SUSY Dark Matter
Loops and Precision
from Particle Physics

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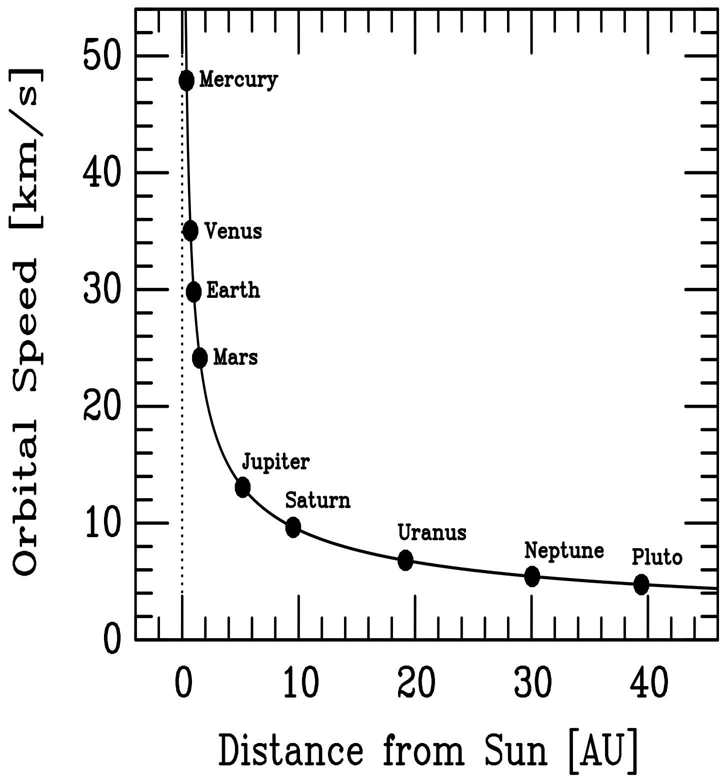
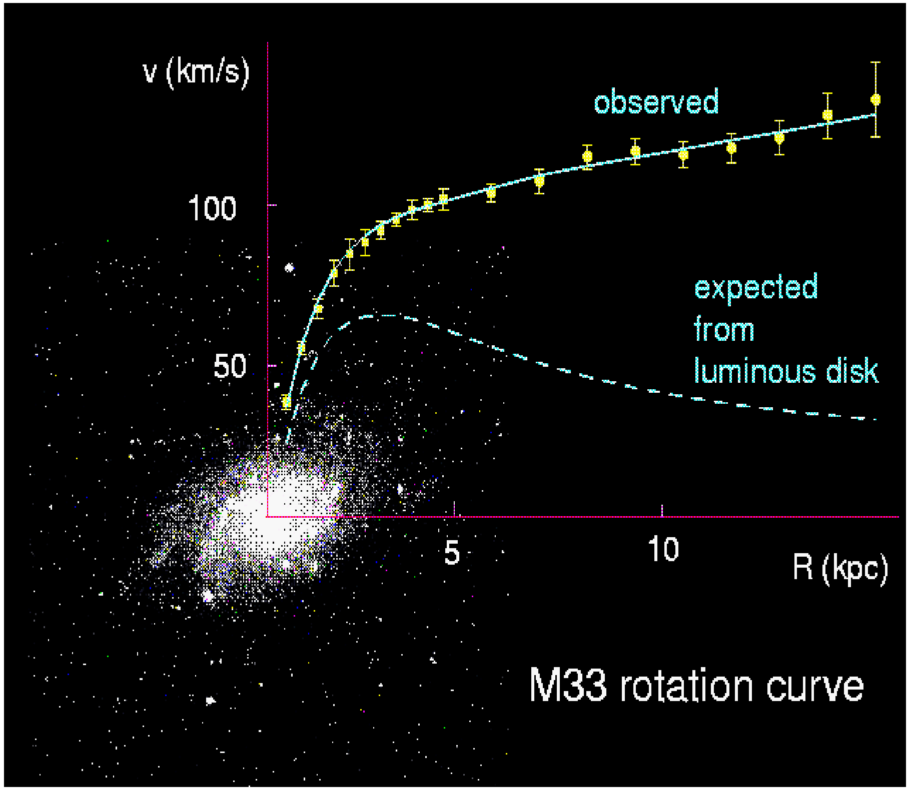
Plan

- Cosmology in the era of precision measurements
  - — Dark Matter is New Physics
  - — Evidence for and Precision on the matter content of the universe
  - — New Paradigm: can LHC/ILC match the precision of the upcoming cosmo/astro experiments and indirectly probe the history of the early universe?
- Relic density: precision annihilation cross sections (I)
  - — From the particle point of view: need for RADCOR (Gram determinant....)
  - — MSSM as a prototype: micrOMEGAs
  - — Precision needed on calculation in some specific SUSY scenarios
- Direct and indirect detection of DM: annihilation cross sections (II)
  - — γ ray line
- Automatisation, modularity, GF, Gram's, results and simulation
- Summary

The need for Dark Matter

Newton's law $\longrightarrow$ $v^2_{\rm rot.}/r = G_N M(r)/r^2$ (tracer star at a distance r from centre of mass distribution)



