

ELEMENTARY PARTICLES AND FIELDS

Theory

Helicity Components of the Cross Section for Double Charged-Pion Production by Real Photons on Protons

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Abstract—The helicity components $\sigma_{1/2}$ and $\sigma_{3/2}$ of the cross section for double charged-pion production by real photons on a nucleon are calculated within a phenomenological approach developed previously. A high sensitivity of the $\sigma_{1/2}$ – $\sigma_{3/2}$ asymmetry to the contribution of nucleon resonances having strongly different electromagnetic helicity amplitudes $A_{1/2}$ and $A_{3/2}$ is demonstrated. This feature is of importance for seeking “missing” baryon states. © 2003 MAIK “Nauka/Interperiodica”.

1. INTRODUCTION

Experiments with polarized electron (photon) beams and polarized proton targets made it possible to determine the cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ corresponding to the total proton and photon helicities of $1/2$ and $3/2$ in the initial states both for the inclusive channel and for various exclusive channels [1–6]. Interest in studying the helicity components $\sigma_{1/2}$ and $\sigma_{3/2}$ is motivated by the following factors.

On the basis of the most general theoretical principles, Gerasimov [7] and Drell and Hearn [8] (GDH in the following) predicted the value of the integral

$$I_{\text{GDH}} = \int (\sigma_{1/2} - \sigma_{3/2}) \frac{d\nu}{\nu}, \quad (1)$$

where $\sigma_{1/2}$ and $\sigma_{3/2}$ are the total photoabsorption cross sections for the case in which the total photon–proton helicity is $1/2$ and $3/2$, respectively, while ν is the photon energy. It follows from [7, 8] that the GDH integral must take the value of $-204 \mu\text{b}$ at the photon point. The experimental results reported in [1, 2] were compared with the predictions made in [7, 8]. The definition of I_{GDH} can be extended to the case of virtual-photon absorption, $I_{\text{GDH}} = I_{\text{GDH}}(Q^2)$, where Q^2 is the sign-reversed square of the virtual-photon 4-momentum. The investigation of the Q^2 dependence of I_{GDH} in [4] revealed that, for $Q^2 > 1.0 \text{ GeV}^2$,

its behavior obeys the $1/Q^2$ law. This behavior follows from the calculations within perturbative QCD in the region $Q^2 > 5 \text{ GeV}^2$, but experimental data are in accord with the asymptotic behavior of photon–proton interaction at lower values of Q^2 down to 1 GeV^2 .

The integral I_{GDH} grows fast from the photon point to $Q^2 \approx 1\text{--}2 \text{ GeV}^2$, and its absolute value decreases approximately by an order of magnitude at $Q^2 = 1.0 \text{ GeV}^2$. Under the assumption of photon interaction with proton partons, the difference $\sigma_{1/2} - \sigma_{3/2}$ is determined by the difference of the probabilities of finding a parton with spin orientation along and against the photon–spin direction. Thus, the integral I_{GDH} is related to the contribution of the parton spin to the total proton spin. According to the analyses performed in [9, 10], the contribution of the parton spin to the total nucleon spin in the asymptotic region ($Q^2 > 1.0 \text{ GeV}^2$) does not exceed 30%; at the photon point ($Q^2 = 0 \text{ GeV}^2$), the main contribution to the nucleon spin comes from constituent quarks.

Thus, the variation in the quantity Q^2 over the interval from 0 to 1.0 GeV^2 leads to a significant variation in the helicity amplitudes of photon–proton interaction and in the contribution of the quark spin to the total proton spin. In order to understand mechanisms behind such strong changes in the spin structure of the photon–proton interaction, it is necessary to analyze contributions of various exclusive channels to the cross-section difference $\sigma_{1/2} - \sigma_{3/2}$.

