hat{\phi}_{12} - \hat{\phi}_{11} \int_{-\infty}^{\theta} \hat{\phi}_{12} \Re F d\theta' + \hat{\phi}_{12} \int_{-\infty}^{\theta} \hat{\phi}_{11} \Re F d\theta', \quad (34)$$

where,  $\delta_1$  and  $\delta_2$  are the arbitrary constants. Using Eqs. (32) and (33) and  $\Re F$  in Eq. (26) and after evaluating the integrals,



**Figure 2** Evolution of velocity of the EM soliton for the various values of amplitude (a)  $\eta = 2.0$ , (b)  $\eta = 3.0$  and (c)  $\eta = 4.0$  with  $c_0 = 0.5$ .

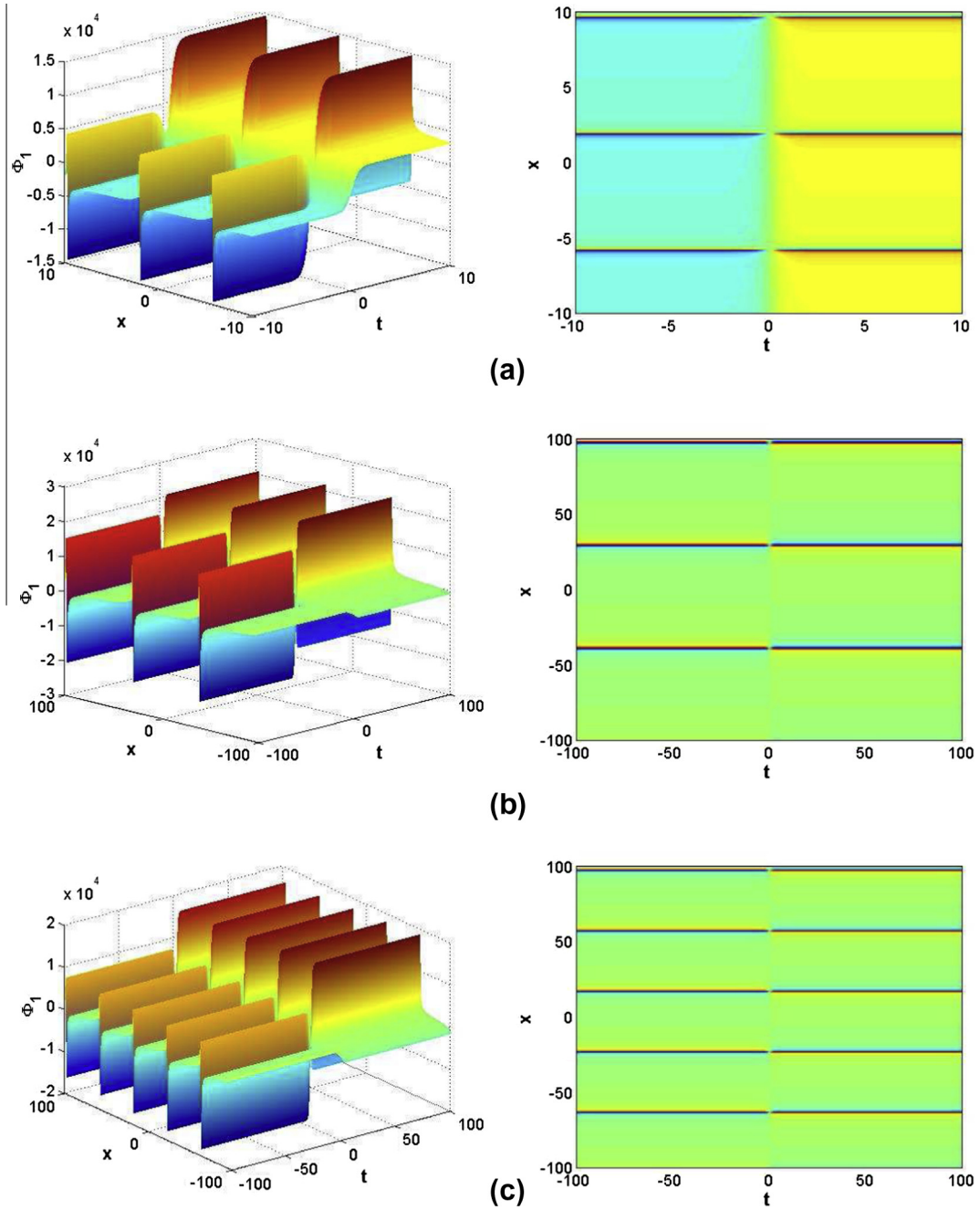
we obtain the general solution for  $\hat{\phi}_1$ . The solution contains the secular term which makes the solution unbounded can be removed by choosing

