Second Order Gauge-Invariant Perturbations during Inflation

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The evolution of gauge invariant second-order scalar perturbations in a general single field inflationary scenario are presented. Different second order gauge invariant expressions for the curvature are considered. We evaluate *perturbatively* one of these second order curvature fluctuations and a second order gauge invariant scalar field fluctuation during the slow-roll stage of a massive chaotic inflationary scenario, taking into account the deviation from a pure de Sitter evolution and considering only the contribution of super-Hubble perturbations in mode-mode coupling. The spectra resulting from their contribution to the second order quantum correlation function are nearly scale-invariant, with additional logarithmic corrections with respect to the first order spectrum. For all scales of interest the amplitude of these spectra depends on the total number of e-folds. We find, on comparing first and second order perturbation results, an upper limit to the total number of e-folds beyond which the two orders are comparable.

I. INTRODUCTION

One of the observational arguments supporting inflation is the evidence of nearly scale-invariant primordial adiabatic fluctuations with a mostly gaussian spectrum, as shown by COBE [1] and WMAP [2, 3]. In its simplest realizations, adiabatic cosmological fluctuations are simply related to linear fluctuations of the inflaton generated during the accelerated stage: such fluctuations have a gaussian spectrum.

Non-linear fluctuations inevitably carry a χ-squared distribution, which may contribute to non-gaussianities in the 3-D power spectrum and in CMB anisotropies depending on the amplitude and spectrum of these non-linearities (see [4] for a review). A study of fluctuations beyond the linear level is therefore interesting not only for the issue of the stability of inflationary dynamics, but also because of its connection with the predictions of non-gaussianities in the matter power spectrum and in CMB anisotropies. Even in absence of non-linearity in the potential for the scalar field, the generality of intrinsic non-gaussianities is guaranteed by the non-linearity of the Einstein equations, a fact which has motivated a great interest both from the theoretical and observational point of view [4]. Our choice of calculating the intrinsic non-gaussianities for a free massive inflationary model is therefore a case of primary interest, since in such a case the whole non-linear sector is given by gravity.

In this paper we shall focus on the perturbative evualuation of *second order gauge-invariant* (GI henceforth) curvature fluctuations. For this purpose we discuss different second order GI measures of curvature perturbations (see also [5, 6, 7, 8, 9, 10, 11]), which to first order coincide with the curvature on flat slicing hypersurfaces. We then compute the correlation function for differents second order GI variables, finding, for all scales of interest, a growth in the spectrum amplitude as the total number of e-folds which inflation lasted increases. A dependence on the total number of e-folds was also found for the production of test scalar fields with a mass smaller than that of the inflaton [12].

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The paper is organized as follows. We present the formalism and different gauge-invariant measures of curvature perturbations in section II. In sections III and IV we evaluate the quantum correlators for different second order GI variables with both analytical and numerical methods. We conclude in section V and in appendices A e B we add useful technical formulae for gauge transformations and for the analytical results given in section III.

II. GENERAL CURVATURE PERTURBATION

Let us consider inflation in a flat universe driven by a classical minimally coupled scalar field with a potential $V(\phi)$ and follow the notation in [13]. The action is given by:

$$S \equiv \int d^4x \mathcal{L} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \tag{1}$$

where $\mathcal{L}$ is the Lagrangian density and we shall study the chaotic model $V(\phi) = \frac{m^2}{2}\phi^2$ [14] (see [15] for a second order calculation in a different inflationary model). We shall consider, up to second order in the fluctuations, both the inflaton:

$$\phi(t, \mathbf{x}) = \phi(t) + \varphi(t, \mathbf{x}) + \varphi^{(2)}(t, \mathbf{x})$$

and the metric of a flat universe ($ds^2 = g_{\mu\nu} dx^\mu dx^\nu$):

$$\begin{aligned} g_{00} &= -1 - 2\alpha - 2\alpha^{(2)} \\ g_{0i} &= -\frac{a}{2}\left(\beta_{,i} + \beta^{(2)}_{,i}\right) \\ g_{ij} &= a^2 \left[\delta_{ij} - 2\delta_{ij}\left(\psi + \psi^{(2)}\right) + D_{ij}\left(E + E^{(2)}\right)\right] , \end{aligned} \tag{2}$$

where $D_{ij} = \partial_i \partial_j - 1/3\, \nabla^2 \delta_{ij}$. As is clear from the above we limit ourselves to the scalar sector of metric perturbations, neglecting vector and tensor perturbations.

In the above formulation the gauge is not fixed, and one can eliminate two scalar functions among the four metric coefficients and the inflaton fluctuation. We wish to extend to second order the GI potential for the curvature perturbation. For this purpose, let us note that to first order the so called curvature (potential) perturbation is given by

$$\hat{\psi} = \psi + \frac{1}{6}\nabla^2 E \tag{3}$$

and this quantity is not GI (for the general gauge trasformation see appendix A). We now consider the comoving slicing [16], which is defined to be the slicing orthogonal to the worldlines of comoving observers: they are free-falling and the expansion defined by them is isotropic. That is the observers do not measure any flux of energy ($T^{0\,(1)}_{\ i} = 0$) and for the universe under consideration this corresponds to having $\varphi = 0$. The transformation from a general slicing to a comoving slicing with $\varphi = 0$ is:

$$\tilde{\varphi}_{com} = \varphi - \epsilon^0_{(1)} \dot{\phi} = 0 \rightarrow \epsilon^0_{(1)} = \frac{\varphi}{\dot{\phi}} \tag{4}$$

and one obtains

$$R^{(1)} = \tilde{\hat{\psi}}_{com} = \psi + \frac{H}{\dot{\phi}}\varphi + \frac{1}{6}\nabla^2 E \,, \tag{5}$$

which is the first order GI comoving curvature (potential) perturbation.

To second order the situation is more involved due to the presence of terms quadratic in the first order. We are interested in the definition of GI quantities including second order. Their construction is not unique and we shall therefore construct three different GI second order curvature (potential) perturbations.

A. The comoving curvature potential perturbation R_A

This variable is obtained by repeating consistently up to second order the procedure used for the first order comoving curvature (potential) perturbation. Starting from

$$\hat{\psi}^{(2)} = \psi^{(2)} + \frac{1}{6}\nabla^2 E^{(2)} \,, \tag{6}$$

to second order the comoving slicing is caracterized by $T^{0\,(1)}_{\ i} = 0$ and $T^{0\,(2)}_{\ i} = 0$, that is $\varphi = 0$ and $\varphi^{(2)} = 0$. In this case, in order to fix all the degrees of freedom, we restrict ourselves to considering only an infinitesimal time trasformation up to second order, which implies taking $\epsilon^s_{(1)}$ and $\epsilon^s_{(2)}$ equal to zero. From the first order we obtain the condition in Eq.(4), while from the second order we have

$$\tilde{\varphi}^{(2)}_{com} = \varphi^{(2)} - \epsilon^0_{(1)}\dot{\varphi} + \frac{1}{2}\left[\epsilon^0_{(1)}\left(\epsilon^0_{(1)}\dot{\phi}\right)^{\cdot} - \epsilon^0_{(2)}\dot{\phi}\right] = 0 \rightarrow \epsilon^0_{(2)} = \frac{2}{\dot{\phi}}\left[\varphi^{(2)} - \epsilon^0_{(1)}\dot{\varphi} + \frac{1}{2}\epsilon^0_{(1)}\left(\epsilon^0_{(1)}\dot{\phi}\right)^{\cdot}\right] . \tag{7}$$

Thus one obtains

$$\begin{aligned} R^{(2)}_A = \tilde{\hat{\psi}}^{(2)}_{com} \;=\; & \psi^{(2)} + \frac{H}{\dot{\phi}}\varphi^{(2)} + \frac{1}{6}\nabla^2 E^{(2)} - \frac{\varphi}{\dot{\phi}}\left(\dot{\psi} + 2H\psi\right) - \frac{H}{\dot{\phi}^2}\varphi\dot{\varphi} + \frac{\varphi^2}{\dot{\phi}^2}\left(-H^2 - \frac{\dot{H}}{2} + \frac{H}{2}\frac{\ddot{\phi}}{\dot{\phi}}\right) \\ & + \frac{1}{6a^2}\frac{1}{\dot{\phi}^2}\left[\partial^i\varphi\partial_i\varphi - \frac{1}{2}\nabla^2\left(\varphi^2\right)\right] - \frac{1}{6a}\frac{1}{\dot{\phi}}\left[\partial^i\beta\partial_i\varphi - \frac{1}{2}\nabla^2\left(\beta\varphi\right)\right] + \frac{1}{4}\frac{\partial^i\partial^j}{\nabla^2}\left[-2H\frac{\varphi}{\dot{\phi}}D_{ij}E \right. \\ & \left. - \frac{\varphi}{\dot{\phi}}D_{ij}\dot{E} + \frac{1}{a^2}\frac{\varphi}{\dot{\phi}^2}D_{ij}\varphi - \frac{1}{2a}\frac{1}{\dot{\phi}}\left(\varphi D_{ij}\beta + \beta D_{ij}\varphi\right)\right] . \end{aligned} \tag{8}$$

Let us note that this quantity is analogous to the second order curvature perturbation on a uniform density hypersurface defined in [7]. These two quantities agree (apart from a sign) in the long wavelength limit (see also [9, 11]).