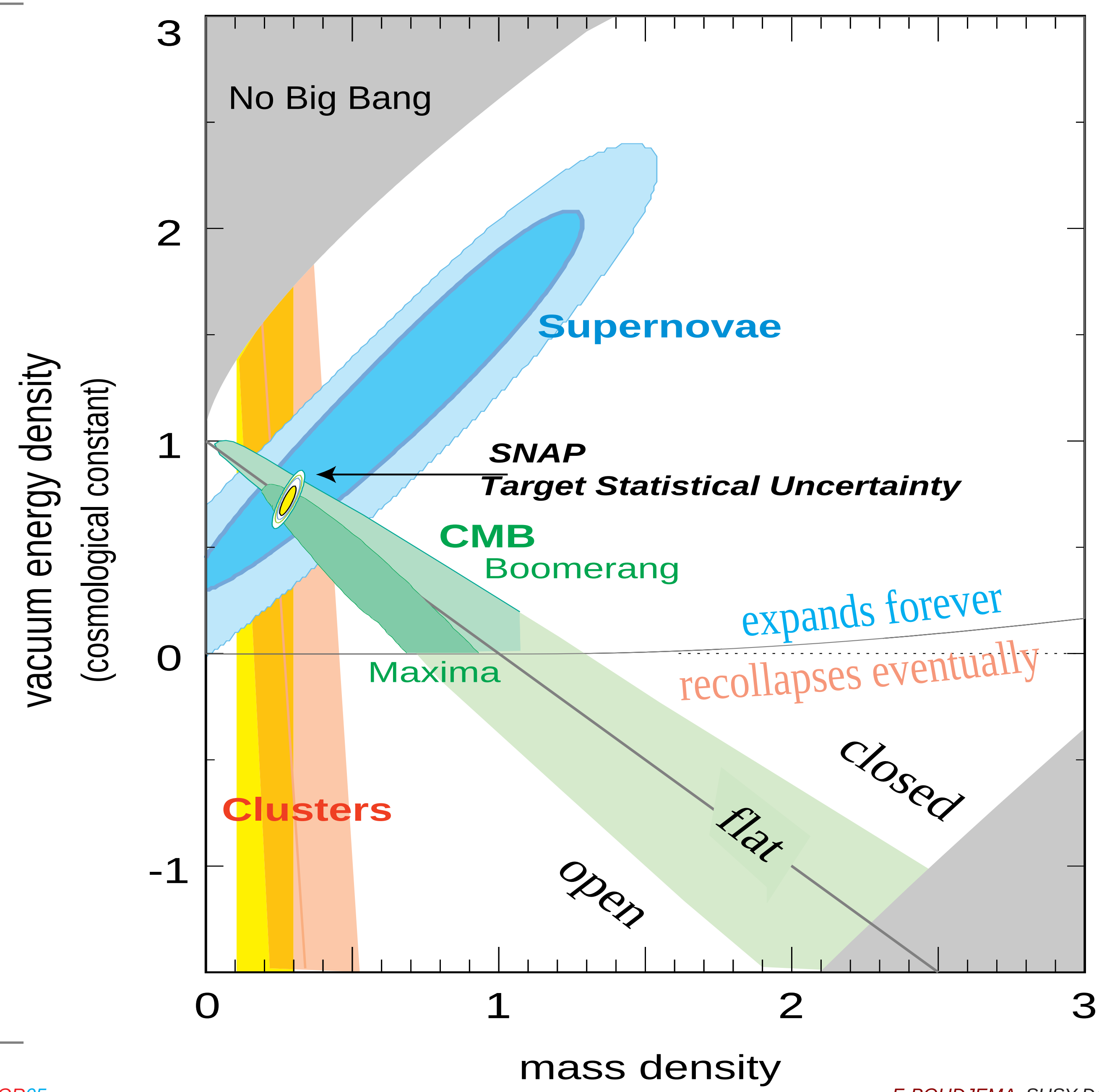


We are not in the centre of the universe

Dark Matter= New Physics

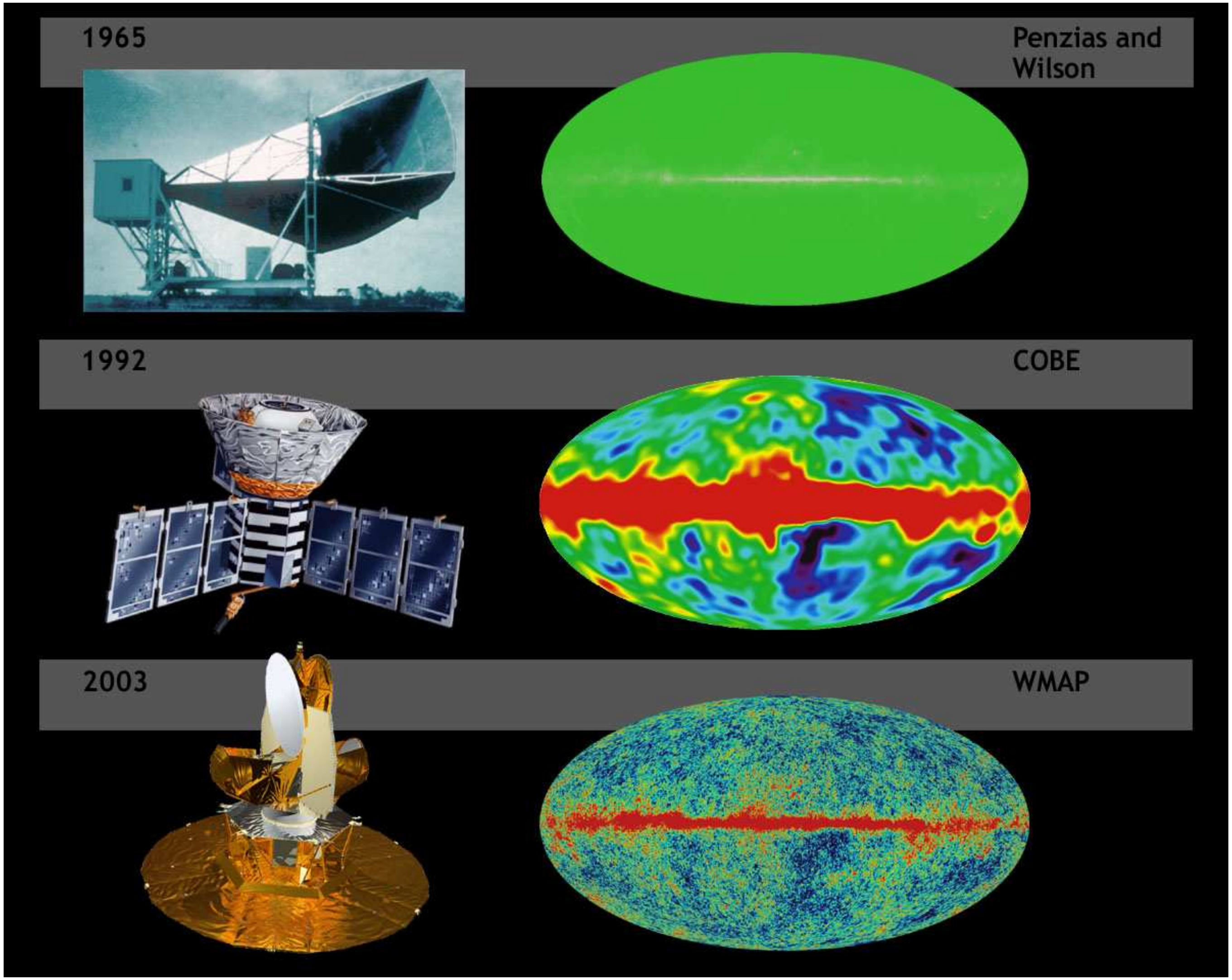
we are not made up of the same stuff as most of our universe

Cosmology in the era of precision measurement I: standard candles



observed luminosity and red-shift exploits the different z dependence of matter/energy density

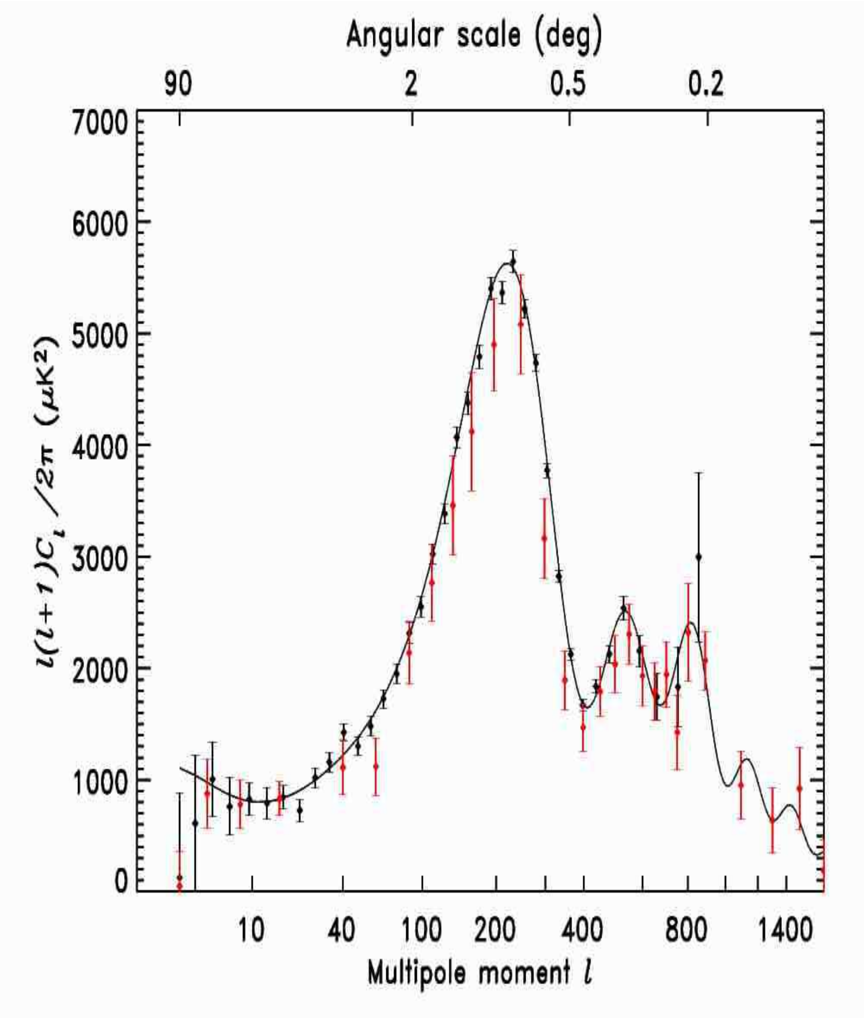
Cosmology in the era of precision measurement II: CMB

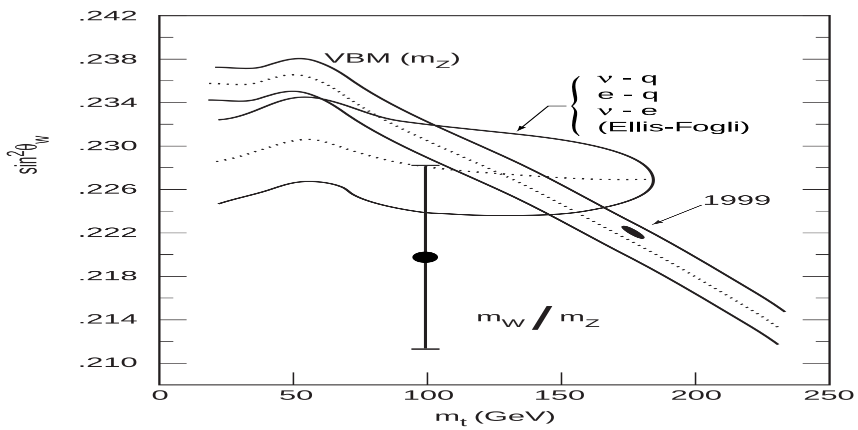


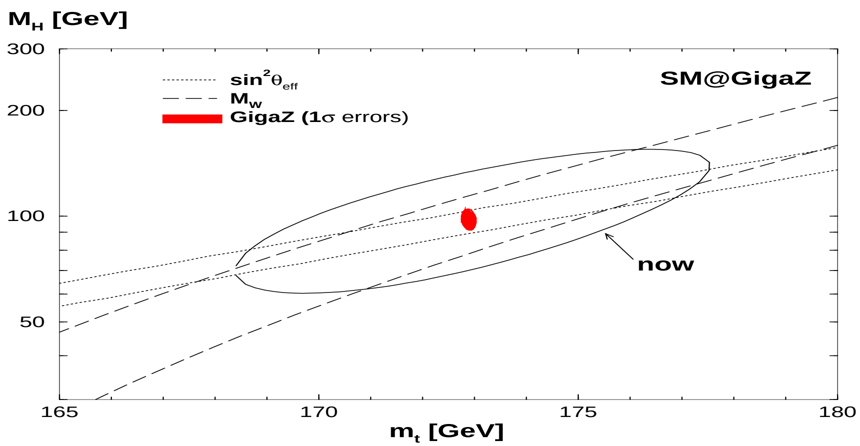
observed temperature anisotropies (related to the density fluctuations at the time of emission) is 10^{-5}

Pre-WMAP and WMAP vs Pre-LEP and LEP

power spectrum of anisotropies, WMAP vs Pre-WMAP





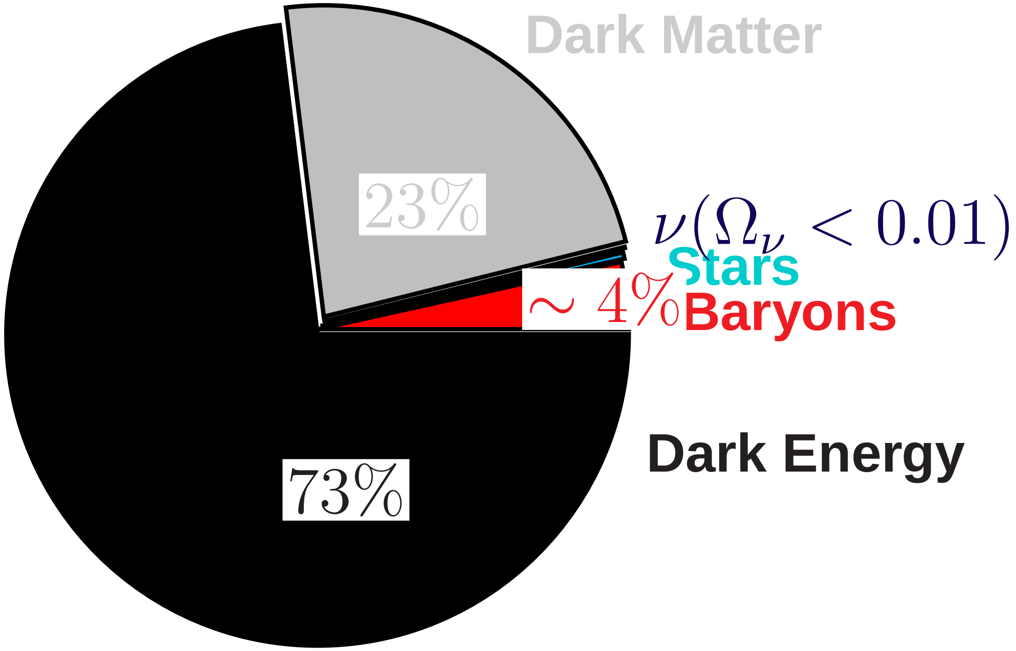


Planck+SNAP will do even better (per-cent precision)

improvement like going from LEP to LHC+ILC

LHC, PLanck ⟶ 2007 ILC,SNAP ⟶ 2015

Matter Budget and Precision 1.

