Calculation for physical and chemical exergy of flows in systems elaborating mixed-phase flows and a case study in an IRSOFC plant

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SUMMARY

The paper deals with the calculation of physical and chemical exergy of flows in systems elaborating mixed-phase flows, such as steam methane reforming and coal gasification systems. The flows involved are mixtures of gases, which can be treated as ideal gases, and steam. The mixtures in which the steam can be treated as ideal gas and those in which the steam cannot be treated as ideal gas are considered separately. As a case study, the calculation is used to evaluate the physical and chemical exergy content of the flows of a system composed by a pressurized internal reforming solid oxide fuel cell (IRSOFC) combined with a gas turbine. Finally, a thermoeconomic analysis of the system is made. Copyright © 2004 John Wiley & Sons, Ltd.

1. INTRODUCTION

Hydrogen, as an interesting option of secondary energy resource, is studied by many researchers. In the study of its production and utilization, exergy and thermoeconomic analysis are often used. The evaluation of the exergy content of the mass flows is a main factor that influence the results of the exergy and economic analysis also. Steam methane reforming and coal gasification are both methods to produce hydrogen. Some researchers have also proposed to integrate methane reforming, fuel cell and gas turbine to produce electricity. In this kind of system, the involved mass flows are often mixtures of gases and steam. The steam can be treated as ideal gas in some conditions, but in some other treating the steam as ideal gas may result in great errors in the evaluation of exergy, which will further influence the results of thermoeconomic analysis. Because of chemical reaction, combustion processes, mixing and

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separation in the system, the composition of the flows are different. In the first part of the paper, both the physical and the chemical exergy are calculated with the steam considered as ideal or real gas according to the state of the steam. In the second part of the paper, the evaluation procedure of the physical and chemical exergy is used to determine the exergy content of the flows in a system (called FCGT4 plant) composed by a pressurized internal reforming solid oxide fuel cell (IRSOFC) integrated with a gas turbine. With the exergy results, a rigorous thermoeconomic analysis of the system is made.

2. PHYSICAL, CHEMICAL, AND TOTAL EXERGY

Considering a system included in the environment (both the kinetic and the potential energy are equal to zero), the physical exergy is the maximum theoretical useful work obtainable as the system passes from its actual state with temperature T and pressure p to the restricted dead state with temperature T_0 and pressure p_0, as stated in Bejan et al. (1996). In this paper 1 atm and 298.15 K are selected for p_0 and T_0. The chemical exergy is the maximum theoretical useful work obtainable as the system passes form the restricted dead state to the dead state where it is in complete chemical equilibrium with the environment (Bejan et al., 1996). The total exergy is the sum of the physical and the chemical exergy (Bejan et al., 1996; Bejan, 1988). For a mixture of N components, it can be expressed as follows:

$$\bar{b}_t = \bar{b}_{ph} + \bar{b}_{ch} \tag{1}$$

where the physical exergy can be calculated according to Bejan (1998):

$$\bar{b}_{ph} = \bar{h} - \bar{h}^* - T_0(\bar{s} - \bar{s}^*) \tag{2}$$

where, the symbols with asterisk represent parameters at the restricted dead state (asterisks will be also used later in the paper for indicating the restricted dead state), and

$$\bar{h} = \sum_{i=1}^{N} \bar{h}_i \tag{3}$$

$$\bar{h}^* = \sum_{i=1}^{N} \bar{h}_i^* \tag{4}$$

$$\bar{s} = \sum_{i=1}^{N} \bar{s}_i \tag{5}$$

$$\bar{s}^* = \sum_{i=1}^{N} \bar{s}_i^* \tag{6}$$

and the chemical exergy can be calculated according to Bejan (1998):

$$\bar{b}_{ch} = \sum_{i=1}^{N} (\mu_i^* - \mu_{0,i}) y_i \tag{7}$$

If the components of the mixture can be considered as ideal gases, Equation (7) becomes

$$\bar{b}_{ch} = \sum_{i=1}^{N} [\mu_i^*(y_i = 1) - \mu_{0,i} + \mu_i^* - \mu_i^*(y_i = 1)]y_i$$

$$= \sum_{i=1}^{N} \left(\bar{b}_{ch,i} + \bar{R}T_0 \ln \frac{p_i^*}{p_0} \right) y_i$$

$$= \sum_{i=1}^{N} y_i \bar{b}_{ch,i} + \bar{R}T_0 \sum_{i=1}^{N} y_i \ln y_i \tag{8}$$

When there is water vapour in the mixture at the restricted dead state, but no liquid water, Equation (8) can also be used to calculate chemical exergy of the mixture (Bejan *et al.*, 1996).

3. CALCULATION FOR MIXTURES OF GASES AND STEAM THAT CAN BE TREATED AS IDEAL GAS

The mixture considered in this paper is composed of gases, which can be treated as ideal gases, and steam (at different state saturated or superheated). The temperature and the pressure of the mixture are T and p. The mole fractions are $y_i (i = 1, 2, \ldots, N)$, where y_1 is the mole fraction of the steam. If y_1 is very small, and then the partial pressure of the steam is very low, then the steam can be treated as an ideal gas. In this paper, when $y_1 \leq 0.1$, the steam is treated as an ideal gas. Otherwise, the steam is treated as a real gas.

