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SENSITIVITY ANALYSIS FOR ROOM THERMAL RESPONSE

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SUMMARY

The sensitivity theory is a suitable approach for assessing the room thermal response. It results in the 'sensitivity coefficients' (*SCs*) which, as derived here, evaluate the variation of the thermal load due to a fluctuation in a given design parameter around its nominal value. In this paper the general method is presented and a number of *SCs* are derived to evaluate the sensitivity of the building energy demand to the window surface area, to the overall transmittance and mass thermal capacity of a given wall, and to other structural data.

INTRODUCTION

For a number of purposes and especially when seeking optimum design, it is required to observe the system response following a modification in a given design parameter. For example, in the case of building thermal analysis, one should wish to know to what extent the thermal load is responsive to fluctuations in the window surface area or in the overall conductance of a given wall or in its thermal capacity, and so on. Once such relationships are available, the designer can manage to achieve the best thermal performance through the slightest of alterations in the design variables.

So far, 'parametric analysis' has been used to this end: it involves tentative changes in some variables being assigned and plenty of computer simulations being carried out until some relationship between the variable in question and the thermal response of the building arises. If optimization is the aim, this process should go on until a satisfactory design is attained.

It is self-evident that such a procedure is cumbersome and labour-intensive.

A more appropriate and straightforward approach comes from 'sensitivity theory'. This is suitable for a linear model (as outlined below) and results in the 'sensitivity coefficients' (*SC*) defined as the (percentage) change in a state variable (e.g. the thermal load) when a given design parameter undertakes a fluctuation around its nominal value (Frank, 1978).

Previous work (Cammarata *et al.*, 1983, 1987) reports on the mathematical model for the building thermal response to be used here and on the sensitivity analysis of rooms not under thermostat constraint (Cammarata and Marletta, 1990).

This paper deals with the thermal load sensitivity to a number of design parameters.

MATHEMATICAL APPROACH

It is well known that the dynamic behaviour of linear systems can be described by a 'state equation' of the following type (Cadzow and Marteus, 1970):

$$[\dot{T}] = [A][T] + [B][U] \tag{1}$$

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In the present case $[T]$ is the state vector and $[\dot{T}]$ its time derivative. Both enclose the state variables. In the lumped parameter models (LPM) for building analysis, the element T_j of vector $[T]$ is the temperature of jth wall. $[U]$ is the input vector, containing the functions of the boundary conditions, i.e. the external forces acting on the system, such as outdoor temperature, solar radiation etc. $[A]$ and $[B]$, respectively referred to as the state array and the input array, include the system structural data.

In the linear system theory (Cadzow and Martens, 1970) it can be shown that equation (1) can be solved through the following relationship:

$$[T]_\tau = [F][T]_{\tau-\Delta\tau} + [D][U]_\tau \tag{2}$$

Here τ is the time and $\Delta\tau$ the temporal step of integration.

Both $[F]$ and $[D]$ are matrices derived from $[A]$ and $[B]$ as given in the literature (Cadzow and Martens, 1970).

In general the item $T_j (j = 1 \div m)$ of the state vector $[T]$ is a time varying r-dimensional function of the system parameters $q_i (i = 1, 2, \cdots, r)$. Then

$$T_j = f_j(q_i, \tau) \qquad (j = 1 \div m;\ i = 1 \div r)$$

The thermal load can be assessed by an equation of the type:

$$Q(\tau) = g(T_j, q_i)$$

Let us introduce the following sensitivity coefficient:

$$\lambda_{ij}(\tau) = \left.\frac{\partial T_j}{\partial q_i}\right|_{\bar{q}_i} \qquad \text{i.e.} \qquad [\lambda]_i(\tau) = \left.\frac{\partial [T]}{\partial q_i}\right|_{\bar{q}_i}$$

This quantifies the change in the temperature of the jth wall due to an arbitrarily small fluctuation in the system parameter q_i around its nominal value $\bar{q}_i$. Similarly we can refer to:

$$\sigma_i(\tau) = \left.\frac{\partial Q}{\partial q_i}\right|_{\bar{q}_i}$$

as the sensitivity coefficient of the thermal load, Q.