A detailed description of the Q^2 and W (W is the total energy in the c.m. frame) dependences of the cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ and of the GDH integral

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in total-photoabsorption and single-pion-production reactions was given in [11]. At $W > 1.6$ GeV, double-pion-production reactions begin to contribute to the cross section for the total-photoabsorption reaction on a proton significantly; at $W > 1.9$ GeV, their contribution becomes dominant. Therefore, it is interesting to investigate the helicity components of the cross section for double pion production by photons on a proton. In [12–14], a model was developed for describing double pion production on a proton by virtual and real photons at $W < 4.0$ GeV and $Q^2, -t < 5.0$ GeV², where t is the square of the difference of the initial- and final-proton 4-momenta. In this kinematical region, the model proposed in [12–14] describes well the entire body of available data on the cross sections for double charged-pion production by photons on a proton. This approach was used to analyze the first data of the CLAS Collaboration on the double production of charged pions by photons on a proton in the energy region corresponding to the excitation of nucleon resonances (E-93-006 experiment at JLAB; spokespersons V.D. Burkert and M. Ripani) [15, 16]. In the present study, we calculate the helicity components $\sigma_{1/2}$ and $\sigma_{3/2}$ of the cross section for double charged-pion production by real photons at $W < 2.0$ GeV, relying on the approach developed in [12–14]. We determine the model parameters from a fit to all available data on the cross sections for double charged-pion production by photons in the energy region of nucleon-resonance excitation.

2. DESCRIPTION OF THE CROSS SECTIONS FOR DOUBLE CHARGED-PION PRODUCTION ON PROTONS AND THEIR HELICITY COMPONENTS

Quasi-two-particle mechanisms involving the production and subsequent decay of Δ and ρ resonances in the intermediate state (see Fig. 1) are known to make the main contribution to the reaction of double charged-pion production on a proton target. The amplitude of each mechanism in Fig. 1 was calculated in the Breit–Wigner approximation as the product of the relevant two-particle amplitude, the amplitudes for the decay of unstable intermediate particles into stable final-state particles, and the corresponding Breit–Wigner propagators [13, 14]. We describe all other processes that contribute to the double-pion-production reaction in the phase-space approximation; that is, their total amplitude is approximated by a quantity $C(W, Q^2)$ that is independent of the final-state kinematical variables and which is a free model parameter to be determined from an analysis of experimental data. The amplitudes for the quasi-two-particle mechanisms in Fig. 2 are described by a superposition of the excitations of

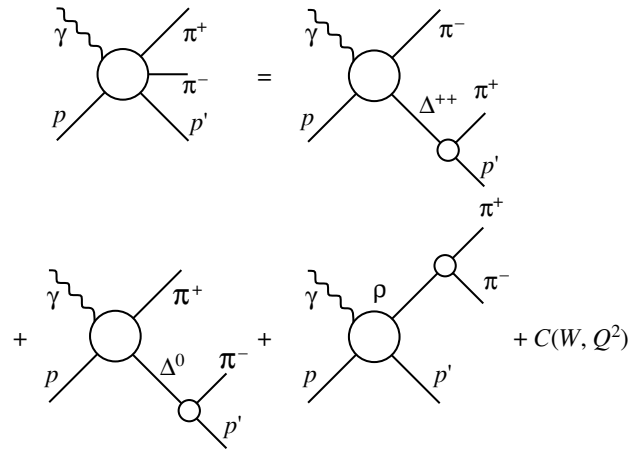


Fig. 1. Mechanisms of the production of $\pi^+\pi^-$ pairs by photons on a proton.

nucleon resonances in photon–proton interaction in the input channel (Figs. 2a, 2f) and nonresonance mechanisms (Figs. 2b–2e and 2g).

In calculating the resonance components in the amplitudes for the quasi-two-particle reactions (Fig. 2), we took into account all well-known nucleon resonances having masses below 2.0 GeV and significant widths with respect to decays through the $\pi\Delta$ and ρp channels [12, 13].

The resonance amplitudes were calculated in the Breit–Wigner approximation. The details of the calculations can be found in [12]. The electromagnetic amplitudes corresponding to the $\gamma p R$ vertex were calculated on the basis of data on the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ from [17]. Here, a nucleon resonance is denoted by R . For these amplitudes, one can also use the results obtained within any quark model.

