Dual Detectors of Gravitational Waves

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ABSTRACT

In a "Dual" gravitational wave (GW) detector a wide band sensitivity is obtained by measuring the differential displacement, driven by the GW, of the facing surfaces of two nested massive bodies mechanically resonating at different frequencies. A "selective readout" scheme, capable of specifically selecting the signal contributed by the vibrational modes sensitive to the gravitational waves, could then reduce the thermal noise contribution from the not sensitive modes. In a dual detector the sensitivity improvement in the displacement transduction could be pursued by means of mechanical amplification systems. This solution is innovative for the resonant GW detectors and we report about preliminary theoretical and experimental study.

Keywords: Gravitational Waves, Transduction, Detector, Dual, Detection, Amplification, Compliance, Noise, Fabry-Perot

1. INTRODUCTION

Resonant GW detectors have been improved by 4 orders of magnitude in energy sensitivity over the last 40 years. Up to now, by using a 2300 kg antenna at cryogenic temperature, the energy sensitivity can be as little as a few thousand of quanta of vibration at about 1kHz. The 5 bar detectors distributed worldwide (ALLEGRO, AURIGA, EXPLORER, NAUTILUS and NIOBE)[1] have operated for a few years as a network, giving for the first time significant upper limits to the yearly rate of violent gravitational wave events in the Galaxy.[2,3] Long baseline interferometric detectors (LIGO, VIRGO, TAMA, GEO)[4–7] that are designed to reach best sensitivity in the range 50 to 500Hz, have already started operating and, complemented by the upgraded bars, they may give rise to a first GW signal detection in a probably very near future. However, it is commonly accepted that to enter in the first "observatory phase" or "GW Astronomy", a significant improvement in GW detection should be achieved. We are actively investigating a novel detection scheme, the "Dual" resonator system,[8–10] which can provide both high sensitivity and wide bandwidth. These new resonant mass detectors are potentially very sensitive in the few kHz frequency range and could work in a complementary role with interferometers.

Up to now, to reduce the effect of the noise of the read-out amplifier, conventional acoustic detectors have made use of resonant transducers, which seriously limit the detector bandwidth. Moreover, resonant transducers have proved so far to lower the mechanical quality factor of the main resonator. The Dual resonator approach could actually exploit the potential sensitivity of resonant detectors and it could enhance their performances to make them complementary to the advanced versions of interferometric detectors.

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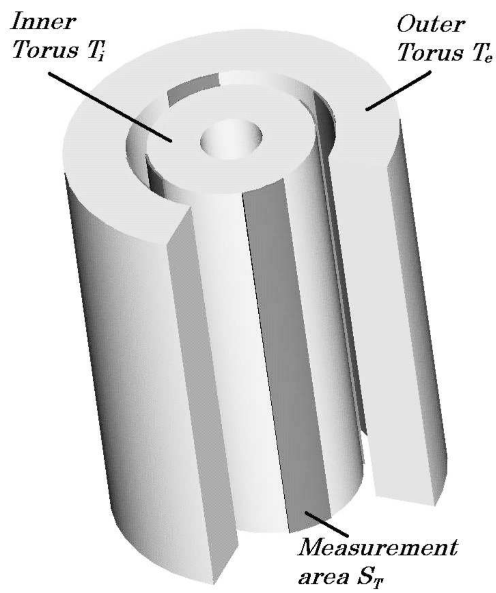

Figure 1. The detector is made of two concentric cylinders, T_e, T_i. The inner cylinder may also have null internal radius. The optimally oriented gravitational wave travels along the z-axis and induces relative displacements among the two cylinders. The output signal is obtained by measuring the relative distance in 4 regions (in black), each of area S_T, for the whole cylinder height.