The relationship between λ_{ij} and σ_i is provided by the chain rule:

$$\sigma_i = \frac{\partial Q}{\partial q_i} = \frac{\partial g}{\partial q_i} + \sum_{j=1}^{m} \frac{\partial g}{\partial T_j}\frac{\partial T_j}{\partial q_i} = \frac{\partial g}{\partial q_i} + \sum_{j=1}^{m} \frac{\partial g}{\partial T_j}\lambda_{ij} \tag{3}$$

Moreover, after equation (1) and since $[U]$ is independent of q_i:

$$\frac{\partial [\dot{T}]}{\partial q_i} = [A]\frac{\partial [T]}{\partial q_i} + \frac{\partial [A]}{\partial q_i}[T] + \frac{\partial [B]}{\partial q_i}[U]$$

Now, by inversion of the order of derivation we get

$$\frac{\partial [\dot{T}]}{\partial q_i} \equiv \frac{\partial}{\partial q_i}\frac{\partial [T]}{\partial \tau} = \frac{\partial}{\partial \tau}\frac{\partial [T]}{\partial q_i} = \frac{\partial [\lambda]_i}{\partial \tau} = [\dot{\lambda}]_i$$

Therefore

$$[\dot{\lambda}]_i = [A][\lambda]_i + \frac{\partial [A]}{\partial q_i}[T] + \frac{\partial [B]}{\partial q_i}[U] \tag{4}$$

Let

$$[\dot{\lambda}] = \frac{\partial [\lambda]}{\partial \tau}; \qquad [A]_i = \frac{\partial [A]}{\partial q_i}; \qquad [B]_i = \frac{\partial [B]}{\partial q_i}$$

$$[L] = \begin{bmatrix} [A] & 0 & 0 & 0 & \cdots & 0 & [A]_1 \\ 0 & [A] & 0 & 0 & \cdots & 0 & [A]_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & [A]_r \\ 0 & 0 & 0 & 0 & \cdots & 0 & [A] \end{bmatrix}; \quad [\lambda] = \begin{bmatrix} [\lambda]_1 \\ [\lambda]_2 \\ \cdots \\ \cdots \\ [\lambda]_r \\ [T] \end{bmatrix}; \quad [M] = \begin{bmatrix} [B]_1 \\ [B]_2 \\ \cdots \\ \cdots \\ [B]_r \\ [B] \end{bmatrix}$$

Hence equation (4) becomes:

$$[\dot{\lambda}] = [L][\lambda] + [M][U] \tag{5}$$

This equation is formally similar to equation (1) and therefore can be solved likewise.

Moreover it can be seen that the state equation (1) is included in (5). As a result, once equation (5) is solved, one gets the dynamic behaviour of the system (i.e. the temperature of walls) and any desired sensitivity coefficient, with considerable savings in computer time. Clearly the λ_{ij} obtained will be used to withdraw σ_i by means of equation (3).

It is to be understood that the *SC*s, as previously derived, are time-dependent and therefore not practicable for design purposes and/or for characterzation of the room thermal response.

To this end it seems more appropriate to introduce the averaged-values over a suitable time period, t:

$$\bar{\sigma}_i = \frac{1}{t}\int_0^t \sigma_i(\tau)\,d\tau$$

Furthermore, as a consequence of the definition, every single *SC* has its own dimensions. On this basis, comparisons among different *SC*s are not possible. Hence it seems reasonable to introduce the normalized value:

$$S_i = \bar{\sigma}_i \frac{\bar{q}_i}{10}$$

which quantifies the change in the thermal load caused by a $\pm$ 10% deviation of q_i from its nominal value $\bar{q}_i$.

ROOM DESCRIPTION AND SIMULATIONS

The above mentioned method was applied to assess the *SC* of the room thermal load with respect to the following design parameters: window surface area (S_w), overall transmittance (U) and thermal capacity (mc) of the external wall, as well as convective coefficients for both internal (α_i) and external (α_e) surface of the same wall.

The investigation deals with a room under thermostat constraints and with regular shape ($6 \times 6 \times 3$ m^3). All walls were supposed to have the same structure and to be adjacent to rooms at the same temperature (internal partitions), except one facing due south with a window on it.

This arrangement seems suitable to make the results easier to understand as it protects the room from heat transfer contributions other than that through the external wall.

The thermal load, Q was evaluated for a number of room configurations: different window size, lightweight or heavyweight wall structure, with and without thermal insulation, for summer and winter conditions (indoor air temperature respectively $T_a = 25\,°C$ and $T_a = 20\,°C$) of a typically mediterranean climate. Selected structural data are summarized in Table 1. The simulations were carried out by means of a lumped parameter model, described in Cammarata and Marletta *et al.* (1983) and Cammarata and Marletta (1990).