When the considered mixture at T and p is brought to the restricted dead state (T_0, p_0), some of the steam may be condensed to liquid water, so the steam may be at superheated or at saturated state. In the latter condition, the steam is composed of saturated vapour and saturated liquid water. In other words, at the restricted dead state, the steam in the mixture could be at different state: superheated and saturated. For the calculation of physical and chemical exergy of the mixture, whether the steam in the mixture at the restricted dead state is superheated or saturated and, in the latter condition, the mole fraction of vapour and liquid water, should be determined first.

3.1. Determination of the state of the steam in the mixture at the restricted dead state

In the mixture at the restricted dead state whether the steam can be treated as an ideal gas should be analysed first. We can obtain that, at the temperature T_0 of the restricted dead state, the saturated pressure of steam is $p_{s0} = 0.03169$ bar, and the specific volume of the saturated steam is $v_{s0} = 43.360 \, \text{m}^3/\text{kg}$. Using equation $Z_{v0} = p_{s0} v_{s0} / R_v T_0$, we can obtain that the compressibility factor Z_{v0} of saturated steam at T_0 is equal to 0.9978, nearly equal to 1, so if the steam in the mixture at the restricted dead state is saturated, it can be treated as an ideal gas. Furthermore, if it is superheated, it can also be treated as an ideal gas.

It is well know that, in the humid air at a specified temperature and pressure, there can exist a maximum amount of water vapour, at the saturated state. In the mixture at the restricted dead state, there can also exist a maximum amount of water vapour, at the saturated state. For the

mixture containing the maximum amount of water vapour, but no liquid water, if the pressure of the saturated water vapour is designated by p_{s0}, then the mole fraction of the water vapour is

$$y_{v,s}^* = \frac{p_{s0}}{p_0} = \frac{0.03169}{1.01325} = 0.031 \tag{9}$$

This is the maximum mole fraction of water vapour in the mixture at the restricted dead state. Therefore, if the mole fraction of the steam in the considered mixture at T and p is greater than this value, i.e. $y_1 > 0.031$, then some steam will be condensed when the mixture is brought to the restricted dead state, while if $y_1 \leq 0.031$, then no steam will be condensed. Therefore, when $y_1 \leq 0.031$, the mixture at the restricted dead state consists of gases and steam, which can be treated as ideal gas as explained at the beginning of this part. When $y_1 > 0.031$, the mixture at the restricted dead state will contain gases, saturated water vapour, and saturated liquid water. The mole fraction of the vapour can be obtained according to the Dalton's law, and then the mole fraction of the liquid water can be easily obtained. In this way, the state of the steam in the mixture at the restricted dead state is determined.

3.2. Calculation of physical exergy

The state of the mixture at the restricted dead state has been already determined, so physical exergy can be calculated according to Equations (2)–(6), in which enthalpy and entropy of the components are calculated as described in the following.

For the gases in the mixture, the enthalpy and the entropy are calculated according to Gyftopoulos and Beretta (1991) and Reid *et al.* (1987):

$$\bar{h}_i = aT + \tfrac{1}{2}\,bT^2 + \tfrac{1}{3}\,cT^3 + \tfrac{1}{4}\,dT^4 \tag{10}$$

$$\bar{s}_i = a\ln T + bT + \tfrac{1}{2}\,cT^2 + \tfrac{1}{3}\,dT^3 - \bar{R}\ln(y_i p) \tag{11}$$

where $\bar{h}_i^*$ and $\bar{s}_i^*$ can be calculated in the same way.

For water and steam, enthalpy of formation and absolute entropy are selected (Bejan *et al.*, 1996). Enthalpy of formation and absolute entropy of the steam in the considered mixture at T and p can be calculated as follows:

$$\bar{h}_v = 10^3\left(H^+ + ay + \tfrac{b}{2}y^2 - cy^{-1} + \tfrac{d}{3}y^3\right) \tag{12}$$

$$\bar{s}_v = S^+ + a\ln T + by - \tfrac{c}{2}y^{-2} + \tfrac{d}{2}y^2 - \bar{R}\ln p_v \tag{13}$$

where, $y = 10^{-3}T$, and H^+, S^+, a, b, c, d are constants for a particular substance (Bejan *et al.*, 1996).

For the water vapour and the liquid water in the mixture at the restricted dead state, the enthalpy and the entropy can be obtained using steam table (Moran and Shapiro, 1995), and then be changed into enthalpy of formation and absolute entropy (Bejan *et al.*, 1996).

3.3. Calculation of chemical exergy

If the mixture at the restricted dead state is composed of gases and water vapour (no liquid water), its chemical exergy can be calculated according to Equation (8), where the standard chemical exergy of every component can be obtained from tabulated data, according to Kotas (1985).

If the mixture at the restricted dead state is composed of gases, water vapour, and liquid water, the chemical exergy of the mixture should be the sum of the chemical exergy of the liquid phase and the gas–vapour phase. The chemical exergy of the gas-vapour phase can be calculated as follows (Bejan *et al.*, 1996):

$$\bar{b}^*_{\text{ch,g+v}} = \sum_{i=1}^{N} y^*_{i,\text{g+v}} \bar{b}_{\text{ch},i} + \bar{R}T_0 \sum_{i=1}^{N} y^*_{i,\text{g+v}} \ln y^*_{i,\text{g+v}} \quad (14)$$

where $y^*_{i,\text{g+v}}$ represents the mole fraction of each component in the gas–vapour phase in the mixture at the restricted dead state. The chemical exergy of the mixture is (Bejan *et al.*, 1996):

$$\bar{b}_{\text{ch}} = y^*_{\text{g+v}} \bar{b}^*_{\text{ch,g+v}} + y^*_{\text{w}} \bar{b}_{\text{ch,w}} \quad (15)$$

where $y^*_{\text{g+v}}$ is the mole fraction of the gas–vapour phase in the mixture at the restricted dead state, while y^*_{w} is that of the liquid water in the same state; the standard chemical exergy of liquid water can be obtained from tabulated data, according to Kotas (1985).