2. NEW KIND OF ACOUSTIC RESONANT DETECTORS: DUAL SYSTEMS

The conventional resonant transducers matched to traditional monolithic bars[11] are used to enhance (of a factor 15 to 30) the displacement induced by the gravitational signal. Thanks to this amplification factor the signal overcomes the amplifier white noise, but only in a frequency range close to the resonance of the main resonator. As disadvantage, they inject their own higher thermal noise in the full system, resulting in a deterioration of the signal to noise ratio a little away from the resonant frequency (about 1 kHz): this results in a narrowing of the sensitive band (about 10% of the operating frequency). To avoid this drawback, "dual" resonators had been designed to be intrinsically sensitive wide band detectors. They consist of two nested mechanical massive resonators whose relative displacement vibrations are measured by *non resonant readouts*. A good compromise between simplicity and sensitivity is a configuration providing two nested cylinders, an inner solid one and a hollow outer one, although it had just been studied the case of a spherical configuration.[9] Figure 1 shows a scheme of a dual detector of cylindrical symmetry. These massive bodies, if forced by a GW, start vibrating as mechanical quadrupolar oscillators; while external resonator acts as an *oscillator driven above its resonance*, the inner one acts as an *oscillator driven below its resonance*. This results in a phase shift by πrad between the two bodies differential displacement. Thus the signal is enhanced in the differential measurement (of the relative position of the facing surfaces). The useful sensitivity bandwidth for gravitational waves *ranges from the fundamental quadrupolar mode of the outer body, at lower frequency to the fundamental quadrupolar mode of the inner body, at higher frequency.*

The situation is different for the response to the readout noise force. Any measurement operation injects noise force through the reading surface. This force acts with opposite sign on the forcing surfaces and so the noise injection is reduced. The overall result can be a very large bandwidth with quite uniform sensitivity (2 to 5kHz).

If we assume the system thermal noise to be negligible, the maximum achievable sensitivity is obtained at the standard quantum limit (SQL) for the readout, when the product of the noise power spectral densities in displacement and in back-action force is $\sim (\hbar^2)/4$. Only a suitably adapted wide band readout can provide the

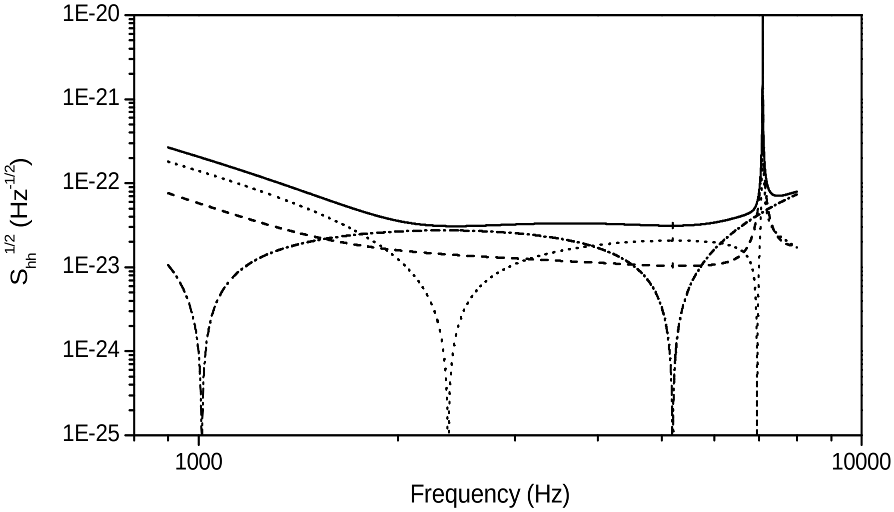

Figure 2. The contribution to the detector noise at the SQL is evaluated for a Molybdenum detector. Continuous line: total noise. Dashed line: thermal noise evaluated at T/Q$\sim$ 2×10^{-7} K^{-1}. Dashed-dotted line: amplifier displacement noise. Dotted line: amplifier back-action noise.

optimal balance between the displacement noise and the back-action force noise. Figure 2 shows the contributions to $\sqrt{S_{hh}}$ at SQL for a cylindrical configuration of dual detector made by Molybdenum, whose external diameter is 0.94m, T/Q$\sim$ 2×10^{-7} K^{-1} (Q is the mechanical quality factor and T the temperature) and the readout displacement noise $\sim 2 \times 10^{-23}$m/$\sqrt{\text{Hz}}$.

