ARTICLES

Hindered decay: Quantum Zeno effect through electromagnetic field domination

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The lifetime of an unstable atom can be extended by watching it closely, i.e., illuminating it with an intense electromagnetic field of appropriate frequency. This is an example of ''dominated evolution'' and is closely related to the so-called ''quantum Zeno effect.'' For a metastable atom bathed in a laser beam at the frequency of *another* of its transitions, we obtain an expression for the modified lifetime as a function of beam intensity. This provides an example of the quantum Zeno effect on a *truly decaying* system, and also should be useful for probing short distance features of atomic wave functions. [S1050-2947(97)00807-X]

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I. INTRODUCTION

This paper has two parallel goals: (1) calculating the results of quantum decay (nondecay, actually) that would allow observation of the so-called quantum Zeno effect in its most dramatic form [1–5], and (2) removing from that calculation undefined concepts such as ''quantum measurements,'' that tend to obscure both the ordinary nature of this quantum phenomenon and the significantly nonclassical behavior that is predicted. A by-product of the second aspect is that there is no need for pulsed laser illumination for the demonstration of the effect [6,7]. We will also show how the experiments we do propose can provide a probe of atomic wave functions at small distance scales.

The temporal evolution of quantum-mechanical systems [8] has a number of subtle and surprising properties; in particular, the short-time behavior and its apparently paradoxical consequences [9] have attracted the attention of several researchers. However, the quantum Zeno effect (QZE) can be given a purely dynamical explanation [10] that does not rely upon the idea of quantum measurement.

As we will show, the phrase ''dominated time evolution'' [11] more accurately describes what is generally known as the quantum Zeno effect, both because the system *can be constrained to move* [12], not only to stand still, and because once one brings the measurement apparatus into the system it is clear that the prevention of decay (or whatever) is the consequence of the domination of the smaller system through its interaction with the larger apparatus—itself considered to be a quantum system. However, in this article we will use the acronym ''QZE'' to refer to the phenomenon.

The system we study is an atom in a relatively long-lived metastable state. Following Zeno, we will show that looking closely at it holds it in its original state, never permitting decay. Breaking the time interval into subintervals of decreasing size will not be necessary (so we have other priorities than Zeno), a steady gaze is sufficient. We will hie more closely to the original paradox, however, in that it is decay that is prevented, rather than Rabi oscillation [6,7].

In the usual description of QZE one notes that for a decaying quantum system, although the *phase* changes by $O(dt)$as it begins to move out of its metastable state, its norm only changes by $O(dt^2)$. In this way, if the system is ''measured'' N times during a fixed time interval its norm only changes by N/N^2, which goes to zero as $N\rightarrow 0$. This business of ''N measurements'' has introduced two confusing issues: first, it was felt that one needed to pulse whatever external apparatus was doing the ''measurement''; and second, questions were raised about how this might differ from ''looking closely'' at a system—steady observation, say by photons. How bright must the light be before QZE stopped decay? (Or should a nucleus not decay because it is embedded in a solid?) And then, if you could answer that, you might still seek an intuitive idea of just how a bright light might stop decay.

In this article we get away from the notion of a ''measurement'' as a separate kind of natural phenomenon. We treat QZE as the interaction of our measured system with a larger world, a world that is nevertheless fully quantum mechanical. Our metastable system also possesses a third, short-lived, level. The ''measurement'' consists of having a strong laser beam at the excitation frequency of the short transition. Thus as soon as our system would decay to the ground state it would be snatched away and put in the third state. So it does not decay to the ground state. This is the ''steady gaze'' and for this system it is enough.

We will also study the intermediate case, where the light is strong, but not strong enough to stop decay. In this case,

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the reduced decay rate depends on the atomic wave functions at large momentum (in the momentum representation). We expect this to provide a probe of these wave functions.

Before making further comment we will present our calculation. In the last section we will review our experimental predictions (including quantitative estimates) and also discuss the conceptual issues raised by our results.

II. THE MODEL HAMILTONIAN

We base our analysis on the following Hamiltonian ($\hbar=1$):

$$H=\begin{pmatrix} E_3 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{pmatrix}+\sum_k \omega_k a_k^\dagger a_k+\sum_k \Omega_k A_k^\dagger A_k + \sum_k (\phi_k a_k^\dagger \sigma_- + \text{H.c.}) + \sum_k (\Phi_k A_k^\dagger \tau_- + \text{H.c.}), \tag{2.1}$$

where the first term is the free Hamiltonian of the three-level atom, the second and third terms are the free Hamiltonians of the photons associated with the $1\leftrightarrow 2$ and $1\leftrightarrow 3$ transitions, respectively ($\omega_k \approx E_2-E_1$ and $\Omega_k\approx E_3-E_1$), and the fourth and fifth terms are the interaction Hamiltonians describing, in the rotating-wave approximation, the $1\leftrightarrow 2$ and $1\leftrightarrow 3$ transitions, respectively. The operators a_k and A_k obey boson commutation relations

$$[a_k, a_{k'}^\dagger]=\delta_{kk'}, \quad [A_k A_{k'}^\dagger]=\delta_{kk'}, \tag{2.2}$$

and the c numbers ϕ_k, Φ_k are matrix elements between the different atomic states

$$|1\rangle \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad |2\rangle \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |3\rangle \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{2.3}$$