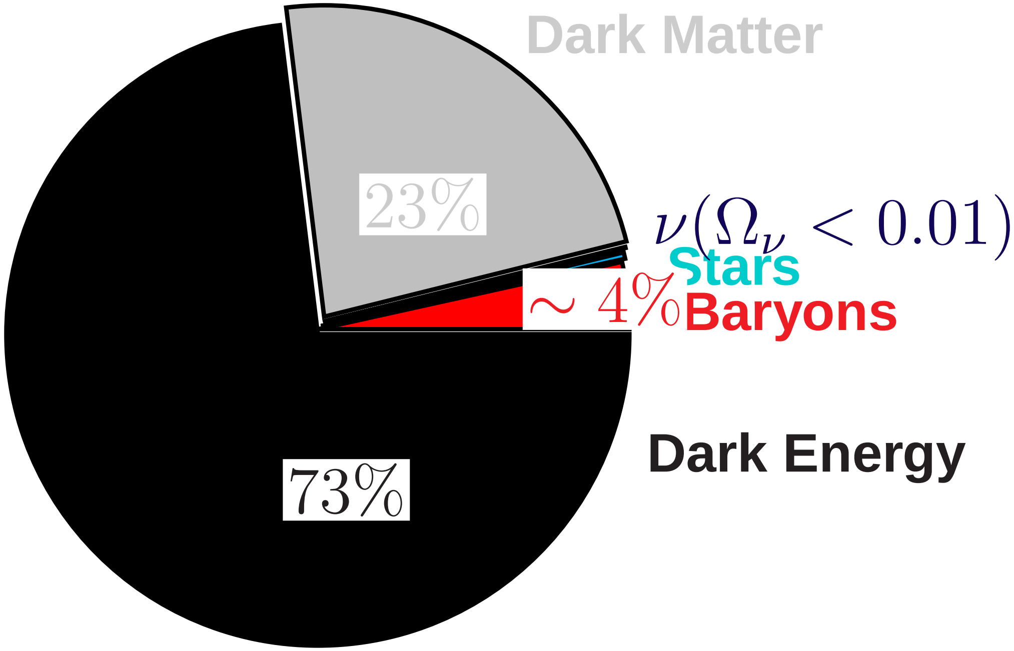


$$t_0 = 13.7 \pm 0.2 \text{ Gyr } (1.5\%)$$

$$\Omega_{\rm tot} = 1.02 \pm 0.02 \ (2\%)$$

$$\Omega_{\rm DM} = 0.23 \pm 0.04 \ (17\%)$$

Matter Budget and Precision 2.



$t_0 = 13.7 \pm 0.2$ Gyr (1.5%)	$\alpha^{-1} = 10 t_0 (10^{-7}\%)$
$\Omega_{tot} = 1.02 \pm 0.02 (2\%)$	$\rho = \Omega_{tot} (\sim 0.1\%)$
$\Omega_{DM} = 0.23 \pm 0.04 (17\%)$	$\sin^2\theta_{eff} = \Omega_{DM} (0.08\%)$

Matter Budget and Precision 3. Testing the cosmology

Present measurement at 2σ $\boxed{0.094 < \omega = \Omega_{\rm DM} h^2 < 0.129}$ (10%)

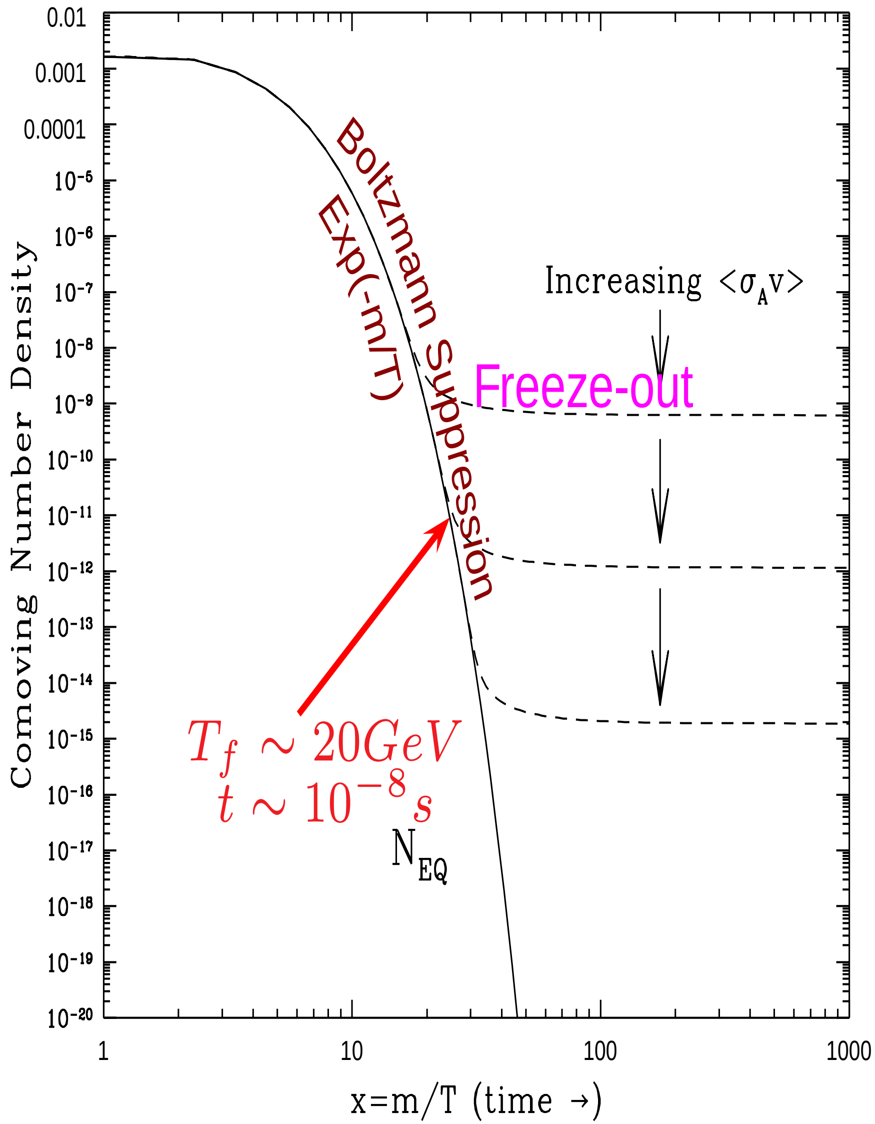
future (SNAP+Planck) $\to$ 2%

Particle Physics $\leftrightarrow$Cosmology through ω

- is wholly New Physics
- But will LHC, ILC see the "same" New Physics?
- New paradigm and new precision: change in perception about this connection
- ω used to: constrain new physics (choice of LHC susy points, benchmarks)
- Now: if New Physics is found, what precision do we require on colliders and theory to constrain cosmology? (Allanach, Belanger, FB, Pukhov JHEP 2004)

strategy/requirements on theory and collider measurements to match the present/future precision on ω

Relic Density: derivation



- At first all particles in thermal equilibrium
- universe cools and expands: interaction rate too small to maintain equilibrium
- (stable) particles can not find each other: freeze out and leave a relic density

dilution due to expansion

$$dN/dt = -3HN - <\sigma v> (N^2 - N_{eq}^2)$$

$$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to X \qquad X \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

$$\Omega_{\tilde{\chi}_1^0} = m_{\tilde{\chi}_1^0} N_{\tilde{\chi}_1^0} / \rho_{\rm cri},$$

$$\rho_{\rm cri} = 3H_0^2 / 8\pi G_N$$

$$\rho_{\rm cri} = h^2 1.9\ 10^{-29} gcm^{-3} \to$$

$$\Omega_{\tilde{\chi}_1^0} h^2 \propto 1/\sigma_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to X}$$

- At early times Universe is radiation dominated: $H(T) \propto T^2$ ▶ ($n_{eq}^{\gamma} \sim T^3$ relativistic particles are not Boltzmann suppressed)
- Expansion rate can be enhanced by some scalar field (kination), extra dimension $H^2 = 8\pi G/3\ \rho(1 + \rho/\rho_5)$, anisotropic cosmology,...
- Entropy non-conservation, e.g., through decays(entropy increase will reduce the relic abundance for example)

Thermal average

must calculate all annihilation, co-annihilation processes. Each annihilation can consist of tens of cross sections...

$$\chi_i^0\chi_j^0 \to X_{SM}Y_{SM}, \chi_1^0\tilde{f}_1 \to X_{SM}Y_{SM}, \ldots$$

$$< \sigma v >= \frac{\sum_{i,j} g_i g_j \int_{(m_i+m_j)^2} ds\sqrt{s}K_1(\sqrt{s}/T)\, p_{ij}^2\sigma_{ij}(s)}{2T\left(\sum_i g_i m_i^2 K_2(m_i/T)\right)^2},$$

p_{ij} is the momentum of the incoming particles in their center-of-mass frame.

$$p_{ij} = \frac{1}{2}\left[\frac{(s-(m_i+m_j)^2)(s-(m_i-m_j)^2)}{s}\right]^{\frac{1}{2}} \to v$$

$$v = 0$$

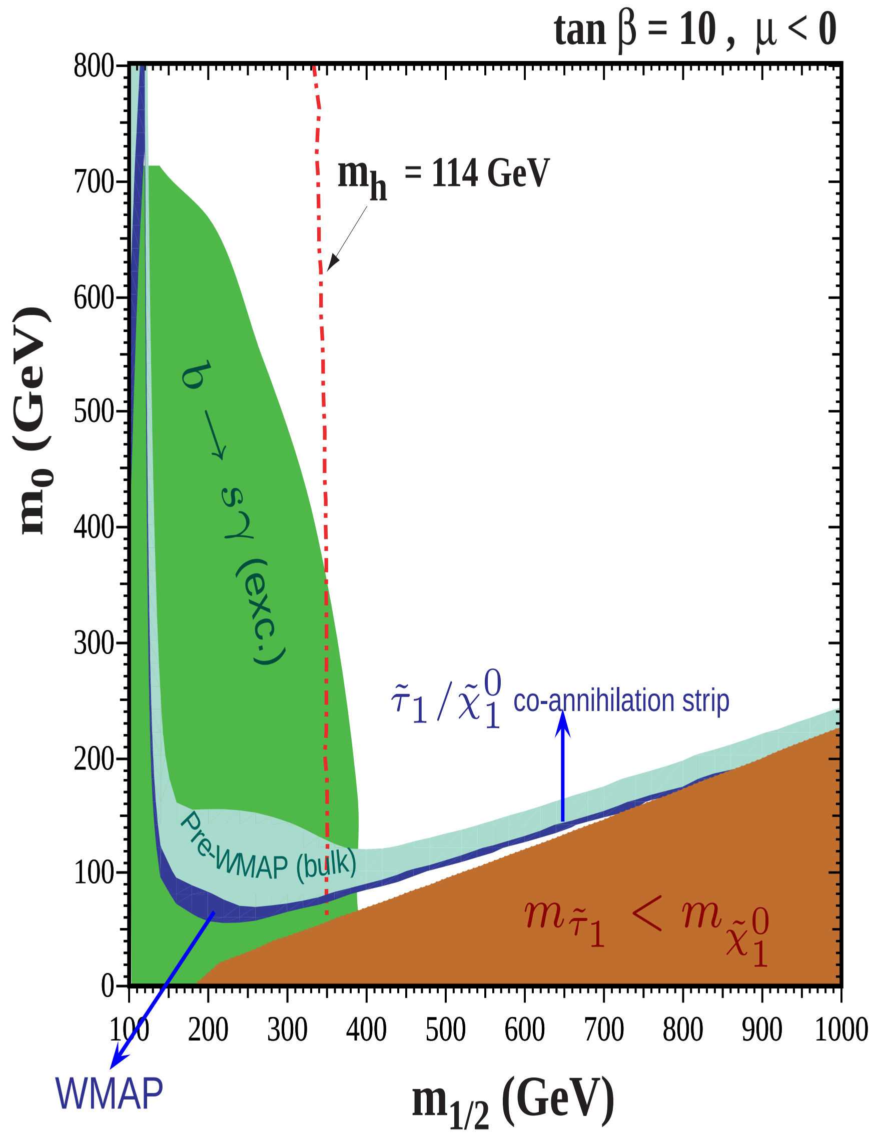
$$v \times \sigma v$$

SUSY as an example

Will concentrate on supersymmetry, SUSY in particular the MSSM (no CP violation)