B. The curvature potential perturbation R_B

We give the second order GI curvature perturbation potential obtained from a form analogous to Eq. (5) (the first three terms of the following equation), and add a minimal set of terms necessary to obtain a second order GI variable. Using the general transformations of Appendix A, we obtain

$$\begin{aligned} R^{(2)}_B \;=\; & \psi^{(2)} + \frac{H}{\dot{\phi}}\varphi^{(2)} + \frac{1}{6}\nabla^2 E^{(2)} \\ & + \frac{1}{2}\left(2H^2 + \dot{H} - H\frac{\ddot{\phi}}{\dot{\phi}}\right)^{-1}\left(\dot{\psi} + 2H\psi + \frac{H}{\dot{\phi}}\dot{\varphi}\right)^2 - \frac{1}{24}\partial_i\beta\,\partial^i\beta \\ & + \frac{1}{6}\nabla^2\left\{\frac{1}{8}\beta^2 + \frac{3}{2}\frac{\partial^i\partial^j}{(\nabla^2)^2}\left[-\frac{1}{4}\beta D_{ij}\beta + 2\psi D_{ij}E + \frac{1}{H}\psi D_{ij}\dot{E}\right]\right\} . \end{aligned} \tag{9}$$

This quantity is the true GI generalization of $\mathcal{R}^{(2)}$ computed in [5], which also takes into account infinitesimal spatial (scalar) transformation in addition to the infinitesimal time translations considered in [5].

C. The comoving intrinsic curvature potential perturbation R_C

In order to introduce the third curvature (potential) perturbation, let us first consider the intrinsic 3-curvature ${}^{(3)}R$ associated with a foliation of space-time with constant proper time t for a flat Universe. Using eq.(49) and eq.(58) of [6] we obtain:

$$\begin{aligned} {}^{(3)}R \;=\; & \frac{1}{a^2}\left[2\partial^i\partial_j C^j_i - 2\partial^i\partial_i C^j_j + 4C^{i\,j}\left(-2\partial_j\partial_k C^k_i + \partial_k\partial^k C_{i\,j} + \partial_i\partial_j C^k_k\right) - \left(2\partial_k C^k_j - \partial_j C^k_k\right)\left(2\partial_i C^{i\,j} - \partial^j C^i_i\right) \right. \\ & \left. + \partial^k C^{i\,j}\left(3\partial_k C_{i\,j} - 2\partial_j C_{i\,k}\right)\right] \end{aligned} \tag{10}$$

with

$$C_{ij} = -\delta_{ij}\left(\psi + \psi^{(2)}\right) + \frac{1}{2}D_{ij}\left(E + E^{(2)}\right) .$$

In our notations the intrinsic curvature is:

$$\begin{aligned}
{}^{(3)}R &= {}^{(3)}R^{(1)} + {}^{(3)}R^{(2)} \\
&= \frac{4}{a^2}\nabla^2\left(\psi + \frac{1}{6}\nabla^2 E\right) + \frac{4}{a^2}\left\{\nabla^2\left(\psi^{(2)} + \frac{1}{6}\nabla^2 E^{(2)}\right) + 4\left(\psi + \frac{1}{6}\nabla^2 E\right)\nabla^2\left(\psi + \frac{1}{6}\nabla^2 E\right)\right. \\
&\quad + \frac{3}{2}\partial^i\left(\psi + \frac{1}{6}\nabla^2 E\right)\partial_i\left(\psi + \frac{1}{6}\nabla^2 E\right) - \frac{1}{2}\nabla^2 E\nabla^2\left(\psi + \frac{1}{6}\nabla^2 E\right) \\
&\quad - \frac{1}{2}\partial^i\partial^j E\partial_i\partial_j\left(\psi + \frac{1}{6}\nabla^2 E\right) - \frac{1}{2}\partial^i\left(\psi + \frac{1}{6}\nabla^2 E\right)\partial_i\nabla^2 E - \frac{1}{16}\partial^i\nabla^2 E\partial_i\nabla^2 E \\
&\quad \left. + \frac{1}{16}\partial^i\partial^j\partial^k E\partial_i\partial_j\partial_k E\right\} .
\end{aligned} \tag{11}$$

Instead of using Eq. (6), we now consider the potential $\hat{\psi}_C^{(2)}$ of the intrinsic curvature to second order ${}^{(3)}R^{(2)}$, given by ${}^{(3)}R^{(2)} = \frac{4}{a^2}\nabla^2\hat{\psi}_C^{(2)}$. We then obtain the comoving GI expression by going from a general slicing to a comoving slicing:

$$\begin{aligned}
R_C^{(2)} &= \psi^{(2)} + \frac{H}{\dot\phi}\varphi^{(2)} + \frac{1}{6}\nabla^2 E^{(2)} - \frac{\varphi}{\dot\phi}\left(\dot\psi + 2H\psi\right) - \frac{H}{\dot\phi^2}\varphi\dot\varphi + \frac{\varphi^2}{\dot\phi^2}\left(-H^2 - \frac{\dot H}{2} + \frac{H}{2}\frac{\ddot\phi}{\dot\phi}\right) \\
&\quad + \frac{1}{6a^2}\frac{1}{\dot\phi^2}\left[\partial^i\varphi\partial_i\varphi - \frac{1}{2}\nabla^2\left(\varphi^2\right)\right] - \frac{1}{6a}\frac{1}{\dot\phi}\left[\partial^i\beta\partial_i\varphi - \frac{1}{2}\nabla^2\left(\beta\varphi\right)\right] + \frac{1}{4}\frac{\partial^i\partial^j}{\nabla^2}\left[-2H\frac{\varphi}{\dot\phi}D_{ij}E\right. \\
&\quad \left. - \frac{\varphi}{\dot\phi}D_{ij}\dot E + \frac{1}{a^2}\frac{\varphi}{\dot\phi^2}D_{ij}\varphi - \frac{1}{2a}\frac{1}{\dot\phi}\left(\varphi D_{ij}\beta + \beta D_{ij}\varphi\right)\right] + \frac{1}{\nabla^2}\left[4\left(\psi + \frac{1}{6}\nabla^2 E + \frac{H}{\dot\phi}\varphi\right)\right. \\
&\quad \nabla^2\left(\psi + \frac{1}{6}\nabla^2 E + \frac{H}{\dot\phi}\varphi\right) + \frac{3}{2}\partial^i\left(\psi + \frac{1}{6}\nabla^2 E + \frac{H}{\dot\phi}\varphi\right)\partial_i\left(\psi + \frac{1}{6}\nabla^2 E + \frac{H}{\dot\phi}\varphi\right) \\
&\quad - \frac{1}{2}\nabla^2 E\nabla^2\left(\psi + \frac{1}{6}\nabla^2 E + \frac{H}{\dot\phi}\varphi\right) - \frac{1}{2}\partial^i\partial^j E\partial_i\partial_j\left(\psi + \frac{1}{6}\nabla^2 E + \frac{H}{\dot\phi}\varphi\right) \\
&\quad - \frac{1}{2}\partial^i\left(\psi + \frac{1}{6}\nabla^2 E + \frac{H}{\dot\phi}\varphi\right)\partial_i\nabla^2 E - \frac{1}{16}\partial^i\nabla^2 E\partial_i\nabla^2 E \\
&\quad \left. + \frac{1}{16}\partial^i\partial^j\partial^k E\partial_i\partial_j\partial_k E\right]
\end{aligned} \tag{12}$$

D. Comparison among curvature perturbations in the long-wavelength limit

The preceding three expressions undergo a considerable simplification in the long-wavelength limit. For large scales we obtain:

$$R_A^{(2)} = \psi^{(2)} + \frac{H}{\dot\phi}\varphi^{(2)} - \frac{\varphi}{\dot\phi}\left(\dot\psi + 2H\psi\right) - \frac{H}{\dot\phi^2}\varphi\dot\varphi + \frac{\varphi^2}{\dot\phi^2}\left(-H^2 - \frac{\dot H}{2} + \frac{H}{2}\frac{\ddot\phi}{\dot\phi}\right) \tag{13}$$

$$R_B^{(2)} = \psi^{(2)} + \frac{H}{\dot\phi}\varphi^{(2)} + \frac{1}{2}\left(2H^2 + \dot H - H\frac{\ddot\phi}{\dot\phi}\right)^{-1}\left(\dot\psi + 2H\psi + \frac{H}{\dot\phi}\dot\varphi\right)^2 \tag{14}$$

$$\begin{aligned}
R_C^{(2)} &= \psi^{(2)} + \frac{H}{\dot\phi}\varphi^{(2)} - \frac{\varphi}{\dot\phi}\left(\dot\psi + 2H\psi\right) - \frac{H}{\dot\phi^2}\varphi\dot\varphi + \frac{\varphi^2}{\dot\phi^2}\left(-H^2 - \frac{\dot H}{2} + \frac{H}{2}\frac{\ddot\phi}{\dot\phi}\right) \\
&\quad + \frac{1}{\nabla^2}\left[4\left(\psi + \frac{H}{\dot\phi}\varphi\right)\nabla^2\left(\psi + \frac{H}{\dot\phi}\varphi\right) + \frac{3}{2}\partial^i\left(\psi + \frac{H}{\dot\phi}\varphi\right)\partial_i\left(\psi + \frac{H}{\dot\phi}\varphi\right)\right]
\end{aligned} \tag{15}$$

in this limit the difference between $R_A^{(2)}$ and $R_B^{(2)}$ is given by [9]:

$$R_B^{(2)} - R_A^{(2)} = \frac{1}{2} \frac{\left(\dot{R}^{(1)} + 2HR^{(1)}\right)^2}{\left(2H^2 + \dot{H} - H\frac{\ddot{\phi}}{\dot{\phi}}\right)} \,. \tag{16}$$

Let us note that, as for the first order $R^{(1)}$, $R_A^{(2)}$ also is constant in time in this limit [8, 9].

Further on using different parametrizations of the metric many other gauge invariant quantities can be constructed. For example from [11] one can define another variable $R_{LR}^{(2)}$, which is related to $R_A^{(2)}$ on large scales by

$$R_A^{(2)} = R_{LR}^{(2)} - \left(R^{(1)}\right)^2 \,. \tag{17}$$

We also consider $R_C^{(2)}$ since it is directly related to a geometrical quantity: the intrinsic 3-curvature.