3. WIDE BAND READOUTS FOR WIDE BAND DETECTORS

In order to achieve a bandwidth of many kHz without loosing sensitivity, we need a broadband readout system, i.e. designed to have its intrinsic resonances above the fundamental quadrupolar inner cylinder resonance. Furthermore, the readout must measure the deformation over a wide area, so to minimize the contribution of the non-quadrupolar higher frequency modes, which do not carry any information about gravitational wave signals. At last, noise reduction can be performed by a readout scheme which geometrically distinguishes the quadrupolar mode displacements from the non-quadrupolar ones (see Figure 1)which are rejected.[12] We stress that a wide-band readout cannot profit from the displacement amplification at resonance of a conventional transducer. The lowest displacement noise achieved experimentally by two different kinds of readout (the opto-mechanical one,[13,15] based on Fabry-Perot cavities, and the capacitive one,[14] based on SQUID amplifiers) is about 5×10^{-20}m/$\sqrt{\text{Hz}}$[14,15] in the kHz range. The properties of these transducer systems are very different and both of them show peculiar advantages and could be implemented in different configurations of dual detectors. In Figure 2 the dual cylinder sensitivity curves are optimized with a quantum limited readout with displacement sensitivity of the order of 3×10^{-23} m/$\sqrt{\text{Hz}}$. To allow a profitable improvement of present readout systems in dual detectors, it is useful to analyze how they work. The efficiency of the signal conversion by the transducer, is proportional to the bias field and the square root of the capacitance in the case of the capacitive transducer and we aim at increasing by at least an order of magnitude (up to $100MV/m$[16]) the static bias electric field between the plates. Correspondingly, in the case of the optical transducer, the efficiency is proportional to the light power injected in the cavity times the finesse of the cavity. For dual detectors, we need to increase the light power by a factor of 1000, up to a few W, but, to do so, it is necessary to improve frequency stabilization (to better than 10^{-9} Hz^2/Hz)and to change the optical configuration.[17]

3.1. The geometry - extension and symmetry - of the readout surface

Readouts with a small surface 'see' many acoustic modes of the detector, up to high frequencies. Each mode has its own thermal noise and is excited by the back action noise. In order to reduce the contribution in noise of higher acoustic modes in a detector, it is useful to extend the sensitive surface, averaging out high order modes. By means of this technique, we can get close to a configuration with only two effective modes, i.e. the two lowest quadrupolar order modes, sensitive to the gravitational wave and necessary for wide-band selective detection.[9, 12] A second major step is the development of a specific readout geometric design that is sensitive to the deformations that have quadrupolar symmetry, thus reducing the response of those with different symmetries that are not sensitive to the gravitational radiation.

For the capacitive transducer a conceptual configuration that realizes this readout selectivity has already been proposed.[12] In this case the selectivity allows to clean the bandwidth from spurious modes and makes possible an effective back-action reduction. As for the optical readout, a new cavity configuration, the Folded Fabry-Perot,[18] was proposed which allows to extend the effective waist size.

3.2. The quantum limit to the noise: state of art

We point out that only if the readout system is itself at the quantum limit, standard quantum limited performances of the detector can be reached . As for the capacitive transducer, the crucial improvement with respect to the state-of-the-art technology regards the energy resolution of the SQUID amplifier. Up to now the best performance achieved in a setup that can be implemented in a bar detector is of about 200 $\hbar$,[19] but energy resolution of about 10 quanta were recently obtained[20] and the single quantum of sensitivity seems to be not so far. As for the optical transducer, on the other hand, the quantum limited sensitivity seems more easily achievable. Major progresses in this case regard the cryogenic operation.

4. MECHANICAL AMPLIFICATION: INCREASING DUAL SENSITIVITY

Mechanical amplifiers based on the elastic deformation of monolithic devices - compliant mechanisms - are well known for their applications in mechanical engineering.[22] Their application to GW detectors seems promising and the contributed noise has been starting to be investigated thoroughly. We notice that this stage will also work as a mechanical impedance matching stage, since it affects the balance force-displacement and in particular the back-action forces due to the readout. Next section describes a such kind of characteristic. This feature could also be helpful to fit the detector mechanical impedance to the noise impedance of the amplifier, in order to obtain the so called "noise matching" condition and to optimize the signal to noise of the system. The improvements described in the previous section could allow to reach, in the near future, sensitivities of a few $10^{-22}\mathrm{m}/\sqrt{\mathrm{Hz}}$ for both capacitive and optical readouts. Then to achieve the needed $10^{-23}\mathrm{m}/\sqrt{\mathrm{Hz}}$ displacement sensitivity range, it may be necessary to develop an alternative and non-resonant device to amplify the differential deformation of the massive bodies by at least a gain factor $\alpha = 10$.[21] In the next sections, we propose a very simple "hybrid" leverage amplifier that can match our requirements.