The matrices $\sigma_\pm, \tau_\pm$, are raising and lowering operators acting on the atomic states. Thus

$$\sigma_- \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \tau_- \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{2.4}$$

We will solve both the time-independent and time-dependent Schrödinger equations, drawing appropriate information from each development. In the first case, the energy-level structure will be shown to be significantly affected by the field. In the second case (for which the formal steps are similar), it is the modified—severely reduced—decay rate that we will evaluate. This will be done using Laplace transforms, *à la* Weisskopf and Wigner [13] (see also [14] and [15]). The results in the two cases will corroborate the point of view expressed in the Introduction.

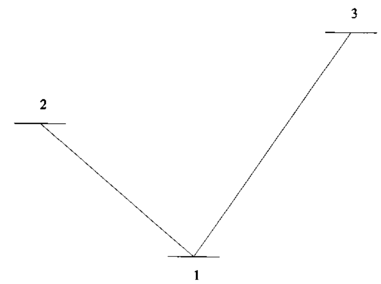

FIG. 1. Energy-level diagram. State 1 is the ground state. State 2 is a long-lived metastable state and state 3 is a short-lived metastable state. The system is illuminated by an intense laser beam at the 1-3 frequency (Ω_0). The 1-2 frequency is ω_0. Coefficients for occupancy of state 3 are denoted x or α, for state 2 y or β and for state 1 z or γ.

A. Energy eigenvalues

The eigenstates of the total system (atom plus photons) have the structure

$$|\Psi\rangle = \begin{pmatrix} \sum_k \alpha_k |3,1_k, N_0-1\rangle \\ \beta|2,0,N_0\rangle \\ \sum_k \gamma_k |1,1_k,N_0\rangle \end{pmatrix}, \tag{2.5}$$

where $\alpha_k, \beta, \gamma_k$ are numerical coefficients ($\Sigma_k|\alpha_k|^2+|\beta|^2+\Sigma_k|\gamma_k|^2=1$), $|j,l_k,M\rangle$ denotes a state in which the atom is in level j ($j=1,2,3$), and there are l ω_k-photon numbers and M Ω_0- photon numbers. An unsubscripted "0" in the second position means there are no "ω" photons in the state. We focus on the case $l_k=0,1$, $N_0 \gg 1$. In our calculation we only take into account the mode Ω_0, where $\Omega_0\equiv E_3-E_1$. This is because the operators A_k and $A_k^\dagger$ give rise to roughly $\sqrt{N_0}$ for the mode Ω_0, and to unity for nearby modes. (Including these other modes would only allow the atom to decay from state 3 to state 1. This is insignificant by comparison with the N_0-enhanced return to state 1.) A diagram of the atomic energy levels together with indications of our notation are shown in Fig. 1.

For convenience we set $E_2+N_0\Omega_0=0$. The stationary Schrödinger equation $H\Psi=E\Psi$ reads

$$E\alpha_k=\nu_k\alpha_k+\sqrt{N_0}\Phi_0^*\gamma_k, \tag{2.6}$$

$$E\beta=\sum_k \phi_k^*\gamma_k, \tag{2.7}$$

$$E\gamma_k=\nu_k\gamma_k+\phi_k\beta+\sqrt{N_0}\Phi_0\alpha_k, \tag{2.8}$$

where $\nu_k\equiv\omega_k-\omega_0$, $\omega_0\equiv E_2-E_1$. Solve for $\alpha_k$in Eq. (2.6), substitute in Eq. (2.8), solve for γ_k, and finally plug the result into Eq. (2.7), to get

$$E=\sum_k |\phi_k|^2 \frac{E-\nu_k}{(E-\nu_k)^2-B^2}, \quad B^2\equiv|\Phi_0|^2N_0. \tag{2.9}$$