III. LONG-WAVELENGTH SECOND ORDER CURVATURE PERTURBATIONS IN UCG

As in one of our previous papers on back-reaction [13], we choose to work in the uniform curvature gauge (UCG), which can be generalized straightforwardly to second order through the conditions: $\psi = \psi^{(2)} = 0$ and $E = E^{(2)} = 0$. In this gauge, the evolution equation for inflaton fluctuations is regular also during the coherent oscillation of the scalar field [17]. In the UCG and in the long wavelenght limit we have, for the given second order curvature perturbation potentials, the following results:

$$R_A^{(2)} = \frac{H}{\dot{\phi}}\varphi^{(2)} - \frac{H}{\dot{\phi}^2}\varphi\dot{\varphi} + \frac{H^2}{\dot{\phi}^2}\varphi^2 \left(-1 - \frac{\dot{H}}{2H^2} + \frac{1}{2H}\frac{\ddot{\phi}}{\dot{\phi}}\right) \tag{18}$$

$$R_B^{(2)} = \frac{H}{\dot{\phi}}\varphi^{(2)} + \frac{1}{2}\left(2H^2 + \dot{H} - H\frac{\ddot{\phi}}{\dot{\phi}}\right)^{-1} \frac{H^2}{\dot{\phi}^2}\dot{\varphi}^2 \tag{19}$$

$$R_C^{(2)} = \frac{H}{\dot{\phi}}\varphi^{(2)} - \frac{H}{\dot{\phi}^2}\varphi\dot{\varphi} + \frac{H^2}{\dot{\phi}^2}\varphi^2 \left(-1 - \frac{\dot{H}}{2H^2} + \frac{1}{2H}\frac{\ddot{\phi}}{\dot{\phi}}\right) + \frac{1}{\nabla^2}\left[4\frac{H^2}{\dot{\phi}^2}\varphi\nabla^2\varphi + \frac{3}{2}\frac{H^2}{\dot{\phi}^2}\partial^i\varphi\partial_i\varphi\right] . \tag{20}$$

On subtracting the average of the quadratic piece (in order to also have a zero average for the non-linear piece), it is useful to introduce the parameter of nonlinearity f_{NL} [11]:

$$R = R_{\rm L} + \frac{3}{5} f_{\rm NL}\left[R_{\rm L}^2 - \langle R_{\rm L}^2\rangle\right] \tag{21}$$

where $R_{\rm L}$ is the Gaussian part of R (that is the first order part $R^{(1)}$). Using the first order comoving curvature perturbation together with the first two second order definitions given in Eqs. (18), (19), we obtain for a $\frac{m^2}{2}\phi^2$ chaotic inflation and using the approximation $\dot{\varphi} = -\frac{\dot{H}}{H}\varphi$ and Eq.(47), to leading order in the slow-roll parameter $\epsilon = -\frac{\dot{H}}{H^2}$, the following results for f_{NL}:

$$f_{\rm NL}^A = -\frac{5}{3}$$

$$f_{\rm NL}^B = \frac{5}{6}\epsilon \,.$$

Let us note that we cannot evaluate $f_{\rm NL}^C$ in analogy with [11] because the non-local terms are not subleading in the long-wavelength approximation.

In the next two sections we proceed with the evaluation of the spectrum of the non-linear corrections to the curvature R_A and of the second order GI field fluctuation $Q^{(2)}$ defined in [18]. The evaluation of the quantum correlator of second-order GI variables (see also [19]) involves a sum over momenta, which is plagued by ultraviolet divergencies. The adiabatic subtraction used in the calculation of the finite part of energy and pressure of fluctuations [12, 13] is roughly equivalent to only considering super-Hubble fluctuations. Motivated by this analogy we shall also use this criterium in the present case: all the integrals which follow have the physical Hubble radius as an upper limit of integration. The same integrals also need a lower limit of integration which defines the region of nearly scale invariant (red tilted) spectra generated by inflation: we choose this lower limit of integration, a comoving momentum ℓ, of order of the Hubble rate when inflation begins, H_0, as previously found in analytical [20] and numerical investigation [12] (see also [21]). At the present state of our knowledge (i.e. without any theoretical input for the choice of ℓ), we believe that this is a natural choice as compared to considering ℓ of the order of the present comoving Hubble radius as is done in [22]. From now on we shall restrict ourselves to the slow-roll stage of $\frac{m^2}{2}\phi^2$ chaotic inflation, during which $\dot{H} \simeq -m^2/3$ holds.

A. Analytic Evaluation of curvature $R_A^{(2)}$

Within the above approximation from eq.(18), to leading order in the slow-roll parameter and using Eq.(28) of our [13], one obtains

$$\frac{H}{\dot{\phi}}\varphi^{(2)} \sim \epsilon R^{(1)\,2} \tag{22}$$

and

$$R_A^{(2)} = R^{(1)\,2}\left[-1 + \mathcal{O}\left(\epsilon\right)\right] . \tag{23}$$

From the above equation and Eq. (17) it is evident that R_{LR} is of order a slow-roll parameter times the square of the first-order perturbation.

Let us consider the quantum correlation function for this second order gauge invariant variable

$$\begin{aligned} \langle 0|R_A^{(2)}(\mathbf{x})R_A^{(2)}(\mathbf{y})|0\rangle &= \left(1 + \mathcal{O}\left(\epsilon\right)\right)\langle 0|\left(R^{(1)}(\mathbf{x})\right)^2\left(R^{(1)}(\mathbf{y})\right)^2|0\rangle \\ &\simeq \frac{H^4}{\dot{\phi}^4}\langle 0|\left(\varphi(t,\mathbf{x})\right)^2\left(\varphi(t,\mathbf{y})\right)^2|0\rangle . \end{aligned} \tag{24}$$

The quantized scalar field variable φ is given by

$$\hat{\varphi}(t,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\int d\mathbf{k}\left[\varphi_k(t)\, e^{i\mathbf{k}\cdot\mathbf{x}}\ \hat{a}_{\mathbf{k}} + \varphi_k^*(t)e^{-i\mathbf{k}\cdot\mathbf{x}}\ \hat{a}_{\mathbf{k}}^\dagger\right] \tag{25}$$

in the slow-roll approximation and on large scales (but $< 2\pi/\ell$) we consider [13]

$$\varphi_k(t) = -\frac{i}{H}\frac{H(t_k)^2}{\sqrt{2k^3}} , \tag{26}$$

with

$$H(t_k) \simeq H_0\left(1 - 2\epsilon_0\ln\frac{k}{H_0}\right)^{1/2} \tag{27}$$

which is the Hubble parameter when the fluctuations crosses the horizon and $\epsilon_0 = \epsilon(t=0)$. The power spectrum of first order curvature perturbation, at large scales and in the UCG, is therefore:

$$P_{R^{(1)}}(k) \equiv \frac{k^3}{2\pi^2}|R_k^{(1)}|^2 = \frac{k^3}{2\pi^2}\frac{H^2}{\dot{\phi}^2}|\varphi_k(t)|^2 = \frac{3}{8\pi^2}\frac{H(t_k)^4}{m^2 M_{pl}^2} . \tag{28}$$

It is important to note that by using the expression (27) for $H(t_k)$ we obtain a power spectrum of curvature perturbations in (28) having the correct value to leading order of the spectral index and exhibiting running during slow-roll.

The physical content of the correlation function for $R_A^{(2)}$ is conveniently described by the power spectrum associated with its fourier transform

$$\langle 0|R_A^{(2)}(\mathbf{x})R_A^{(2)}(\mathbf{y})|0\rangle = \frac{1}{(2\pi)^3}\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}|R_{A\,k}^{(2)}(t)|^2 \tag{29}$$

which becomes, using Eq.(24)

$$\langle 0|R_A^{(2)}(\mathbf{x})R_A^{(2)}(\mathbf{y})|0\rangle \simeq \frac{H^4}{4\dot{H}^2}\frac{1}{M_{pl}^4}\frac{1}{(2\pi)^6}\left\{\int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\frac{1}{2H^4}\Theta(aH-k_1)\right.$$
$$\Theta(aH-k_2)\Theta(k_1-l)\Theta(k_2-l)\exp^{i(\mathbf{k_1}+\mathbf{k_2})\cdot(\mathbf{x}-\mathbf{y})} + \int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\frac{1}{4H^4}$$
$$\left.\Theta(aH-k_1)\Theta(aH-k_2)\Theta(k_1-l)\Theta(k_2-l)\right\}\,, \tag{30}$$

where we have considered only modes which are super-Hubble, but greater then comoving infrared cut-off l, using the Heaviside functions Θ. We can perform a Fourier transform with respect to $\mathbf{r} = \mathbf{x} - \mathbf{y}$ of Eq.(30) and obtain

$$|R_{A\,p}^{(2)}(t)|^2 \simeq \frac{H^4}{4\dot{H}^2}\frac{1}{M_{pl}^4}\frac{1}{(2\pi)^3}\int d^3k \frac{H(t_k)^4}{k^3}\frac{H(t_{p-k})^4}{|\mathbf{p}-\mathbf{k}|^3}\frac{1}{2H^4}\Theta(aH-k)\Theta(aH-|\mathbf{p}-\mathbf{k}|)$$
$$\Theta(k-l)\Theta(|\mathbf{p}-\mathbf{k}|-l) + (2\pi)^3\delta^{(3)}(p)\tilde{Q}_0 \tag{31}$$

where

$$\tilde{Q}_0 = \frac{H^4}{4\dot{H}^2}\frac{1}{M_{pl}^4}\frac{1}{(2\pi)^6}\int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\frac{1}{4H^4}\Theta(aH-k_1)\Theta(aH-k_2)\Theta(k_1-l)\Theta(k_2-l)\,. \tag{32}$$

For $p \neq 0$ and on evaluating the integral in Eq.(31) to leading order and in the momentum window given by the condition $l << p << a(t)H(t)$ one obtains the expression:

$$|R_{A\,p}^{(2)}(t)|^2 \simeq \frac{1}{32\pi^2}\frac{1}{M_{pl}^4}\frac{H_0^4}{\epsilon_0^2}\frac{1}{p^3}\left\{\sum_{i=0}^{3} A_i\left(\ln\frac{p}{l}\right)^i + g\left(\frac{l}{p}\right) + h\left(\frac{p}{a(t)H(t)}\right)\right\} \tag{33}$$

which is time independent in our approximation. The coefficients A_i are given in appendix B, $g\left(\frac{l}{p}\right) = \mathcal{O}\left(\left(\ln\frac{p}{l}\right)^2\frac{l}{p}\right)$ and $h\left(\frac{p}{a(t)H(t)}\right) = \mathcal{O}\left(\left(\ln\frac{a(t)H(t)}{p}\right)^4\left(\frac{p}{a(t)H(t)}\right)^3\right)$.