Having calculated the cross section for double charged-pion production within the model proposed in [12–14] with the model amplitudes $A_{1/2}$ and $A_{3/2}$ estimated on the basis of a quark model, we can compare the results with experimental data and, in this way, confirm that the model description of the electromagnetic form factors for nucleon resonances is adequate. On the other hand, we can treat the electromagnetic form factors as free parameters of the model and extract their values from a fit of the calculated cross sections to experimental data. We determined the amplitudes of the strong nucleon-resonance decays corresponding to the Rpp ($R\pi\Delta$) vertex from the results obtained in [18] by analyzing the cross sections for $\pi N \rightarrow \pi\pi N$ reactions.

We described nonresonance processes in the quasi-two-particle reaction $\gamma p \rightarrow \rho p$ in the diffraction approximation [13, 14]. The common factor in the nonresonance amplitude is a free parameter that is

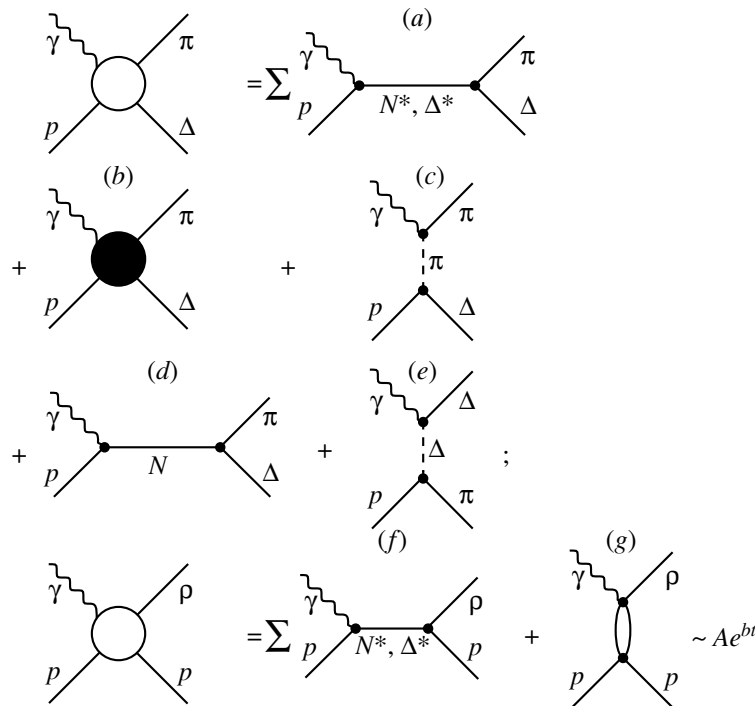


Fig. 2. Diagrams describing the quasi-two-particle reactions $\gamma p \rightarrow \pi\Delta$ [resonance mechanisms (a) and Born terms: (b) contact term, (c) on-flight pion, (d) nucleon term, and (e) on-flight delta] and $\gamma p \rightarrow \rho p$ [(f) resonance mechanisms and (g) diffractive production of a ρ meson].

independent of W and which must be determined from experimental data.

We described the nonresonance amplitude for the channel $\gamma p \rightarrow \pi\Delta$ by the set of gauge-invariant Born amplitudes corresponding to the diagrams in Figs. 2b–2e. Off-shell pion formation in the mechanism represented by the diagram in Fig. 2c was taken into account by introducing the $\pi N\Delta$ vertex function determined from data on NN scattering and the pion electromagnetic form factor [12]. The product of the strong and electromagnetic vertices for the diagram involving the exchange of a Δ isobar (Fig. 2e), as well as the t and Q^2 dependence of the contact term (Fig. 2b), was determined from the condition requiring that the sum of Born terms be gauge-invariant [12]. The possibility of the exchange of various mesons with pion quantum numbers via the mechanism in Fig. 2c—this possibility is of particular importance at $W > 1.7$ GeV—is effectively described by means of the substitution of the pion Regge trajectory for the one-pion propagator, as was proposed in [19]. To restore the gauge invariance of Born terms, we follow the procedure used in [19]. All Born amplitudes corresponding to one-pion exchange are multiplied by the common factor

