

Helioseismology and screening of nuclear reactions in the Sun

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Abstract

We show that models for screening of nuclear reactions in the Sun can be tested by means of helioseismology. As well known, solar models using the weak screening factors are in agreement with data. We find that the solar model calculated with the anti screening factors of Tsytovitch is not consistent with helioseismology, both for the sound speed profile and for the depth of the convective envelope. Moreover, the difference between the no-screening and weak screening model is significant in comparison with helioseismic uncertainty. In other words, the existence of screening can be proved by means of helioseismology.

I. INTRODUCTION

The solar neutrino problem is so important that any aspect of solar, plasma and nuclear physics pertinent to it has to be deeply investigated before definitive conclusions can be drawn. In this respect, screening of the charges of the reacting nuclei due to free charges in the solar plasma is of some interest.

The study of screened nuclear reaction rates was started with the pioneering work of Salpeter [1], who discussed both the extreme cases of "weak" and "strong" screening, providing suitable expressions for the screening factors

$$f_{ij} = \langle \sigma v \rangle_{ij, plasma} / \langle \sigma v \rangle_{ij, bare} . \quad (1)$$

The solar core is not far from the weak screening case, however it does not satisfy the usual conditions under which the weak screening approximation holds. This is the reason why the problem has been investigated by several authors, see e.g. [2-7].

Gruzinov and Bahcall [8] calculated the electron density in the vicinity of fusing nuclei using the partial differential equation for the density matrix that is derived in quantum statistical mechanics. Their numerical result agrees, within small uncertainties, with Salpeter's weak screening formula. Furthermore, Bahcall et al. [9] recently provided several arguments that demonstrate the validity of the Salpeter formula near the solar center with insignificant errors.

The conclusions of Gruzinov and Bahcall [8] are not unanimously accepted. According to Shaviv and Shaviv [10], the weak screening formula does not hold in the Sun. Some nuclear reactions are enhanced by the surrounding plasma whereas some others are suppressed. According to Tsytovitch and Bornatici [11,12] a kinetic description of collective plasma effects results in a decrease of the thermonuclear reaction rates in contrast to the Salpeter's enhancement.

We observe that solar models are built by using stellar evolutionary codes which include specific expressions for the nuclear reaction rates. If one uses different formulas for the screening factors f_{ij} one obtains different solar models. On the other hand, helioseismology provides precise information on the sound speed profile and on the properties of the convective envelope, see e.g. [13], which have to be reproduced by the correct solar model. The main purpose of this paper is to test the screening models by means of helioseismology. We build several solar models corresponding to different screening factors and compare the results with helioseismic data.

We also comment on the predicted neutrino fluxes. Many of the attempts to modify the screening factors have been produced as an effort to avoid or mitigate the so called solar neutrino puzzle, by reducing the predicted 8B neutrino flux. We will show that a reduction of the screening factors does not generally imply a reduction of the 8B neutrino flux.

II. RESULTS OF SOLAR MODEL CALCULATIONS FOR DIFFERENT SCREENING PRESCRIPTIONS

By using FRANEC [14], a stellar evolutionary code including diffusion of helium and heavy elements [15], we constructed solar models based on four different assumptions:

i) The weak screening approximation (WES). The screening factors f_{ij} are given by:

$$\ln f_{ij}^{\text{WES}} = Z_i Z_j e^2 / (a_D kT) \quad (2)$$

where Z_i, Z_j are the charges of the interacting nuclei, T is the temperature and a_D is the Debye radius. As clear from equation above, the screening factors are always larger than unity, i.e. the plasma provides enhancement of the thermonuclear reaction rates.

ii) The Mitler result [4] (MIT), obtained with an analytical method which goes beyond the linearized approach and which correctly reproduces both the limits of weak and strong screening. Neglecting the small effects of a radial dependence in the effective potential, see [5], the enhancement factors are given now by:

$$\ln f_{ij}^{\text{MIT}} = -\frac{8}{5} (\pi e n_e a_D^5)^2 [(\zeta_i + \zeta_j + 1)^{5/3} - (\zeta_i + 1)^{5/3} - (\zeta_j + 1)^{5/3}] / (kT) \quad (3)$$

where $\zeta_{i,j} = 3Z_{i,j}/4\pi n_e a_D^3$ and n_e is the electron number density.