Substituting for the coefficients A_i we obtain the following result:

$$|R_{A\,p}^{(2)}|^2 \simeq \frac{1}{32\pi^2}\frac{1}{M_{pl}^4}\frac{1}{p^3}\left\{-2.8405\frac{1}{\dot{H}}H(t_p)^6 + 19.3975H(t_p)^4 - 14.9341\dot{H}H(t_p)^2 + 9.7273\dot{H}^2 + 4\frac{1}{\dot{H}^2}H(t_p)^8\right.$$
$$\left.\ln\frac{p}{l} + 16H(t_p)^4\left(\ln\frac{p}{l}\right)^2 + \frac{16}{3}H(t_p)^4\left(\ln\frac{p}{l}\right)^3 + g\left(\frac{l}{p}\right) + h\left(\frac{p}{a(t)H(t)}\right)\right\}\,. \tag{34}$$

On taking only the leading term for the coefficents of the different powers in $\ln(p/l)$, we obtain:

$$|R_{A\,p}^{(2)}|^2 \simeq \frac{1}{32\pi^2}\frac{1}{M_{pl}^4}\frac{H(t_p)^4}{p^3}\left\{8.5215\frac{H(t_p)^2}{m^2} + 36\frac{H(t_p)^4}{m^4}\ln\frac{p}{l} + 16\left(\ln\frac{p}{l}\right)^2 + \frac{16}{3}\left(\ln\frac{p}{l}\right)^3\right\}$$
$$\simeq \frac{1}{24\pi^2}|R_p^{(1)}|^2\left\{8.5215\,\frac{H(t_p)^2}{M_{pl}^2} + 36\frac{H(t_p)^4}{m^2M_{pl}^2}\left(\ln\frac{p}{l}\right) + 16\frac{m^2}{M_{pl}^2}\left(\ln\frac{p}{l}\right)^2 + \frac{16}{3}\frac{m^2}{M_{pl}^2}\left(\ln\frac{p}{l}\right)^3\right\}\,, \tag{35}$$

where in the last equality we have used Eq.(28). This last expression shows compactly one of the main results of our investigation: the spectrum of this second order GI curvature perturbation is proportional to the first order spectrum through logarithmic corrections which encode the scale l at which inflation started. Without these logarithmic corrections the second order curvature perturbation would be completely local and much smaller than first order terms since for the scales of interest $H(t_k) << M_{pl}$.

B. Analytic Evaluation of $Q^{(2)}$

Let us consider the second order GI scalar field fluctuation $Q^{(2)}$ (which is a possible second order generalization of the first order Mukhanov variable [23]) defined in [18] as the second order field fluctuations on uniform curvature hypersurfaces in the longwavelength limit. The uniform curvature hypersurfaces is defined as the slicing with $\hat{\psi} = \hat{\psi}^{(2)} = 0$, and one obtains the following result [7, 18]:

$$Q^{(2)} = \varphi^{(2)} + \frac{\dot{\phi}}{H}\psi^{(2)} + \frac{1}{H}\psi\dot{\varphi} + \left(1 + \frac{1}{2H}\frac{\ddot{\phi}}{\dot{\phi}} - \frac{1}{2}\frac{\dot{H}}{H^2}\right)\frac{\dot{\phi}}{H}\psi^2 + \frac{\dot{\phi}}{H^2}\psi\dot{\psi} \,. \tag{36}$$

In the UCG, as for the first order, this GI variable is simply the second order field fluctuations $\varphi^{(2)}$, studied in our previous paper [13]. Thus the equation of motion for this second order GI field fluctuation in the UCG, to leading order in the slow-roll parameter and in the long-wavelength approximation is given by [13]:

$$\ddot{Q}^{(2)} + 3H\dot{Q}^{(2)} + 3\dot{H}Q^{(2)} = m^2 \frac{\dot{\phi}}{2H}\frac{\varphi^2}{M_{pl}^2} \,. \tag{37}$$

It is interesting to comment on the self-interaction mediated by gravity corrected for the inflaton. Just as the feedback of metric perturbation vanishes in first order perturbation theory for $\dot{\phi} = 0$ (see Eq. (7) of [13]), it also vanishes for the self-interaction to second order in Eq. (37).

Integrating the above in two steps we obtain:

$$\dot{Q}^{(2)} + 3HQ^{(2)} = m^2 \int dt' \frac{\dot{\phi}}{2H(t')}\frac{\varphi(t',\mathbf{x})^2}{M_{pl}^2} \,, \tag{38}$$

and

$$Q^{(2)}(t,\mathbf{x}) = \frac{1}{a(t)^3}\int^t dt' a(t')^3 \int^{t'} dt'' \frac{\dot{\phi}}{2H(t'')} m^2 \frac{\varphi(t'',\mathbf{x})^2}{M_{pl}^2} \,. \tag{39}$$

Let us consider the quantum correlation function of this second order $Q^{(2)}$ gauge invariant variable $\langle 0|Q^{(2)}(t,\mathbf{x})Q^{(2)}(t,\mathbf{y})|0\rangle$. The quantized scalar variable $Q^{(2)}$ is given by

$$\hat{Q}^{(2)}(t,\mathbf{x}) \;=\; \frac{1}{(2\pi)^{3/2}}\int d\mathbf{k}\left[Q_k^{(2)}(t)\, e^{i\mathbf{k}\cdot\mathbf{x}}\;\hat{b}_{\mathbf{k}} + Q_k^{(2)*}(t) e^{-i\mathbf{k}\cdot\mathbf{x}}\;\hat{b}_{\mathbf{k}}^{\dagger}\right] \,. \tag{40}$$

The physical content of the correlation function for $Q^{(2)}$ is better described by the power spectrum associated with its fourier transform

$$\langle 0|Q^{(2)}(t,\mathbf{x})Q^{(2)}(t,\mathbf{y})|0\rangle = \frac{1}{(2\pi)^3}\int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}|Q_k^{(2)}(t)|^2 \tag{41}$$

and using Eq.(39) and Eq.(26), one obtains

$$\begin{aligned}
\langle 0|Q^{(2)}(t,\mathbf{x})Q^{(2)}(t,\mathbf{y})|0\rangle = \frac{1}{a(t)^6}\left(\frac{\dot{\phi}}{2}\frac{m^2}{M_{pl}^2}\right)^2 \frac{1}{(2\pi)^6}\Bigg\{\int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\frac{1}{2}\Bigg[\int^t dt' a(t')^3 \\
\int^{t'} dt'' \frac{1}{H(t'')^3}\Theta(a(t'')H(t'') - k_1)\Theta(a(t'')H(t'') - k_2)\Bigg]^2 \Theta(k_1 - l)\Theta(k_2 - l)\exp^{i(\mathbf{k_1}+\mathbf{k_2})\cdot(\mathbf{x}-\mathbf{y})} \\
+ \int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\frac{1}{4}\left[\int^t dt' a(t')^3 \int^{t'} dt'' \frac{1}{H(t'')^3}\Theta(a(t'')H(t'') - k_1)\right] \\
\left[\int^t dt' a(t')^3 \int^{t'} dt'' \frac{1}{H(t'')^3}\Theta(a(t'')H(t'') - k_2)\right]\Theta(k_1 - l)\Theta(k_2 - l)\Bigg\} \,.
\end{aligned} \tag{42}$$

On performing the Fourier transformation w.r.t. $\mathbf{r} = \mathbf{x} - \mathbf{y}$ of Eq.(42) one finds

$$|Q_p^{(2)}(t)|^2 = \frac{1}{a(t)^6}\left(\frac{\dot\phi}{2}\frac{m^2}{M_{pl}^2}\right)^2 \frac{1}{(2\pi)^3}\int d^3k \frac{H(t_k)^4}{k^3}\frac{H(t_{p-k})^4}{|\mathbf{p}-\mathbf{k}|^3}\frac{1}{2}\left[\int^t dt' a(t')^3 \int^{t'} dt'' \frac{1}{H(t'')^3}\Theta(a(t'')H(t'') - k)\right.$$
$$\left.\Theta(a(t'')H(t'') - |\mathbf{p}-\mathbf{k}|)\right]^2 \Theta(k-l)\Theta(|\mathbf{p}-\mathbf{k}|-l) + (2\pi)^3\delta^{(3)}(p)Q_0 \qquad (43)$$

where

$$Q_0 = \frac{1}{a(t)^6}\left(\frac{\dot\phi}{2}\frac{m^2}{M_{pl}^2}\right)^2 \frac{1}{(2\pi)^6}\int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\frac{1}{4}\left[\int^t dt' a(t')^3 \int^{t'} dt'' \frac{1}{H(t'')^3}\Theta(a(t'')H(t'') - k_1)\right]$$
$$\left[\int^t ds' a(s')^3 \int^{s'} ds'' \frac{1}{H(s'')^3}\Theta(a(s'')H(s'') - k_2)\right]\Theta(k_1 - l)\Theta(k_2 - l)\,. \qquad (44)$$