THE RESULTS

Figure 1 shows the temporal evolution of the *SC* of the thermal load, Q, to window area, dQ/dS_w, overall transmittance, dQ/dU and mass thermal capacity, $dQ/d(mc)$ for the south wall of a room with lightweight

Table 1. Structural data

External wall	Structure weight: Light	Medium	Heavy
U (W/m^2 °C)	2·03	1·07	0·82
mc (kJ/m^2 °C)	122	200	282
a_e (W/m^2 °C)		20	
a_i (W/m^2 °C)		8	

structure and 25% window to wall ratio. Figure 1(a) refers to summer and Figure 1(b) to winter operation. (The same division into part (a) representing summer and part (b) representing winter performance applies to Figures 2 to 8 also.)

The dynamic behaviour of *SC*s can be understood looking at Figure 2, where the following functions are shown: thermal load, Q, temperature difference between indoor and outdoor air, $(T_a - T_e)$, time derivative $dT/d\tau$ of the wall internal temperature.

It should not escape the reader that $dT/d\tau$ is proportional to the energy storage in the wall, $mcdT/dt$.

Apparently both dQ/dS_w and dQ/dU behave rather similarly to $(T_a - T_e)$ as well as $dQ/d(mc)$ to $(dT/d\tau)$; this means that $(T_a - T_e)$ acts as a driving force with respect to dQ/dS_w and dQ/dU. Similarly one can think about $dQ/d(mc)$ versus the thermal energy stored in the wall $(mcdT/d\tau)$.

A similar approach can be used for the analysis of Figures 3 and 4. They both refer to a room of heavyweight structure and allow us to perceive the dumping effect of the thermal capacity on the room thermal response.

The same effect can be seen in Figure 5, which gives dQ/dU as a function of the window surface area S_w. It is realized that for any given typology (lightweight or heavyweight structure) a diminishing amount of solid wall surface area makes dQ/dU decrease as S_w increases.

From Figure 6 one can assess the relationship between dQ/dS_w and the window overall transmittance (K_w). As one can observe, doubling K_w makes dQ/dS_w almost double as well.

In Figure 7 the sensitivity function of thermal load, Q, to the convective coefficient a_e is given. As a general remark one can see that the values attained by dQ/da_e are rather high, as compared with those of other *SC*s. That implies the high sensitivity of Q to a_e and ultimately to the wind conditions of the site. Furthermore, this time the heavyweight structure is affected by the highest fluctuations in dQ/da_e. This happens because heavy walls have lower overall transmittance, U, in comparison to light walls (see Table 1).

For comparison, the sensitivity of Q to the internal convective coefficient a_i is reported in Figure 8.

Now let us consider the mean values of the *SC*. Primarily it must be observed that, because of the wavy shape of *SC* curves, it would not be suitable to get them by averaging over the whole day (24 hours). In this case indeed such values will be next to zero, resulting in misleading information. It is necessary to adopt more appropriate criteria; for example, one can average over the time period in which the thermal load is either negative or positive.

In Figures 9 and 10, mean values are given for winter ($Q > 0$) and summer conditions ($Q < 0$). It can be seen that mc curves in Figure 9 referred to as 'lightweight', 'mediumweight' and 'heavyweight' lie extremely close to each other, which means that dQ/dS_w is nearly independent of the thermal mass. As to dQ/dU (Figure 10), it is realized that it attains in general much lower values than dQ/dS_w and does depend on S_w.

In conclusion, the thermal load, Q, of the room is much more sensitive to the window surface area than to the wall transmittance and even less to the wall thermal capacity. The dependence of Q upon a_e is also not negligible.

Similar information can be valuable in optimum design strategies.

In Table 2 the standard deviations are summarized for winter and summer operation.