4. CALCULATION FOR MIXTURES OF IDEAL GASES AND STEAM THAT CANNOT BE TREATED AS IDEAL GAS

For calculating physical and chemical exergy, the state of the steam must be determined first.

4.1. Determination of the state of the steam in the mixture

As in the mixture at the restricted dead state, in the mixture at (T, p) there can exist a maximum amount of water vapour, represented by the amount at the saturated state. This maximum amount will be calculated with Equations (16)–(20):

Additive pressure rule (Moran and Shapiro, 1995):

$$p = p_{\text{s}} + \sum_{i=2}^{N} p_i \quad (16)$$

Relation between saturated pressure and saturated temperature for water:

$$p_{\text{s}} = f_1(T) \quad (17)$$

p–v–T relations for gases and steam (Moran and Shapiro, 1995):

$$\bar{v} = f_2(y_{\text{s}}, p_{\text{s}}, T) \quad (18)$$

$$p_i \bar{v} = y_i \bar{R} T \quad (i = 2, 3, \ldots, N) \quad (19)$$

$$p_{\text{s}} \bar{v} = y_{\text{s}} z_{\text{v}} \bar{R} T \quad (20)$$

where $\bar{v}$ represents the molal volume of the mixture. Equations (17) and (18) can be obtained using the steam table. In these equations, the variables p_{s}, p_i ($i = 2, 3, \ldots, N$), $\bar{v}$, y_{s}, and z_{v}, are unknown. There are $N + 3$ unknown variables. Equations (16)–(20) represent $N + 3$ equations. Therefore y_{s} can be obtained. It is the maximum mole fraction of water vapour in the mixture.

If the mole fraction of the steam in the mixture is less than the maximum value, i.e. $y_1 < y_{\text{s}}$, then the steam is superheated, and the partial pressure of the steam can be obtained using Equations (16), (18)–(20), in which p_{s} is substituted by p_{v}, and y_{s} is substituted by y_1. If $y_1 = y_{\text{s}}$,

then the steam is saturated water vapour. If $y_1 > y_s$, then the steam is at saturated state with purity less than 1, i.e. there are saturated water vapour and saturated liquid water in the mixture. The mole fraction of saturated water vapour y_v can be obtained using Equations (16)–(20), in which y_s is substituted by the mole fraction of the water vapour in the gas–vapour phase, i.e. $y_v/(1-y_1+y_v)$. The mole fraction of liquid water is $y_w = y_1 - y_v$.

4.2. Calculation of physical and chemical exergy

The state of the mixture at the restricted dead state can be determined as described in Section 3.1.

The enthalpy and the entropy of steam and water in the considered mixture are obtained using the steam table, and then the values are changed into enthalpy of formation and absolute entropy.

The physical exergy is calculated according to Equations (2)–(6).

Chemical exergy can be calculated as described in Section 3.3.

5. CASE STUDY: THERMOECONOMIC ANALYSIS OF AN FCGT4 SYSTEM

The above calculation of physical and chemical exergy can be used to evaluate the exergy content of flows composed of ideal gases and steam. It can be applied to systems containing this typology of flows, and it is useful in the thermoeconomic analysis of the system. A thermoeconomic analysis of one of these systems has been performed in the following.

5.1. Description of the system

The system is a model of advanced plant, based on solid oxide fuel cell technology (proposed as FCGT4 plant in Massardo and Lubelli, 2000): it is composed of a pressurized internal reforming solid oxide fuel cell (IRSOFC) combined with a gas turbine. The plant could represent a target of new procedures of power generation based on electrochemical conversion with low CO_2 emissions. Figure 1 shows the physical scheme of the plant. Water flow F33 is pumped to the economizer 13, heated and then evaporated in the evaporator 12 and the superheater 11 by the turbine exhaust gases. The superheated steam is then separated into two streams: one is sent to the steam/CH_4 mixer 2, and the other is sent to the steam/gas mixer 9. Methane flow F21 is heated by the turbine exhaust gases in the CH_4 heater 1, and then it is mixed with superheated steam in steam/CH_4 mixer 2. The methane /steam mixture is sent to the reforming unit 3, where it is reformed into a syngas (flow F24), containing a large amount of hydrogen. The syngas enters the solid oxide fuel cell (SOFC) 5 as the anode fuel flow. The air flow F1 is compressed in the compressor 7, heated in the recuperator 8 and sent to the cathode of the fuel cell 5. The two exhaust flows leaving the fuel cell enter the combustion chamber 6. The combustion gas F5 is mixed with the steam flow F39 and then enters the gas turbine to produce electricity. The exhaust gas F8 from the turbine enters the combustion chamber 4, where a methane flow F27 is introduced to increase the gas temperature before the reforming reaction. The flue gas F9 from the chamber is used to supply the heat needed for the steam methane reforming, and also to pre-heat the feed water and the methane used for the reforming. The main operating conditions of the plant are summarized in Table I, and the main parameters are listed in Table II.

