MICROSCOPIC CALCULATION OF THE ENHANCEMENT FACTOR IN THE ELECTRIC DIPOLE SUM RULE

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Correlated basis function perturbation theory with state-dependent correlations is used to calculate the nuclear photoabsorption enhancement factor K in the electric dipole sum rule for some realistic models of nuclear matter. The contribution due to 2p–2h admixtures in the ground state wave function turns out to be only a few percent of the unperturbed value. The values obtained for K are about 1.8 at experimental equilibrium density and increase almost linearly with density. We also give estimates of K for finite nuclei, obtained within the local density approximation framework. The surface effects give a contribution which is ~ 20% of the volume term.

Recently, microscopic calculations of the enhancement factor K in the electric dipole sum rule, given by [1]

$$K = (A/NZ)(m/\hbar^2)\langle 0|[D_z, [V, D_z]]|0\rangle , \tag{1}$$

have been performed [2] for some realistic models of nuclear matter. In the above equation $|0\rangle$ is the ground state of the non-relativistic hamiltonian having V as nuclear potential and $D_z = \frac{1}{2}\Sigma_{i=1,A}\tau_{iz}z_i$ is the z component of the electric dipole operator, with τ_{iz} being the third component of the isospin opertor for the ith nucleon. The theoretical estimates [2] of K do not depend very much on the realistic interaction adopted, and are more than a factor of two larger than the experimental value [3], $K_{\mathrm{exp}} = 0.76 \pm 0.10$, obtained from the integrated photonuclear cross sections up to the π-meson production threshold. It is important to know how much of this discrepancy is due to effects not explicitly taken into account in the Bethe–Levinger sum rule, like tail corrections of the integrated cross section, higher multipoles and dipole retardation effects, and how much is due to the inadequacy of the variational wave function used in the calculation.

In this letter we present the results obtained for K when the variational ground state is corrected by adding 2p2h correlated basis functions (CBF) components to it. The 2p2h admixtures are calculated by using second order CBF perturbation theory [4–6]. The CBF states are normalized but not orthogonal, and are given by

$$|\Psi_i\rangle = F|\Phi_i\rangle/\langle\Phi_i|F^+F|\Phi_i\rangle^{1/2} , \tag{2}$$

where $|\Phi_i\rangle$ are Fermi gas states and $F = S\,\Pi_{i<j=1,A}F_{ij}$, with S being the symmetrizer and F_{ij} a two-particle correlation operator. In our calculations F_{ij} contains central, tensor and spin–orbit components and is determined variationally, by minimizing the expectation value of the hamiltonian in the state $|\Psi_0\rangle$. The correlation operator F formally defines the unperturbed hamiltonian H_0 of CBF perturbation theory [4], which is given by

$$H_{0,ij} = \delta_{ij}\langle\Psi_i|H|\Psi_i\rangle/\langle\Psi_i|\Psi_i\rangle . \tag{3}$$

The consequent perturbation series based upon the states $|\Psi_i\rangle$ has been proved [6] to have the linked cluster property.

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Recent analyses on the optical potential [6] and the momentum distribution [7] of a nucleon in nuclear matter have shown that the correlation operator F does not realistically represent the correlations of particles close to the Fermi surface. In fact, second order CBF perturbation theory leads to corrections which are quite substantial for the effective mass m^* and the momentum distribution at $k \sim k_F$. For an independent particle hamiltonian the enhancement factor K is given by $K = m/m^*(k_F) - 1$. For an unperturbed CBF hamiltonian of the type given in eq. (3), if only the 1p1h CBF states are included in the summed oscillator strength, one gets [8] $1 + K = m/m^*(k_F)[1 + \chi(k_F)]^2$, where $\chi(k_F)$ is directly related to the dynamical correlation. This structural property suggests that 2p2h state admixtures in the ground state might be important in the evaluation of K. In this approximation, second order CBF perturbation theory provides for $|0_2\rangle$ the following unnormalized wave function

$$|0_2\rangle = |\Psi_0\rangle + \tfrac{1}{4} \sum_{h_1,h_2,p_1,p_2} \alpha(h_1, h_2, p_1, p_2)|\Psi_{h_1 h_2 p_1 p_2}\rangle , \tag{4}$$

where both $|\Psi_0\rangle$ and $|\Psi_{h_1 h_2 p_1 p_2}\rangle$ are normalized and

$$\alpha(h_1, h_2, p_1, p_2) = \langle\Psi_{h_1 h_2 p_1 p_2}|H - H_{00}|\Psi_0\rangle/[e^v(h_1) + e^v(h_2) - e^v(p_1) - e^v(p_2)] . \tag{5}$$