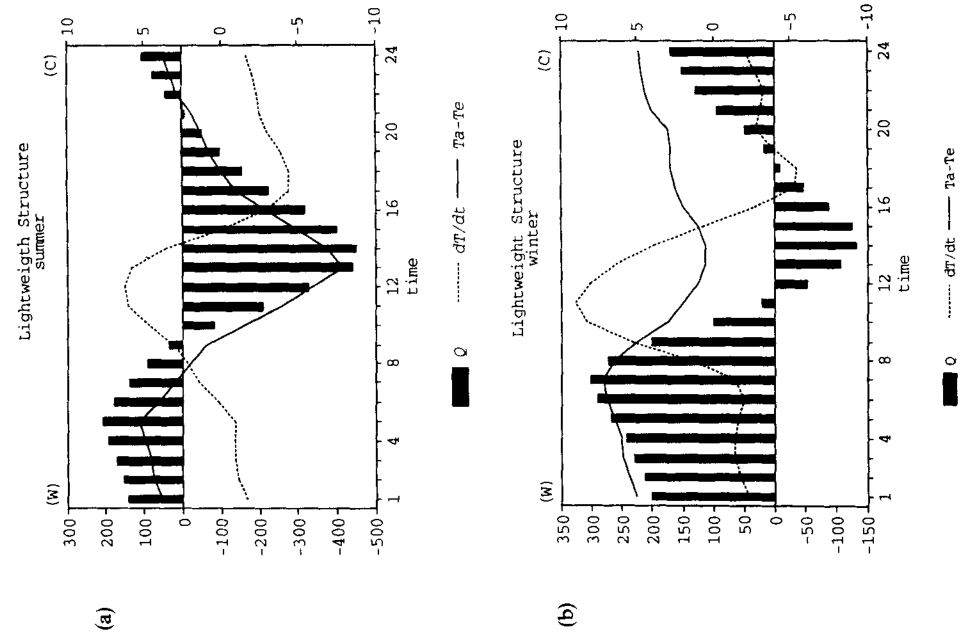

Figure 2

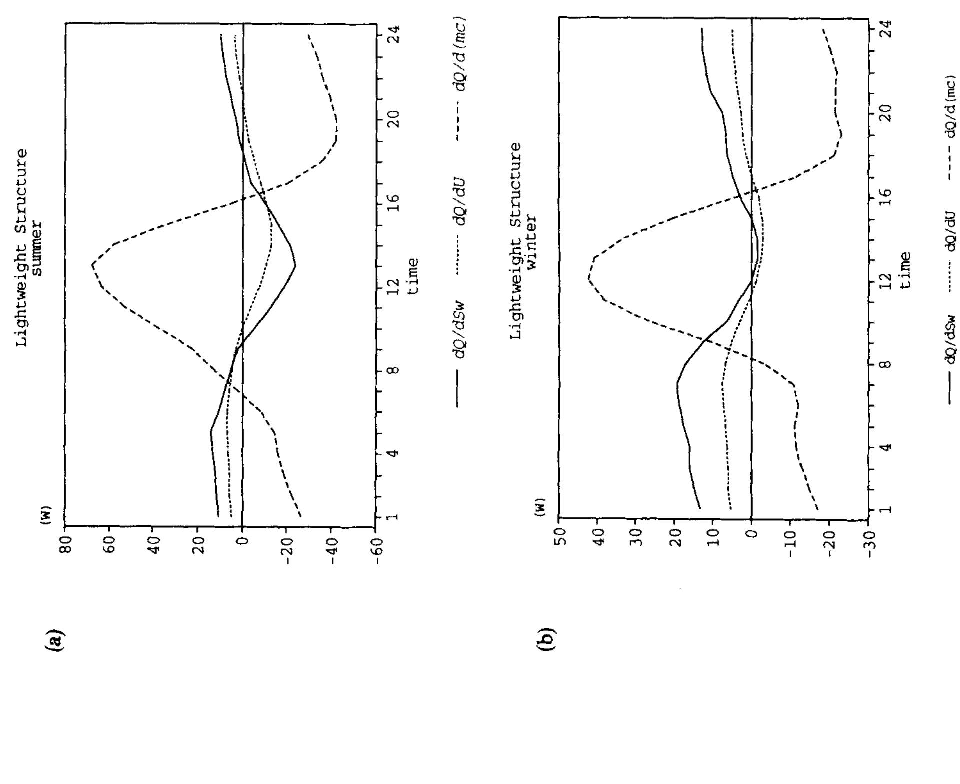

Figure 1

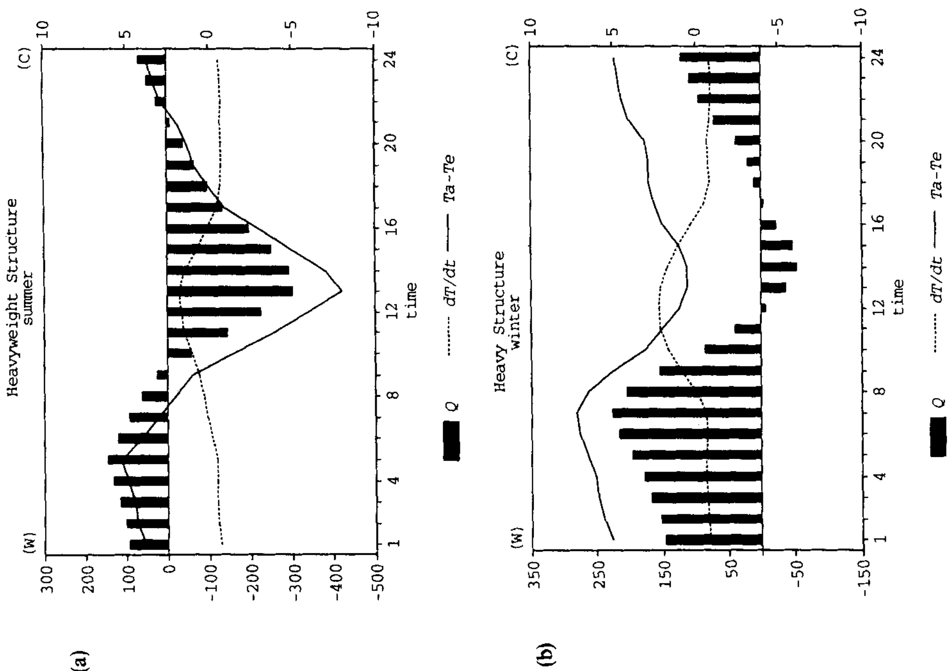

Figure 4

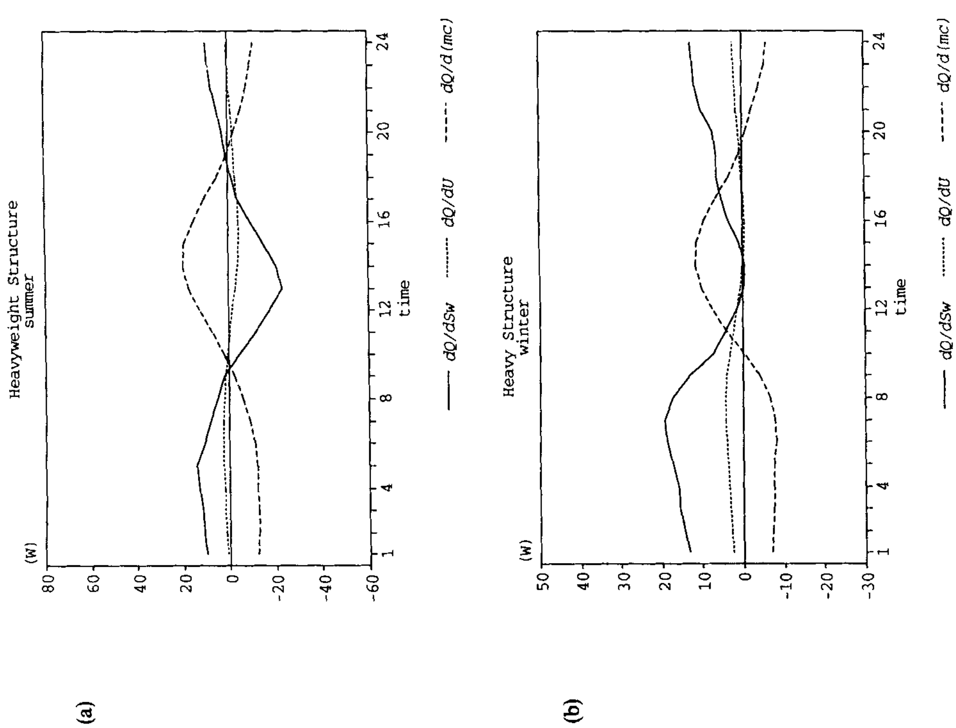

Figure 3

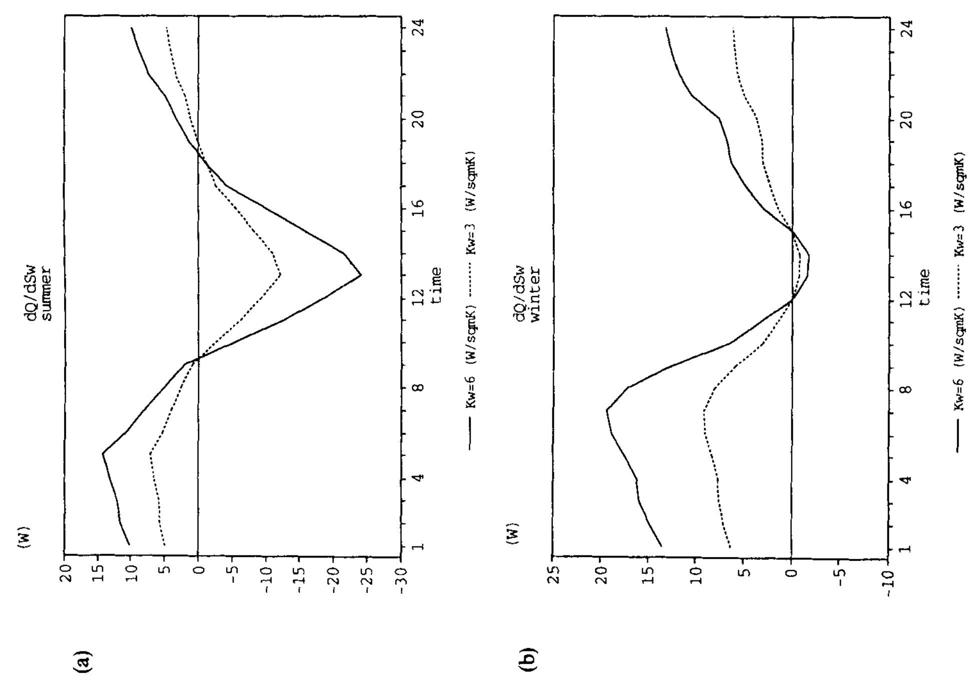

Figure 6

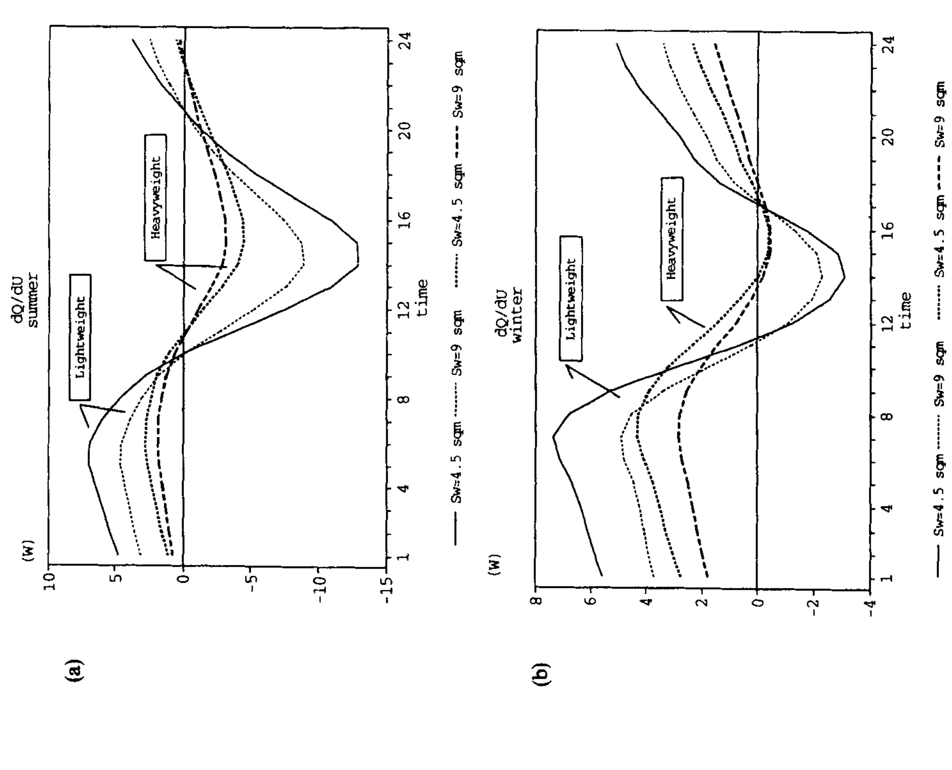

Figure 5

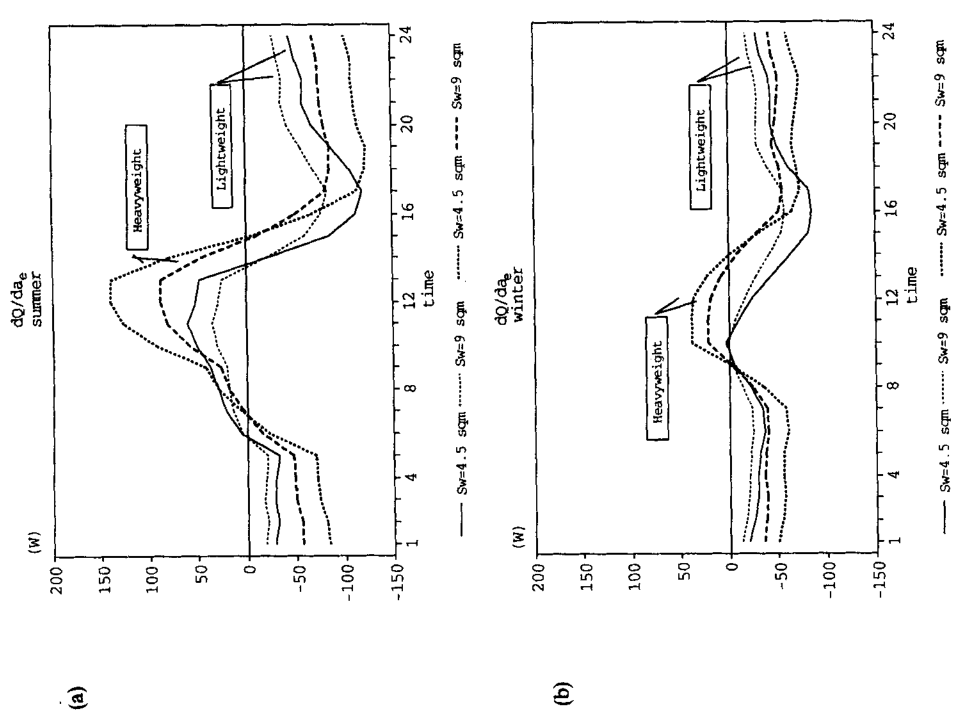

Figure 7

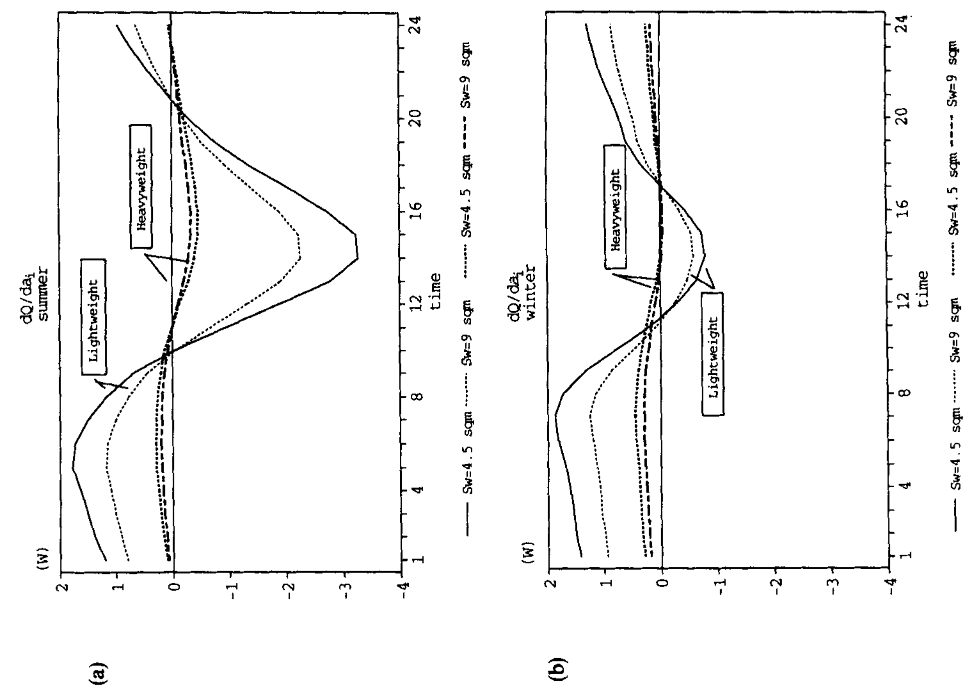

Figure 8

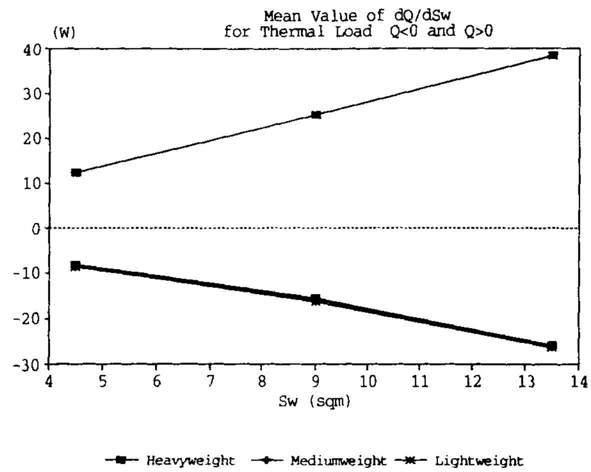

Figure 9

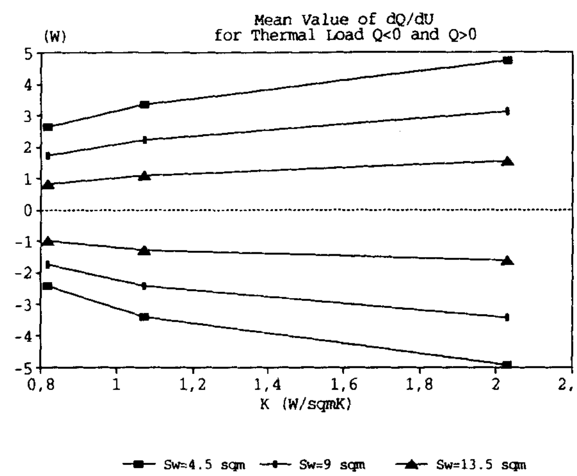

Figure 10

Table 2. Standard deviation of dQ/dS_w and dQ/dU