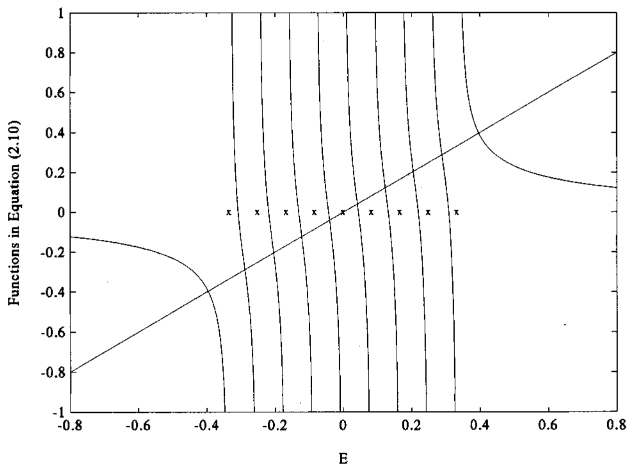

FIG. 2. Energy levels for a level interacting with a continuum. Energy levels are solutions of Eq. (2.10). The crosses on the x axis are the unperturbed energy levels. The positive-slope straight line is the left-hand side of this equation while the curved vertical lines are the right-hand side. Intersections are the energy levels. All but two lie between unperturbed levels.

When $B=0$, the above result leads to the familiar coupling of a single level (number 2, in our case) to a continuum. Let us review this case:

$$E=\sum_{k=1}^{M} |\phi_k|^2 \frac{1}{E-\nu_k}, \tag{2.10}$$

where the frequencies ν_k form a quasicontinuum. There are $M+1$energy levels: one between each pair of frequencies ν_k, ν_{k+1} $(k=1,\ldots,M)$, and two levels outside the ν_k band. See Fig. 2. This is a familiar situation in a variety of physical problems.

Now consider the case that B is large and assume that $\{\nu_k\}$ has a finite bandwidth, smaller than B; that is, $\phi_k=0$ for $\nu_k>B$. See Fig. 3. There appear two separated bands $\{\nu_k\}\pm B$. There is still one energy level between each pair of frequencies ν_k,ν_{k+1} and three levels (at $E=0$, $E<\nu_M$ and $E>\nu_M$) outside these bands. However, level 2, whose energy is still E_2, has no continuum to which to decay.

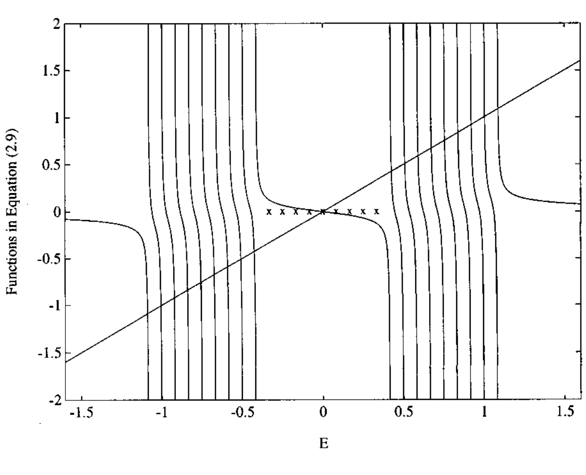

FIG. 3. Energy levels for large B are found as described in Fig. 2, except that now Eq. (2.9) is used. The band is doubled and displaced away from $E=0$, which with our conventions is the energy of the original decaying state.

What is happening is the following: If the pumping field between levels 1 and 3 is very intense, that is, $B>|\nu_k|,\forall k$, level 2 becomes quasistable, in the sense that it becomes energetically isolated and ceases to belong to a continuum. Notice that this is not ''metastability'' in the ordinary sense, for it essentially depends on the state of the pumping field A_k.

In practice, although the coefficients ϕ_k will be reduced for large ν_k, they are not zero. The calculation of this will occupy us below and will provide estimates of the residual decay even in the presence of a large field.

B. Temporal evolution

This calculation parallels that given above, with E replaced by $i\partial/\partial t$. In this context, however, one must deal with boundary conditions and features of the complex energy plane. Again, the state of the total system is written as

$$|\Psi\rangle=\begin{pmatrix} \Sigma_k x_k|3,1_k,N_0-1\rangle \\ y|2,0,N_0\rangle \\ \Sigma_k z_k|1,1_k,N_0\rangle \end{pmatrix}, \tag{2.11}$$

where $\Sigma_k|x_k|^2+|y|^2+\Sigma_k|z_k|^2=1$ and the notation is the same as in Eq. (2.5). We prepare our system in the state

$$|\Psi_0\rangle=\begin{pmatrix} 0 \\ |2,0,N_0\rangle \\ 0 \end{pmatrix} \Leftrightarrow y(0)=1, \quad x_k(0)=z_k(0)=0 \;\; \forall k, \tag{2.12}$$

and solve the time-dependent Schrödinger equation $i\partial\Psi/\partial t=H\Psi$. The equations of motion read $(E_2+N_0\Omega_0=0,\ \nu_k\equiv\omega_k-\omega_0,\ \omega_0\equiv E_2-E_1)$

$$i\dot{x}_k=\nu_k x_k+\sqrt{N_0}\Phi_0^* z_k, \tag{2.13}$$

$$i\dot{y}=\sum_k \phi_k^* z_k, \tag{2.14}$$

$$i\dot{z}_k=\nu_k z_k+\phi_k y+\sqrt{N_0}\Phi_0 x_k. \tag{2.15}$$

The Laplace transform and its inverse are given by

$$\hat{f}(s)=\int_0^\infty e^{-st}f(t)dt, \quad f(t)=\frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty} e^{st}\hat{f}(s)ds. \tag{2.16}$$