H_{00} is the variational ground state energy given by $\langle\Psi_0|H|\Psi_0\rangle/\langle\Psi_0|\Psi_0\rangle^{1/2}$ and $e^v(k)$ are the variational single particle energies [9], which satisfy the relation $e^v(p) - e^v(h) = \langle\Psi_{hp}|H|\Psi_{hp}\rangle/\langle\Psi_{hp}|\Psi_{hp}\rangle$. Retaining up to the quadratic terms in α, one gets the following correction to the variational K^v:

$$\delta K = \delta K_1 + \delta K_2 , \tag{6}$$

where

$$\delta K_1 = (m/\hbar^2 A) \sum_{h_1,h_2,p_1,p_2} \langle\Psi_0|DC - DC_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle\alpha(h_1, h_2, p_1, p_2) + \text{c.c.} , \tag{7}$$

and the second order δK_2 is approximated by neglecting the non-diagonal matrix elements, with the result

$$\delta K_2 \cong (m/h^2 A) \sum_{h_1,h_2,p_1,p_2} \langle\Psi_{h_1 h_2 p_1 p_2}|DC - DC_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle\alpha^2(h_1, h_2, p_1, p_2) . \tag{8}$$

The above approximation is justified by the fact that the calculated δK_2 is an order of magnitude smaller than δK_1

The double commutator $DC = [D_z, [V, D_z]]$ is given by the sum of the two-body operator DC_2 and the three-body operator DC_3, which correspond to the two-nucleon and three-nucleon parts of the interaction considered. DC_{00} is the expectation value of the operator DC for the wave function $|\Psi_0\rangle$. The presence of DC_{00} in eqs. (7) and (8), as well as the prescence of H_{00} in eq. (5), guarantees the linked cluster property of the corresponding matrix element, so that it is of order $1/A$ and the matrix elements of $DC - DC_{00}$ appearing in eqs. (7) and (8) are of the order of $1/A$ and unity respectively. The cluster expansion of the non-diagonal matrix elements of $DC - DC_{00}$ is very similar to that [5,7] of $H - H_{00}$, and the procedure to calculate its cluster terms is the same as that discussed in ref. [7]. Similarly, the cluster expansion of the diagonal matrix elements of $DC - DC_{00}$ in eq. (8) is analogous to that [5,6,9] of

$$\langle\Psi_{h_1 h_2 p_1 p_2}|H - H_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle = e^v(p_1) + e^v(p_2) - e^v(h_1) - e^v(h_2) + O(1/A) ,$$

and leads to the result

$$\langle\Psi_{h_1 h_2 p_1 p_2}|DC - DC_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle = d(p_1) + d(p_2) - d(h_1) - d(h_2) + O(1/A) , \tag{9}$$

where the function $d(k)$ can be easily calculated by following the procedure devised in ref. [9] to compute the single particle energies $e_v(k)$.

Diagonal matrix elements have been calculated within the FHNC/SOC approximation [2,9], whereas only the two-body (TB) and the three-body separable (TBS) cluster diagrams have been retained in the evaluation of the non-diagonal matrix elements of the hamiltonian [6] which are necessary to compute the coefficients $\alpha(h_1, h_2,$

p_1, p_2) given in eq. (5). The non-diagonal matrix elements of the double commutator operator have been computed at the lowest order of the cluster expansion, namely

$$\langle\Psi_0|DC - DC_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle \approx \tfrac{1}{2}A(A-1)\langle\Phi_0|F_{12}W(12)F_{12}|\Phi_{h_1 h_2 p_1 p_2}\rangle\,, \tag{10}$$

where W is defined [2] by $DC_2 = \Sigma_{i<j=1,A} W(ij)$. Some of the approximations used in the present calculation might be too crude. Terms, like $\langle\Psi_{h_1' h_2' p_1' p_2'}|DC - DC_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle\alpha^*(h_1' h_2' p_1' p_2')\alpha(h_1 h_2 p_1 p_2)$, as well as the neglected many body cluster contributions may play some role [6], and we do not expect an accuracy better than 15% in the evaluation of δK.

