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Acoustic GW detectors in the 2010 timeframe

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Abstract
I consider the spectral sensitivities and bandwidths, in the standard quantum limit, of the narrowband spherical detectors, which would evolve from the present bar detectors and the wideband novel 'dual' detectors that have been proposed recently. If appropriate advanced fabrication and read-out technologies are developed, both kinds of GW acoustic detectors would play a relevant role in the near-kHz frequency region.

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1. Introduction

Since Weber [1] invented and first operated GW detectors, based on the GW excitation of kHz frequency quadrupolar modes of massive cylinders, 'bars', the field has evolved following one specific track: reduce the thermal noise in the bar by lowering the temperature down to sub-Kelvin temperatures and use secondary resonant masses tuned to the bar resonant mode of choice, to mechanically amplify the displacement of the bar end faces, before the electromechanical transduction to a final amplifier. With one secondary resonator in the transducer, as in all bar detectors operated upto now, one has a system of two tightly coupled mechanical resonators, typically in the vicinity of 1 kHz, with the two modes separated by a few tens of Hz. Such a GW detector is said to be *intrinsically narrowband*. It is of interest to consider, in view of future developments, how broadband and sensitive can acoustic GW detectors be.

2. The bandwidth of acoustic GW detectors

In principle the analysis of an acoustic GW detector is simple. One has a linear system, which translates the GW force excitation of the massive resonator into an output electric signal, introducing three sources of noise which, referred to the output, are: (i) the *narrowband* thermal noise, as the input white band stochastic dissipative force, due to internal friction in the resonant mass(es), appears at the output after passing through the mechanical transfer

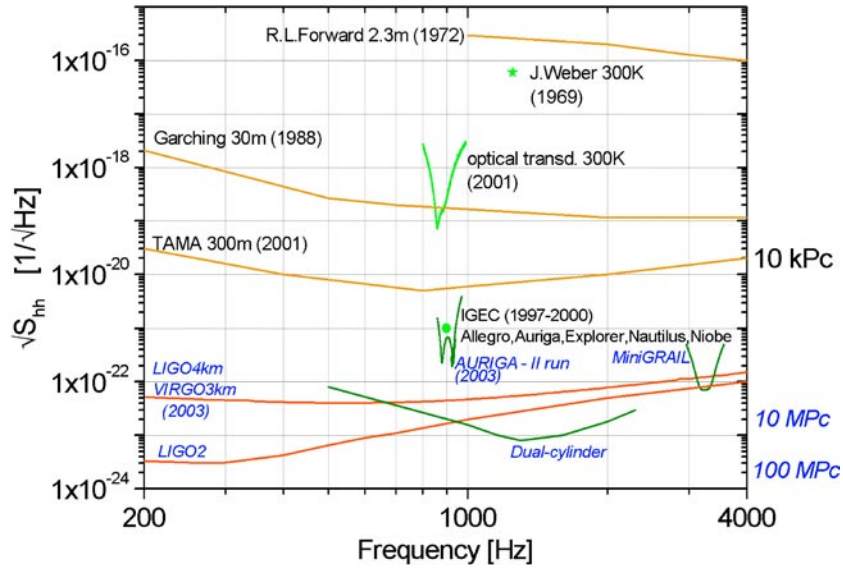

Figure 1. The evolution in time of ground based GW detectors in their spectral sensitivities and bandwidths. The detectors of the future, in the bottom section, are assumed to operate at the relevant quantum limit. On the right their typical reach-out for violent events, such as the merger of ns and bh binaries. The graph is intended to be indicative and by no means exhaustive. Spectral sensitivities are from [11, 15–18], www.igec.lnl.infn.it, www.auriga.lnl.infn.it, www.minigrail.nl, www.ligo.caltech.edu and http://tamago.mtk.nao.ac.jp.