We first Laplace transform Eq. (2.13), incorporating the relevant boundary data. Then substitute the solution for $\hat{x}_k$ into the Laplace transform of Eq. (2.15). Finally, we plug the solution for $\hat{z}_k$ into the Laplace transform of Eq. (2.14) and take the inverse transform. The result is

$$y(t)=\frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{e^{st}}{s+Q(s)}ds, \tag{2.17}$$

$$Q(s)\equiv\sum_k |\phi_k|^2 \frac{s+i\nu_k}{(s+i\nu_k)^2+B^2}. \tag{2.18}$$

We now look for the poles of the integrand.

As before, we first consider the situation $B=0$. We have to find the zeros of the denominator

$$s+Q(s)|_{B=0}=s+\sum_k |\phi_k|^2 \frac{1}{s+i\nu_k}=0. \tag{2.19}$$

This leads us back to Eq. (2.10) with $E=is$, as expected. This is also the first step in a derivation of Fermi's golden rule. We will carry out this derivation since it will be extended below. In the continuum limit,

$$\nu_k \to E,$$

$$\sum_k |\phi_k|^2 \to \int dE\ \rho(E)|\varphi(E)|^2 \equiv \int dE\ g(E), \tag{2.20}$$

where ρ is the density of states, $\varphi(E)$ are the scaled matrix elements, and we define $g\equiv\rho|\varphi|^2$. The pole condition (2.19) becomes

$$s+\int dE\ g(E)\frac{-i}{E-is}=0. \tag{2.21}$$

Since s, due to Eq. (2.17), is almost purely imaginary and has a small negative real part, the above equation yields

$$s-i\left[P\int dE\ \frac{g(E)}{E}+i\pi g(0)\right]=0. \tag{2.22}$$

By setting

$$s=-\gamma/2+i\Delta E \quad (\gamma,\Delta E\in \mathbf{R}) \tag{2.23}$$

one gets

$$\Delta E=P\int dE\frac{g(E)}{E},$$

$$\gamma=2\pi g(0)=2\pi\rho(0)|\varphi(0)|^2, \tag{2.24}$$

which is Fermi's "golden rule" [16]. Substitution in Eq. (2.17) yields the exponential decay law:

$$y(t)\simeq e^{(-\gamma/2+i\Delta E)t}. \tag{2.25}$$

Consider now the case $B\neq 0$. If the band of energy levels has been split, as in Fig. 3, then for finite times there can be some transitions, although for large times energy conservation will prevent them. In any case this is nothing like an exponential decay and the Laplace transform formalism is not useful for this case. (Below we also check that the integral predicts infinite lifetime in this case.) However, cutting off the band in this way does not describe the physics. For these electromagnetic transitions there is some nonzero matrix element, although, as we indicate below, it is small. For this case, we proceed as we did in the "golden rule" derivation; take the continuum limit and look for poles with small s. In the continuum limit,

$$Q(s)=\int dE\ g(E)\frac{s+iE}{(s+iE)^2+B^2}, \tag{2.26}$$

where the integration range is essentially the whole real axis (neglecting the effects of a lower cutoff, as in the Breit-Wigner method [13,17,18]). We rewrite Eq. (2.26) as

$$\begin{aligned}Q(s)&=\int_{-\infty}^{\infty} dE\ g(E)\frac{-i(E-is)}{E^2-B^2-2iEs-s^2}\\ &\simeq\int_{-\infty}^{\infty} dE\ g(E)\frac{-iE}{E^2-B^2-2iEs},\end{aligned} \tag{2.27}$$

where, as indicated, s is small. This gives rise to two poles, $E_0=\pm B+is$. Notice that both have a small positive imaginary part, in the complex E plane.