$$\delta_2 = 0. \quad (35)$$

Further, using the boundary conditions

$$\hat{\phi}_1(0)|_{\theta_0=0} = 0; \quad \hat{\phi}_{1\theta}(0)|_{\theta_0=0} = 0, \quad (36)$$

we obtain



**Figure 3** Real part ( $\hat{\phi}_1$ ) of the perturbed EM spin soliton for the parametric values (a)  $\eta = 1.5$ , (b)  $\eta = 0.8$  (c)  $\eta = 0.8$ .  $\lambda = 0.1$  on all plots.

$$\delta_1 = \frac{5}{12}. \quad (37)$$

Using Eqs. (35) and (37), the explicit form of  $\hat{\phi}_1$  is constructed and is given by

$$\hat{\phi}_1 = \frac{1}{6}\eta(\theta - \theta_0)\text{sech}\eta(\theta - \theta_0) - \left[ \frac{1}{4}(\xi\theta_{0T} + \sigma_{0T})(\theta - \theta_0) + \frac{2}{3}\eta(\theta - \theta_0) \right] \times \text{sech}\eta(\theta - \theta_0)\tanh\eta(\theta - \theta_0). \quad (38)$$

In a similar fashion, the solution for  $\psi_1$  can be evaluated by solving Eq. (25). The homogeneous part of Eq. (25) admits two particular solutions  $\hat{\psi}_{11}$  and  $\hat{\psi}_{12}$  which are of the form

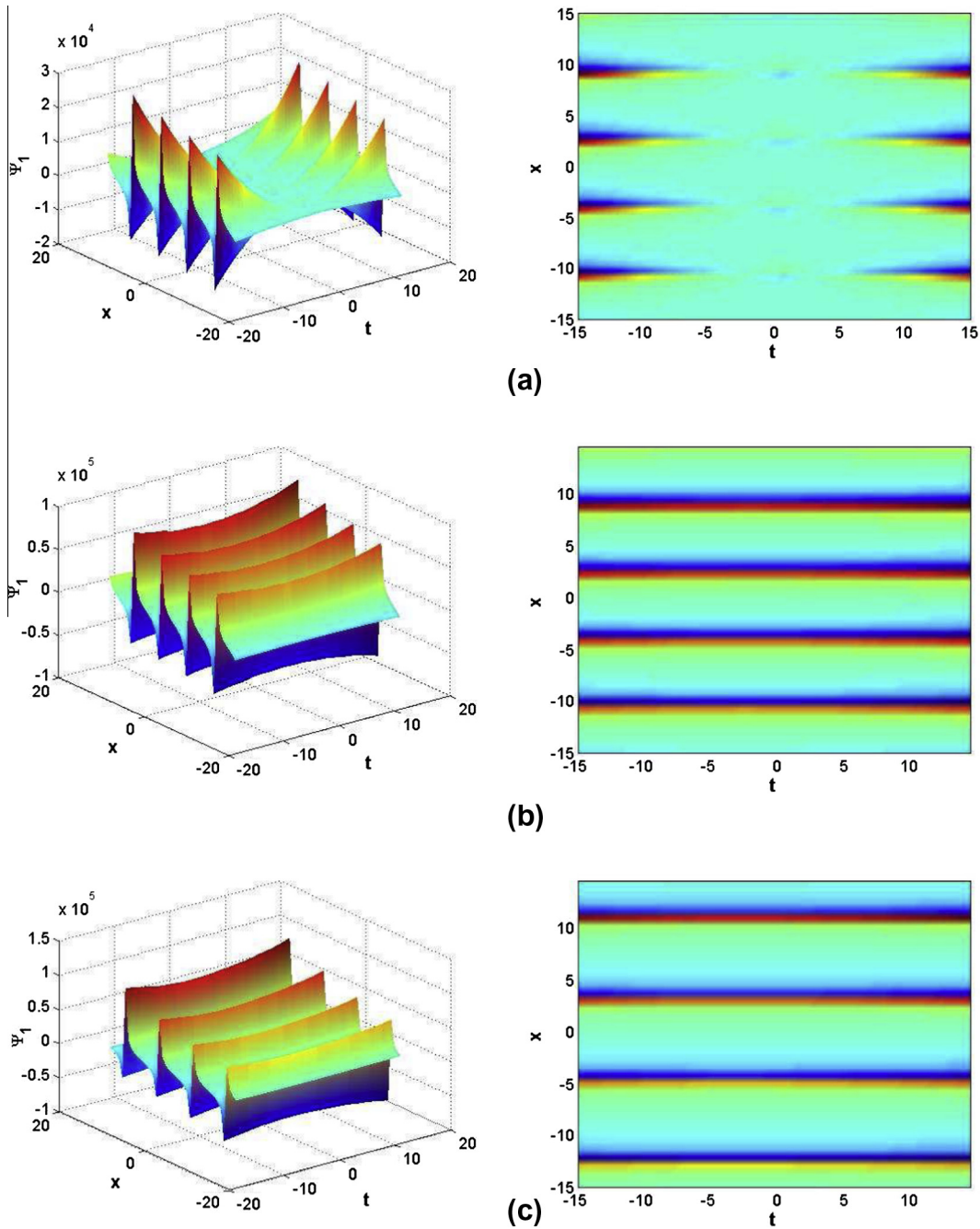
$$\hat{\psi}_{11} = \text{sech}\eta(\theta - \theta_0), \quad (39)$$