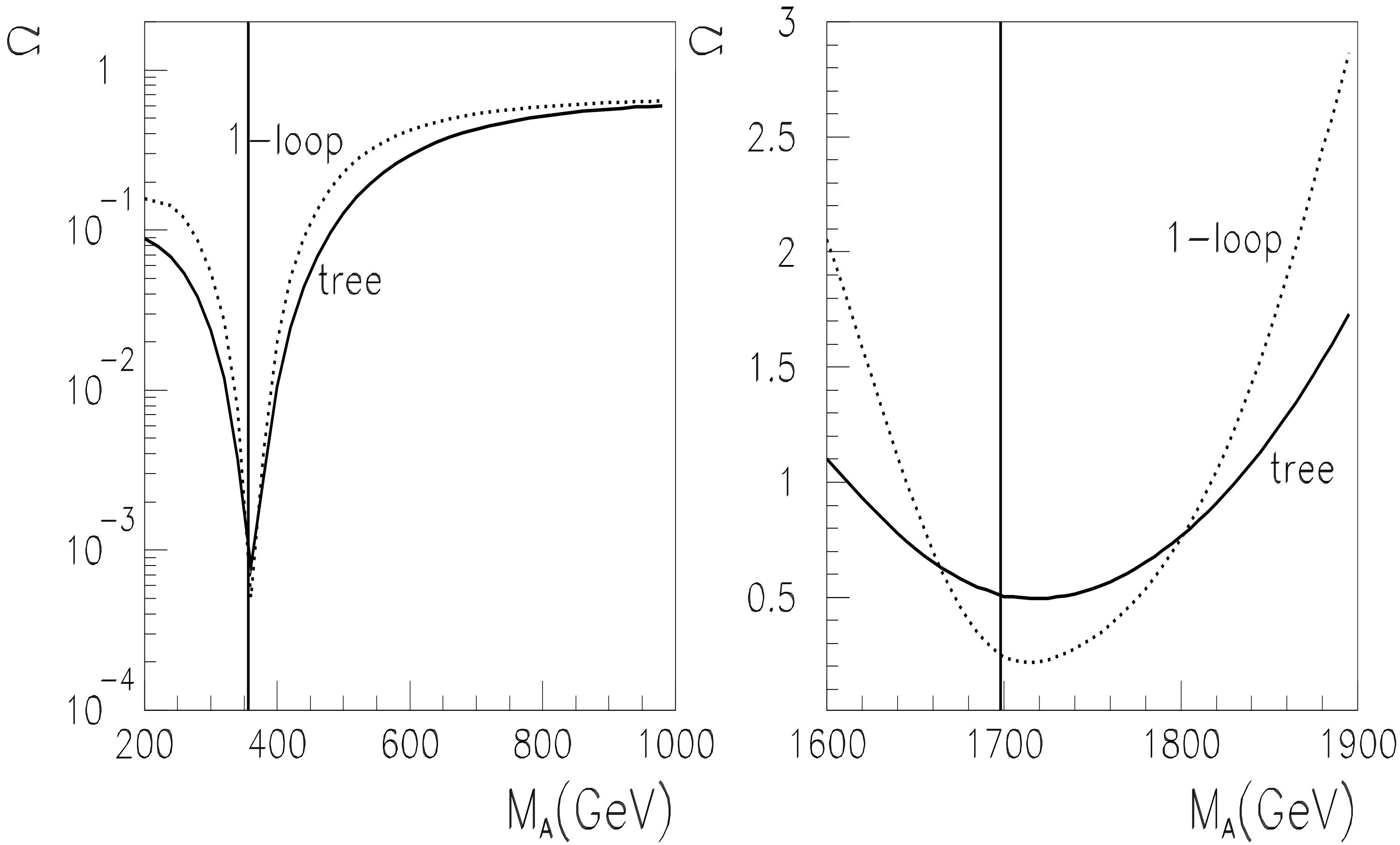
sometimes assumes the mSUGRA scenario (bring down the number of parameters to $4+1/2$, but relies on RGE

- given any set of parameters it can identify LSP, NLSP, generate and calculate ω
- Model defined in Lanhep (more later)
- Fed into CompHEP ....tree-level, some 3000 processes could be needed.
- Higgs sector: improved Higgs masses/mixings (read from FeynHiggs, for example) but interpreted in terms of an effective scalar potential (GI), following FB and A. Semenov (PRD 02)
- Effective Lagrangian also includes important RC (Higgs couplings, Δm_b effects,..)
- Interfaced with Isajet, Suspect, SoftSUSY parameters at high scale run down to the ew scale
- $(g-2)_\mu$, $b \to s\gamma$, $B_s \to \mu^-\mu^+$
- NMSSM done (with C. Hugonie), JHEP 05
- CP violation in progress
- "open source": procedure to define your own model, soon



- Bulk region: bino LSP, $\tilde{l}_R$ exchange, (small $m_0, M_{1/2}$)
- $\tilde{\tau}_1$ co-annihilation: NLSP thermally accessible, ratio of the two populations $exp(-\Delta M/T_f)$ small m_0, $M_{1/2}: 350-900$GeV
- Higgs Funnel: Large $\tan\beta$, $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to A \to b\bar{b}, (\tau\bar{\tau})$, $M_{1/2}: 250-1100$GeV, $m_0: 450-1000$GeV
- Focus region: small $\mu \sim M_1$, important higgsino component, requires very large TeV m_0

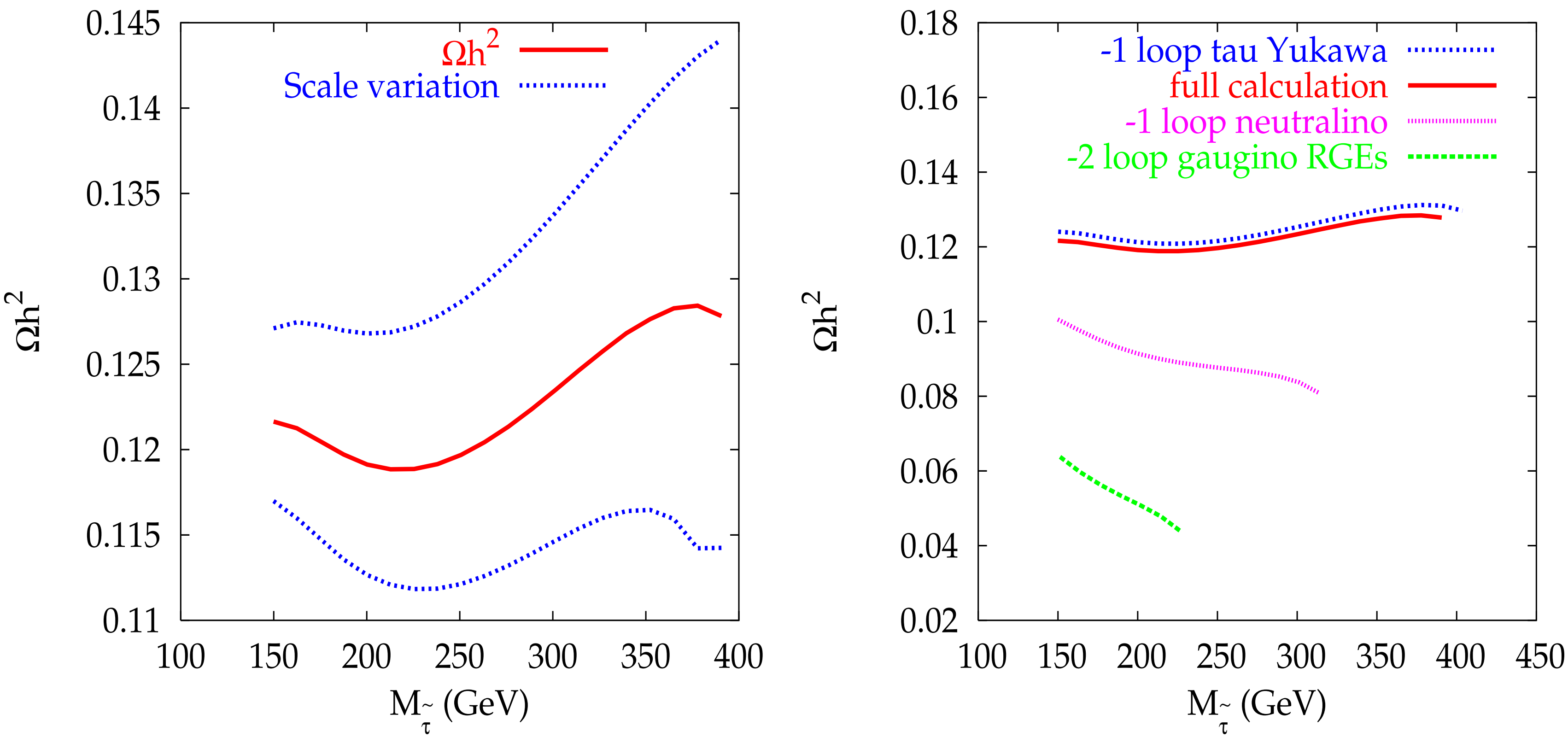
Relic density around Higgs Pole with and without RC



Use micrOMEGAs+SOFTSUSY $\rightarrow$...Softmicro..