On considering $p \neq 0$ and replacing the Θ function by the domain of integration, it is possible to perform the time integrals exactly:

$$|Q_p^{(2)}(t)|^2 = \left(\frac{\dot\phi}{2}\frac{m^2}{M_{pl}^2}\right)^2 \frac{1}{(2\pi)^2}\frac{1}{2}\int_l^{a(t)H(t)} dk \int_{-1}^{+1} dy \Theta(a(t)H(t) - (p^2+k^2-2kpy)^{1/2})\Theta((p^2+k^2-2kpy)^{1/2} - l)$$
$$\frac{1}{k}\frac{1}{(p^2+k^2-2kpy)^{3/2}}H_0^6\left(1+2\frac{\dot H}{H_0^2}\ln\frac{k}{H_0}\right)^2\left(1+2\frac{\dot H}{H_0^2}\ln\frac{(p^2+k^2-2kpy)^{1/2}}{H_0}\right)^2$$
$$\frac{1}{36\dot H^2}\mathrm{e}^{-\frac{3}{2}\frac{H(t)^2}{\dot H}}\left\{\frac{1}{f(k,p,y)^2}\left(-\frac{3}{2\dot H}\right)^{1/2}\left[\Gamma\left(\frac{1}{2},-\frac{3}{2}\frac{H(t)^2}{\dot H}\right) - \Gamma\left(\frac{1}{2},-\frac{3}{2}\frac{f(k,p,y)^2}{\dot H}\right)\right]\right.$$
$$\left. - \left(-\frac{3}{2\dot H}\right)^{3/2}\left[\Gamma\left(-\frac{1}{2},-\frac{3}{2}\frac{H(t)^2}{\dot H}\right) - \Gamma\left(-\frac{1}{2},-\frac{3}{2}\frac{f(k,p,y)^2}{\dot H}\right)\right]\right\}^2, \qquad (45)$$

where $f(k,p,y)$ is given by

$$f(k,p,y) = Min\left[H_0\left(1+2\frac{\dot H}{H_0^2}\ln\frac{k}{H_0}\right)^{1/2}, H_0\left(1+2\frac{\dot H}{H_0^2}\ln\frac{(p^2+k^2-2kpy)^{1/2}}{H_0}\right)^{1/2}\right].$$

It is also possible to obtain an explicit approximate espression for the integral starting from Eq.(43) subject to the condition $l << p << a(t)H(t)$ and the approximation $l << p << a(t'')H(t'')$. In particular we find

$$|Q_p^{(2)}(t)|^2 \simeq \frac{1}{a(t)^6}\left(\frac{\dot\phi}{2}\frac{m^2}{M_{pl}^2}\right)^2 \frac{1}{8\pi^2}H_0^2\frac{1}{p^3}\left\{\sum_{i=0}^{3}A_i\left(\ln\frac{p}{l}\right)^i + g\left(\frac{l}{p}\right) + h\left(\frac{p}{a(t)H(t)}\right)\right\}\frac{1}{36}\frac{1}{\dot H^2}e^{-3\frac{H_0^2}{\dot H}}$$
$$\left\{\left(-\frac{3}{2}\frac{H_0^2}{\dot H}\right)^{1/2}\left[\Gamma\left(\frac{1}{2},-\frac{3}{2}\frac{H^2}{\dot H}\right) - \Gamma\left(\frac{1}{2},-\frac{3}{2}\frac{H_0^2}{\dot H}\right)\right] - \left(-\frac{3}{2}\frac{H_0^2}{\dot H}\right)^{3/2}\left[\Gamma\left(-\frac{1}{2},-\frac{3}{2}\frac{H^2}{\dot H}\right) - \Gamma\left(-\frac{1}{2},-\frac{3}{2}\frac{H_0^2}{\dot H}\right)\right]\right\}^2 \qquad (46)$$

where, as in the previous subsection, the coefficients A_i are discussed in the appendix, $g\left(\frac{l}{p}\right) = \mathcal{O}\left(\left(\ln\frac{p}{l}\right)^2\frac{l}{p}\right)$ and $h\left(\frac{p}{a(t)H(t)}\right) = \mathcal{O}\left(\left(\ln\frac{a(t)H(t)}{p}\right)^4\left(\frac{p}{a(t)H(t)}\right)^3\right)$.

Let us also investigate a much more crude approximation, obtained on neglecting $\dot Q^{(2)}$ in Eq.(38) and considering the infrared limit $\dot\varphi = -\frac{\dot H}{H}\varphi$, which leads to the expression

$$Q^{(2)} \simeq \frac{1}{4}\frac{\dot\phi}{H}\frac{1}{M_{pl}^2}\varphi^2 = \frac{\dot\phi}{H}\frac{\epsilon}{2}R^{(1)\,2}\,. \qquad (47)$$

The correlation function for $Q^{(2)}$ turn simplifies to

$$\langle 0|Q^{(2)}(t,\mathbf{x})Q^{(2)}(t,\mathbf{y})|0\rangle \simeq -\frac{\epsilon^3 M_{pl}^2}{2}\langle 0|R_A^{(2)}(t,\mathbf{x})R_A^{(2)}(t,\mathbf{y})|0\rangle = \left(\frac{\dot{\phi}}{4H}\frac{1}{M_{pl}^2}\right)^2 \frac{1}{(2\pi)^6}\left\{\int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\right.$$
$$\frac{1}{2H^4}\Theta(aH-k_1)\Theta(aH-k_2)\Theta(k_1-l)\Theta(k_2-l)\exp^{i(\mathbf{k_1}+\mathbf{k_2})\cdot(\mathbf{x}-\mathbf{y})} + \int d^3k_1 d^3k_2 \frac{H(t_{k_1})^4}{k_1^3}\frac{H(t_{k_2})^4}{k_2^3}\frac{1}{4H^4}$$
$$\left.\Theta(aH-k_1)\Theta(aH-k_2)\Theta(k_1-l)\Theta(k_2-l)\right\}\,. \tag{48}$$

So, on proceeding as the previous subsection, for $p \neq 0$, to the leading order and in an "approximate" fashion with the condition $l << p << a(t)H(t)$ one obtains

$$|Q_p^{(2)}(t)|^2 \simeq \left(\frac{\dot{\phi}}{4H}\frac{1}{M_{pl}^2}\right)^2 \frac{1}{8\pi^2}\frac{H_0^8}{H^4}\frac{1}{p^3}\left\{\sum_{i=0}^{3} A_i\left(\ln\frac{p}{l}\right)^i + g\left(\frac{l}{p}\right) + h\left(\frac{p}{a(t)H(t)}\right)\right\}\,. \tag{49}$$

Substituting the coefficients A_i we obtain the following result:

$$|Q_p^{(2)}(t)|^2 \simeq \left(\frac{\dot{\phi}}{4H}\frac{1}{M_{pl}^2}\right)^2 \frac{1}{8\pi^2}\epsilon^2\frac{1}{p^3}\left\{-2.8405\frac{1}{\dot{H}}H(t_p)^6 + 19.3975H(t_p)^4 - 14.9341\dot{H}H(t_p)^2 + 9.7273\dot{H}^2 + 4\frac{1}{\dot{H}^2}H(t_p)^8\right.$$
$$\left.\ln\frac{p}{l} + 16H(t_p)^4\left(\ln\frac{p}{l}\right)^2 + \frac{16}{3}H(t_p)^4\left(\ln\frac{p}{l}\right)^3 + g\left(\frac{l}{p}\right) + h\left(\frac{p}{a(t)H(t)}\right)\right\}\,. \tag{50}$$

On taking only the leading term for the coefficents of the different powers in $\ln(p/l)$, we obtain:

$$|Q_p^{(2)}(t)|^2 \simeq \frac{1}{96\pi^2}\epsilon^2|\varphi_p(t)|^2\left\{8.5215\frac{H(t_p)^2}{M_{pl}^2} + 36\frac{H(t_p)^4}{m^2M_{pl}^2}\left(\ln\frac{p}{l}\right) + 16\frac{m^2}{M_{pl}^2}\left(\ln\frac{p}{l}\right)^2 + \frac{16}{3}\frac{m^2}{M_{pl}^2}\left(\ln\frac{p}{l}\right)^3\right\}\,. \tag{51}$$

Analogous considerations to those given after Eq.(35) are also valid in this case. Also, as we can see from Eq. (50), one obtains a spectrum which, according to Eqs. (23) and (47), slowly increases in time during slow-roll as ϵ^3, whereas $R_A^{(2)}$ is constant. Again, we note that both the GI second order field fluctuation $Q^{(2)}$ and the second order curvature $R_A^{(2)}$ have a nearly scale invariant spectrum with a logarithmic corrections.