Figure 1. FCGT4 plant layout.
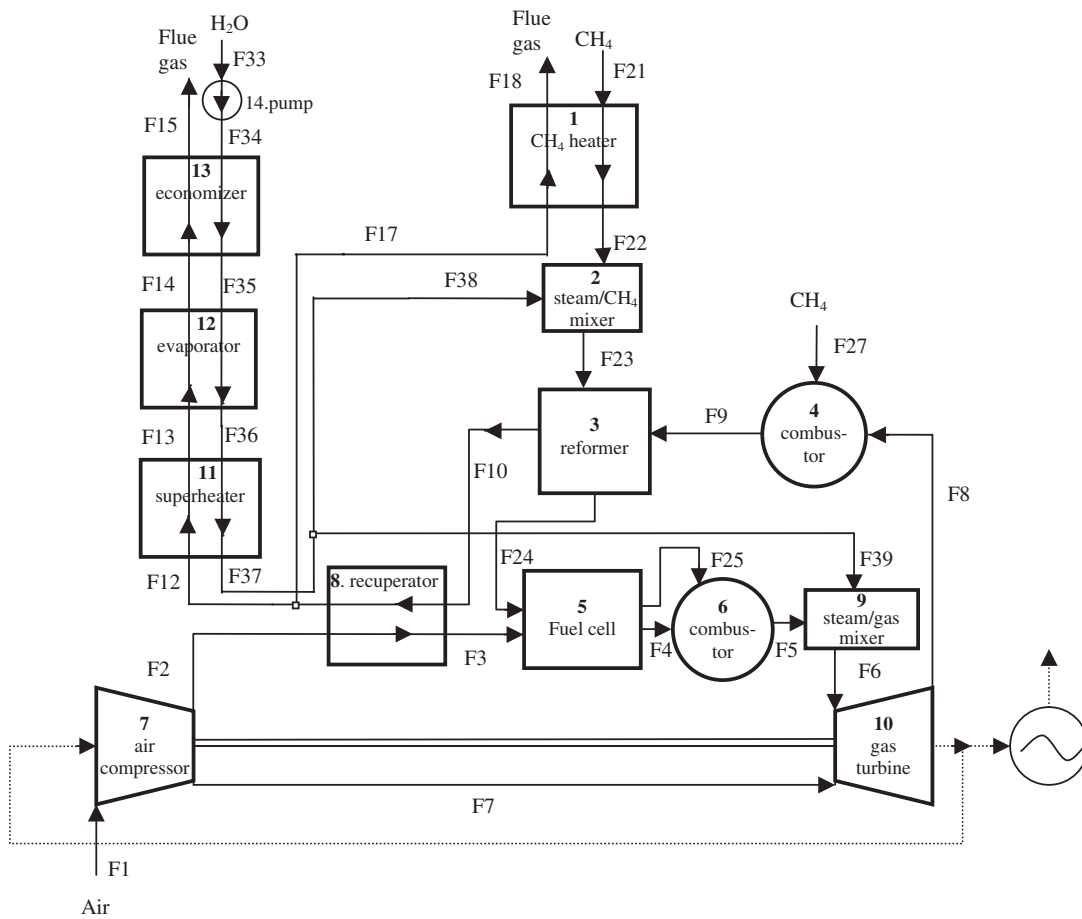
flows	F1—F3: air	F8: flue gas, air	F23: CH_4, H_2O	F25: outlet gas of fuel cell
	F4: outlet gas of fuel cell	F9—F18: flue gas	F24: H_2, CO	
	F6: flue gas	F21,F22,F27: CH_4	F33—F39: H_2O	Dot lines: power flow

Table I. Operating conditions of FCGT4 plant.

Fuel cell		Gas turbine		Total plant	
W_{el} (MW)	110	W_{el} (MW)	32	W_{el} (MW)	142
T (°C)	1000	Pressure ratio	7.5		
p (MPa)	0.56	TIT (T_6) (°C)	1196		
$G_{CH4/steam}$ (G_{23})(kg/s)	11.5	Exhausts temperature T_8 (°C)	767		
G_{air} (G_3) (kg/s)	89.3	G_{gas} (G_6)(kg/s)	105.9		

Table II. Parameters of the system.

Flows	1	2	3	4	5	6	7	8	9	10
Mass flow (kg/s)	100	89.3	89.3	77.4	100.8	106	10.7	116.6	117	117
Temperature (K)	288	548.6	900	1290	1527	1469	548.7	1040	1160	928
Pressure (bar)	1.013	7.597	7.37	5.363	5.202	5.202	7.597	1.129	1.095	1.062
	12	13	14	15	17	18	21	22	23	24
Mass flow (kg/s)	107.1	107.1	107.1	107.1	9.93	9.93	3.5	3.5	11.5	11.5
Temperature (K)	702	654	451	394	702	398	298	675	673	1119
Pressure (bar)	1.031	1.029	1.017	1.013	1.031	1.013	6	5.82	5.82	5.645
	25	27	33	34	35	36	37	38	39	
Mass flow (kg/s)	23.37	0.42	13.11	13.11	13.11	13.11	13.11	8	5.11	
Temperature (K)	1295	298	298	298	433	433	683	683	683	
Pressure (bar)	5.363	1.581	1.013	9	8.1	8.1	7.452	7.45	7.45	