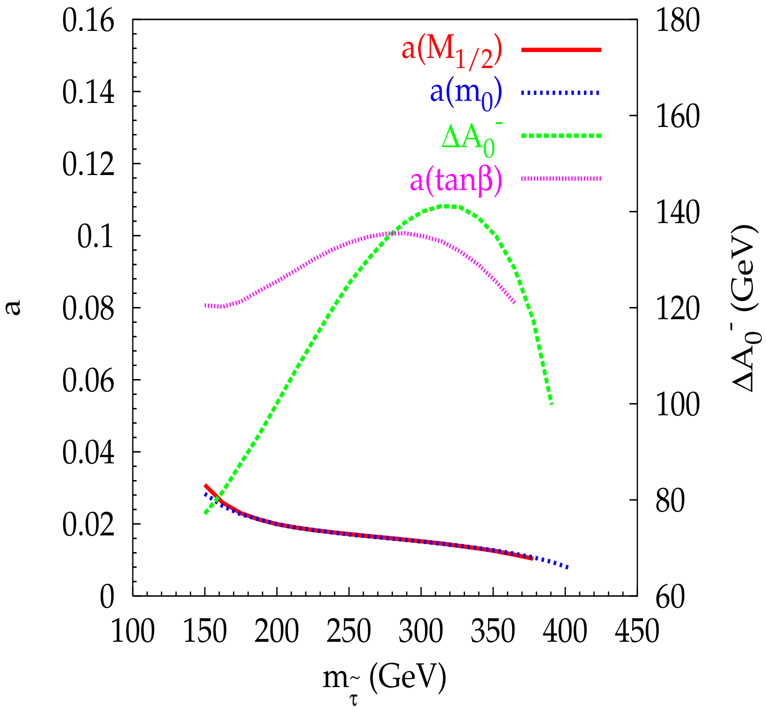
- fix $A_0, \tan\beta, sgn(\mu)$ but scan on $M_{1/2}$
- WMAP strips imply $m_0 = f(M_{1/2})$: slopes
- RGE also needs SM input parameters!
- scale dependence of relic: default $M_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$: scale of EWSB conditions
- theoretical uncertainty: effect of different refinements in RGE and threshold corrections
- derive accuracy within mSUGRA, relying completely on mSUGRA. accuracies on high scale parameters and SM inputs
- model independent approach: find out most relevant parameters and extract accuracy on these (weak scale parameters)
- accuracies derived in an iterative procedure and refer to the 10% WMAP precision

$\tilde{\tau}_1$ co-annihilation region: Theory uncertainty



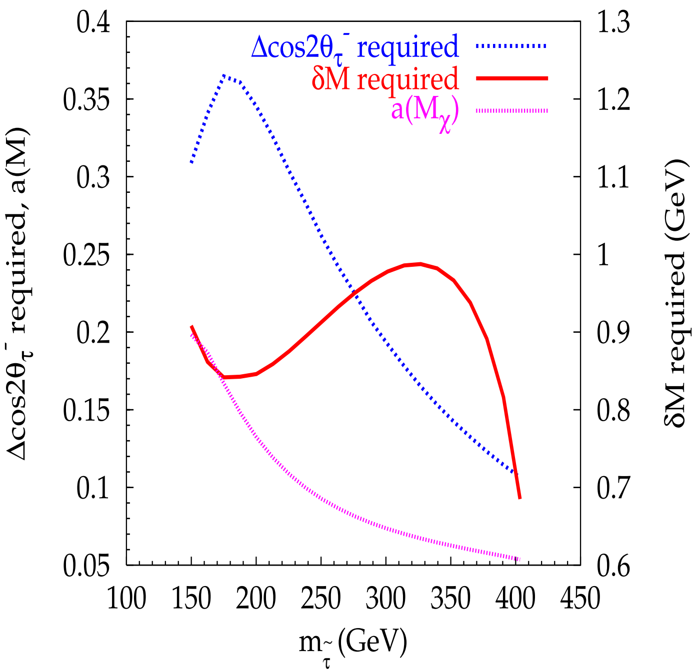
- Scale variation: From 5% (at small $m_{\tilde{\tau}_1}$) to 20% at large $m_{\tilde{\tau}_1}$)
- 2-loop gaugino RGE's ESSENTIAL as is 1-loop threshold correction to $\tilde{\chi}_1^0$

$\tilde{\tau}_1$ co-annihilation region: accuracy within mSUGRA

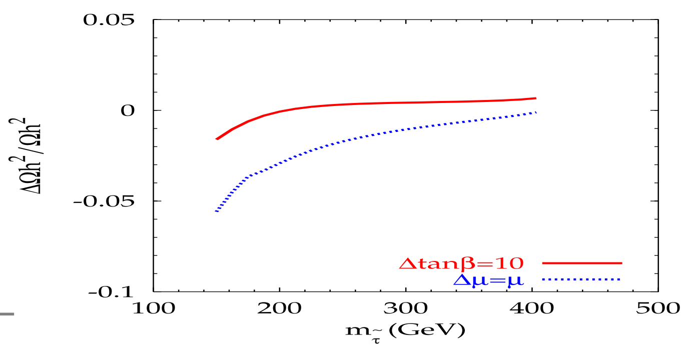


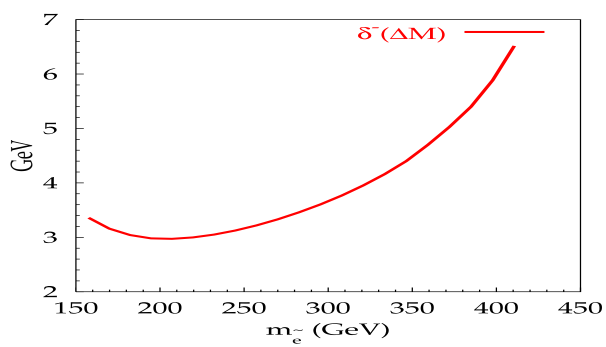
- accuracy on $m_0, M_{1/2}$ demanding: 3% : 1% may be achievable at LHC
- for $\tan\beta$ require 10percent.

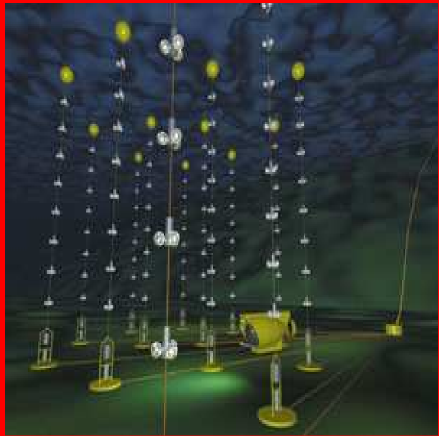
$\tilde{\tau}_1$ co-annihilation region: Model Independent



- ΔM must be measured to less than 1GeV
- mixing angle accuracy should be feasible at ILC
- accuracy on LSP mass not demanding but this is because we have constrained ΔM.
- other slepton masses need also be measured
- in terms of physical parameters residual $\mu \tan\beta$ accuracies not demanding
- Preliminary studies indicate these accuracies will be met for the lowest $m_{\tilde{\chi}_1^0}$

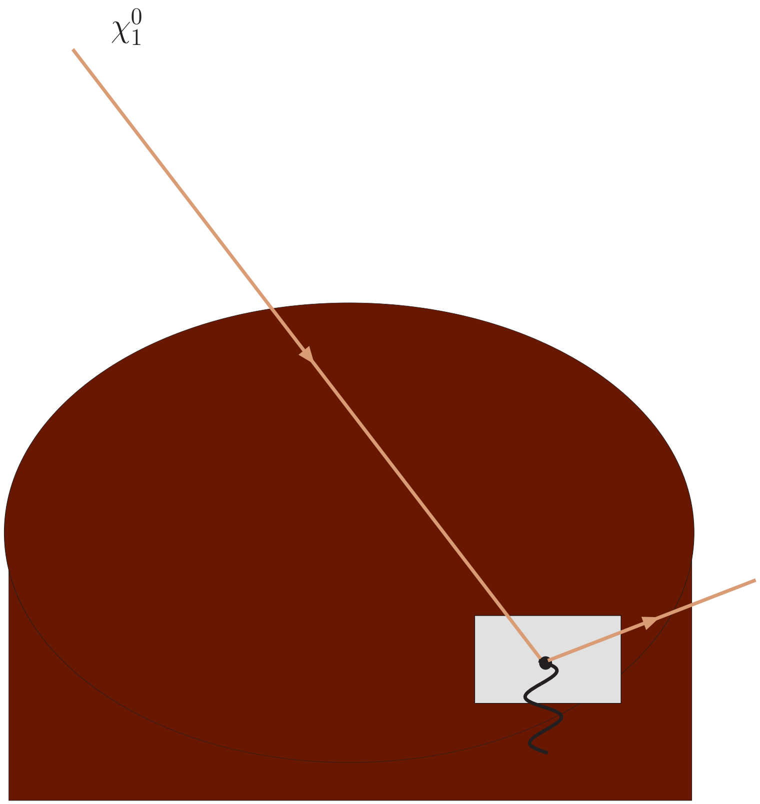




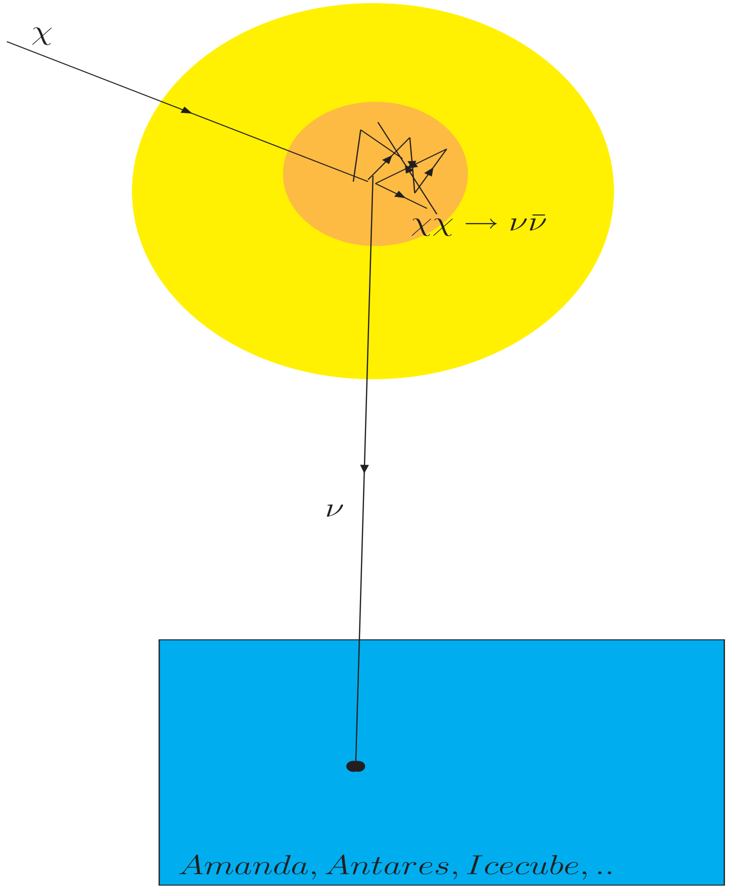


Direct and Indirect Searches

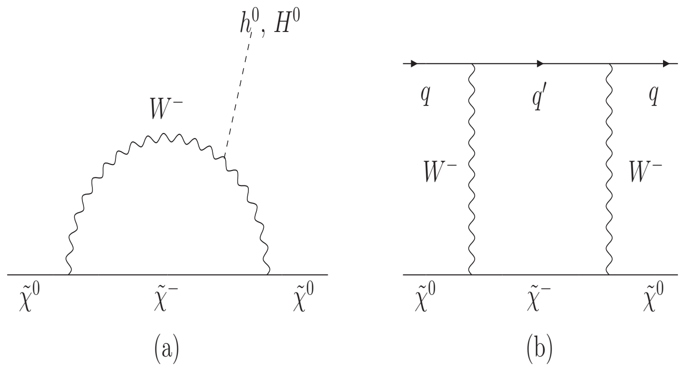
$$\bar{p},\ e^{+},\ \gamma,\ \nu, \ldots$$