4.1. Mechanical amplifiers: a first opto-mechanical prototype

The improvements described in the previous section could allow to reach, in the near future, sensitivities of a few $10^{-22}\mathrm{m}/\sqrt{\mathrm{Hz}}$ for both capacitive and optical readouts. Then to achieve the needed $10^{-23}\mathrm{m}/\sqrt{\mathrm{Hz}}$ displacement sensitivity range, we need to exploit others physical devices: mechanical leverages could be one possibility.[21] Mechanical amplifiers based on the elastic deformation of monolithic devices - compliant mechanisms - are well known for their applications in mechanical engineering.[22] We notice that this stage will also work as a mechanical impedance matching stage, since it affects the balance force-displacement and in particular the back-action forces due to the readout. This feature could be also helpful to fit the detector mechanical impedance to the noise impedance of the amplifier, in order to obtain the so called "noise matching" condition and to optimize the signal to noise of the system.

The application of a mechanical amplifier to GW detectors seems promising but the contributed noise needs to be investigated thoroughly. For this reason we started a research program to develop a mechanical leverage and experimentally characterize its mechanical transfer function and thermal noise. Our design goal is a mechanical

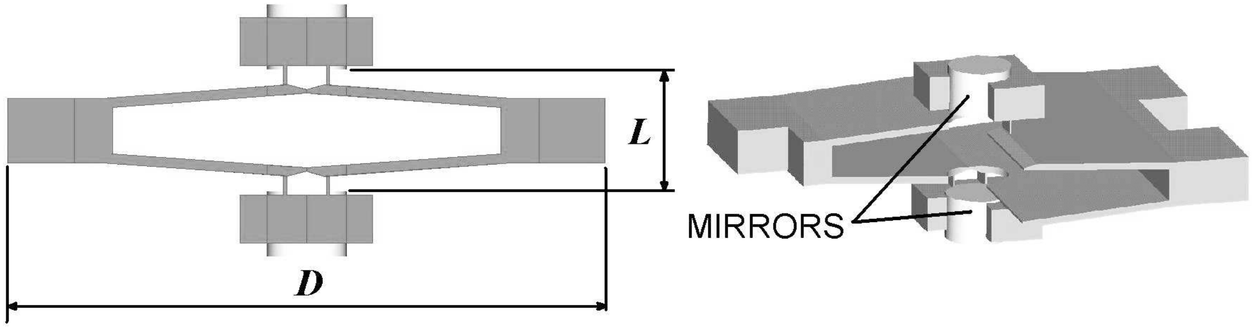

Figure 3. A leverage prototype: the variation on its length D represents the displacement signal and are converted in variations on the distance L. The displacement L can be measured by an optical readout, and two mirror holders are placed on to the amplifier device to make the optical cavity.