iii) Neglect completely any screening effect (NOS), i.e. nuclear reactions occur with rates $\langle \sigma v \rangle_{\text{bare}}$. This case is considered in connection with the suggestions that screening can be much smaller than Salpeter's estimate, see e.g. [16].

iv) The Tsytovich model (TSY) [11,12], which provides a decrease of all the thermonuclear reaction rates with respect to the case of bare nuclei. Screening factors are taken from Table

1b of ref. [12] and, for the ${}^7\text{Be}$ electron capture, from Table 2 of the same reference (all factors are assumed constant along the solar profile).

In Table I we report the screening factors at the solar center for the various models. One sees that the weak screening approximation always yields the largest enhancement factors, as physically clear due to the fact that electrons and ions are assumed to be free and capable of following the reacting nuclei. We also remind that in this model the electron cloud is allowed to strongly condense around the nuclei. In the Mitler model, where electron density at the nuclear site is fixed at n_e , the enhancement factor is smaller. By definition there is no enhancement in the NOS model, whereas in TSY model there is a decrease of the reaction rate, as already remarked.

The main features of the solar models we obtained are presented in Table II. When moving from WES to solar models where nuclear reactions are less favoured one observes the following effects:

- i) The central temperature T_c increases. In fact the hydrogen burning rate is fixed by the solar luminosity and a decrease of f_{11} has to be compensated with a temperature increase;
- ii) The isothermal sound speed, $u = P/\rho$ near the center increases. This is due to the increase of temperature whereas the “mean molecular weight” remains approximately constant;
- iii) The properties of the convective envelope are affected. In particular the border between the radiative and the convective region moves outwards and the photospheric helium abundance decreases.

III. HELIOSEISMOLOGY AND ELECTRON SCREENING

As well known several properties of the sun can be determined accurately by helioseismic data, see e.g. [17,18]. The photospheric helium abundance Y_{ph} and the depth of the convective zone R_b are given by:

$$Y_{ph} = 0.249(1 \pm 1.4\%) \quad (4)$$

$$R_b/R_\odot = 0.711(1 \pm 0.2\%). \quad (5)$$

The quoted errors are the so called “statistical” or “ 1σ ” errors of [13,17]. This error estimate was obtained by adding in quadrature each contribution to the uncertainty. Similar error estimates are given in [19,20]. A more conservative approach corresponds to add linearly all known individual uncertainties. This gives the so called “conservative” errors studied in [13], which are about a factor three larger than those in eqs. (4,5).

Moreover, by inversion of helioseismic data one can determine the sound speed profile in the solar interior. This analysis can be performed either in terms of the isothermal squared sound speed, $u = P/\rho$, or in terms of the adiabatic squared sound speed $c^2 = \partial P/\partial \rho|_{ad} = \gamma P/\rho$, as the coefficient $\gamma = \partial \log P/\partial \log \rho|_{adiab}$ is extremely well determined by the equation of state of the stellar plasma. The typical “ 1σ ” error on u is about $1.3 \text{ } \text{‰}$ in the intermediate solar region and increases up to $7 \text{ } \text{‰}$ near the solar center, see [13]. A similar error estimate is obtained in [18]. The “conservative” error estimate of ref [13] is about a factor three larger.

Recent Standard Solar Models calculated by using the weak screening prescription are in agreement with helioseismic constraints on the properties of the convective envelope and on the sound speed profile, see e.g. the BP98 model of ref [21] and BP2000 model of ref [22]. As an example, we present in Fig. 1. the comparison between the prediction of BP2000 and helioseismic data for $u = p/\rho$.