$$\hat{\psi}_{12} = \frac{1}{2\eta}[\eta(\theta - \theta_0)\text{sech}\eta(\theta - \theta_0) + \sinh\eta(\theta - \theta_0)]. \quad (40)$$

Knowing the two particular solutions, the general solution for  $\hat{\psi}_1$  can be obtained through the following

$$\hat{\psi}_1 = \delta_3\hat{\psi}_{11} + \delta_4\hat{\psi}_{12} - \hat{\psi}_{11} \int_{-\infty}^{\theta} \hat{\psi}_{12} \Im F d\theta' + \hat{\psi}_{12} \int_{-\infty}^{\theta} \hat{\psi}_{11} \Im F d\theta', \quad (41)$$

where,  $\delta_3$  and  $\delta_4$  are the arbitrary constants. By substituting Eqs. (39) and (40) in Eq. (41) and after evaluating the integrals, we obtain the general solution for  $\hat{\psi}_1$ . However, the solution



**Figure 4** Imaginary part ( $\hat{\psi}_1$ ) of the perturbed EM spin soliton for the parametric values (a)  $\eta = 0.9$ , (b)  $\eta = 0.75$ , (c)  $\eta = 0.91$ .  $\lambda = 0.695$  on all plots.

contains the secular term which makes the solution unbounded is removed by choosing

$$\delta_4 = 0. \quad (42)$$

On using the boundary conditions

$$\hat{\psi}_1(0)|_{\theta_0=0} = 0; \quad \hat{\psi}_{1\theta}(0)|_{\theta_0=0} = 0, \quad (43)$$

we get

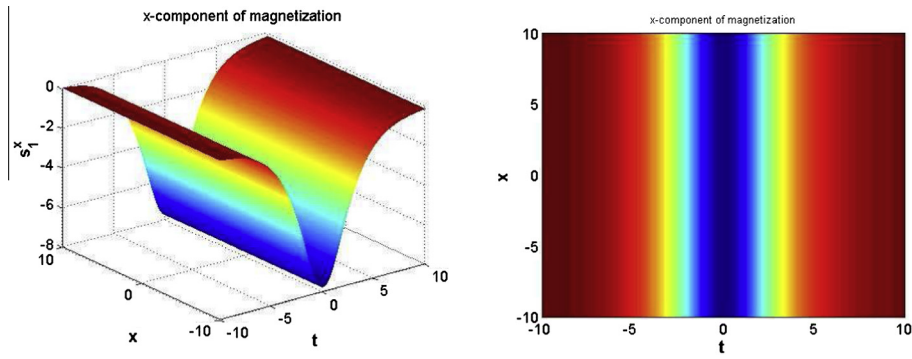
$$\delta_3 = 0. \quad (44)$$

Using Eqs. (42) and (44), the explicit form of  $\hat{\psi}_1$  is constructed and is given by