5.2. Thermoeconomic analysis

5.2.1. Thermodynamic analysis. A thermoeconomic functional analysis of the system has been performed according to the methods described in Frangopoulos (1987), Valero *et al.* (1994), Lozano and Valero (1993). The functional diagram of the system is described in Figure 2. In this figure there are real components, which are represented by rectangular blocks, and model components: junctions and branchings. Junctions are represented by rhombus, while branchings are represented by small circles. For each component, there are input (Fuel) and output (Products) flows, which are exergy or negentropy flows. The real components 1–14 correspond to those in Figure 1, while the real components 15 and 16 represent the alternator and the chimney, respectively. The junctions are used to 'join' several flows from a thermo-economic point of view, while the branchings are used to separate one flow into several. In the functional diagram, the symbol B followed by a number represents the exergy of the flow designated by the number. The symbol S represents negetropy: S with a number before represents the negentropy entering a component designated by the number, while S with a number after represents the negentropy exiting a component designated by the number. The symbol W and a number following it represents the work produced or needed by the component designated by the number.

The first step of the thermoeconomic analysis is a thermodynamic analysis including the calculation of the parameters listed in Table III.

The results of the thermodynamic analysis are listed in Table IV. In the bottom of the table, the total fuel, product, irreversibility, and the overall exergetic efficiency of the plant are listed.

From Table IV, we can see that heat exchangers 11, 12, 13, and combustor 4 have low efficiencies. The efficiency of the economizer 13 is the lowest, about 59%, while that of the evaporator is about 71%, and the value of the superheater is about 79%. The average logarithmic temperature difference of these three components are, respectively, 46.72, 78.53, and 79.93°C. The above data shows that the temperature difference is not sufficient to compare the exergetic performance of different heat exchangers. For example, the temperature difference of
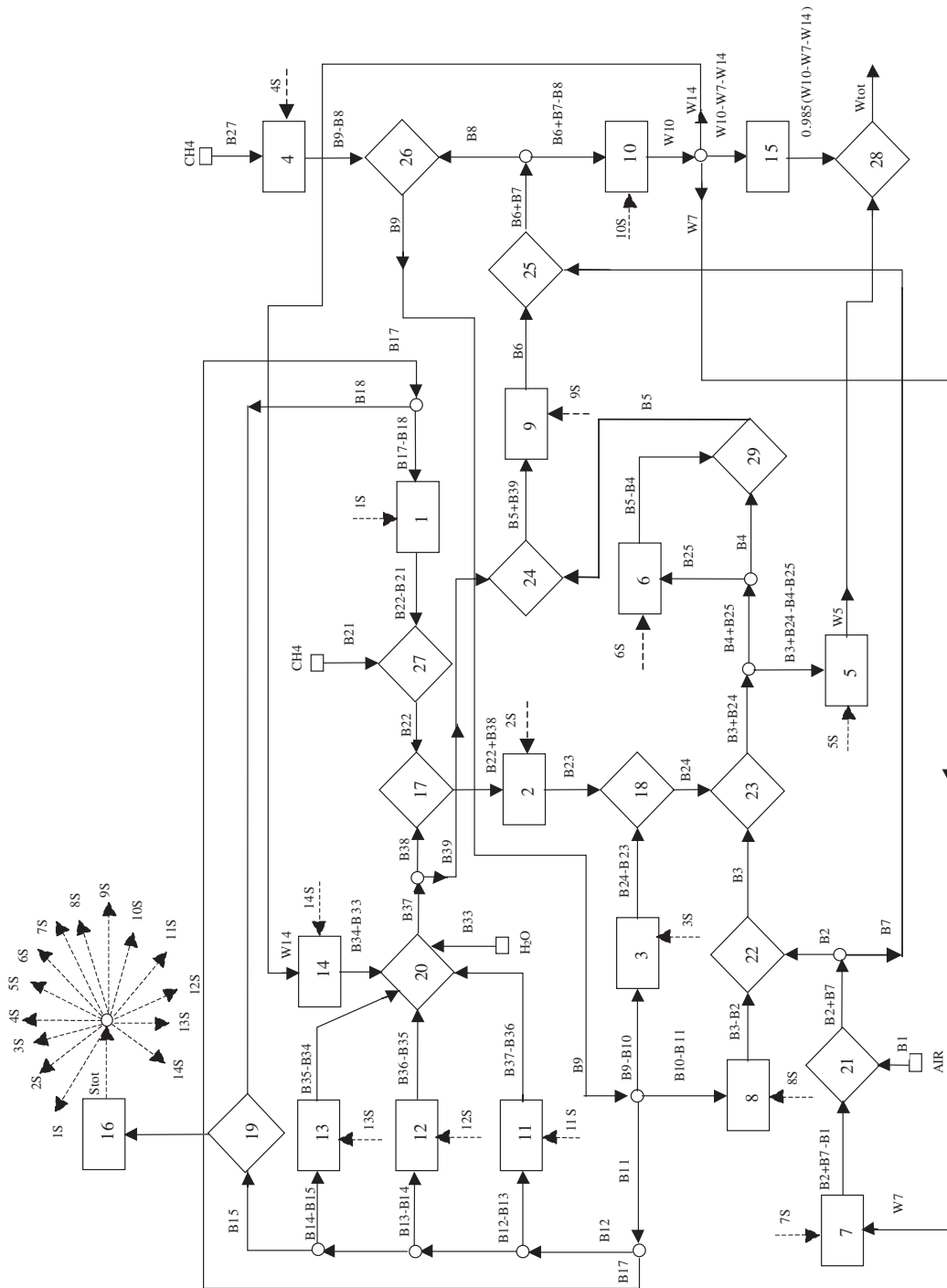
Figure 2. The functional diagram of the FCGT4 system.