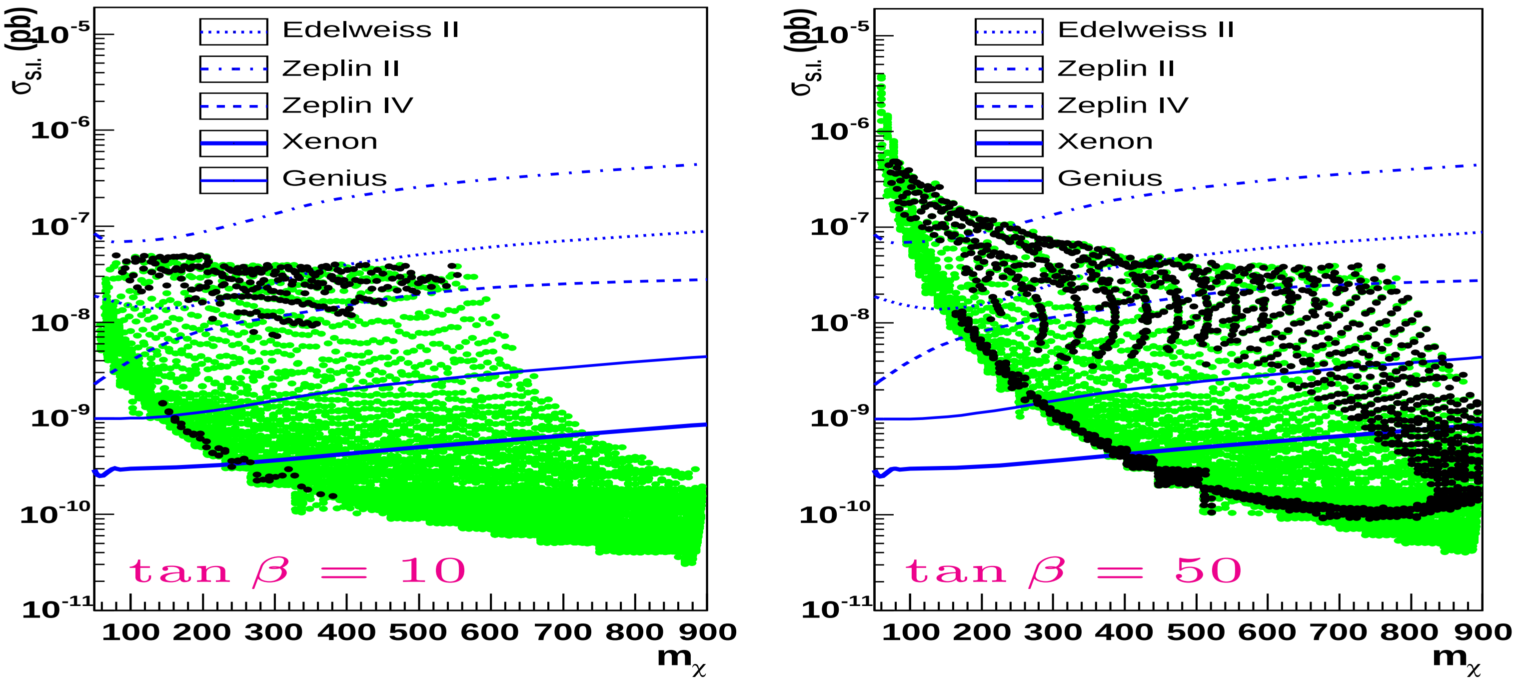
$CDMS, Edelweiss, DAMA, Genuis, ..$



Loops for direct detection



- **within WMAP**

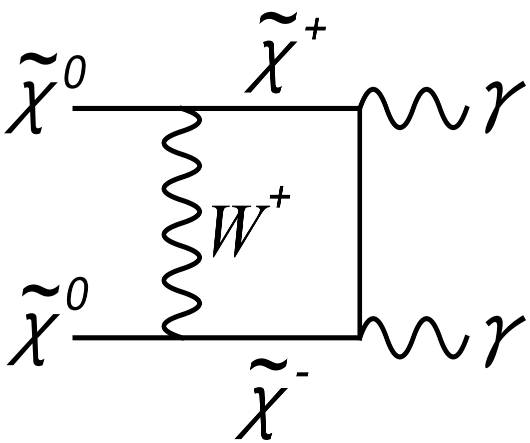


Annihilation into photons

$$\frac{d\Phi_\gamma}{d\Omega dE_\gamma} = \sum_i \underbrace{\frac{dN^i_\gamma}{dE_\gamma}\sigma_i v \frac{1}{4\pi m^2_\chi}}_{\text{Physique des Particules}} \underbrace{\int \rho^2 dl}_{\text{Astro}}$$

$\gamma's$: Point to the source, independent of propagation model(s)

- continuum spectrum from $\tilde{\chi}^0_1\tilde{\chi}^0_1 \rightarrow f\bar{f},\ldots,$ hadronisation/fragmentation ($\rightarrow \pi^0 \rightarrow \gamma$) done through isajet/herwig
- Loop induced mono energetic photons,$\gamma\gamma$, $Z\gamma$ final states



ACT: HESS, Magic, VERITAS, Cangoroo, ...

Space-based: AMS, GLAST, Egret,...

SLOOPS

FB, A. Semenov, D. Temes

- Need for an automatic tool for susy calculations
- handles large numbers of diagrams both for tree-level
- and loop level
- able to compute loop diagrams at $v = 0$: dark matter, LSP, move at galactic velocities, $v = 10^{-3}$
- ability to check results: UV and IR finiteness but also gauge parameter independence for example
- ability to include different models easily and switch between different renormalisation schemes

Non-linear gauge implementation

$$\mathcal{L}_{GF} = -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu\,+} + \xi_W\frac{g}{2}(v + \tilde{\delta}_h h + \tilde{\delta}_H H + i\tilde{\kappa}\chi_3)\chi^+|^2$$
$$-\frac{1}{2\xi_Z}(\partial.Z + \xi_Z\frac{g}{2c_W}(v + \tilde{\epsilon}_h h + \tilde{\epsilon}_H H)\chi_3)^2 - \frac{1}{2\xi_\gamma}(\partial.A)^2$$

- quite a handful of gauge parameters, but with $\xi_i = 1$, no “unphysical threshold”
- more important: no need for higher (than the minimal set)for higher rank tensors and tedious algebraic manipulations

Strategy: Exploiting and interfacing modules from different codes

Lagrangian of the model defined in LanHEP

- particle content
- interaction terms
- shifts in fields and parameters
- ghost terms constructed by BRST

$\downarrow$ $\downarrow$

Generic Model
-kinematical structures

Classes Model
-Feynman rules, including CT

$\Downarrow$

Evaluation via FeynArts-FormCalc

LoopTools modified!!
tensor reduction inappropriate for small relative velocities
(Zero Gram determinants)

$\Uparrow$

Renormalisation scheme

- definition of renorm. const. in the classes model

Non-Linear gauge-fixing constraints, gauge parameter dependence checks

From the Lagrangian to the Feynman Rules

```
vector
    A/A: (photon, gauge),
    Z/Z:('Z boson', mass MZ = 91.1875, gauge),
    'W+'/'W-': ('W boson', mass MW = MZ*CW, gauge).
scalar  H/H:(Higgs, mass MH = 115).



transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
    'W+'->'W+'*(1+dZW/2),'W-'->'W-'*(1+dZW/2).
transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
    'W+.f'->'W+.f'*(1+dZWf/2),'W-.f'->'W-.f'*(1+dZWf/2).

let pp = { -i*'W+.f',  (vev(2*MW/EE*SW)+H+i*'Z.f')/Sqrt2 },
PP=anti(pp).



lterm -2*lambda*(pp*anti(pp)-v**2/2)**2
     where
    lambda=(EE*MH/MW/SW)**2/16, v=2*MW*SW/EE .

let Dpp^mu^a = (deriv^mu+i*g1/2*B0^mu)*pp^a +
     i*g/2*taupm^a^b^c*WW^mu^c*pp^b.
let DPP^mu^a = (deriv^mu-i*g1/2*B0^mu)*PP^a
    -i*g/2*taupm^a^b^c*{'W-'^mu,W3^mu,'W+'^mu}^c*PP^b.
lterm DPP*Dpp.
```

Gauge fixing and BRS transformation

```
let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

lterm -'Z.C'*brst(G_Z).
```

```
RenConst[ dMHsq ] := ReTilde[SelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZH ] := -ReTilde[DSelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZZf ] := -ReTilde[DSelfEnergy[prt["Z.f"] -> prt["Z.f"],
MZ]] RenConst[ dZWf ] := -ReTilde[DSelfEnergy[prt["W+.f"] ->
prt["W+.f"], MW]]
```

Output of Feynman Rules with Counterterms !!

```
M$CouplingMatrices = {

  (*------     H   H   ------*)
   C[ S[3], S[3] ] == - I *
{
 { 0 , dZH },
 { 0 , MH^2 dZH + dMHsq }
},
  (*------     W+.f  W-.f  ------*)
   C[ S[2], -S[2] ] == - I *
{
 { 0 , dZWf },
 { 0, 0 }
}, (*------     A   Z   ------*)
   C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
{
 { 0, 0 },
 { 0 , dZZA },
 { 0, 0 }
},


(*------     H   H   H   ------*)
   C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
{
 { 2 MH^2 , 3 MH^2 dZH -2 MH^2 / SW dSW - MH^2 / MW^2 dMWsq
},
  (*------     H   W+.f   W-.f   ------*)
   C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
{
 { 2 MH^2 , MH^2 dZH + 2 MH^2 dZWf -2 MH^2 / SW dSW - MH^2 /
},


  (*------     W-.C   A.c   W+   ------*)
   C[ -U[3], U[1], V[3] ] == - I EE *
{
 { 1 },
 { - nla }
},
```

Avoiding zero Gram determinants: Segmentation (FB, A. Semenov, D. Temes ,hep-ph/0507127, PRD...)