(This figure is in colour only in the electronic version)

function of the system; (ii) the *wideband* final amplifier noise, showing up directly at output and (iii) the *narrowband* back-action noise, originating from the back-energy flow from the amplifier, which excites the resonant mass(es). The width of the narrowband contributions is that of the mechanical resonance(s), a few mHz with the mechanical quality factor $Q \sim 10^6$ of current bar detectors. What then are the spectral sensitivity and bandwidth of a bar detector? The floor of lowest spectral noise is found in a region where thermal and back-action noises dominate the amplifier noise, and there the noise level is the sum of the two. Such a frequency region is delimited by the lower and higher frequency values, at which the narrowband thermal plus back-action noise contributions start to disappear into the wideband noise of the amplifier. The bandwidth is thus related to the intrinsic bandwidth of the mechanical resonances, but does *not* coincide with that. In fact, the lower the wideband amplifier noise, the more separated in frequency will be the points where the narrowband band noise goes under it, and the wider will be the detector bandwidth, easily many orders of magnitude wider than the mechanical ones.

This can be seen in figure 1, in particular comparing the spectral sensitivities of the 'best' room temperature bar detector with those of the cryogenic ones: at high temperature the thermal noise is large and dominates the amplifier noise on a wide band of ~50 Hz, while at low temperatures, despite the use of SQUIDs as amplifiers with noise as low as 10^4 quanta, the situation reverses and the spectral sensitivity appears as a dot in the graph, since the bandwidth is ~1 Hz. On the other hand, one can appreciate the reduction in noise floor in lowering the temperature.

What are the limits in all this? A general analysis has been given long ago [2]. The limit is encountered when, after reducing the thermal noise to negligible, the so-called dissipationless limit, one hits the quantum limit of the final amplifier, which can also be given quite in general [3]. In fact, even if the thermal noise can be neglected, still the amplifier, to perform the measurement, must, in a quantum-mechanical sense, react back on the system: the back-action noise, at the quantum limit, comes to be an unavoidable narrowband quantum noise, which limits the floor sensitivity *and* the bandwidth, depending on the mechanical transfer function and the amplifier input impedance.

Then one sees [2] that, for a two-mode GW acoustic detector, the best performance one can get is such that the widest band is some 10% of the main resonant frequency, i.e., $\sim$100 Hz for a 1 kHz detector. For this reason acoustic detectors are said to be *intrinsically* narrowband. Still in their bandwidth they promise to be extremely sensitive. Solid spherical detectors with one resonant transducer for each independent signal channel have been studied extensively, also with successful experimentation, to demonstrate their basic features in terms of signal reconstruction. A hollow sphere would be extremely sensitive at a frequency of $\sim$1 kHz. The result of these studies is that a spherical GW detector could give an omnidirectional GW detector that is able to reconstruct the direction of propagation and polarization of the signal. Two of them at a distance would do for a complete observatory, giving the direction of propagation and testing the velocity of the signal. I will describe below the merits and prospects for 'advanced' *narrowband* acoustic detectors of this kind.

At the beginning, Weber's design had a passive non-resonant transducer. He then designed a one-mode resonator, as only the fundamental longitudinal mode of the bar was used for detection; all other modes were well separated in frequency and made independent by the high mechanical factor of merit. The piezoelectric elements used for that purpose were very noisy and gave a low coupling to the strain excitation of the bar, so the spectral sensitivity was low and the band narrow; see figure 1. A different design was already attempted to provide a wider band: the 'split bar', developed and operated by the Glasgow team. They used non-resonant piezoelectric transducers to join together end-face to end-face two identical bars and, at the same time, to the relative displacement of two identical bars [4]. However, as the analysis in [2] shows, to get a bandwidth as wide as the order of the resonant frequency, one would need a signal coupling many orders of magnitude stronger than attainable.