IV. NUMERICAL ANALYSIS

Let us introduce a few parameters which are useful for the numerical analysis. The first is $N_{tot} = \ln\left(a(t_f)/a(t_i)\right)$, the total number of e-folds, thus we need to specify the time associated with the end of inflation. This definition may be obtained on requiring the Hubble parameter $H(t)$ to be equal, for example, to the inflaton mass m. This happens, in our chaotic model for the time t_f when the slow-roll parameter ϵ is equal to 1/3. We then solve the equation

$$\epsilon(\tilde{t}) = \tilde{\epsilon} \tag{52}$$

obtaining

$$\tilde{t} = -\frac{H_0}{\dot{H}} - \sqrt{-\frac{1}{\tilde{\epsilon}\dot{H}}} \tag{53}$$

and the final time, defined by $\tilde{\epsilon} = 1/3$, is

$$t_f = -\frac{H_0}{\dot{H}} - \sqrt{-\frac{3}{\dot{H}}} \tag{54}$$

corresponding to a total number of e-folds given by

$$N_{tot} = \frac{3}{2}\frac{H_0^2}{m^2} - \frac{3}{2}\,. \tag{55}$$

It is also useful to define the variable $N(t)$, which is equal to the number of e-folds from the end of inflation,

$$N(t) = \ln\frac{a(t_f)}{a(t)} = \frac{3}{2}\frac{H(t)^2}{m^2} - \frac{3}{2}\,. \tag{56}$$

In our analysis we are interested in the fluctuations that cross the horizon at $N_* = 55$ e-folds from the end of inflation. The time t_* associated with $N_* \equiv \ln\left(a(t_f)/a(t_*)\right) = 55$ is:

$$t_* = -\frac{H_0}{\dot{H}} - \sqrt{-\frac{3}{\dot{H}} - \frac{2}{\dot{H}}N_*} \tag{57}$$

and the comoving wave-number which crosses the Hubble radius at N_* is:

$$k_* = a(t_*)H(t_*) = e^{-\frac{H_0^2}{2\dot{H}} - \frac{3}{2} - N_*}\sqrt{-3\dot{H} - 2\dot{H}N_*}\,. \tag{58}$$

If we wish to take a "photograph" of a fluctuation with k_* on super-Hubble scales during inflation we should take:

$$\frac{1}{3} > \tilde{\epsilon} > \frac{1}{2N_* + 3}\,. \tag{59}$$

We now proceed with the numerical analysis taking $m = 10^{-5}M_{pl}$. In particular we are interested in the comparison of second order fluctuations with first order ones, in order to understand for which condition first order perturbation theory breaks down. We shall also plot the shape, in k, of the spectrum of fluctuations.

A. Second order GI scalar field fluctuation $Q^{(2)}$

In figure 1 we consider, for two different values of the initial Hubble parameter H_0 ($7m$ and $14m$, respectively), the logarithm of the second order contribution to the power spectrum of $Q^{(2)}$:

$$F^{(2)}(k,t) = \log_{10}\left(\frac{1}{2\pi^2}\frac{k^3}{M_{pl}^2}|Q_k^{(2)}(t)|^2\right),$$

as given by Eq.(50), Eq.(46) and the numerical solution of Eq.(45), in order to see the improvement of the roughest analytical approximation, with respect to the exact numerical calculation with increasing H_0, and the shape of the spectrum. We give the figure at an instant during inflation for which $\tilde{\epsilon} = 1/10$.

Corresponding to those two values of H_0 we have the following data

- For $H_0 = 7m$ we have $N_{\rm tot} = 72$, $t_f = 18/m$, $t_* = 2.58805/m$ and $k_*/m = 1.48247\ 10^8$. The time at which we "photograph" the inflation is $\tilde{t} = 15.5228/m$.
- For $H_0 = 14m$ we have $N_{tot} = 292.5$, $t_f = 39/m$, $t_* = 23.588/m$ and $k_*/m = 8.56876\ 10^{103}$. The time at which we "photograph" the inflation is $\tilde{t} = 36.5228/m$.

In order to compare the $F^{(2)}(k,t)$ given by the numerical solution of Eq.(45) with the first order contribution to the power spectrum

$$F^{(1)}(k,t) = \log_{10}\left(\frac{1}{2\pi^2}\frac{k^3}{M_{pl}^2}|\varphi_k(t)|^2\right)$$

given by Eq.(26), we give two different types of figures. In figure 2 we compare the values calculated at k_* as a function of the total number of e-folds N_{tot}, at the end of inflation $t = t_f$. From the figure on the right (which differs from the figure on the left only because of the bigger range of N_{tot}) we see that second order effects are of the same order as first order effects for N_{tot} near 30000 which corresponds to a H_0 near $141m$. Therefore for such values first order perturbation theory breaks down

In figure 3 we show the dependence of the fluctuations on the modes, crossing the horizon N e-folds before the end of inflation, as seen at the end of inflation. We consider three different cases corresponding to values of H_0/m equal to 7, 14 and 30, compare $F^{(1)}(k_N, t_f)$ and $F^{(2)}(k_N, t_f)$, and vary N from 55 to 1 with:

$$k_N = e^{-\frac{H_0^2}{2\dot{H}} - \frac{3}{2} - N}\sqrt{-3\dot{H} - 2\dot{H}N} \tag{60}$$

which is the mode of the fluctuation that crosses the horizon at N e-folds before the end of inflation. As before the first and second order fluctuations are of the same order of magnitude for $H_0 = 141m$.

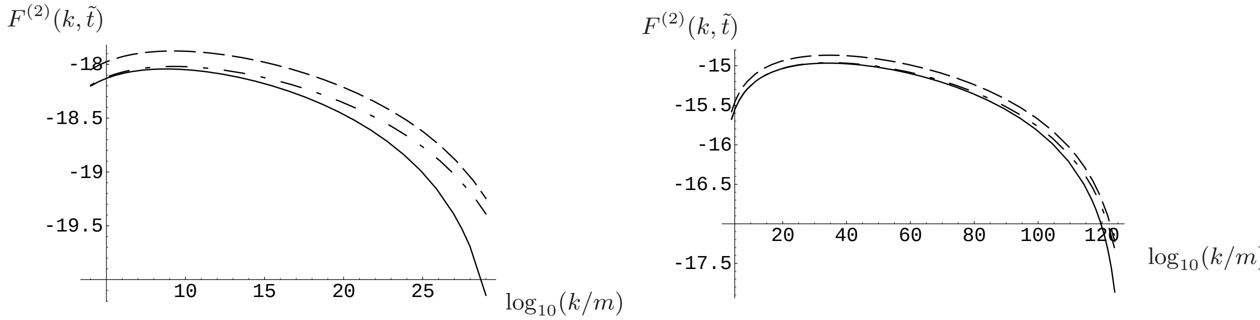

FIG. 1: We plot for $H_0 = 7m$ (on the left) and $H_0 = 14m$ (on the right) the evolution with respect to $\log_{10}\left(\frac{k}{m}\right)$ of $F^{(2)}(k, \tilde{t})$ using the analytic approximations Eq.(50)(dashed line) and Eq.(46) (dot-dashed line) and the numerical solution of Eq.(45) (solid line).

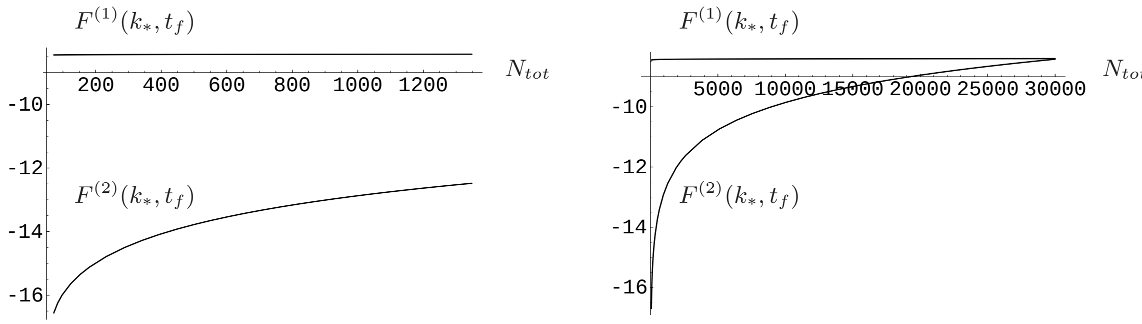

FIG. 2: We plot $F^{(1)}(k_*, t_f)$ and $F^{(2)}(k_*, t_f)$ with respect the total number of e-folds N_{tot}.

B. Comoving curvature perturbation $R_A^{(2)}$

Let us proceed in a analogous way for the second order curvature perturbation (potential) $R_A^{(2)}$. As before in figure 4 we shall consider two different cases for inflation with H_0 equal to 7m and 14m and we shall plot

$$S^{(2)}(k) = \log_{10}\left(\frac{k^3}{2\pi^2}|R_{A\,k}^{(2)}|^2\right)$$

the logarithm of a second order contribution to the power spectrum, as given by the Eq.(34), to exhibit the shape of the spectrum. We give the picture an instant during inflation for which $\tilde{\epsilon} = 1/10$.

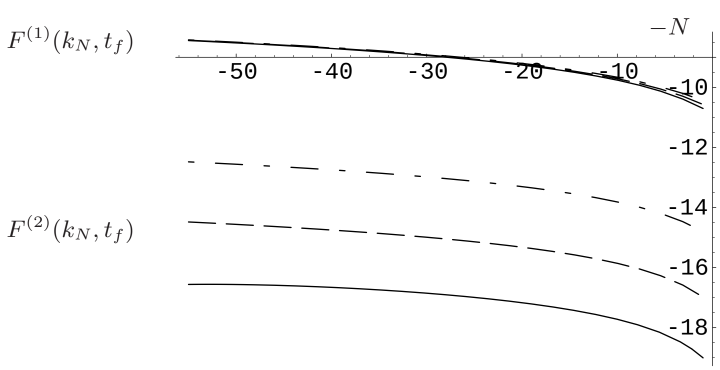

FIG. 3: We plot $F^{(1)}(k_N, t_f)$ and $F^{(2)}(k_N, t_f)$ for $H_0 = 7m$ (solid line), $H_0 = 14m$ (dashed line) and $H_0 = 30m$ (dot-dashed line) with respect N.