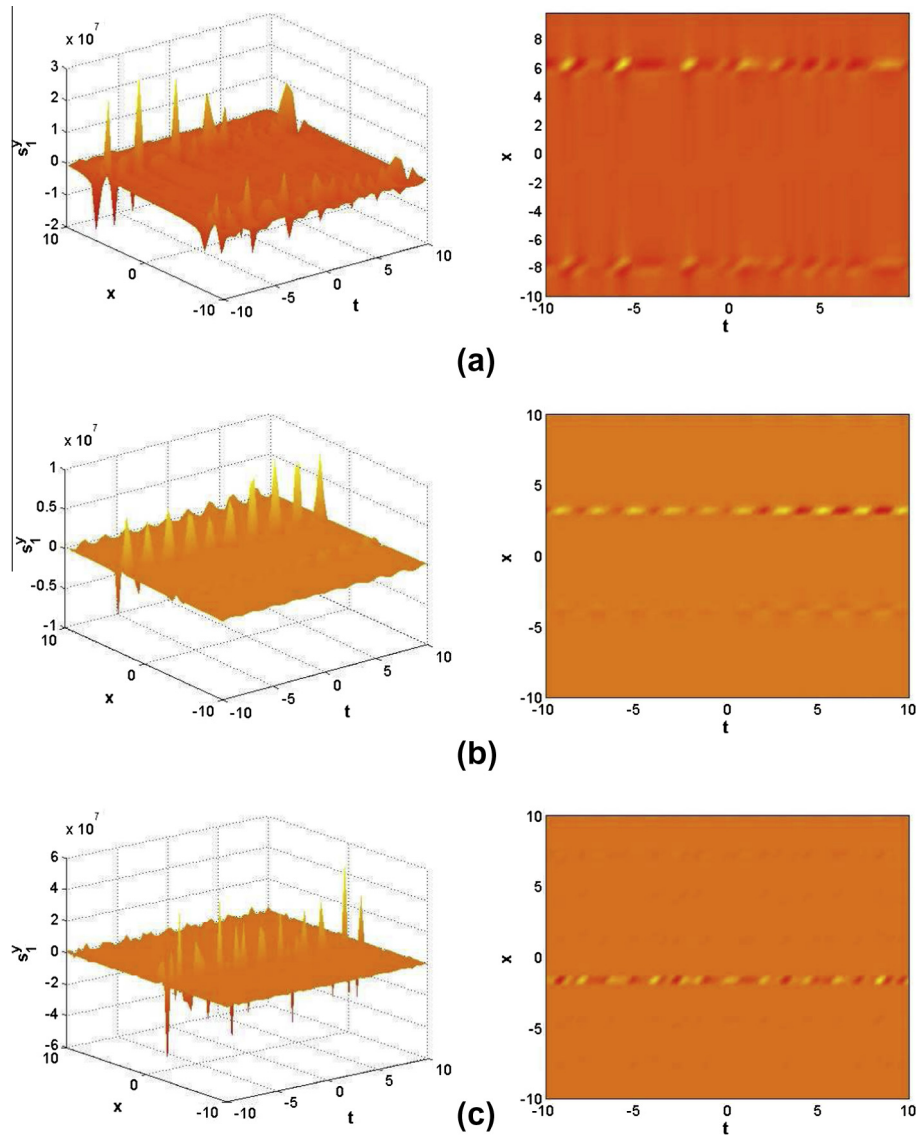
$$\hat{\psi}_1 = \left[ \frac{1}{2} \xi \eta (\theta - \theta_0)^2 - \frac{\eta}{4} (\theta - \theta_0)_T (\theta - \theta_0) \right] \text{sech} \eta (\theta - \theta_0). \quad (45)$$