The integral in Eq. (2.27) has a singular part. Its calculation is straightforward:

$$\begin{aligned}Q(s)&=\int_{-\infty}^{0} dE\ g(E)\frac{-iE}{(E-B)(E+B-is)}\\ &\quad+\int_{0}^{\infty} dE\ g(E)\frac{-iE}{(E+B)(E-B-is)}\\ &=-i\left[P\int_{-\infty}^{0} dE\ g(E)\frac{E}{E^2-B^2}+i\pi\frac{g(-B)}{2}\right.\\ &\quad\left.+P\int_{0}^{\infty} dE\ g(E)\frac{E}{E^2-B^2}+i\pi\frac{g(B)}{2}\right]\\ &=P\int_{-\infty}^{\infty} dE\ g(E)\frac{-iE}{E^2-B^2}+\pi\frac{g(B)+g(-B)}{2}.\end{aligned} \tag{2.28}$$

Observe that both singular contributions to Q are positive, due to the causality requirements for the inverse Laplace transform (2.17) [causality is reflected in the well-known formula $1/(x\pm i\epsilon)=P(1/x)\mp i\pi\delta(x)$]. The poles in Eq. (2.17) are the solution of the equation

$$s+Q(s)=0. \tag{2.29}$$

By setting, as usual,

$$s=-\gamma'/2+i\Delta E' \quad (\gamma',\Delta E'\in\mathbf{R}) \tag{2.30}$$

we obtain

$$\Delta E'=P\int dE\ g(E)\frac{E}{E^2-B^2},$$

$$\begin{aligned}\gamma'&=\pi[g(B)+g(-B)]\\ &=\pi[\rho(B)|\varphi(B)|^2+\rho(-B)|\varphi(-B)|^2].\end{aligned} \tag{2.31}$$

This result is to be compared with Eq. (2.24). The presence of the Φ field (which is associated with the $1\leftrightarrow 3$ transition) modifies the lifetime of level 2. In particular,

$$\frac{\gamma}{\gamma'}=\frac{2\rho(0)|\varphi(0)|^2}{\rho(B)|\varphi(B)|^2+\rho(-B)|\varphi(-B)|^2}. \tag{2.32}$$

A quantitative estimate of the above ratio requires evaluation of the matrix elements φ and the phase-space factor ρ. We will treat this in the next section. However, one obviously expects that $\gamma' \ll \gamma$, as B becomes large.

It is interesting to remark that, unlike the usual QZE analysis, we did not consider the solution for very short times (yielding a vanishing derivative for the ''survival probability'' of the initial state). By contrast, we investigated the temporal behavior of our system *after* it settled into its exponential regime. This is the ''dominated temporal evolution'' to which we referred [11]. It is a purely dynamical effect, and does not involve hypothetical constructs such as projection operators or quantum measurements.

In the above derivation, which follows the Breit-Wigner method, it is assumed that s is ''small'' in order to extract a singular contribution from the relevant integrals. However, one could also obtain the same result in a way that takes into account the lower boundedness of the spectrum. It is not difficult to show that γ', in Eq. (2.31), arises from a discontinuity of $Q(s)$ in the complex s plane. Indeed, if E_0 and E_g are the initial energy of our total system and the ground state of the Hamiltonian, respectively, the cut in the complex s plane runs from $i(E_0 - E_g)$ to $i\infty$. Across this cut one has $Q(\epsilon) - Q(-\epsilon) = \gamma'$, in full agreement with Eq. (2.31).

In this context, it is also worth noting that if B were outside the range of the integration domain in Eq. (2.27) (i.e., if the energy band were split, as in Fig. 3), one would obtain

$$Q(s) = \int_{\text{finite range}} dE\ g(E) \frac{-iE}{E^2 - B^2}, \tag{2.33}$$

which is *not* a singular integral, and yields a purely imaginary result. In this case, by setting $s = -\gamma'/2 + i\Delta E'$ one gets

$$\Delta E' = -\int dE \frac{g(E)}{E^2 - B^2}, \qquad \gamma' = 0. \tag{2.34}$$

The atom does not decay.

III. MATRIX ELEMENTS OF THE HAMILTONIAN

For physical prediction we require several quantities. The matrix elements ϕ_k are essentially

$$\phi_k \sim \langle \psi_1 | \vec{p} \cdot \vec{A} | \psi_2 \rangle, \tag{3.1}$$

with ψ_j the atomic wave functions and $\vec{A}$ the electromagnetic vector potential. The ϕ_k (or more precisely their continuum limits) are related to the (usual) lifetime and density of states by $1/\tau_{21} = \gamma = 2\pi |\varphi|^2 \rho \equiv 2\pi g$, as shown above. What we need now is the way $g(E)$ behaves far from k_0. The important dependence is the k dependence. For large B, $\nu_k \equiv \omega_k - \omega_0$ will be large. This means that the integral

$$\phi_k \sim \langle \psi_1 | \vec{p} \cdot \vec{A} | \psi_2 \rangle \sim \frac{1}{\sqrt{\omega_k}} \int d^3x\ \psi_1^*(x) \frac{1}{i} \frac{d}{dx} \exp(ikx) \psi_2(x) \tag{3.2}$$

(where the $\sqrt{\omega_k}$ factor is due to the normalization of the vector potential) will go to zero because of the rapid oscillations of the integrand. This is because large B implies large energy displacement, which translates to large k through the relation $\omega_{\text{photon}} = ck$. This should be contrasted to the usual situation where $\exp(ikx)$ is expanded in powers of k because of the long wavelength of light on the atomic scale. As we will see, aside from the issue of dominated (non)decay this should be a useful probe of atomic matrix elements for intermediate values of B.