Most of our calculations have been done for the model interaction Urbana-v_{14} + TNI(V) [10,11]. Urbana-v_{14} is a realistic two-body interaction [10], which reproduces the deuteron properties and the NN scattering data for E_{lab} up to 400 MeV and is given by a sum of 14 operator components (central, tensor, spin–orbit, L^2 and quadratic spin–orbit). TNI(V) is the semi-phenomenological model V of ref. [11] for the three-nucleon interaction and gives reasonable results for light nuclei and nuclear matter. TNI is explicitly included in the calculation of the diagonal matrix elements [2], whereas in the evaluation of the non-diagonal ones it is not. The effect of TNI in the single particle energies is ~5%, hence the above approximation should lie within the accuracy of the present calculation. Table 1 gives the results obtained for δK_1, δK_2 and δK for the Urbana-v_{14} + TNI(V) model at $k_F = 1.33$ fm^{-1} for several values of the healing distance d_t of the tensor correlation. The other variational parameters of the pair correlation opertor $F(ij)$ have been kept fixed at their "optimal" values as given in ref. [2]. The table displays also the second-order corrections to the energy [6], given by

$$\delta E = (1/4A)\sum_{h_1,h_2,p_1,p_2} |\langle\Psi_0|H - H_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle|^2/[e^v(h_1) + e^v(h_2) - e^v(p_1) - e^v(p_2)]\,, \tag{11}$$

and the quantity κ_B, defined by

$$\kappa_B = (1/2A)\sum_{h_1,h_2,p_1,p_2} |\langle\Psi_0|H - H_{00}|\Psi_{h_1 h_2 p_1 p_2}\rangle|^2/[e^v(h_1) + e^v(h_2) - e^v(p_1) - e^v(p_2)]^2\,, \tag{12}$$

which provides an unbiased measure of the magnitude of $\alpha(h_1, h_2, p_1, p_2)$ and measures the additional depletion of the Fermi sea due to 2p2h CBF state admixtures in the ground state [7]. The last two columns of the table give the second-order corrections to the energy and to the enhancement factor, when only the first four central components are retained in both $H - H_{00}$ and $DC - DC_{00}$.

The "optimal" value d_{tp} of the parameter d_t for CBF perturbation theory is considered to be the one for which κ_B is minimum. Generally, δE is also minimum at $d_t = d_{tp}$; d_{tp} does not necessarily coincide with the "optimal" variational value d_{tv} of d_t, since low-lying states do not contribute much to H_{oo} [6]. For the case displayed in the table, $d_{tv} = 3.2r_0$ and $d_{tp} = 3.17r_0$. For the RSC [12,2] model, in its v_{12} reduction, at $k_F = 1.33$ fm^{-1} we get $d_{tp} = 3.44r_0$ and $d_{tv} = 2.89r_0$. The suppression of the tensor components in $H - H_{00}$ and $DC - DC_{00}$ largely reduces the perturbative corrections, expecially for $DC - DC_{00}$, indicating that most of the long

Table 1
Break-up of the second order perturbative corrections to the energy and to the enhancement factor for the Urbana-v_{14} + TNI(V) model of nuclear matter at $k_F = 1.33$ fm^{-1} for several values of the tensor healing distance d_t in units of $r_0 = (3/4\pi\rho)^{1/3}$. The energies are in MeV.

d_t/r_0	κB	δE	δK_1	δK_2	δK	$\delta E^{p\leq 4}$	$\delta K^{p\leq 4}$
2.8	0.036	−2.20	0.26	−0.023	0.24	−1.63	0.05
3.2	0.026	−1.96	−0.04	−0.014	−0.06	−0.43	0.04
3.6	0.040	−3.50	−0.42	−0.015	−0.44	−2.97	−0.003

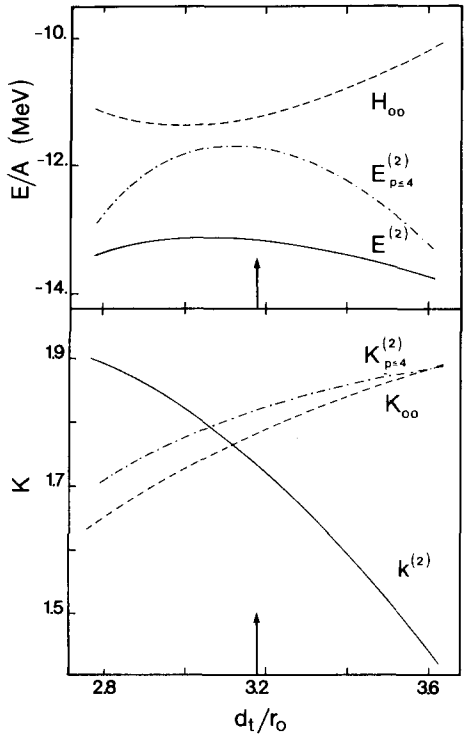

Fig. 1. Binding energy and enhancement factor of nuclear matter in 0th and 2nd order of CBF perturbation theory as a function of d_t at $k_F = 1.33$ fm^{-1}. The arrows indicate the value $d_t = d_{tp}$ at which κ_B is minimum.