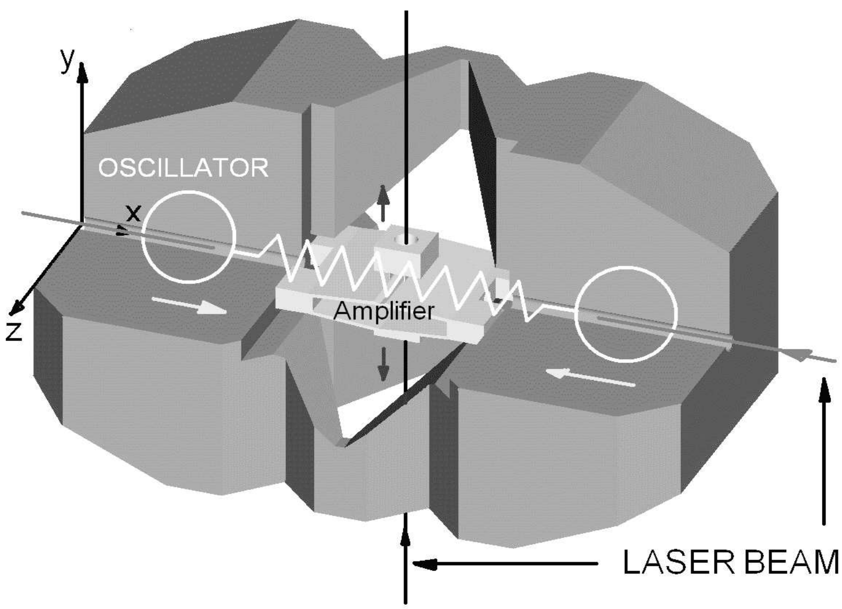

Figure 4. The overall assembly of the testing oscillator (dark gray) with the inner mechanical amplifier (gray). Two laser beams are shown and drive two orthogonal optical cavities which allow the measurement of the relevant quantities: the test resonator length D variation and the corresponding variation in the length L of the mechanical amplifier. In principle the two cavities can be used to measure at the same time the two distances to obtain the system transfer function, but for the moment we only planned to separately measure the thermal noise of these quantities.

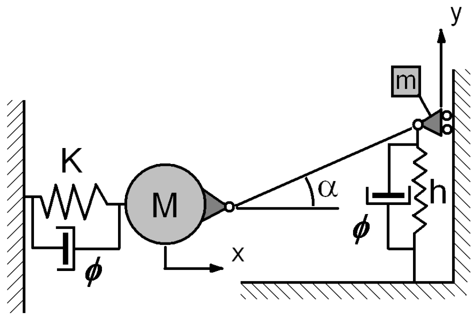

Figure 5. a) Lumped elements mechanical model of the amplifier+test resonator system. The mass M is the oscillators effective mass. The mass m represents the mirror+support effective mass. K is the stiffness of the oscillator and h is the amplifier one. The beam is considered rigid and of negligible mass. The same structural dissipation (ϕ) was considered for both amplifier and oscillator. b) System transfer function predicted by FEM simulation.

leverage able to perform at least a gain factor of 10 on the displacement in a wide bandwidth. A prototype of this device is shown in Fig. 3: the variation on its length D represents the displacement signal and are converted in variations on the distance L. The amplifier gain factor depends on the frequency of the displacement signal and was evaluated within a modeling software framework (ANSYS) by the use of properly developed macros. As expected, the static gain is essentially maintained up to the frequency of the internal resonance modes of the device. At higher frequencies the system stiffness is reduced and the leverage gain is consequently spoiled off. The proposed leverage has a rhombic structure with localized rigid hinges (see Fig. 3). The displacement gain factor is $\alpha \simeq 10$ and the structure is free from internal resonance up to about 1.5 kHz. We are aware that the localized rigid hinges could be a big source of thermoelastic noise[24] if used in a sensitive detector and then further evaluation of this noise component will be necessary.

In order to establish the amplifier thermal noise contribution in a resonant detector, we studied a system made of the displacement amplifier nested with an hollow mechanical test oscillator (see Fig. 4). The input variable is the test oscillator length D, that should be measured as an amplified distance at the leverage output L. The system transfer function $L = T(D)$ was analytically evaluated by the approximate lumped model shown in Fig. 5a (which gives a constant gain $\alpha = 10$ for all frequencies) and then predicted by FEM as shown in Fig. 5b. FEM simulation predicts a gain factor dominated by the leverage configuration below the fundamental resonance, while above the resonance the gain factor increases its value due to the non-ideal rigidity of the leverage, and it is driven by the dynamic displacement of the fundamental normal mode. So at resonance, the oscillator-amplifier system behaves like the resonant transducer presently used for the AURIGA detector.