$$(t - m_\pi^2) R_\pi(t), \quad (2)$$

where $R_\pi(t)$ is the Regge propagator proposed in [19] for the pion trajectory, t is the Mandelstam variable corresponding to the amplitude in Fig. 2c, and m_π is the pion mass. The multiplication of the one-pion-exchange amplitude (Fig. 2c) by the factor in (2) corresponds to the substitution of the Reggeized propagator $R_\pi(t)$ for the one-pion propagator in this amplitude. Since the gauge-invariant sum of Born amplitudes involving one-pion exchange is multiplied by a common factor, the gauge invariance of the sum is preserved upon the substitution of the Reggeized propagator $R_\pi(t)$ for the one-pion-exchange propagator.

The use of the Reggeized gauge-invariant Born amplitudes made it possible to obtain a satisfactory description of experimental data from [20] on the angular distributions of π^- mesons in the quasi-two-particle channel $\gamma p \rightarrow \pi\Delta$ (see Fig. 3). Thereby, problems concerning the description of the π^- -meson angular distribution in the reaction $\gamma p \rightarrow \pi\Delta$, which were discussed in [12], have been successfully solved; at the present stage, the Born terms in the channel $\gamma p \rightarrow \pi\Delta$ are gauge-invariant in our model.

Coupling to open inelastic channels in the initial and final states is of importance in describing the nonresonance Born amplitudes for the reaction $\gamma p \rightarrow \pi\Delta$. In [12], a dedicated approach was developed according to which this coupling is effectively taken into

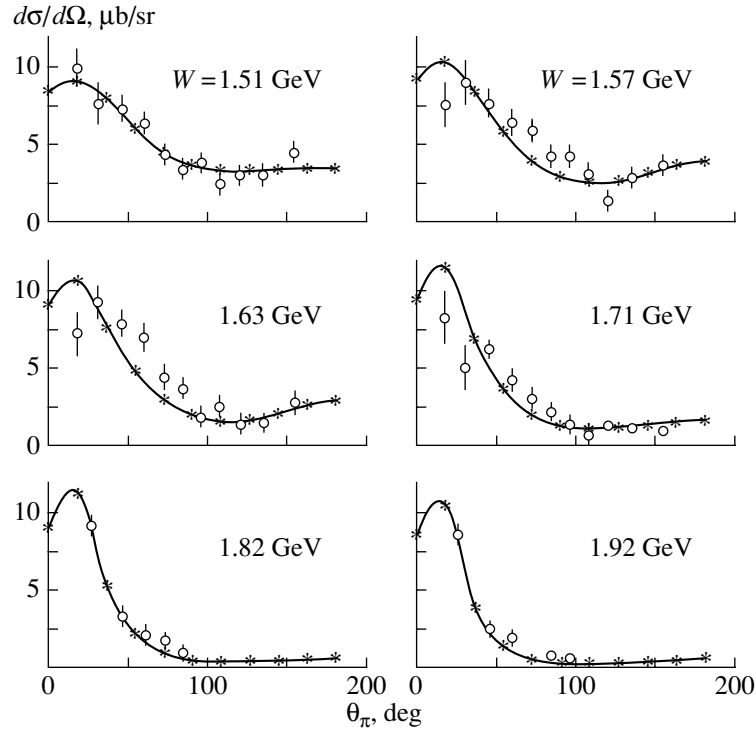


Fig. 3. Angular distribution of π^- mesons in the reaction $\gamma p \rightarrow \pi^- \Delta^{++}$ according to our calculations with Reggeized Born terms (see main body of the text), along with experimental data from [20]. The angle between the γ and π^- momenta in the c.m. frame is plotted along the abscissa.

account as the absorption of projectile particles in the initial state and the absorption of emitted particles in the final state. The absorption factors are determined from data on πN scattering.