Recently a novel concept has been proposed for what appears, with respect to 'bars' and 'interferometers', a *third* kind of ground based GW detector, the 'dual' resonator. While bars work best *at* the frequencies of GW-sensitive mechanical modes and (wideband operated) interferometers work best *far* from the frequencies of suspensions and mirrors' internal mechanical modes, the duo would work best *in between* the frequencies of specific GW-sensitive mechanical modes. In that frequency band, the 'dual' would show an *additive* effect on signal coupling and a *reduction* effect on back-action noise, which both crucially contribute, together with the use of *non-mechanically resonant* differential displacement transducers, to open wide and sensitize the whole band. I will give in the following a brief presentation of the ideas on which the dual concept is based and discuss the interest in the realization of such a GW detector.

3. Narrowband advanced acoustic detectors

For a long time [5] the 'bar' community has considered spherical resonators of great interest, as 'advanced' detectors, for the following relevant features:

- Given the linear dimensions, sphere diameter and bar length, the cross section is larger for the ratio of the masses, which for the same material is a factor about 20 [5]. In general, the spectral strain noise density, in the useful band at the quantum limit, varies with the frequency f and the material properties sound velocity v_s and density ρ simply as $S_h \sim f^3/\rho v_s{}^5$.

- The detector is omnidirectional in that the total energy absorbed from the incoming signal is independent of the incident direction; in addition one can reconstruct the direction and polarization of the incoming signal [6]. Two spherical detectors at a distance make up an 'observatory' [7], which would also test whether the propagation velocity is that of light and thus make viable a powerful veto against spurious signals. By properly choosing such a distance, the correlation in the detection of the stochastic background would be maximized [8].
- The cross section of the second quadrupolar resonant mode is of the same order as that of the first. An intriguing scheme has been proposed for detecting in a distinctive way the final coalescence of neutron star binaries, as the signal excites the two modes at subsequent times [9].

Hollow spheres, with resonant transducers at the quantum limit [10], are of particular interest, as their first quadrupole mode can be at few hundred Hz and their second mode at about 1 kHz. With the use of materials such as Al 5056 and $CuAl^{6\%}$, already tested for high Q at low temperatures, the spectral sensitivity would be of the order of $\sim 10^{-24}$ $Hz^{-1/2}$. A $CuAl^{6\%}$ solid sphere of 60 cm diameter and 1 ton weight, MiniGRAIL, has been successfully cooled to ultracryogenic temperatures; its predicted sensitivity, when equipped with quantum limited resonant transducers, is shown in figure 1. In this symposium deWaard presented in detail the MiniGRAIL project [11].

Here, I briefly consider what would be the performance of an 'advanced' version of this kind of acoustic detector. For the 2010 time frame it is reasonable to make a few, possibly still conservative, extrapolations: (i) the final amplifier, possibly a double SQUID, will further approach the quantum limit after the recent performance at the $\sim$100 quanta level [12] in a configuration fully coupled to a resonant load; (ii) a material such as Be or SiC, with ρv_s^5 larger by a factor $\sim$75 with respect to Al5056, will become available for fabrication of a hollow sphere of a few tens of tons (note that Be produced from powders by hot pressing and SiC like ceramics have already been used for $\sim$1.5 m diameter mirror substrates), showing a mechanical quality factor $Q > 10^7$ at sub-Kelvin temperatures.

Then I envision a hollow sphere of diameter $D = 1.8$ m, inner-to-outer diameter ratio $r = 0.7$, $M = 20$ ton, with the first quadrupole at $f_1 = 2000$ Hz, cooled at 100 mK, equipped with five or six one-secondary-mass resonant transducers (capacitive or inductive) in a configuration respectively like PHC or TIGA [6], read out by quantum limited double SQUIDs to get 'maximally flat' [2] frequency response in a 10% band around f_1. The predicted sensitivity $S_h^{1/2} = 5 \times 10^{-24}$ $Hz^{-1/2}$ and bandwidth $\Delta f = 200$ Hz are not much different from those of a narrowband 'advanced' interferometer at the same frequency [13]. Of course, while in an interferometer the frequency at which narrowbanding is achieved can be easily modified, in such an acoustic detector it is a built-in feature.