By taking the ratios of density of states and transition matrix elements at $E = \pm B$ there is sufficient information (using the lifetime) to get the suppressed lifetime in the presence of strong fields. From Eq. (2.32), the typical term we require is ($k_0 = \omega_0/c, k_B$ satisfies $\hbar\omega_{k_B} = B$)

$$\frac{\rho(k_B)|\phi(k_B)|^2}{\rho(k_0)|\phi(k_0)|^2}, \tag{3.3}$$

which is to say the nontrivial part is $\phi(k_B)/\phi(k_0)$. (Note that we now use k as argument for both ρ and ϕ.) We use q for the momentum variable of the electron coordinate and for $\phi(k_0)$ use an expansion in which the wavelength of the light is considered long. Let $\tilde{\psi}$ represent a momentum space wave function. This gives

$$\frac{\phi(k_B)}{\phi(k_0)} = \frac{\sqrt{\omega_{k_0}}}{\sqrt{\omega_{k_B}}} \frac{\int dq\ \tilde{\psi}_1^*(q) q \exp[k_B(d/dq)] \tilde{\psi}_2(q)}{\int dq\ \tilde{\psi}_1^*(q) q \exp[k_0(d/dq)] \tilde{\psi}_2(q)}. \tag{3.4}$$

Since state 2 is long lived, presumably the dipole expansion would be inappropriate in the denominator. However, some low power of $k_0 d/dq$ should suffice, and as usual the coordinate representation should prove more useful. On the other hand, for the numerator, we can write $\exp[k_B(d/dq)]\tilde{\psi}(q) = \tilde{\psi}(q + k_B)$. For idealized hydrogenic wave functions of angular momentum l, $\tilde{\psi}$ drops off for large k_B as k_B^{4+l}. This, together with the k_B built into the normalization of the vector potential [cf. the $\sqrt{\omega_{k_B}}$ in Eq. (3.2)] overwhelms the density-of-states factor, so that $\rho(k_B)|\phi(k_B)|^2$ will in fact go to zero with increasing B, providing the dominated evolution that we seek. Note that the above formula neglects possible changes in the wave functions due to the strong field itself. This will be discussed in our final section.

To compute the relevant ratio, $\phi(k_B)/\phi(k_0)$, we look at

$$f(k) \equiv \int dq\ \tilde{\psi}_1^*(q) q\ \exp[k(d/dq)] \tilde{\psi}_2(q). \tag{3.5}$$

Given the asymptotic forms of $\tilde{\psi}$, this will drop roughly as $k^{-(4+l)}$ for large k, and be (comparatively) near to unity for small k, i.e., k_0. Thus, up to factors of order 1, $f(k_B)/f(k_0) \sim k_B^{-(4+l)}$. If B is taken to have energy units, the value of k_B is approximately $k_B \sim B/\hbar c$. Collecting terms,

$$\frac{\gamma'}{\gamma} \sim \frac{\rho(k_B)|\varphi(k_B)|^2}{\rho(k_0)|\varphi(k_0)|^2} \sim \frac{\rho(k_B) k_0 |f(k_B)|^2}{\rho(k_0) k_B |f(k_0|^2}, \tag{3.6}$$

which is in turn approximately

$$\frac{\gamma'}{\gamma} \sim \frac{k_B^2 k_0 O(k_B^{-2(4+l)})}{k_0^2 k_B O(1)} \sim \left(\frac{k_B}{k_0}\right)^{-7-2l} \tag{3.7}$$

With $k_B \propto B$ this will give rapid decline with B.

An estimate of reasonable B values in terms of experimental data involves quantities for the 3→1 transition. The typical photon occupation number (''N_0'') depends on the strength of the laser field. Although one often expresses this in terms of a coherent state label, since we are contemplating only large fields there should not be too much of a distinction. This is because with a large field the relative size of fluctuations, hence the distinction between various characterizations of the field, will be small.

An important potential limitation of our method is that for sufficiently large B, there can be a significant effect on the atom being studied. This effect can be anything from distorting the atomic wave function to completely destroying the atom. Recall that large B means large photon number, implying, for example, large electric fields, $\vec{E}$. If $\vec{d}_{13}$ is the dipole moment for the 1↔3 transition, then we have roughly

$$\Phi_0 \sim \vec{d}_{13} \cdot \vec{E}. \tag{3.8}$$

Thus large B means large $|\vec{E}|$. On the other hand, if $\vec{d}_{13} \cdot \vec{E}$ is larger than $\hbar \Omega_0$ or $\hbar \omega_0$ (namely, about 1 eV, for optical transitions), one must be concerned about possible ionization of the atom being studied [19]. However, once the atom is successfully placed in state 2, the disruptive effects of strong Ω_0 irradiation (i.e., the 1→3 frequency) should be less. This issue will depend on experimental details as well as on relative bandwidths (hence required values of B) for the transitions being studied.