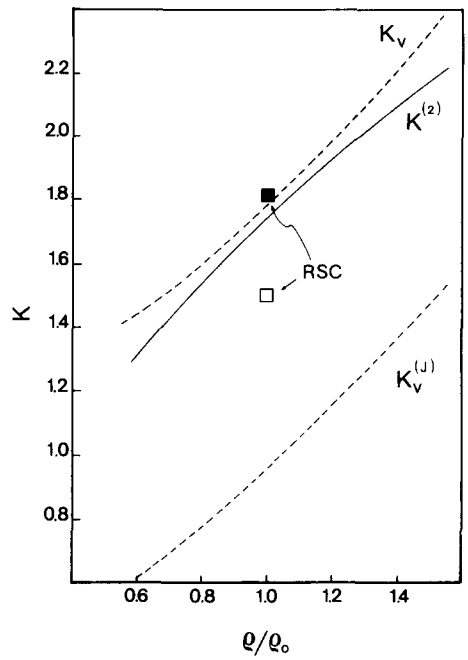

Fig. 2. Enhancement factor for nuclear matter as a function of the density ρ in units of $\rho_0 = 2k_F^3/3\pi^2$. The dashed lines give the variational estimates of refs. [2,8] with (K_V) or without ($K_V^{(J)}$) state dependent correlations. The solid line gives the second order CBF evaluation of K. The open and full squares correspond to the variational and 2nd order estimates of K for the RSC model.

range correlations perturbatively introduced in $|\Psi_0\rangle$ by the 2p2h admixtures are induced by the tensor interaction. This implies also that d_t is a critical parameter for the perturbative corrections to K.

The results for the energy and for the enhancement factor are summarized in fig. 1. The zeroth order estimates H_{00} and $K_{00} = (4/A)(m/\hbar^2)DC_{00}$ are compared with the "fully corrected" values $E^{(2)} = H_{00} + \delta E$ and $K^{(2)} = K_{00} + \delta K$ and the "partially corrected" values $E^{(2)}_{p\leq 4} = H_{00} + \delta E^{p\leq 4}$ and $K^{(2)}_{p\leq 4} = K_{00} + \delta K^{p\leq 4}$. The rather strong dependence of $K^{(2)}$ on d_t shows that $|\delta K|$ overestimates $|K - K_{00}|$ especially when this quantity is large.

The dependence of the enhancement factor on the density is shown in fig. 2, where the variational estimates [‡1] K_V of ref. [2] are also reported. The curve labeled with $K_V^{(J)}$ corresponds to the results obtained [8] for K_V when only the scalar component ($p = 1$) is retained in the pair correlation operator. The zeroth order estimates K_{00} ($d_t = d_{tp}$) almost coincide with K_V at all the density values considered. The results obtained for the enhancement factor including the perturbative corrections are shown by the curve labeled $K^{(2)}$, and they are given by $K_{00}(d_{tp}) + \delta K(d_{tp})$ for each value of the density. The substantial difference between $K_V^{(J)}$ and K_V stresses the importance of the state dependent correlations, and primarily of the tensor correlation [2,8,13–15] in photonuclear reactions. The perturbative corrections $\delta K(d_{tp})$ never exceed a 5% of the corresponding $K_V \approx K_{00}$, hence it is likely

[‡1] An accurate minimization shows that the optimal values of d_{tv} are slightly lower than those given in ref. [11] and used in ref. [2] also. For instance $d_{tv}(k_F = 1.13) = 2.86r_0$, $d_{tv}(k_F = 1.33) = 3.02r_0$ and $d_{tv}(k_F = 1.53) = 3.20r_0$.

that the present calculation of the enhancement factor in nuclear matter is fairly accurate.

In the case of RSC model the perturbative corrections to K_V are larger than for the Urbana model. At $k_F = 1.33\ \mathrm{fm}^{-1}$ we get the result $\delta K = 0.3$, which is a 20% correction to the variational estimate. The corrected value $K_{00}(3.44r_0) + \delta K(3.44r_0) = 1.81$ almost coincides with the corresponding result obtained for the Urbana-v_{14} + TNI(V) model. However the TNI accounts for about 10% of the total contribution to K, hence the RSC potential gives an enhancement factor which is larger than the one provided by the Urbana-v_{14} potential, in accord with the fact that the RSC potential has a larger D-state percentage [16].