Table III. Parameters in thermodynamic analysis.

Parameters	Equation of definition	Parameters	Equation of definition
Irreversibility	$I_i = F_i - P_i$	Fuel depletion rate	$\delta_i = \dfrac{I_i}{F_{Tot}}$
Exergy efficiency	$\eta_i = \dfrac{P_i}{F_i} = 1 - \dfrac{I_i}{F_i}$	Productivity lack	$\xi_i = \dfrac{I_i}{P_{Tot}}$
Relative irreversibility	$\chi_i = \dfrac{I_i}{I_{Tot}}$	Exergetic factor	$f_i = \dfrac{F_i}{F_{Tot}}$

Table IV. Results of the thermodynamic analysis of the plant.

Component	F(kW)	P(kW)	I(kW)	η	χ (%)	δ (%)	ξ (%)	f (%)
1. Heater CH$_4$	1718	1393	325	0.811	0.6	0.2	0.2	0.8
2. Mixer	194 364	192 731	1634	0.992	3.1	0.8	1.2	93.3
3. Reformer	27 276	24 535	2742	0.899	5.2	1.3	1.9	13.1
4. Combustor	21 928	15 575	6354	0.710	11.9	3.1	4.5	10.5
5. Fuel cell	126 228	109 900	16 328	0.871	30.7	7.8	11.5	60.6
6. Combustor	72 698	63 373	9325	0.872	17.5	4.5	6.6	34.9
7. Compressor	26 310	24 460	1850	0.930	3.5	0.9	1.3	12.6
8. Recuperator	22 581	20 044	2537	0.888	4.8	1.2	1.8	10.8
9. Mixer	130 266	129 368	898	0.993	1.7	0.4	0.6	62.5
10. Turbine	642 39	58 840	5399	0.916	10.1	2.6	3.8	30.8
11. Superheater	3798	2987	811	0.786	1.5	0.4	0.6	1.8
12. Evaporator	12 623	9006	3616	0.713	6.8	1.7	2.5	6.1
13. Economizer	2237	1316	922	0.588	1.7	0.4	0.6	1.1
14. Pump	12.43	10.65	1.78	0.857	0.003	0.001	0.001	0.006
15. Alternator	32 517	32 030	488	0.985	0.92	0.23	0.34	15.61
	F_{Tot}(kW)	P_{Tot}(kW)	I_{Tot}(kW)	P/F_{Tot}	I/F_{Tot}	I/P_{Tot}		
	208 317	141 930	5230	0.681	0.256	0.375		

the economizer is 46.72°C, smaller than that of the evaporator and superheater, but its efficiency is lower than the other two. Therefore, a heat exchanger with a low exergy efficiency may have also a low temperature difference, and it does not worth to decrease it. Although the three heat exchangers have lower efficiencies, they have very low relative irreverisibility, as they handle small amount of exergy, (they have very low exergetic factor f_i). Large values of relative irreversibility lies in the fuel cell 5, combustor 6, combustor 4, and gas turbine 10. This is mainly caused by the high amount of the exergy handled by the components, except for combustor 4, which has a low efficiency. Through the analysis, it is outlined that the components with high irreversibility rates have also high efficiency values (except component 4), while the components with low efficiency values have low relative irreversibility; therefore, the low efficiency components have little influence on the overall efficiency of the plant.

5.3. Thermoeconomic analysis

5.3.1. Unit exergetic costs and unit thermoeconomic costs. The unit exergetic costs and the unit thermoeconomic costs are evaluated. In the calculation of the thermoeconomic cost, both the

cost of the external resources consumed and the recovery investment and maintenance cost of the components are considered. In the analysis of the system the cost of the residual exergy is assigned to every component in proportion to its entropy generation. Instead of exergy flow, negentropy flow is used for the distribution of the cost of the residue exergy.

Through the cost balance of every component, the matrix equation for the evaluation of the unit exergetic cost and the unit thermoeconomic cost is obtained.

The matrix equation for the evaluation of the unit exergetic costs of the products of every component is

$$K_P^* = [U_D - (kp)^t]^{-1} K_{ext} \tag{21}$$

where U_D is an identity matrix, and kp is an $n \times n$ matrix, with the elements $k_{ij}^* = \Psi_{ij}/\Psi_{pj}$. n is the number of real components and junctions: in this case it is equal to 29. Ψ_{ij} is the exergy flow from component i to j, while Ψ_{pj} is the product exergy of component j. K_{ext} is an $n \times 1$ matrix, in which the elements are $k_{0j} = \Psi_{0j}/\Psi_{pj}$. Ψ_{0j} is the exergy flow from the environment to component j.

The matrix equation for the evaluation of the thermoeconomic costs of the products of every components is

$$C_P^* = [U_D - (k_p)^t]^{-1} (C_{Pext} K_{ext} + R_{ext}) \tag{22}$$

where C_P^* is an $n \times 1$ matrix, with elements c_{pj}^* representing the unit thermoeconomic cost of the product of component j. U_D, k_p, and K_{ext} have the same meaning as in the equation of unit exergetic cost. C_{pext} is a $1 \times n$ matrix, with the elements c_{p0j}^* representing the unit cost of the exergy flow entering the component j from the environment. R_{ext} is an $n \times 1$ matrix, with the elements $R_j = Z_j/\Psi_{pj}$, where Z_j is the recovery of investment and maintenance cost of component j, while Ψ_{pj} is the exergetic product of the component j.