The helicity differential-cross-section components $d\sigma_{1/2}$ and $d\sigma_{3/2}$ are defined as

$$d\sigma_{1/2(3/2)} = \frac{1}{2} 4\pi\alpha \frac{1}{4K_L M_N} \times \sum_{\lambda_\gamma \lambda_p \lambda_{p'}} |\langle \pi \pi \lambda_{p'} | T | \lambda_\gamma \lambda_p \rangle|^2 d\tau, \quad (3)$$

$$|\lambda_\gamma + \lambda_p| = 1/2(3/2),$$

where $\langle \pi \pi \lambda_{p'} | T | \lambda_\gamma \lambda_p \rangle$ are the helicity amplitudes for the reaction of double charged-pion production at the incident-photon helicity of λ_γ , the target-proton helicity of λ_p , and the final-state-proton helicity of $\lambda_{p'}$.

The quantity K_L is the equivalent-photon wave vector

$$K_L = \frac{W^2 - M_N^2}{2M_N}, \quad (4)$$

where M_N is the nucleon mass and $\alpha = 1/137$ is the fine-structure constant.

The element $d\tau$ of the final-state three-particle phase space is

$$d\tau = \frac{1}{32W^2} \frac{1}{(2\pi)^5} dS_{\pi^+p} dS_{\pi^+\pi^-} d\Omega d\alpha, \quad (5)$$

where S_{π^+p} and $S_{\pi^+\pi^-}$ are the squares of the invariant masses of, respectively, the π^+p and the $\pi^+\pi^-$ system in the final state; Ω is the solid angle of proton (or π^- -meson) emission; and α is the angle between the plane spanned by the photon and proton (or photon and π^- -meson) 3-momenta and the plane spanned by the π^+ - and π^- -meson (or π^+ -meson and proton) 3-momenta.

In calculating the helicity components $d\sigma_{1/2(3/2)}$, the terms characterized by the total helicities $\lambda_\gamma + \lambda_p$ are included in the sum in (3) according to the lower line in (3) and the conventions in [21]. The sum $d\sigma_{1/2} + d\sigma_{3/2}$ is related to the total unpolarized cross section as follows:

$$d\sigma_{1/2} + d\sigma_{3/2} = 2d\sigma. \quad (6)$$

The amplitudes parameterized in the phase-space approximation, $C(W, Q^2)$, are shared between the helicity components of 1/2 and 3/2 under the assumption of their equal contributions to all helicity states.

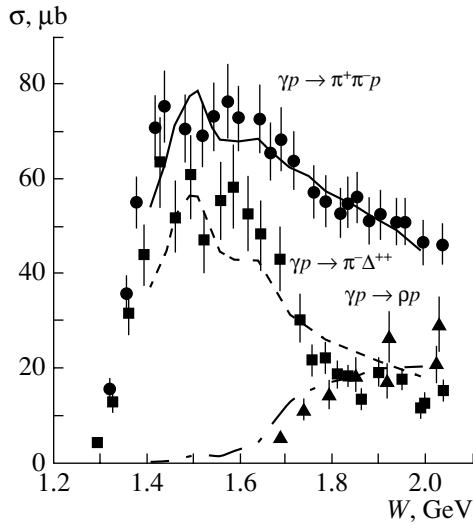


Fig. 4. Experimental data from [20] on the W dependence of the integrated cross sections for the reaction $\gamma p \rightarrow \pi^+ \pi^- p$ (\bullet) and contributions of the quasi-two-particle channels $\gamma p \rightarrow \pi^- \Delta^{++}$ (\blacksquare) and $\gamma p \rightarrow \rho p$ (\blacktriangle), along with the results of the calculations based on the model described in the main body of the text.