The enhancement factor increases almost linearly with density, hence we expect that surface effects will reduce the nuclear matter K. We estimated these effects by employing the local density approximation (LDA) [17]. We have used the prescription $\rho_{av} = \rho(R)$ with $R = \frac{1}{2}(r_1 + r_2)$ in the calculation of $\langle DC_2 \rangle$ and $R = \frac{1}{3}(r_1 + r_2 + r_3)$ in evaluating $\langle DC_3 \rangle$. The corresponding expression for K is given by

$$K = (4m/\hbar^2 A) \int \mathrm{d}R \, \langle \widetilde{DC}_2 + \widetilde{DC}_3 \rangle_{\rho_{av}} , \tag{13}$$

where

$$\widetilde{DC}_2(r_{12}, R) = [DC_2(R_{12})/2\rho(R)] \int_{-1}^{+1} \mathrm{d}|\cos\theta_R| \, \rho(r_1)\, \rho(r_2) , \tag{14}$$

$$DC_3(r_{12}, r_{13}, r_{23}, R) = [DC_3(r_{12}, r_{13}, r_{23})/8\pi^2\rho^2(R)] \int \mathrm{d}\Omega_R \, \mathrm{d}\phi_{13} \, \rho(r_1)\, \rho(r_2)\, \rho(r_3) , \tag{15}$$

where the integrations are done by taking r_{12} as the z-axis. In eq. (13) the expectation value is calculated for symmetrical nuclear matter, which means that we assume K to be strictly independent on the symmetry parameter $\beta = (N - Z)/A$. We have taken for $\rho(r)$ the charge density distribution of ref. [18] normalized so as to be the particle density. We found very little dependence on the different types of model parametrization given in ref. [18]. The results obtained are displayed by the solid line of fig. 3 and they are very well fitted by the "mass formula" $K_\infty - 2.3A^{-1/3}$ where the nuclear matter enhancement factor $K_\infty = 1.74$ represents the volume term. The experimental data reported on the figure correspond to

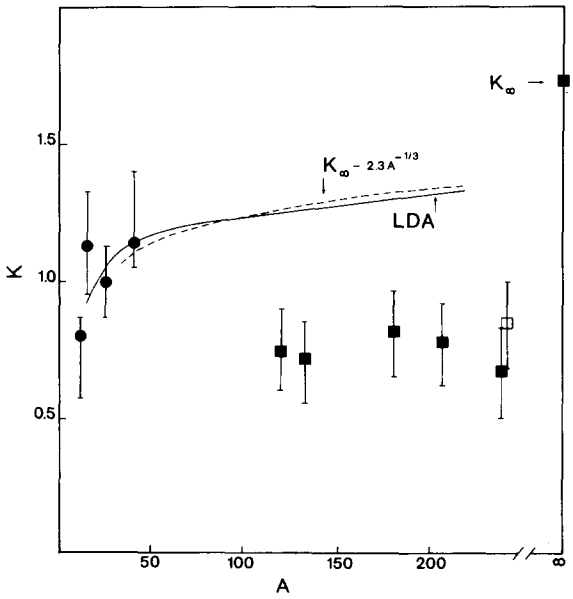

Fig. 3. Enhancement factor for finite nuclei as obtained in the LDA approximation. The experimental data are from ref. [3] (squares) and ref. [20] (dots).

$$1 + K_{\mathrm{exp}} = \sigma_0^{-1} \int^{140\ \mathrm{MeV}} \sigma(E_\gamma)\, \mathrm{d}E_\gamma \,, \tag{16}$$

where $\sigma_0 = 0.06\ (NZ/A)$ MeV b. The data for light nuclei ($A \leqslant 40$) have been obtained [19] by measuring the total absorption cross section, those for heavy nuclei ($A \geqslant 100$) by the inclusive multiple photoneutron production cross section [3]. The results shown in fig. 3 deserve the following comments: (i) the depletion (~20%) due to surface effects is quite substantial even in the lead region. (ii) the contribution from TNI us reduced with respect to that of nuclear matter and it accounts for only few per cent of the total K; (iii) the LDA values are in reasonably good agreement with previous theoretical estimates [16,20] except for light nuclei where LDA is too crude. Our results for the enhancement factor should not be directly compared with the experimental data. Actually the difference $K - K_{\mathrm{exp}} \simeq 0.6$ in the heavy nuclei region is a measure of tail contributions and higher multipoles and dipole retardation effects. It could be wiser to compare K with that extracted from the forward Compton scattering which is related to the absorption cross section by means of dispersion relations [21]. A recent analysis [22] of this type indicates a value of $K = 1.6 \mp 0.2$ for ^{208}Pb.

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