All this shows that the weak screening model is in agreement with data and deviations from WES cannot be too large. From the comparison of different models, see Table II and Fig 2, one obtain the following results:

i) The difference between the Tsytovitch model (TSY) and the weak screening model (WES) exceeds the “conservative” uncertainty on u in a significant portion of the solar profile. We remark that also the depth of the convective envelope is significantly altered. In other words the anti-screening predictions of ref. [11,12] can be excluded by means of helioseismology.

ii) Also the difference between the no-screening model (NOS) and WES is significant for both u and R_b in comparison with helioseismic uncertainty. In other words the existence of a screening effect can be proved by means of helioseismology.

iii) The Mitler model of screening (MIT) cannot be distinguished from the weak screening model within the present accuracy of helioseismology.

IV. NEUTRINO FLUXES

A reduction of the screening factors does not automatically mitigate the “solar neutrino problem”. As an example the TSY model predicts a larger 8B flux, Chlorine and Gallium signals than the WES model, see last column of Tab. II.

As discussed extensively in ref [6], the behaviour of neutrino fluxes can be understood by considering that a decrease of the screening factors has the following effects on the solar structure:

i) The hydrogen burning rate is fixed by the solar luminosity and a decrease of f_{11} has to be compensated with a temperature increase, being approximatively [6]:

$$T_c \propto f_{11}^{-1/8} ; \quad (6)$$

ii) The rate of the ${}^3He + {}^4He$ reaction, which is responsible for the PP-II chain and for Beryllium neutrino production, is changed. This results both from the increase in central temperature and from the variations in the screening factors f_{34} and f_{33} , see [6]. As a consequence, one expects a variation in the beryllium neutrino flux given by :

$$\Phi_{Be} \propto \frac{f_{34}}{f_{33}^{1/2}} \cdot f_{11}^{-10/8} , \quad (7)$$

iii) The 8B neutrino flux in addition depends on the ratio of the proton to electron capture rates on 7Be :

$$\Phi_B \propto \frac{f_{17}}{f_{e7}} \frac{f_{34}}{f_{33}^{1/2}} \cdot f_{11}^{-3} . \quad (8)$$

These scaling laws account for the numerical results of Table II and provide an explanation for the 8B neutrino flux increase of the TSY model. As clear from eq.(8) effects on the proton and electron capture almost compensate ($f_{17}^{WES}/f_{17}^{TSY} = 2.213$; $f_{e7}^{WES}/f_{e7}^{TSY} = 2.166$), and the increase is essentially due to the f_{11}^{-3} term, which corresponds to the temperature effect.

V. CONCLUSIONS

We recall here the main points of our discussion:

- i) The anti-screening predictions of ref [11,12] can be excluded by means of helioseismology, since both u and R_b are significantly altered.
- ii) We find that the a no-screening solar model is not completely consistent with helioseismic data on u and R_b , in other words the existence of a screening effect can be proved by means of helioseismology.

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TABLES

TABLE I. Screening factors in solar center, for weak screening (WES) [1], Mitler model (MIT) [4], no screening (NOS) and Tsytovitch model (TSY) [11].

	WES	MIT	NOS	TSY
$p + p$	1.049	1.045	1	0.949
${}^3\text{He} + {}^3\text{He}$	1.213	1.176	1	0.814
${}^3\text{He} + {}^4\text{He}$	1.213	1.176	1	0.810
${}^7\text{Be} + p$	1.213	1.171	1	0.542

TABLE II. Comparison among solar models with different screening factors. We show the fractional differences, (model -WES)/WES, for the photospheric helium abundance (Y_{ph}), depth of the convective envelope (R_b), central temperature (T_c), isothermal sound speed squared at the solar center (u_c), neutrino fluxes (Φ_i) and predicted signals for Chlorine (Cl) and the Gallium (Ga) experiments. All variations are in per cent.

	MIT	NOS	TSY
Y_{ph}	-0.076	-0.86	-1.4
R_b	+ 0.037	+0.34	+0.59
T_c	+ 0.45	+0.54	+1.4
u_c	+ 0.10	+1.0	+1.4
Φ_{pp}	+0.033	+0.45	-0.35
$\Phi_{\tau\text{Be}}$	-0.19	-2.4	-5.9
Φ_{sB}	-2.7	-12.	+11
Cl	-2.5	-11.	+9.7
Ga	-0.76	-2.9	+2.3

FIGURES

FIG. 1. Comparison between the BP2000 model [22] and helioseismic data for $u = P/\rho$. The “statistical” and “conservative” helioseismic uncertainties [13] correspond to the dark and light areas respectively.