In the thermoeconomic analysis the following assumptions have been made: (1) hours of operation per year(h/yr): 8000; (2) years of plant operation: 20; (3) rate of interest i (%): 17.5; (4) fuel unit cost (€/kJ): 4.0; (5) the cost of the solid oxide fuel cell (SOFC) has been assumed to be 1538 €/kW; (6) the other components have market costs.

The results of the thermoeconomic analysis are listed in Tables V and VI, where f_p represents the exergoeconomic factor, which is defined by

$$f_{p,i} = Z_i/(Z_i + c_{F,i}^* I_i) \tag{23}$$

The economizer 13, the evaporator 12, the combustor 4, and the superheater 11 have high absolute and relative increase of unit exergetic cost, mainly for their low efficiency. Fortunately, the impact of these components on the cost of the final product of the system is small because of their very small exergetic factor.

The larger increase of unit thermoeconomic cost lies in fuel cell, pump, and economizer. The fuel cell has the largest increase of unit thermoeconomic cost: 4.042 c€/kWh, this is mainly due to its high investment cost. The pump has also a very large unit thermoeconomic cost: 3.041 c€/ kWh; this is because, although the total investment of the pump is not high, its product is small, but this has little influence on the product of the system. The economizer has also a large unit thermoeconomic cost, mainly because of its low efficiency, but it has little impact on the final product of the system because it handles a small amount of exergy.

Table V. Unit exergetic costs of fuels and products of every component.

Components	K_F^* (kJ/kJ)	K_p^* (kJ/kJ)	ΔK^* (kJ/kJ)	$\Delta K^*/k_F^*$
1. Heater CH_4	1.433	1.850	0.416	0.291
2. Mixer	1.055	1.067	0.012	0.011
3. Reformer	1.433	1.648	0.215	0.150
4. Combustor	1.000	1.577	0.577	0.577
5. Fuel cell	1.222	1.436	0.214	0.175
6. Combustor	1.222	1.465	0.243	0.199
7. Compressor	1.554	1.716	0.163	0.105
8. Recuperator	1.433	1.661	0.227	0.159
9. Mixer	1.377	1.394	0.017	0.013
10. Turbine	1.400	1.554	0.153	0.109
11. Superheater	1.433	1.942	0.510	0.356
12. Evaporator	1.433	2.172	0.739	0.515
13. Economizer	1.433	2.725	1.292	0.902
14. Pump	1.554	1.890	0.336	0.216
15. Alternator	1.554	1.577	0.024	0.015
28. Joint ELC	1.468	1.468	0	0

Table VI. Unit thermoeconomic costs of fuels and products of every component.

Components	Z(c€/s)	f_p	c_F^* (c€/kWh)	c_P^* (c€/kWh)	Δc^* (c€/kWh)	$\Delta c^*/c_F^*$
1. Heater	19.933	0.501	2.195	3.347	1.152	0.525
2. Mixer	0.000	0.000	1.527	1.544	0.017	0.011
3. Reformer	4.782	0.741	2.195	3.226	1.031	0.470
4. Combustor	8.626E−2	0.033	1.447	2.294	0.847	0.585
5. Fuel cell	1.133	0.929	1.897	5.939	4.042	2.130
6. Combustor	5.834E−2	0.012	1.897	2.276	0.379	0.200
7. Compressor	1.078	0.460	2.462	2.877	0.415	0.168
8. Recuperator	6.273E−1	0.289	2.195	2.656	0.461	0.210
9. Mixer	0.000	0.000	2.133	2.159	0.026	0.013
10. Turbine	8.567E−1	0.208	2.173	2.462	0.289	0.133
11. Superheater	1.535E−1	0.237	2.195	3.161	0.966	0.440
12. Evaporator	3.054E−1	0.122	2.195	3.449	1.254	0.571
13. Economizer	1.412E−1	0.201	2.195	4.560	2.365	1.077
14. Pump	7.428E−3	0.859	2.462	5.503	3.041	1.235
15. Alternator	8.035E−1	0.707	2.462	2.590	0.126	0.052
28. Joint ELC	0	0	2.105	2.105	0	0

6. CONCLUSIONS

In this paper, the evaluation of the physical and chemical exergy of flows in systems elaborating mixed-phase flows has been described. It has been used to develop a thermoeconomic analysis of a pressurised internal reforming solid oxide fuel cell + gas turbine plant (FCGT4). In conclusion, the following points could be underlined:

(1) In a mixed-phase flow, if the steam content is very small, and then its partial pressure is very low, then the steam can be treated as an ideal gas; otherwise, it is better to treat the steam as a real gas;

(2) the steam in the mixture at the reference state is treated as an ideal gas because its compressibility factor is nearly equal to 1;

(3) the determination of the state of the steam in the mixture at the restricted dead state is necessary for the calculation of physical and chemical exergy of the mixture;

(4) in the FCGT4 system the components with high irreversibility rates have high efficiency values, while the components with low efficiency rates have low relative irreversibility, so the low efficiency components have little influence on the overall efficiency of the plant; therefore from the thermodynamic point of view, the system is well arranged;

(5) the components with large increase of unit thermoeconomic cost are the fuel cell, the pump, and the economizer, but the last two components have little impact on the cost of the final product of the system because they handle a small quantity of exergy flow; the fuel cell greatly influence the cost of the final product; the large increase of unit thermoeconomic cost of the fuel cell is mainly caused by its high investment cost.