Upon including, in the model developed in [12–14], the calculations of the polarization components $d\sigma_{1/2(3/2)}$ of the cross sections, the values of the electromagnetic form factors for nucleon resonances can be extracted from a simultaneous fit to unpolarized differential cross sections $d\sigma$ and their helicity components $d\sigma_{1/2(3/2)}$. Such an extension of the range of fitted data may significantly improve the accuracy with which one extracts the electromagnetic form factors for nucleon resonances. In addition, a calculation of the helicity components $d\sigma_{1/2(3/2)}$ with the amplitudes $A_{1/2}$ and $A_{3/2}$ from various quark models and a comparison of the results obtained in this way with experimental data extend considerably the possibilities for validating the description of the structure of nucleon resonances as objects formed by interacting quarks and gluons.

The approach developed in the present study can also provide information about the contributions of resonance and nonresonance mechanisms and various two-quasi-particle channels to the cross-section components $d\sigma_{1/2(3/2)}$. This information is of great value for obtaining deeper insight into the spin structure of nucleons and into the dynamics of processes that are responsible for the evolution of the helicity structure of photon–proton interaction.

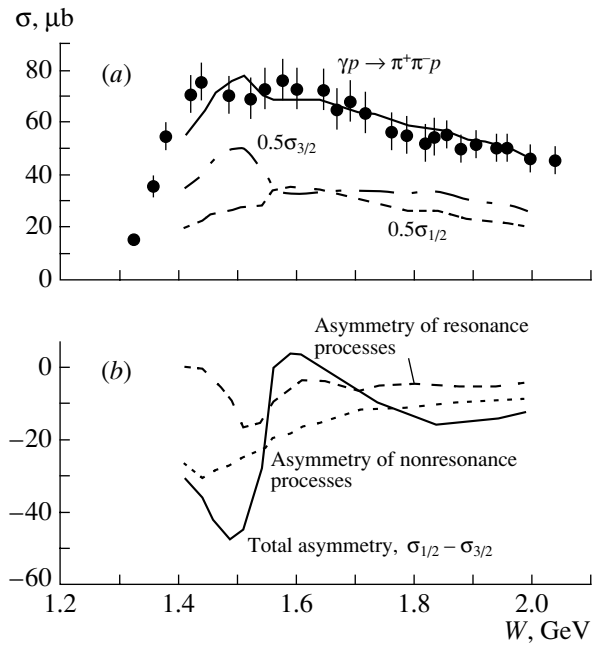


Fig. 5. (a) Total cross section (solid curve) and its helicity components for the reaction $\gamma p \rightarrow \pi^+ \pi^- p$ along with experimental data from [20]; (b) $\sigma_{1/2} - \sigma_{3/2}$ asymmetry.

3. RESULTS OF THE CALCULATIONS FOR THE HELICITY COMPONENTS OF THE CROSS SECTIONS FOR DOUBLE CHARGED-PION PRODUCTION BY PHOTONS

Within the approach described above, we have calculated the W dependences of the helicity cross-section components $\sigma_{1/2}$ and $\sigma_{3/2}$ for the exclusive channel of double charged-pion production by photons. The electromagnetic form factors $A_{1/2}$ and $A_{3/2}$ for nucleon resonances were taken in accordance with data presented by the Particle Data Group [17]. The amplitudes of strong resonance decays were extracted from [18] by using the method described in [12]. The following parameters of the model were taken to be free: the amplitude of the three-particle phase space, $C(W, Q^2)$; the effective coupling constant for the π Regge trajectory; and the multiplicative factor of the diffractive nonresonance amplitude for ρ -meson production. The free parameters were determined from a fit to data of the ABBHM Collaboration [20] on the W dependences of the integrated cross sections for double charged-pion photoproduction.