The above extrapolation appears viable, given a sufficient effort in technology, mainly on the grounds of experiments with a TIGA prototype, equipped with one-secondary-mass resonant transducers, which gave an actual demonstration of all the distinctive features predicted for this kind of detector [14].

A less conservative extrapolation concerns the use of two secondary masses or making resonant the coupling electric circuit in the transducer. Both these features would further broaden the band, as predicted in [2]. Several attempts have been made in the past in this direction. Recently the ALLEGRO and AURIGA teams have made significant progress on such two-mode transducers; the first team developed a two-mechanical-mode transducer, and the second team developed a transducer with one mechanical and one electrical mode.

It might be considered that, while a two-mode transducer on a bar gives all in all a three-mode system, on a sphere one wants six transducers in a TIGA or five in a PHC configuration [6], and thus the overall number of modes would become so large that the outcome in terms of mode mixing is not obvious.

4. Wideband acoustic detectors

A new scheme has been proposed recently to obtain a *wideband* performance in acoustic detectors [15]. This comparatively new subfield is still in its infancy, as the studies are in no way as well focused as those on the hollow and solid (single) spheres described above.

To get wideband operation, as discussed above, one wants to use *non-resonant* transducers. Then one has to find a way to get in some other manner the enhancement in displacement, which the resonant transducers provide. In the 'dual' resonators system this is accomplished by reading with a non-resonant transducer the differential surface displacements between two concentric freely suspended resonators, spheres or cylinders, as they vibrate independently under the GW excitation. Systematically, the outer (hollow) resonator would have the quadrupolar GW-sensitive resonant modes at lower frequencies with respect to the inner (solid or hollow) resonator. Then *in between* any two corresponding quadrupolar modes, each one coming respectively from each resonator, as the GW excites the resonators in phase, but they respond $\sim 180^\circ$ out of phase, the differential read-out gives an *addition* effect on the output signal. Moreover as the back-action force, through the transducer, pushes the surfaces of the two resonators $\sim 180^\circ$ out of phase, the back-action noise, for the same reason, tends to *subtract*, again just *in between* the said two resonant modes of the system. The result of these two effects, signal addition and back-action noise subtraction, is a signal-to-noise enhancement, which opens up the band, to be practically flat in the whole frequency interval between the said resonant modes of the system.

These are the two distinctive novel features of the dual resonators concept, which put the 'dual' in a sort of 'third' class of ground-based GW detectors, and both help to render the 'dual' a wideband detector. In fact, while interferometric detectors work *far* from the internal mechanical resonances and bar detectors work *at* the sensitive mass, the 'dual' would work *in between* two resonant modes of the system. The limiting frequency values of the flat sensitive region(s) can be decided by properly choosing the appropriate geometry and dimensions of the dual system. The overall diameter controls the lower frequency.

At the bottom of figure 1 and, in an expanded version, in figure 2 the *wideband* spectral sensitivity of a SiC 'dual' cylinder system of 2.5 m diameter and 2.5 m length, equipped with a non-resonant quantum limited transducer, is shown. The inner diameter of the outer torus is 1.4 m. The inner torus is also hollow, with an inner diameter of 0.2 m. The band would be as wide as 1.0–2.5 kHz, with a spectral sensitivity in between nearly flat at the level of $S^{1/2}{}_{\mathrm{h}} \sim 10^{-23}\ \mathrm{Hz}^{-1/2}$. It should be noted that the one shown in figures 1 and 2 is the S_{h} actually predicted: no *ad hoc* cancellation of unwanted non-GW sensitive resonant modes has been given, as, with that choice of geometry, there are no other resonant modes in the 'in between' frequency interval. This is a distinctive feature of the 'dual', which is obtained thanks to the many degrees of freedom in the choice of the geometry, in this case the possibility of choosing the appropriate thickness of the two hollow cylinders.