We now turn to the evaluation of ''B.'' Consider

$$B^2 \equiv N_0 |\Phi_0|^2 = \frac{\hbar N_0}{2\pi\rho} \frac{1}{\tau_{13}}, \tag{3.9}$$

with ρ the density of states for the 1-3 transition, and the correct power of $\hbar$ has been inserted to give B the dimensions of energy. The density of states, integrated over angle, is [20]

$$\rho = \frac{V}{2\pi^2 \hbar c^3} \Omega_0^2, \tag{3.10}$$

with V volume. Inserting this above we have

$$B^2 = \frac{1}{8\pi^2} \left[\frac{N_0 \hbar \Omega_0}{V} \right] \lambda_0^3 \frac{\hbar}{\tau_{13}}, \tag{3.11}$$

where $\lambda_0 = 2\pi c/\Omega_0$ is the wavelength for the 3→1 transition.

For orientation purposes we provide an estimate of characteristic values taken by the quantity ''B.'' In this case we will treat the quantity $N_0 \hbar \Omega_0 / V$ as energy per unit volume. Note, however, that this is only a rough estimate and may not reflect the actual quantum number associated with the state in Eq. (2.5). For the correct N_0 we expect that the coherence length of the laser would enter the calculation.

For a typical excimer ArF laser, the energy in one pulse is about 500 mJ and the pulse width is about 100 ns. The wavelength is roughly 200 nm and the beam diameter about 2 mm. We require the energy in a volume λ_0^3 which is about 5×10^{-17} J. It follows that $B \sim 1$ meV. Since each photon has an energy of about 1×10^{-18} J, there will be about five photons in each such volume. There are also more powerful lasers with considerably higher-energy density. Thus a Cr^{3+}:Al_2O_3 laser has energy 50 J in one pulse, wavelength about 700 nm, and a pulse width of 3 ps. With this laser, the energy found in our previous calculation would be about 10 eV. (Note that our proposal does *not* involve pulsing. We are only giving these examples to get a feeling for the numbers.)

Finally we point out that for B values insufficient to halt decay there will nevertheless be an extension of the 2→1 lifetime. A measurement of this lifetime would provide information about atomic wave functions through expressions involving the function $f(k)$ defined in Eq. (3.5). In particular, large k_B would give information at small distance scales.

IV. COMMENTS AND OUTLOOK

We have provided an explicit calculation showing how staring very hard at a metastable state can keep it from decaying. There was no need to invoke special notions of quantum ''measurements,'' and to the extent that one might wish to think in those terms the intense laser field with which one ''stares'' can be considered a fully quantum model of a measurement apparatus [21]. This is an example of dominated evolution, a general concept going back to von Neumann that includes what is often called the quantum Zeno effect. Like QZE our experimental proposal calls for stopping an atom from decaying; unlike QZE there is no need for pulsing. It will be interesting to see the relation between the two approaches: we expect that a characteristic time associated with our external field will correspond to the pulsing interval that often is used in discussions of QZE. We mention too that we are certainly not the first to raise the issue of continuous versus pulsed measurement for QZE. An elegant and extreme example of the halting of transitions by continuous observation is the explanation in [22] of the chiral nature of certain molecules that might have been expected to appear in their nonchiral theoretical ground state. It is the continuous monitoring by the environment, a phenomenon that Harris and co-workers [22] explicitly relate to the QZE, that maintains them in their chiral state.

Although a sufficiently strong field can substantially stop the decay, in fact our formula covers weaker fields in which the lifetime is simply extended. Since the formula in turn depends on atomic wave functions at small distances, the lifetime measurements we propose could provide a probe of those wave functions.

We expect that actual experimental implementation of our proposal could involve relatively large numbers of atoms, initially excited to the level that we have been calling ''2'' (see Fig. 1). This could be accomplished by a laser pulse at frequency ω_0, a pulse much briefer than the 1→2 lifetime. The subsequent (hindered) decay would be monitored by looking for isotropic radiation at the frequency ω_0.

We emphasize that our proposal is a direct test of QZE on a truly decaying system. This is to be contrasted to previous work [6,7,10,23]. Moreover, our proposal prevents repopulation effects that affect experiments performed on oscillating two-level systems [5].