FIG. 2. Comparison of different screening models with the WES model for $u = P/\rho$, same notation as in Table I. The “statistical” and “conservative” helioseismic uncertainties [13] correspond to the dark and light areas.

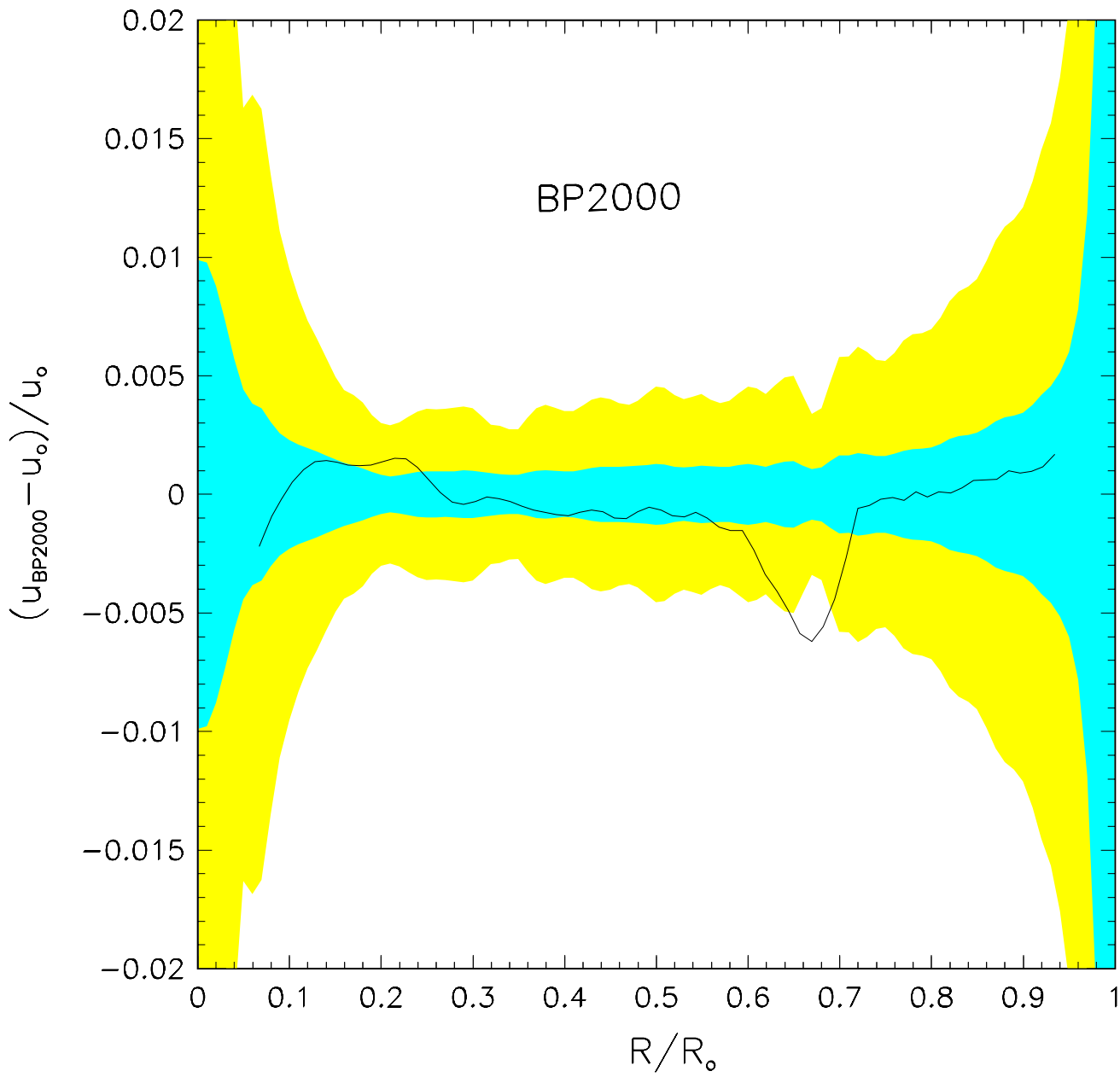


Fig. 1

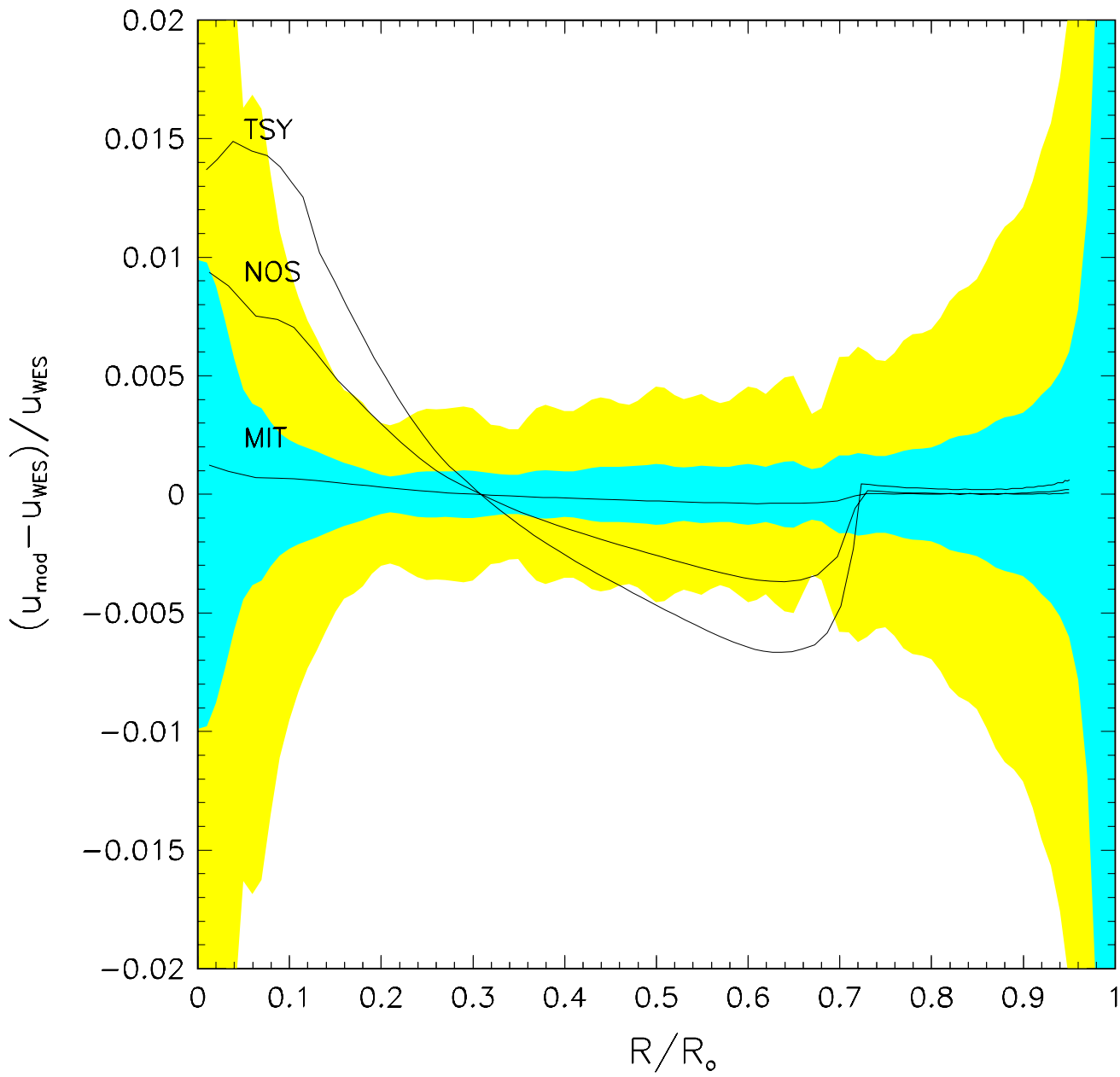


Fig. 2