For the problems at hand:

$$DetG = M^6_{\tilde{\chi}^0_1} v^2 \frac{\sin^2\theta}{(1-v^2/4)^3}(1-z^2), \quad z^2 = \frac{M_Z^2}{4M^2_{\tilde{\chi}^0_1}}(1-v^2/4)$$

$$DetG(p_1,p_2) = -M^4_{\tilde{\chi}^0_1} v^2 \frac{1}{(1-v^2/4)^2}.$$

Segmentation

$$\frac{1}{D_0 D_1 D_2 D_3} = \left(\frac{1}{D_0 D_1 D_2} - \alpha\frac{1}{D_0 D_2 D_3} - \beta\frac{1}{D_0 D_1 D_3} + (\alpha+\beta-1)\frac{1}{D_1 D_2 D_3}\right) \times \frac{1}{A + 2l.(s_3 - \alpha s_1 - \beta s_2)}$$

$$A = (s_3^2 - M_3^2) - \alpha(s_1^2 - M_1^2) - \beta(s_2^2 - M_2^2) - (\alpha+\beta-1)M_0^2.$$

$$(D_i = (l+s_i)^2 - M_i^2, \quad s_i = \sum_{j=1}^{i} p_j)$$

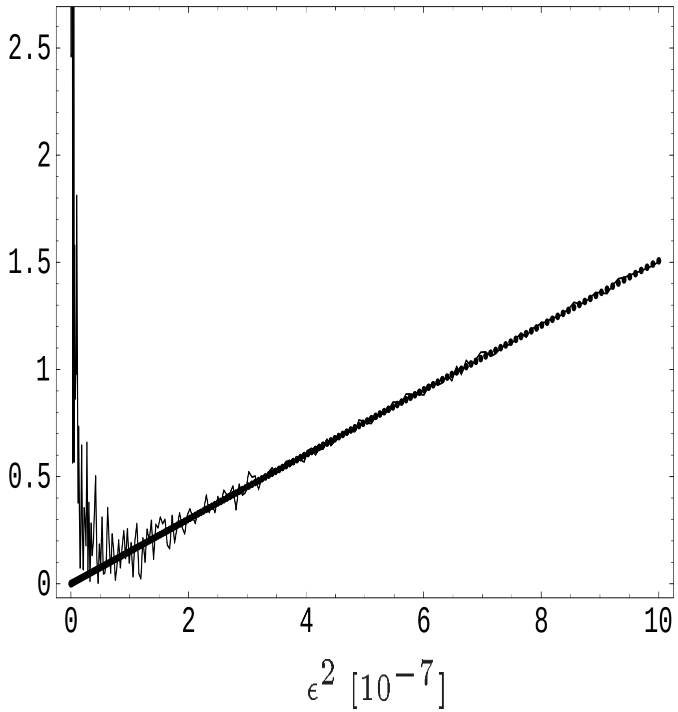
For any graph if $DetG(s_1,s_2,s_3) = 0$ (or "small"), construct all 3 sub-determinants $DetG(s_i,s_j)$and take the couple s_i, s_j (as independent basis) that corresponds to $Max\,|Det(s_i,s_j)|$, then write

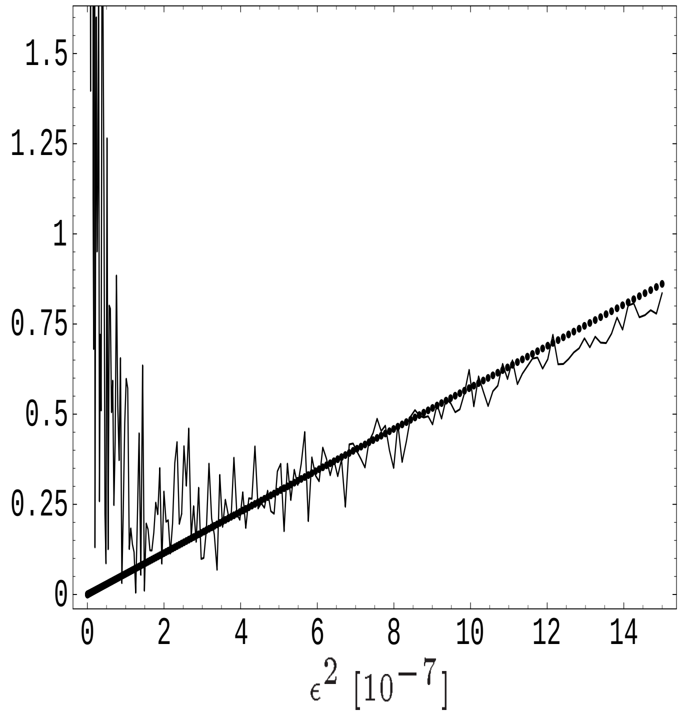
$$s_3 = \alpha s_1 + \beta s_2 + \varepsilon_T \quad \text{with} \quad s_1.\varepsilon_T = s_2.\varepsilon_T = 0 \quad \text{meaning}$$

$$\varepsilon_{T,\mu} = \epsilon_{\mu\alpha\beta\delta} s_1^\alpha s_2^\beta t^\delta, \quad \alpha = \frac{s_2^2 s_3.s_1 - s_1.s_2 s_2.s_3}{DetG(s_1,s_2)}, \quad \beta = \alpha(s_1 \leftrightarrow s_2).$$

$$DetG(s_1,s_2,s_3) = \varepsilon_T^2 DetG(s_1,s_2).$$

Expanding around small $\mathcal{V}$



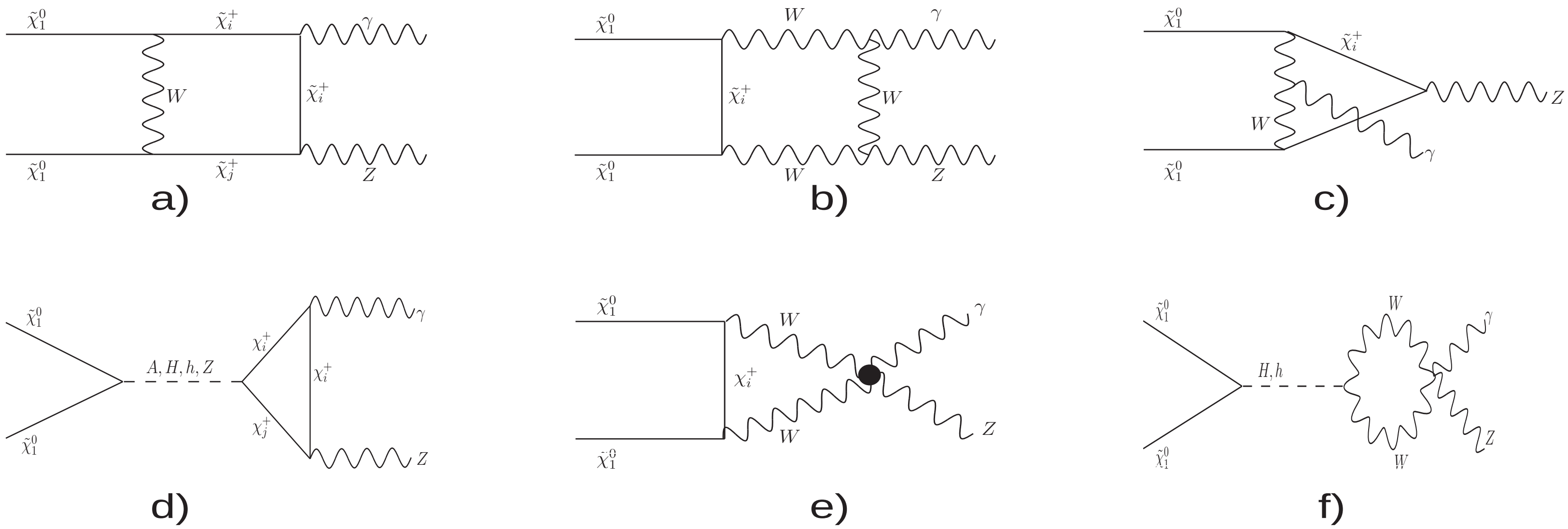


From Nans BARO, Diploma , LAPTH/ENS-Lyon

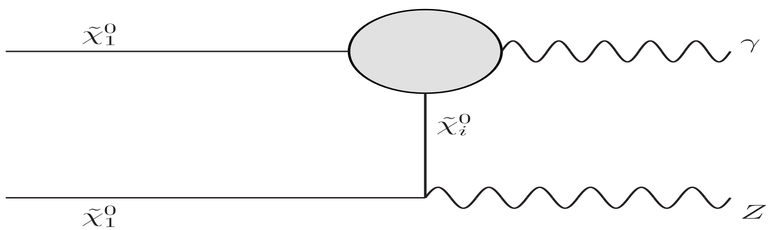
Application to $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \gamma\gamma, Z\gamma, gg$ (FB, A. Semenov, D. Temes ,hep-ph/0507127, PRD...)

Computing the cross-sections

More than a thousand diagrams including



New contribution found in $Z\gamma$!