In order to test our models we plan to measure the relevant lengths D and L by the use of a displacement optical readout.[13] For this sake two mirror holders are placed on to the amplifier device to make an optical cavity. The Pound-Drever[25] signal of the Fabry-Perot cavity inside the amplifier will then be compared with the one of a reference frequency stabilized Fabry-Perot cavity. In the same way two mirrors can be placed on the test resonator and its length D can be measured by a second FP optical cavity. To reduce the vibrational noise the test resonator will be suspended by a mechanical isolator inside a vacuum chamber. To prevent the Fabry-Perot cavities on the prototype from unlocking due to the thermal expansion of the material (we make use of an Al 7075 alloy for both oscillator and amplifier), the full system will be housed in a thermally stabilized box that will limit the temperature variations below 10 μK. The thermal control system will be made by two nested feed-back stabilized thermal shields.

We evaluated the system thermal noise by two independent methods. This simple model shown in Fig. 5 describes the system dynamical behavior and can be used to infer the corresponding thermal noise by the Fluctuation Dissipation Theorem.[23] On the other side the thermal noises can be evaluated by FEM calculation provided that the measuring surface is given. As shown in Fig. 6, the results are in good agreement and very encouraging. In fact the thermal noise at the amplifier output is essentially the thermal noise at the input multiplied by the amplifier gain. In other words the amplifier own thermal noise contribution is evaluated as

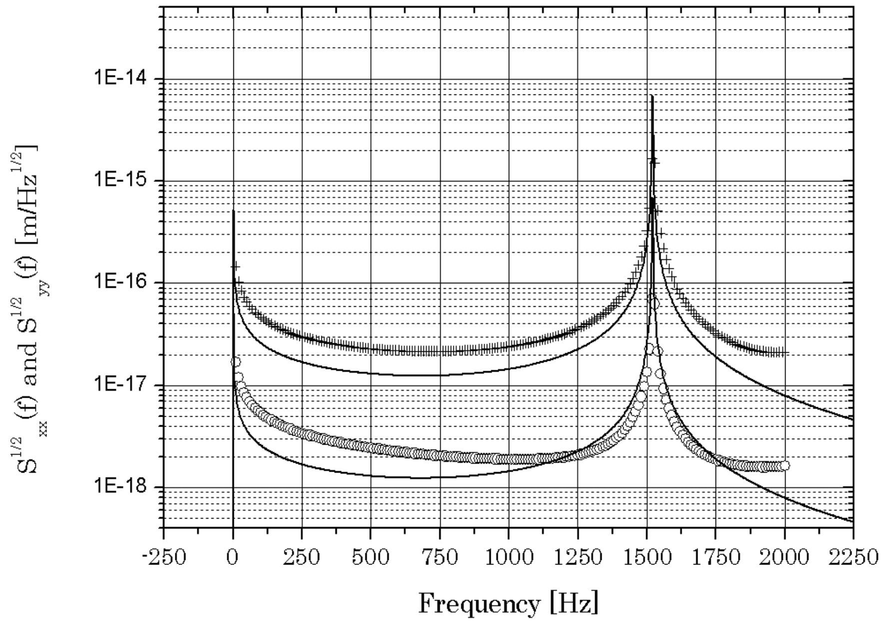

Figure 6. FEM prediction compared with the analytical ones (continuous line): The crosses represent thermal noise read by the cavity L, with a 365 μm waist Gaussian profile on a 2mm wide circular surface on the amplifier; white circles represent thermal noise on the test oscillator length D, as read by a 365 μm waist Gaussian profile on a 2mm wide circular surface (nested amplifier mounted). The analytical curves are obtained by applying the Fluctuation Dissipation Theorem to the transfer function of our model. The temperature is T=300 K and the mechanical Q=10000

negligible and the signal to noise ratio on the length D (that is our signal) should remain unchanged at the amplifier output. This system could then be used to raise the signal displacement over the noise floor of the readout with only a small additional thermal noise contribution.

We planned to realize this prototype and fully characterize its performances in terms of mechanical transfer function and thermal noise (at room temperature) within 2 years. A cryogenic test of this system is in principle possible, provided that the cryogenic compliant FP cavities will be developed and installed. For a low temperature test the amplifier could be also equipped with a SQUID based capacitive readout. We notice in the end that the amplifier bandwidth, determined by the first vibrational mode at 1.5 kHz in our aluminum prototype, could be increased up to 5 kHz by the use of special materials, as beryllium or SiC.

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