NOMENCLATURE

$\bar{b}_{\mathrm{ph}}, \bar{b}_{\mathrm{ch}}, \bar{b}_{\mathrm{t}}$	= physical, chemical and total exergy of the mixture (kJ/kmol)
$\bar{b}_{\mathrm{ch},i}, \bar{b}_{\mathrm{ch,w}}$	= standard chemical exergy of component i, and water (kJ/kmol)
$\bar{b}_{\mathrm{ch,g+v}}^{*}$	= chemical exergy of gas and vapour phase in the mixture at the restricted dead state (kJ/kmol)
c_{F}^{*}, c_{P}^{*}	= unit thermoeconomic cost of the fuel and the product of a component (c€/kWh)
Δc^{*}	= increase of unit thermoeconomic cost (c€kWh)
C_{p}	= $n \times 1$ matrix, with elements c_{Pj}^{*}
$c_{P_{0}j}^{*}$	= cost of unit exergy flow entering component j from the environment (c€/kWh)
C_{pext}	= $1 \times n$ matrix, with elements $c_{P_{0}j}^{*}$
Ψ_{ij}, Ψ_{0j}	= exergy flow from component i, and from the environment to component j (kW)
Ψ_{pj}	= product exergy of component j (kW)
F	= exergy of the fuel of a component (kW)
f	= exergetic factor
f_{p}	= exergoeconomic factor
G	= mass flow (kg/s)
$\bar{h}, \bar{h}^{*}$	= enthalpy of mixture at (T, p), and at the restricted dead state (kJ/kmol)
$\bar{h}_{i}^{*}$	= enthalpy of component i in the mixture at the restricted dead state (kJ/kmol)
$\bar{h}_{i}, \bar{h}_{\mathrm{v}}$	= enthalpy of component i and water vapour in the mixture (kJ/kmol)
I	= irreversibility (kW)
$k_{\mathrm{F}}^{*}, k_{\mathrm{P}}^{*}$	= unit exergetic cost of the fuel and the product of a component (kJ/kJ)
Δk^{*}	= increase of unit exergetic cost (kJ/kJ)
kp	= $n \times n$ matrix, with elements $k_{ij}^{*} = E_{ij}/E_{pj}$

K_{ext}	=	$n \times 1$ matrix, with elements $k_{0j} = E_{0j}/E_{pj}$
N	=	number of components of a mixture
n	=	number of real components and junctions
P	=	exergy of the product of a component (kW)
p	=	pressure of the mixture (bar)
p_0	=	pressure of the environment (bar)
p_i	=	partial pressure of component i in the mixture at (T, p) (bar)
p_i^*	=	partial pressure of component i in the mixture at the restricted dead state (bar)
p_s, p_{s0}	=	saturated pressure of water at T and T_0 (bar)
p_v	=	partial pressure of water vapour in the mixture at (T, p) (bar)
$\bar{R}$	=	universal gas constant (kJ/kmol K)
R_v	=	gas constant of steam (kJ/kg K)
R_{ext}	=	$n \times 1$ matrix, with elements $R_j = Z_j/E_{Pj}$
$\bar{s}, \bar{s}^*$	=	entropy of mixture at (T, p), and at the restricted dead state (kJ/kmol K)
$\bar{s}_i^*$	=	entropy of component i in the mixture at the restricted dead state (kJ/kmol)
$\bar{s}_i, \bar{s}_v$	=	entropy of component i and water vapour in the mixture at (T, p) (kJ/kmol K)
T	=	temperature of the mixture (K)
T_0	=	temperature of the environment (K)
v_{s0}	=	specific volume of saturated steam at T_0 (m^3/kg)
U_D	=	identity matrix of $n \times n$
$\bar{v}$	=	molal volume of the mixture at (T, p) (m^3/kmol)
W_{el}	=	power (MW)
y_i, y_1	=	mole fraction of component i and water in the mixture
$y_{i,g+v}^*$	=	mole fraction of component i in the gas and vapour phase in the mixture at the restricted dead state
y_{g+v}^*, y_w^*	=	mole fraction of gas and vapour phase and of liquid water in the mixture at the restricted dead state
y_s	=	maximum mole fraction of water vapour in the mixture at (T, p)
y_v, y_w	=	mole fraction of water vapour and liquid water in the mixture at (T, p)
$y_{v,s}^*$	=	maximum mole fraction of water vapour in the mixture at the restricted dead state
Z	=	recovery cost (c€/s)
Z_v	=	compressibility factor of steam
Z_{v0}	=	compressibility factor of saturated steam at T_0

Greek letters

μ_i^*	=	chemical potential of component i in the mixture at the restricted dead state (kJ/kmol)
$\mu_{0,i}$	=	chemical potential of component i at the dead state (kJ/kmol)
$\mu_i^*(y_i = 1)$	=	chemical potential of pure substance i at the restricted dead state (kJ/kmol)
η	=	exergy efficiency
δ	=	fuel depletion rate
χ	=	relative irreversibility
ξ	=	productivity lack

Subscript letters

Tot	=	total value of the plant
i, j	=	real components and junctions of the system

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