Having obtained the explicit form of  $\hat{\phi}_1$  and  $\hat{\psi}_1$ , the first order perturbed soliton  $\hat{u}_1$  can be constructed through the relation  $\hat{u}_1 = \hat{\phi}_1 + i\hat{\psi}_1$ . The solution for  $\hat{u}_1$  is

$$\begin{aligned} \hat{u}_1 = & \frac{1}{6} \eta (\theta - \theta_0) \text{sech} \eta (\theta - \theta_0) - \left[ \frac{1}{4} (\xi \theta_{0T} + \sigma_{0T}) (\theta - \theta_0) \right. \\ & \left. + \frac{2\eta}{3} (\theta - \theta_0) \right] \text{sech} \eta (\theta - \theta_0) \times \tanh \eta (\theta - \theta_0) \\ & + i \left\{ \left[ \frac{\xi}{2} \eta (\theta - \theta_0)^2 - \frac{\eta}{2} (\theta - \theta_0)_T (\theta - \theta_0) \right] \text{sech} \eta (\theta - \theta_0) \right\}. \quad (46) \end{aligned}$$



**Figure 5** Propagation of  $x$ -component ( $S_1^x$ ) of EM spin component with  $\eta = 0.5$  and  $\lambda = 0.001$ .

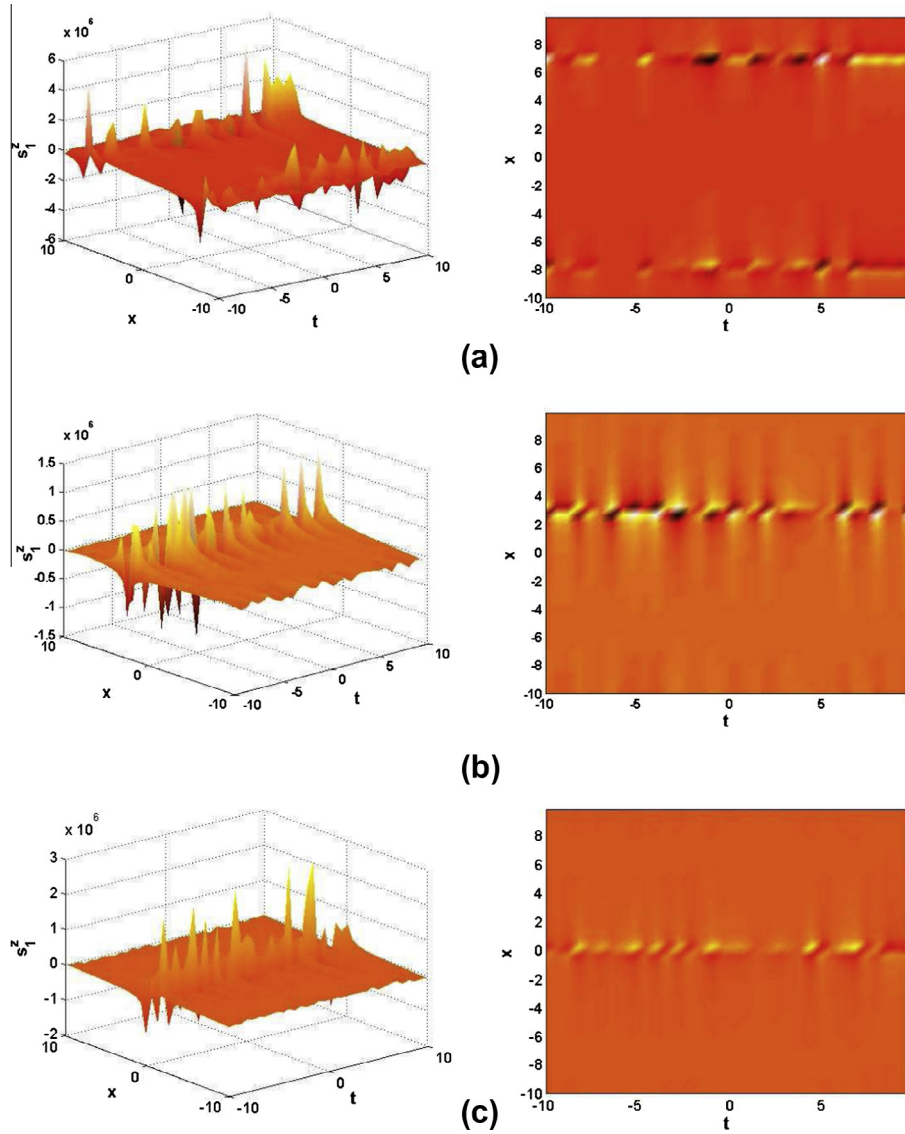


**Figure 6** Propagation of  $y$ -component ( $S_1^y$ ) of EM spin component with (a)  $\eta = 3.9$ , (b)  $\eta = 2.87$ , (c)  $\eta = 2.87$  with  $\lambda = 0.652$ .