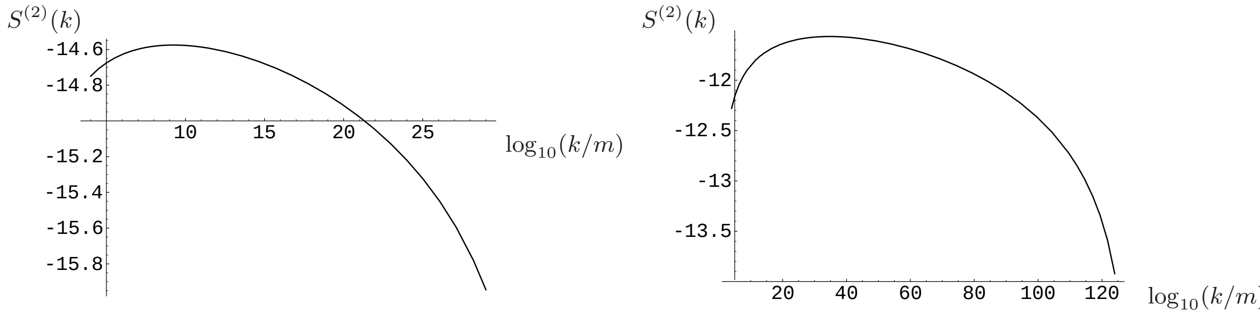

FIG. 4: We plot, for $H_0 = 7m$ (on the left) and $H_0 = 14m$ (on the right), the evolution with respect $\log_{10}\left(\frac{k}{m}\right)$ of $S^{(2)}(k)$.

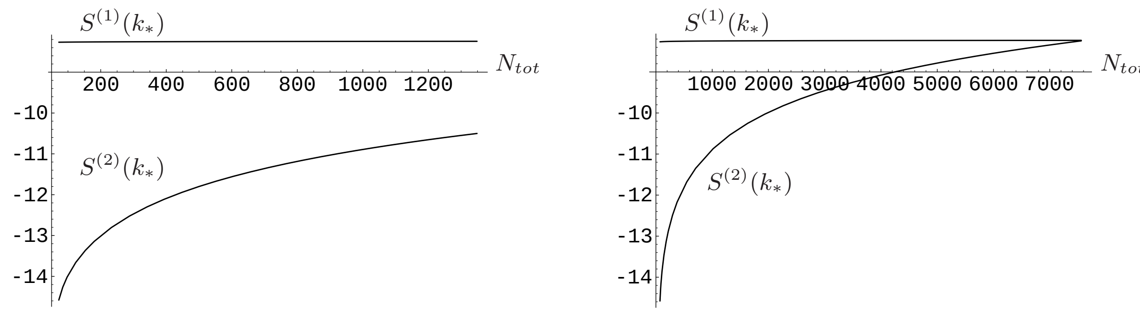

FIG. 5: We plot $S^{(1)}(k_*)$ and $S^{(2)}(k_*)$ with respect to the total number of e-folds N_{tot}.

For the sake of comparing $S^{(2)}(k)$ with the first order contribution to the power spectrum

$$S^{(1)}(k) = \log_{10}\left(\frac{k^3}{2\pi^2}|R_k^{(1)}|^2\right)$$

given by Eq.(28), we give two different kinds of figures. In figure 5 we compare those values calculated at k_*, as a function of the total number of e-folds N_{tot}, at the end of inflation t_f. From the figure on the right (which again differs from the figure on the left only because of the larger range of N_{tot}) we see that for this case the second order effect is of the same order as the first order effect for N_{tot} near 7560 which corresponds to a H_0 near $71m$. Therefore first order perturbation theory breaks down for a value of H_0 smaller than that obtained before and we have a stronger limit.

In figure 6 we again consider three different value of H_0/m, namely 7, 14 and 30, compare $S^{(1)}(k_N)$ and $S^{(2)}(k_N)$, and vary N from 55 to 1 at the end of inflation. As before the first and second orders will be of the same order of magnitude if we take $H_0 = 71m$.

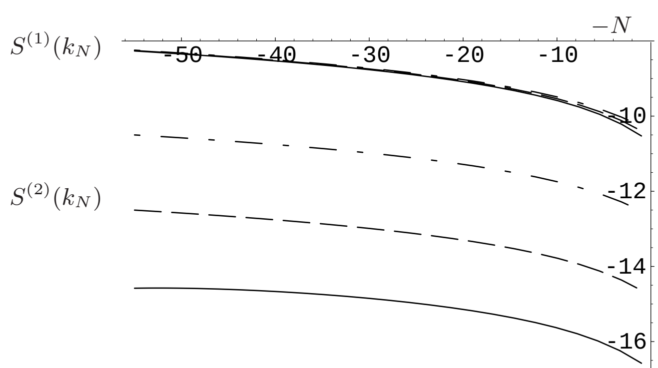

FIG. 6: We plot $S^{(1)}(k_N)$ and $S^{(2)}(k_N)$ for $H_0 = 7m$ (solid line), $H_0 = 14m$ (dashed line) and $H_0 = 30m$ (dot-dashed line) with respect to N.

Finally we wish to compare the critical initial values of H_0 which we obtain in this paper, with two different limiting values. The first one is the value $H_{\rm br}$, obtained by requiring that back-reaction in the UCG gauge will become important by the end of inflation [13, 24], when $H \sim m$. Such a value is:

$$H_{br} \sim \left(16\pi^2\right)^{1/6} m^{2/3} M_{pl}^{1/3} \,. \tag{61}$$

The second one is the value obtained by the self-reproduction argument [25]

$$H_{sr} \simeq (8\pi)^{3/4} \left(\frac{1}{6}\right)^{1/2} m^{1/2} M_{pl}^{1/2} \,. \tag{62}$$

This comparison is given in Table I.

H_0 limit for $R_A^{(2)}$	71 m
H_0 limit for $Q^{(2)}$	141 m
$H_{\rm br}$	108 m
$H_{\rm sr}$	1449 m

TABLE I: Comparison of the values of H_0 for which first order perturbation theory break down with $H_{\rm br}$ and $H_{\rm sr}$.

We stress that neither of the two values of H_0 or of $H_{\rm br}$ are related to the self-reproduction scale [25]. The values of H_0 obtained by considering the breakdown of linear perturbation theory in this paper are of the same order of magnitude as $H_{\rm br}$ (see also [26] for an analogous value). This fact is not surprising: both these calculations include second-order cosmological perturbations. The results for back-reaction were considered ambiguous since it is not simple to demonstrate the slow-down of the inflationary expansion in terms of a GI quantity. On the other hand in the present paper, the breakdown of first order perturbation theory is found for GI variables in the long-wavelength limit.

The results summarized in the Table I and derived from figures 2, 3, 5 and 6 can also be obtained directly from Eq.(35) and Eq.(51). In those cases and for $H_0 >> m$ on using Eq.(60) the leading logarithm values (that is the term with $\left(\ln \frac{p}{l}\right)^3$) of $|R^{(2)}_{A\,p_N}|^2$ and $|Q^{(2)}_{p_N}(t_f)|^2$ are given by:

$$|R^{(2)}_{A\,p_N}|^2 \simeq \frac{3}{4\pi^2}|R^{(1)}_{p_N}|^2\frac{H_0^6}{M_{pl}^2 m^4} \tag{63}$$

$$|Q^{(2)}_{p_N}(t_f)|^2 \simeq \frac{3}{16\pi^2}\epsilon(t_f)^2|\varphi_{p_N}(t_f)|^2\frac{H_0^6}{M_{pl}^2 m^4} \tag{64}$$

and the first and second orders are comparable for

$$H_0 \simeq m\left(\frac{2\pi}{\sqrt{3}}\frac{M_{pl}}{m}\right)^{1/3} \tag{65}$$

and

$$H_0 \simeq m\left(\frac{12\pi}{\sqrt{3}}\frac{M_{pl}}{m}\right)^{1/3} \tag{66}$$

respectively, where Eq.(65) nearly give the same value as Table I (namely near $71m$), while Eq.(66) only gives the same order of magnitude (actually $130m$), which is not surprising considering the approximations made.

V. CONCLUSIONS

We have studied second order cosmological perturbations for a chaotic $\frac{m^2}{2}\phi^2$ inflationary model, and considered three second order GI measures of curvature perturbations.

For the GI curvature perturbation studied and for the second order GI scalar field flcutuation presented in [18], we have found, for all scales of interest (see, for example, Eq.(63) and Eq.(64)), that the amplitude of the spectrum of quadratic corrections grows with the total number of e-folds $N_{\rm tot}$. For $Q^{(2)}$ we also obtain a mild time dependence associated with the growth of the slow-roll parameter ϵ. Since both the quantities studied here are GI, this dependence on $N_{\rm tot}$ cannot be a gauge artifact.

The spectrum of those second order GI variables obtained by Fourier trasforming the curvature-curvature quantum correlation function is nearly scale-invariant, with additional logarithmic corrections with respect to the first order spectrum. On comparing the first and the second order contributions for the two different cases considered, one finds two different values for the initial Hubble parameter H_0 beyond which first order perturbation theory breaks down. We found limits which are of the same order of magnitude as that one found by back-reaction in the UCG [13]. Neither the back-reaction limit nor those found here have anything to do with the self-reproduction scale.

In the curvature-curvature quantum correlation function, the cross terms involving first and third order perturbation may also contribute. The third order curvature perturbation is clearly beyond the scope of the present paper and will be the subject of future work. Unless such a term cancels or dominates the growth we have found, our results imply that in single field models the amplitude of the spectrum of intrinsic non-gaussianities generated during the slow-roll stage will depend on the total number of e-folds which inflation has lasted.