The integrated cross sections that were calculated for double charged-pion photoproduction on the basis of our model with the parameter values corresponding to the best fit are displayed in Fig. 4 along with experimental data from [20]. The calculations are seen to reproduce these data well. In order to test our model, we

calculated the integrated cross sections for the quasi-two-particle reactions $\gamma p \rightarrow \pi^- \Delta^{++}$ and $\gamma p \rightarrow \rho p$, employing the parameters determined from a fit to the total cross section (solid curve in Fig. 4). The W dependences of the integrated cross sections for these reactions (dashed and dash-dotted curves in Fig. 4) are contrasted against experimental data from [20]. It can be seen that these theoretical results faithfully reproduce the data from [20] on the W dependences of the integrated cross sections for these quasi-two-particle channels.

The calculated helicity components $\sigma_{1/2}$ and $\sigma_{3/2}$ and their difference for the integrated cross sections are shown in Fig. 5 versus W . The lower panel in Fig. 5 displays the contributions of the resonance mechanisms (dashed curve) and nonresonance processes (dotted curve) to the difference $\sigma_{1/2} - \sigma_{3/2}$.

A feature peculiar to the difference $\sigma_{1/2} - \sigma_{3/2}$ is the formation of a dip at $W = 1.50$ GeV. Such a structure was observed in the MAMI experimental data [3] on the difference $\sigma_{1/2} - \sigma_{3/2}$ in double charged-pion production. The predicted structure is in qualitative agreement with the data from [3]. The W dependence for nonresonance processes is smooth, while the W dependence for the excitation of nucleon resonances features a structure at $W = 1.51$ GeV (Fig. 5b). The structure in the W dependence of the difference $\sigma_{1/2} - \sigma_{3/2}$ results from the contribution of the $D_{13}(1520)$ state, which is characterized by significantly different values of the electromagnetic helicity amplitudes $A_{1/2}$ and $A_{3/2}$ (-0.024 and 0.166 GeV $^{-1/2}$, respectively). So great a distinction between these helicity amplitudes leads to strongly different values of the helicity components of the resonance part of the cross section; this in turn leads to a dip in the W dependence of the difference of the helicity total-cross-section components $\sigma_{1/2}$ and $\sigma_{3/2}$ at a W value close to the mass of the $D_{13}(1520)$ resonance. Thus, measurement of the helicity components of the cross section for double pion photoproduction is sensitive to the existence of resonances characterized by significantly different values of the electromagnetic form factors $A_{1/2}$ and $A_{3/2}$ and can be used as an effective tool in searches for such states. In particular, measurements of the helicity components of the cross section will play an important role in seeking “missing” baryon states having significantly different electromagnetic form factors.

From the theoretical results presented in Fig. 5b, it follows that, in the energy region corresponding to the excitation of nucleon resonances, there is a strong interference between resonance and nonresonance amplitudes. Measurement of the helicity components

of the cross section can aid in determining the relative phase of resonance and nonresonance amplitudes from experimental data, and this is of importance because it is difficult to do this theoretically.

4. CONCLUSION

On the basis of the model developed in [12–14] for describing double charged-pion photoproduction on a proton, we have predicted here the W dependence of the helicity cross-section components $\sigma_{1/2}$ and $\sigma_{3/2}$ and of their difference. The results exhibit a rather high sensitivity of the difference $\sigma_{1/2} - \sigma_{3/2}$ to the excitation of resonance states characterized by significantly different values of the electromagnetic form factors $A_{1/2}$ and $A_{3/2}$. The excitation of such resonances leads to the formation of resonance structures in the W dependence of the difference of the helicity cross-section components $\sigma_{1/2}$ and $\sigma_{3/2}$. Thus, measurement of the helicity components of the cross section for double charged-pion photoproduction will play an important role in the investigation of nucleon resonances having significantly different values of $A_{1/2}$ and $A_{3/2}$ —in particular, in searches for “missing” baryon states that possess such properties.

Our approach makes it possible to extract the electromagnetic form factors for nucleon resonances from a global fit to data on the differential cross sections for double charged-pion photoproduction and their helicity components. The development of the model from [12–14] in the course of the present investigation, along with the extension of the range of data subjected to analysis, significantly enhances its potential for determining the electromagnetic form factors for nucleon resonances and improves the reliability of the results obtained in this way.

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