To validate the presence of magnetization reversal through the EM spin soliton, we have plotted the real and imaginary part of the perturbed soliton solutions for the specific choices of parameters in Figs. 3 and 4. Fig. 3 exploits the real part ( $\phi_1$ )

of the perturbed soliton through multiple flipping and its amplitude exhibits an oscillating behavior both in positive and negative directions. In the corresponding contour plots shown aside, the yellow and blue regions represent the zero amplitude of the





**Figure 7** Propagation of  $z$ -component ( $S_z^1$ ) of EM spin component with (a)  $\eta = 1.22$ , (b)  $\eta = 1.22$ , (c)  $\eta = 1.372$  with  $\lambda = 0.3$ .

soliton and dark line represents the flipping region. Again when the amplitude of the flipping soliton is higher, it enhances the speed of the switching soliton as portrayed in Fig. 3b and c. A similar trend is observed for the imaginary part ( $\hat{\psi}_1$ ) of the perturbed EM soliton for the parametric choices  $\lambda = 0.1$  and  $\eta = 0.9, 0.75, 0.91$  as shown in Fig. 4 which depict the existence of multiple peaks of the switching soliton supporting the reversal in the ferromagnetic medium. The imaginary part of the perturbed soliton indicates that the amplitude of the soliton increases to negative maximum and then flips to the positive maximum representing a 3-dimensional equivalent version of the velocity evolution as shown in Fig. 2. This soliton flipping occurs in the medium indefinitely thereby confirming the magnetization reversal in the spin lattice.

$$S_1^x = -\frac{1}{2S_0} (\hat{u}_0^2 + 2\lambda\hat{u}_0\hat{\phi}_1), \quad (47)$$

$$S_1^y = (\hat{u}_0 + \lambda\hat{\phi}_1) \cos \alpha - \lambda\hat{\psi}_1 \sin \alpha, \quad (48)$$

$$S_1^z = -(\hat{u}_0 + \lambda\hat{\phi}_1) \sin \alpha - \lambda\hat{\psi}_1 \cos \alpha \quad (49)$$

where,  $\alpha = \xi(\theta - \theta_0) + (\sigma - \sigma_0)$  and  $\hat{u}_0 = \eta \text{sech} \eta(\theta - \theta_0)$ . The spin density distribution of the magnetization is shown in Fig. 5. The  $x$  component of magnetization shows antikink soliton profile (Fig. 5a) whereas the  $y$  component of the EM spin soliton shows multiple switching of soliton admitting a periodic behavior with amplitudes fluctuating in the positive and negative directions as depicted in Fig. 6. In the contour plots, the dark red and yellow spots show the positive and negative amplitude fluctuating regions in contrast, the uniform plain region indicates the zero amplitude of the switching EM spin soliton. The  $z$  component of the EM spin soliton (Fig. 7) depicts a similar trend as predicted for the  $y$  component of the EM soliton. The dark spots shown in contour plots of Fig. 7 represent the switching EM spin soliton whereas the plain uniform region shows the zero amplitude of the soliton. Thus from the above EM spin soliton flipping phenomenon which leads to magnetization reversal in a ferromagnetic medium is expected to have potential applications in magnetic memories and recording.

## 5. Conclusion

In summary, the nonlinear spin dynamics of the one-dimensional site-dependent bilinear anisotropic ferromagnet under the influence of electromagnetic field is well established through perturbed nonlinear Schrödinger equation. Using an effective reductive perturbation method and multiscale perturbation analysis, we have unambiguously demonstrated the magnetization reversal dynamics in the ferromagnetic medium and find that the velocity of soliton undergoes magnetization reversal behavior in the nanosecond regime due to the presence of inhomogeneity in the form of linear function, whereas the amplitude of the soliton remains constant. The reversal process is further confirmed by the perturbed soliton solution obtained through perturbation analysis which shows amplitude fluctuations in both positive and negative directions. In addition, the spin component of the magnetization exhibits the variation of magnetization in the form of EM spin soliton.

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