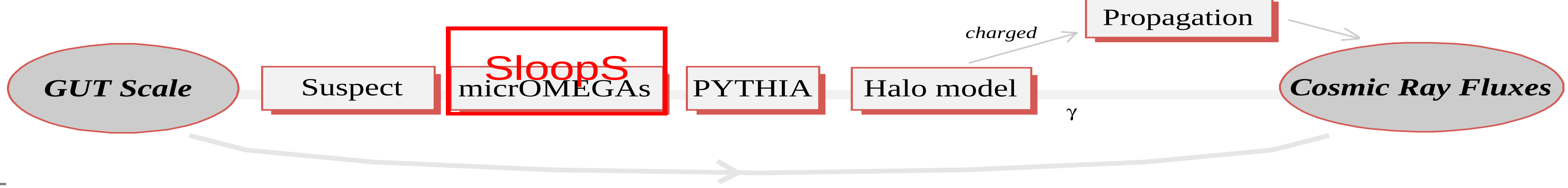


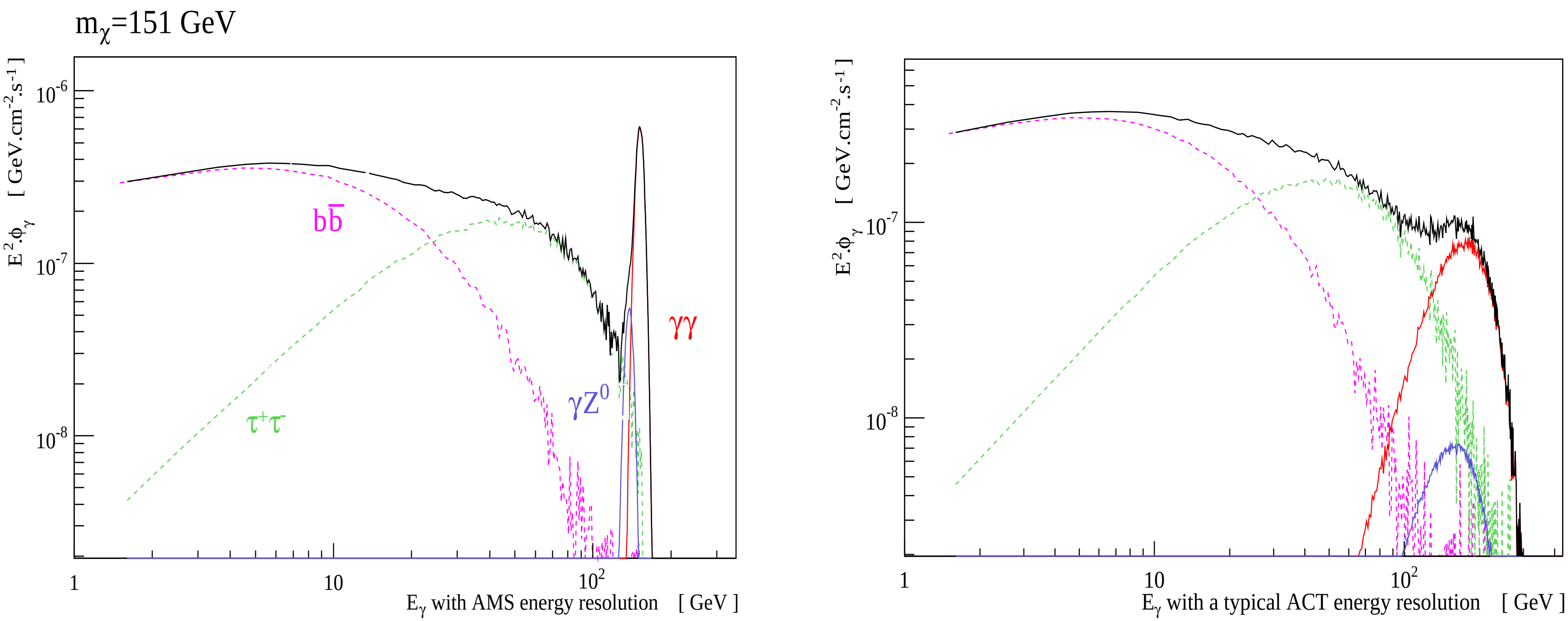
Counterterm contribution:

- Obtained from tree-level $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to ZZ$ and $\delta Z_{Z\gamma}$
- Since $\delta Z_{Z\gamma} \sim (1 - \tilde{\alpha})$ CT can be put to zero if $\tilde{\alpha} = 1$

Choosing a proper gauge simplifies the computation

▶ 1 ▶ 2





SIMULATION:

Parameterising the halo profile:

$(\alpha, \beta, \gamma) = (1, 3, 1)$, $a = 25$kpc. (core radius), $r_0 = 8kpc$ (distance to galactic centre), $\rho_0 = 0.3\, GeV/cm^3$ (DM density), opening angle cone 1^o

SUSY parameterisation

$m_0 = 113$GeV, $m_{1/2} = 375$ GeV, $A = 0$, $\tan\beta = 20$, $\mu > 0$

γ lines could be distinguished from diffuse background

Summary

- Precise calculations for the relic density are necessary with the foreseen precision on some cosmological parameters
- Rates for direct and direct detection also need some loop calculations, here measurements most probably will constrain the astrophysical parameters
- A general code is being developed for minimal susy, but could be extended, to provide annihilation rates for cosmo/astro and corrected cross sections for colliders
- but still much to be done and to be improved

Extra Expansion of Universe, Einstein Equations

$$\text{Einstein} \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{\Lambda}{8\pi G} \right)$$

Isotropic and Homogeneous

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$\text{conservation} \quad H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2}$$

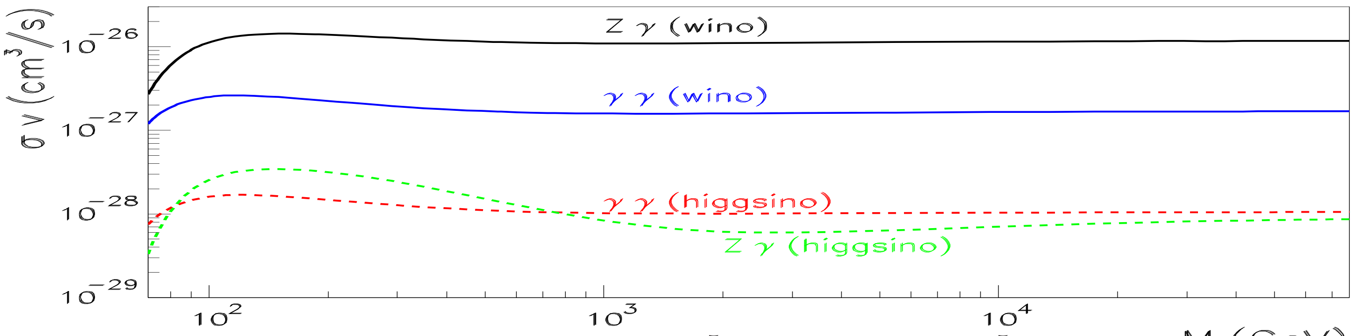
$$\to \sum_M \Omega_M + \Omega_\Lambda + \Omega_k = 1 \quad \Omega_M = \frac{\rho_M}{\rho_c} \quad \rho_c = \frac{3H^2}{8\pi G}$$

$$\text{Acceleration} \quad \left(\frac{\ddot{a}}{a} \right) = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i) \quad p = w\rho$$

$$\rho(a) \propto \frac{1}{a(t)^{3(1+w)}} \quad w_{rad} = 1/3 \;\; w_M = 0 \; w_\Lambda = -1$$

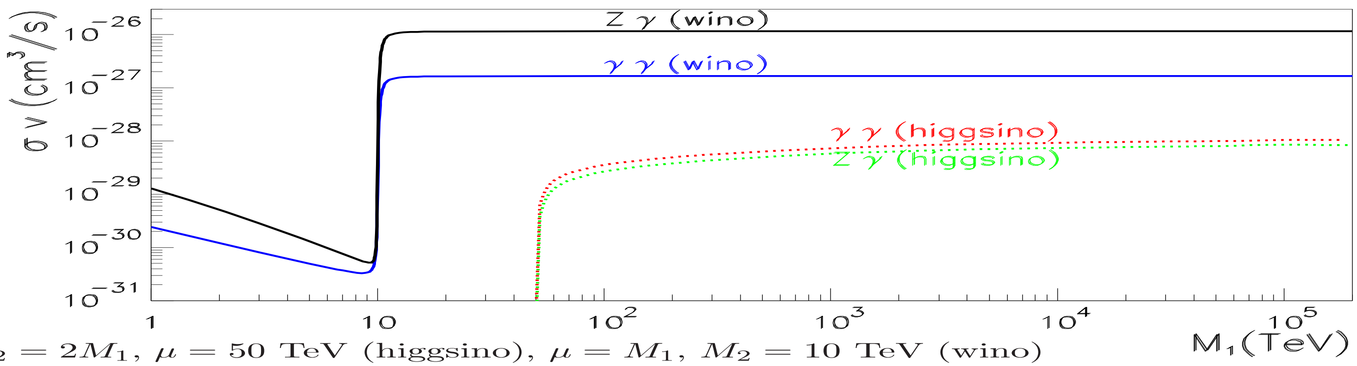
Higgsino and Wino limits

- σv vs. $M_{\tilde{\chi}_1^0}$ $(v = 0)$



$\tan\beta = 10$, $A = 0$, $m_A = 100$ TeV, $m_{\tilde{f}} = 4\,10^5$ TeV, $M_1 = 2\,10^5$ TeV
$M_2 = 2M_1$ (higgsino), $\mu = M_1$ (wino)

- σv vs. % higgsino/wino $(v = 0)$



$M_2 = 2M_1$, $\mu = 50$ TeV (higgsino), $\mu = M_1$, $M_2 = 10$ TeV (wino)
$\delta M = M_{\chi_1^+} - M_{\chi_1^0}$

Asymptotic value for large $M_{\tilde{\chi}_1^0}$, $\sigma v \sim 1/M_W^2$

Largest cross section for $Z\gamma$ in wino case

Smooth behaviour in higgsino case, $\delta M \sim m_z^2/M_1$

Constant value after transition in wino case, $\delta M \sim m_z^4/M_1^3$

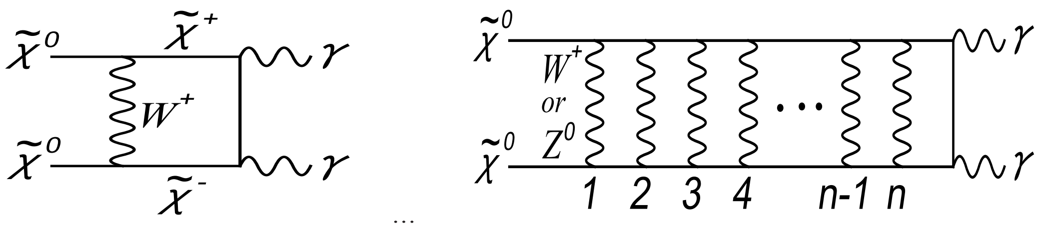
Numerical results reproduce analytical behaviour

Matching with non-perturbative computation

Hisano et al.'04

In the extreme higgsino and wino limits,

- one-loop treatment breaks unitarity
- non-perturbative non-relativistic approach



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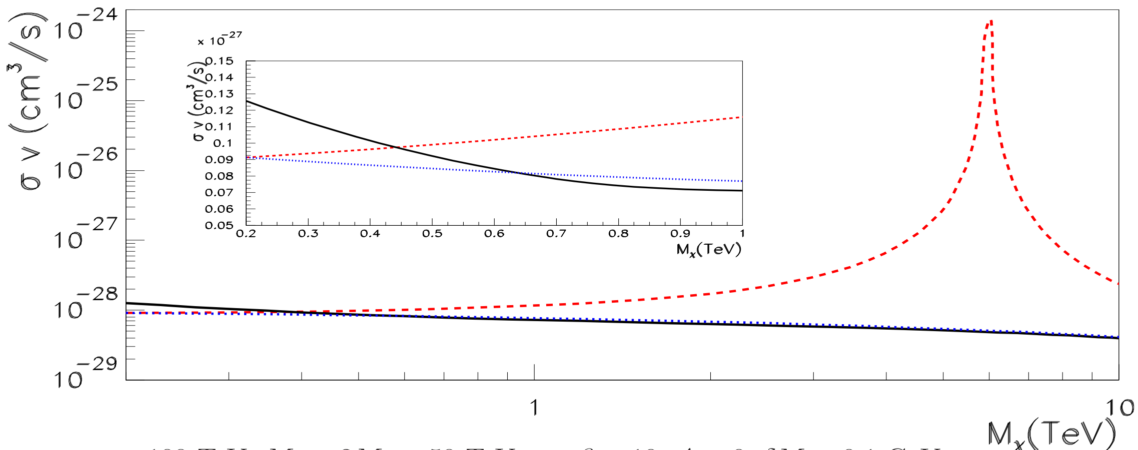
Non-perturbative computation and one-loop results

higgsino case $(v = 0)$



$m_{\tilde{f}} = m_A = 100$ TeV, $M_2 = 2M_1 = 50$ TeV, $\tan\beta = 10$, $A = 0$, $\delta M = 0.1$ GeV

Resonances can enhance result several orders of magnitude

Matching will take place around $400 - 500$ GeV