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VI. APPENDIX A: GAUGE TRANSFORMATIONS

The gauge in our formulation (see Eq.(2)) is not fixed, since one can eliminate two scalars among the metric and the inflaton degrees of freedom. After a general analysis we shall be interested in making a gauge choice in order to match previously studied situations. An infinitesimal coordinate trasformation to second order is [27]:

$$x^\mu \to \tilde{x}^\mu = x^\mu + \epsilon^\mu_{(1)} + \frac{1}{2}\left(\epsilon^\mu_{(1),\nu}\epsilon^\nu_{(1)} + \epsilon^\mu_{(2)}\right) , \tag{67}$$

where $\epsilon_{(1)}$ and $\epsilon_{(2)}$ are the coordinate changes to first an second order, respectively, and $\epsilon^i_{(2)} = \partial^i \epsilon^s_{(2)}$ and $\epsilon^i_{(1)} = \partial^i \epsilon^s_{(1)}$. It induces the following change in a geometric object $T = T^{(0)} + T^{(1)} + T^{(2)}$:

$$T^{(1)} \to \tilde{T}^{(1)} = T^{(1)} - \mathcal{L}_{\epsilon_{(1)}} T^{(0)} \tag{68}$$

$$T^{(2)} \to \tilde{T}^{(2)} = T^{(2)} - \mathcal{L}_{\epsilon_{(1)}} T^{(1)} + \frac{1}{2}\left(\mathcal{L}^2_{\epsilon_{(1)}} T_0 - \mathcal{L}_{\epsilon_{(2)}} T_0\right) \tag{69}$$

which leads to the following general gauge trasformation for our scalar perturbations. To first order we have:

$$\tilde{\varphi} = \varphi - \epsilon^0_{(1)}\dot{\phi} \tag{70}$$

$$\tilde{\alpha} = \alpha - \dot{\epsilon}^0_{(1)} \tag{71}$$

$$\tilde{\beta} = \beta - \frac{2}{a}\epsilon^0_{(1)} + 2a\dot{\epsilon}^s_{(1)} \tag{72}$$

$$\tilde{\psi} = \psi + H\epsilon^0_{(1)} + \frac{1}{3}\nabla^2 \epsilon^s_{(1)} \tag{73}$$

$$\tilde{E} = E - 2\epsilon^s_{(1)} \, . \tag{74}$$

To second order one finds:

$$\tilde{\varphi}^{(2)} = \varphi^{(2)} - \epsilon^0_{(1)}\dot{\varphi} + \frac{1}{2}\left[\epsilon^0_{(1)}\left(\epsilon^0_{(1)}\dot{\phi}\right)^{\cdot} - \epsilon^0_{(2)}\dot{\phi}\right] - \partial_k\varphi\partial^k\epsilon^s_{(1)} \, , \tag{75}$$

$$\tilde{\alpha}^{(2)} = \alpha^{(2)} - \epsilon^0_{(1)}\dot{\alpha} - 2\alpha\dot{\epsilon}^0_{(1)} - \frac{1}{2}\dot{\epsilon}^0_{(2)} + \frac{1}{2}\epsilon^0_{(1)}\ddot{\epsilon}^0_{(1)} + \left(\dot{\epsilon}^0_{(1)}\right)^2 \tag{76}$$

$$\tilde{\beta}^{(2)} = \beta^{(2)} - \frac{1}{a}\epsilon^0_{(2)} + a\dot{\epsilon}^s_{(2)} + \frac{1}{2a}\frac{d}{dt}(\epsilon^0_{(1)})^2 + \frac{\partial^i}{\nabla^2}\left[\frac{2}{a}\partial_i\epsilon^0_{(1)}\dot{\epsilon}^0_{(1)} - \epsilon^0_{(1)}\partial_i\dot{\beta} - H\epsilon^0_{(1)}\partial_i\beta - \dot{\epsilon}^0_{(1)}\partial_i\beta - \frac{4}{a}\alpha\partial_i\epsilon^0_{(1)}\right] \tag{77}$$

$$\begin{aligned}\tilde{\psi}^{(2)} &= \psi^{(2)} - \epsilon^0_{(1)}\left(2H\psi + \dot{\psi}\right) + \frac{H}{2}\epsilon^0_{(2)} - \frac{H}{2}\epsilon^0_{(1)}\dot{\epsilon}^0_{(1)} - \frac{\epsilon^{0\,2}_{(1)}}{2}\left(\dot{H} + 2H^2\right) + \frac{1}{6a^2}\partial^i\epsilon^0_{(1)}\,\partial_i\epsilon^0_{(1)} - \frac{1}{6a}\partial^i\beta\partial_i\epsilon^0_{(1)} + \frac{1}{6}\nabla^2\epsilon^s_{(2)} - \\ &-\frac{1}{12}\nabla^2\left(\partial_k\epsilon^s_{(1)}\partial^k\epsilon^s_{(1)}\right) - \frac{2}{3}H\epsilon^0_{(1)}\nabla^2\epsilon^s_{(1)} - \frac{1}{6}\epsilon^0_{(1)}\nabla^2\dot{\epsilon}^s_{(1)} - \frac{1}{2}H\partial^k\epsilon^s_{(1)}\partial_k\epsilon^0_{(1)} - \frac{1}{6}\partial^k\dot{\epsilon}^s_{(1)}\partial_k\epsilon^0_{(1)} - \\ &-\partial^k\epsilon^s_{(1)}\partial_k\psi - \frac{2}{3}\psi\nabla^2\epsilon^s_{(1)} + \frac{1}{3}D_{kj}E\partial^j\partial^k\epsilon^s_{(1)}\end{aligned} \tag{78}$$

$$\begin{aligned}\tilde{E}^{(2)} &= E^{(2)} + \frac{1}{2a}\beta\epsilon^0_{(1)} - \frac{1}{2a^2}(\epsilon^0_{(1)})^2 - \epsilon^s_{(2)} + \frac{1}{2}\partial^k\epsilon^s_{(1)}\partial_k\epsilon^s_{(1)} + \frac{1}{2}\dot{\epsilon}^s_{(1)}\epsilon^0_{(1)} - \partial_kE\partial^k\epsilon^s_{(1)} + \\ &+\frac{3}{2}\frac{\partial^i\partial^j}{(\nabla^2)^2}\left[-2H\epsilon^0_{(1)}D_{ij}E - \epsilon^0_{(1)}D_{ij}\dot{E} + \frac{1}{a^2}\epsilon^0_{(1)}D_{ij}\epsilon^0_{(1)} - \frac{1}{2a}\left(\epsilon^0_{(1)}D_{ij}\beta + \beta D_{ij}\epsilon^0_{(1)}\right) + 4H\epsilon^0_{(1)}D_{ij}\epsilon^s_{(1)} + \right. \\ &\left.+\frac{1}{2}\left(\epsilon^0_{(1)}D_{ij}\dot{\epsilon}^s_{(1)} - \dot{\epsilon}^s_{(1)}D_{ij}\epsilon^0_{(1)}\right) + 4\psi D_{ij}\epsilon^s_{(1)} + \partial_kED_{ij}\partial^k\epsilon^s_{(1)} + \frac{2}{3}\nabla^2ED_{ij}\epsilon^s_{(1)}\right].\end{aligned} \tag{79}$$

VII. APPENDIX B: A_i COEFFICIENT

The coefficients A_i are given by:

$$\begin{aligned}A_0(k) &= 16\,\epsilon_0^2\,(\zeta(3)-1)C_1(k) - 10.198\,\epsilon_0^2\,C_2(k) + 24.656\,\epsilon_0^2\,C_3(k) + \epsilon_0\left(8 - \frac{2}{3}\pi^2\right)C_1(k)C_4(k) \\ &\quad -9.30635\,\epsilon_0\,C_2(k)C_4(k) + 33.333\,\epsilon_0\,C_3(k)C_4(k) + \left(\frac{\pi^2}{3} - 4\right)C_2(k)C_4(k)^2 + 6.07344\,C_3(k)C_4(k)^2 \\ A_1(k) &= 4\,C_1(k)C_4(k)^2 - 8\,C_2(k)C_4(k)^2 \\ A_2(k) &= 4\,\epsilon_0\,C_1(k)C_4(k) - 8\,\epsilon_0\,C_2(k)C_4(k) + 2\,C_2(k)C_4(k)^2 + 8\,C_3(k)C_4(k)^2 \\ A_3(k) &= \frac{8}{3}\,\epsilon_0^2\,C_1(k) - \frac{16}{3}\,\epsilon_0^2\,C_2(k) + \frac{8}{3}\,C_3(k)\,C_4(k)^2\end{aligned} \tag{80}$$

where

$$\begin{aligned}C_1(k) &= \frac{H(t_k)^4}{H_0^4} - 4\epsilon_0\frac{H(t_k)^2}{H_0^2} + 8\epsilon_0^2 \\ C_2(k) &= -2\epsilon_0\left(\frac{H(t_k)^2}{H_0^2} - 2\epsilon_0\right) \\ C_3(k) &= \epsilon_0^2 \\ C_4(k) &= \frac{H(t_k)^2}{H_0^2}\end{aligned} \tag{81}$$

so one obtains

$$\begin{aligned}A_0(k) &= \left(16 - \frac{4}{3}\pi^2\right)\epsilon_0\frac{H(t_k)^6}{H_0^6} + \left(4\pi^2 + 16\,\zeta(3) - 39.3139\right)\epsilon_0^2\frac{H(t_k)^4}{H_0^4} + \left(144.5036 - \frac{16}{3}\pi^2 - 64\,\zeta(3)\right) \\ &\quad \epsilon_0^3\frac{H(t_k)^2}{H_0^2} + (128\,\zeta(3) - 144.136)\,\epsilon_0^4 \\ A_1(k) &= 4\frac{H(t_k)^8}{H_0^8}\end{aligned}$$

$$A_2(k) = 16\epsilon_0^2 \frac{H(t_k)^4}{H_0^4}$$
$$A_3(k) = \frac{16}{3}\epsilon_0^2 \frac{H(t_k)^4}{H_0^4} \tag{82}$$

with the relation $3\epsilon_0 < \frac{H(t_k)^2}{H_0^2} < 1$, which is valid during the inflationary era (which, for us, ends at $\epsilon(t) = 1/3$).

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