Another degree of freedom in design means that the 'dual' can be sort of 'narrowbanded': using different materials for the inner and outer resonators one can achieve a spectral sensitivity better by as much as one order of magnitude, at the price of narrowing the band. This may look similar to the narrowbanding one can achieve with interferometers. However, in the 'dual'

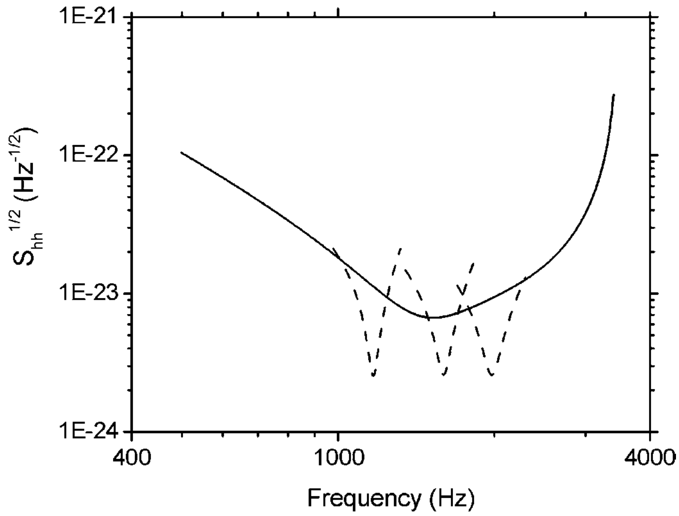

Figure 2. *Wideband* and *narrowband* 'dual' cylinders (see Bonaldi *et al* [15]). Solid line: all SiC system (see text). Dashed lines: three SiC (external)–Mo (internal) systems of overall diameter 2.6 m and length 2.6 m. The different frequencies at which narrowbanding occurs are obtained by using different ratios for the external-to-internal diameter of each cylinder.

case the frequency around which the narrowbanding is achieved would be, as far as presently the system is understood, a built-in feature, not to be changed at will as with interferometers. Figure 2 shows an example: three 'dual' systems, with the external cylinder made of SiC and the internal cylinder made of Mo, to narrow the band around specific frequencies. The spectral sensitivity increases by as much as a full order of magnitude, with respect to the all SiC wideband 'dual', at the expense of the bandwidth.

Both a wideband and a narrowband 'dual' would have spectral sensitivities in their band, comparable, if not a little higher, to those of respectively a broadband and a narrowband 'advanced' interferometer [13]. However, it must be noted that the 'advanced' intereferometers have a different bandwidth shape [13], so that in the end they would show, both in the wideband and in the narrowband versions, bandwidths which are much wider than respectively the wideband and narrow band 'dual'. As the studies on 'dual' systems have just started, a comprehensive comparison has possibly to wait for a fuller understanding of the ultimate capabilities of 'dual' systems.

5. Concluding remarks

The high-frequency region beyond 1 kHz is a most difficult one, as the GW signal amplitudes are expected to intrinsically decrease with increasing frequency, while at the same time the detector spectral noise systematically increases with frequency (we have seen above the f^3 behaviour of the quantum limited spheres spectral noise). Therefore, I believe it is of great interest to better understand what performance, in spectral sensitivity and bandwidth, acoustic GW detectors may offer, both the narrowband and the 'dual' wideband ones. The feasibility should be considered of those which promise spectral sensitivities in the range $S_{\mathrm{h}}^{1/2} \sim 10^{-24}$ Hz$^{-1/2}$ for the narrowband ones and $S_{\mathrm{h}}^{1/2} \sim 10^{-23}$ Hz$^{-1/2}$ for the wideband ones. In fact such a sensitivity can be regarded as sort of a typical value, needed to reach out to cosmological distances for the most violent GW emission events and to perform, in correlation with other detectors of any kind, relevant searches for stochastic background.

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