		SUMMER ($Q < 0$)			WINTER ($Q > 0$)		
		Wall structure			Wall structure		
	S_w (m^2)	Light $U = 2{\cdot}03$ ($W/m^2\,°C$)	Medium $U = 1{\cdot}07$ ($W/m^2\,°C$)	Heavy $U = 0{\cdot}82$ ($W/m^2\,°C$)	Light $U = 2{\cdot}03$ ($W/m^2\,°C$)	Medium $U = 1{\cdot}07$ ($W/m^2\,°C$)	Heavy $U = 0{\cdot}82$ ($W/m^2\,°C$)
dQ/S_w	4·5	9·5	9·1	8·9	4·6	4·5	4·4
	9·0	18·5	18·0	17·8	8·9	8·8	8·7
	13·5	25·5	25·2	25·0	12·9	12·8	12·7
dQ/dU	4·5	4·1	2·3	1·6	1·9	1·5	1·2
	9·0	2·8	1·5	1·1	1·3	1·0	0·8
	13·5	1·5	0·8	0·6	0·7	0·5	0·4

FINAL REMARKS

It is worth mentioning that, in most cases, numerical checks came out with

$$\sum_{j=1}^{m} \frac{\partial g}{\partial T_j} \frac{\partial T_j}{\partial q_i} \ll \frac{\partial g}{\partial q_j}$$

This implies that whenever the parameter under investigation appears in both the following equations

$$Q(\tau) = g(T_j, q_i) \tag{6}$$

$$T_j = f_j(q_i, \tau) \quad (j = 1 \div m; \quad i = 1 \div r) \tag{7}$$

the building sensitivity to the thermal load may be studied with reasonable approximation through equation (6), which is algebraic, instead of equation (7), which is indeed a set of differential equations.

Provided that such a result is also obtained from a DPM (distributed parameter model) (Cammarata and Marletta, 1987)—which is a much more accurate prediction tool than the LPM (lumped parameter model) used here—problems such as the '*model reduction and parameter identification*' for building analysis could be faced on the basis of a LPM instead of a DPM, with a consistent reduction in computational efforts.

At the present a great deal of research work is being devoted to this aim from our group and other scientific communities.

Results in this topic will be reported in a future note.

CONCLUSIONS

The *sensitivity analysis* can be stated from a linear model and allows the sensitivity coefficients to be drawn, which can assess how much a fluctuation in a given design parameter affects the system performance. As derived here, the approach is quite general and highly rigorous and, as such, it is a sound alternative to the traditional *parametric analysis*. Finally, by means of *SC*s, it is possible to consciously lead the design process and especially the optimization strategies.

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