This article focuses on QZE, and the external electromagnetic field plays the role of a measurement device. Nevertheless, our calculation also provides an example of a light-atom interaction that is similar to some of the interesting and at times dramatic effects that have been discussed in the recent literature. *Induced transparency* is one such effect, in which intense irradiation prevents absorption by an atom [24–26]. This phenomenon can be considered inverse to our effect, in the sense that we speak of the suppression of emission, they of absorption. As for the theoretical description of induced transparency, our Sec. II A can be thought of as describing ''dressed'' states that are no longer receptive to the usual quantum transition. It is not clear at present how far the analogy can be pushed, since our effect seems to require rather strong fields (in particular, the fields involved in ''quantum jump'' experiments [27] did not seem to change the lifetime significantly). It will be interesting to apply our method to the already well-understood induced transparency phenomenon.

In addition to induced transparency, other recently explored atom-light phenomena are worth recalling at this point. One that involves coherent interplay of three levels is *accumulated echo*, in which echo effects can be discerned through what might have appeared to be rather indirect, multilevel processes [28]. Other fascinating quantum-optical effects for which our methods may be relevant are nonlinear effects close to resonance [29] and the recently reported experimental observation of laser oscillations without population inversion [30,25]. Finally, a recent calculation suggesting routes to extremely short and intense laser pulses may be relevant to variations of our approach in which high-intensity laser light, in short pulses, might be used [31].

We comment on various technical points and caveats.

(1) In our matrix element estimates we have not included second-order terms in the electromagnetic field, e^2A^2. These represent two-photon processes and are pretty much irrelevant for our story. Although such terms might access other levels of the atom, that would not interfere with the suppression of the $2\rightarrow1$ decay unless there were other levels of the atom (accessed by the e^2A^2 terms) accidentally sensitive to this effect.

(2) Since we do not look at the extreme early times, we do not need to contend with the lower bound of energy nor with the branch cuts that must be included in the Laplace transform inversion when one looks at those times.

(3) Since we do not consider experiments involving pulsing (performing MN measurements in a finite time interval T), we do not encounter conceptual difficulties related to the impossibility (in principle and in practice) of taking the $N\rightarrow\infty$ limit [32].

(4) The use of photon occupation-number eigenstates, rather than coherent states of the electromagnetic field, should not affect our results. This is because for such strong fields the spread in photon number implicit in the coherent state is small relative to the average photon number.

(5) The effect of the intense field on the atom. The intense bath of Ω_0 photons could have a big effect *if* the atom were in state 3. But it is not, and it is precisely that field that prevents this from happening. Incidentally, even if the initial excitation to state 2 were inefficient or incomplete, the Ω_0 field would soon excite or destroy atoms that were not in state 2. If the signal of interest is the frequency- ω_0 photons, then this should have no effect on the measurement. However, there may be changes in the atomic wave functions or ionization-inducing multiphoton processes that result from the strong field.

A theoretical point of some interest concerns the pre-exponential time regime that is an essential feature in many discussions of QZE. In such presentations, the nonexponential ''$1-O(t^2)$'' early-time behavior plays a central role. This is not our approach. We are thus relieved of certain problems associated with that method (although this was not our primary goal). For example, we do not have to confront the critical issue of deciding the exact initial moment of excitation nor the yet more subtle question of defining the metastable state.

How then does our approach relate to the more usual one? As remarked above, we are including the ''apparatus'' in our description. The ''projection'' that conventionally takes place in QZE descriptions, and which represents a kind of *deus ex machina*, for us is an intrinsic part of ordinary quantum evolution. Thus in our setup the virtual emission of a $1\leftrightarrow2$ photon is followed by an immediate transition of the atom to its *other* excited state. Without the intense laser field such virtual emission represents a gradual passage of amplitude to the $|1,1_k\rangle$ state, a passage that leaves this state coherently connected with the original one, and allows gradual decay. It is our expectation (as yet not rigorously established) that the time scale for the usual pre-exponential regime corresponds in our system to a characteristic time for the $1\rightarrow3$ transition, which in turn should relate to $\hbar/$''B''. This expectation is supported by the fact that gradual intensifying of B plays the same role as increase of the frequency of measurements (projections) in the usual QZE description. (Of course, since some of our results are derived by a pole approximation to a Laplace transform, there will be a pre-exponential regime for that as well. We expect this to be *much* shorter, corresponding to a shrunken version of the $1\leftrightarrow3$ pre-exponential time scale—shrunk because of the N_0 enhancement. This time scale does not play a role in our description and should be shorter than anything relevant to the usual description as well.)

It should also be clear that our proposal is not in its general principles confined to atomic decay phenomenon, nor does the level ''1'' need to be the ground state. Any system with the same three-level structure that has been so rich in atomic studies may be used. It may even turn out that because of narrow bandwidths in some of these systems the effect would be easier to observe.

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