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Intrinsic Mode Cross Correlation: a novel technique to identify scale-dependent lags between two signals and its application to ionospheric science

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***Abstract*—In this work we address the following question: can we use modern, cutting edge techniques conceived for the analysis of nonlinear non-stationary signals to measure scale-wise lags? To this scope, we propose a novel technique, called Intrinsic Mode Cross Correlation method, which leverages on the decomposition of nonlinear non-stationary signals by the Multivariate Fast Iterative Filtering (MvFIF) technique and the computation of a scale by scale cross correlation. We evaluate this technique on artificial signals (whose ground truth is known) and plasma density data provided by the Langmuir probes onboard the Swarm satellites. We show that this technique allows indeed to reconstruct the lag dependence on the involved spatio/temporal scales for the artificial data set (even in presence of high levels of noise), and to estimate them in a real life signal. This can pave the way to future uses of this technique in contexts in which the causation chain can be hidden in a complex, multiscale coupling of the investigated features.**

***Index Terms*—I.5.4.l Sciences I.5.4.m Signal processing I.5.4.o Waveform analysis**

I. INTRODUCTION

The development of modern techniques for the analysis of multiscale systems has become of paramount importance, especially for the investigation of natural phenomena whose complexity manifests on a wide range of spatial and temporal scales. This is the case of the Earth's ionosphere, featured by a complex behavior due to its nonlinear coupling with the solar wind-magnetosphere system from above and with the lower atmosphere from below (see, e.g. [1]). In this context, we present a novel approach, called Intrinsic Mode Cross Correlation (IMXC spelled I-M-cross-C) method, for the scale-wise measurement of lags between two complex and non-stationary signals. We leverage on the Multivariate Fast Iterative Filtering (MvFIF) [2] technique, being the multivariate implementation of the Fast Iterative Filtering (FIF) technique [4]. The lags are then identified on a scale-by-scale basis by using the maximum cross correlation among homogeneous modes. The scale-wise lags identification abilities are tested on artificial signals, for which the ground truth is known *a priori*, under different levels of additive noise. The proposed approach is then used on plasma density data provided by the Langmuir probes onboard the Swarm satellites, addressing a simple case of interest.

II. METHODS

A. Fast Iterative Filtering (FIF)

Fast Iterative Filtering (FIF) [3] is a decomposition method that splits a non-stationary multi-component signal into simple oscillatory components, named intrinsic mode components (IMCs). Recently FIF has been extended to handle multivariate signals in what is called Multivariate FIF (MvFIF) [2]. When two measurements, assumed to be associated to the "cause" and the "effect" of some physical phenomenon, are analyzed as two channels of the MvFIF technique, the respective IMCs produced are guaranteed to posses the same frequency ranges. This is a fundamental feature required to ensure a suitable scale-wise comparison of the IMCs. The IMCs produced via MvFIF are comparable with results of Hilbert-Huang transform or other Empirical Mode Decomposition-based techniques, as well as alternative methods. Nevertheless, the MvFIF method proved to have several advantages. In particular, we mention here its low computational complexity which makes it the fastest technique of its kind; the guaranteed uniqueness of the derived decompositions; a complete mathematical

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framework; and a complete adaptivity to the signal under investigation ensuring that there is no need to set a priori neither the number of components to be extracted nor the basis to be used in the process. Interested readers can find more details in [2], [4], [9].

B. *Lag estimation*

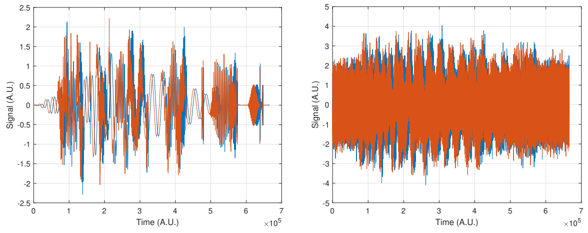
To estimate the lag for every specific frequency (scale) range, in what we call the IMXC technique, we perform the best match filtering using the maximal cross correlation (XC) of the corresponding IMCs of the two signal decompositions. As we mentioned before, the MvFIF guarantees that corresponding IMCs of the two channels correspond to the same frequency range. We are aware that "correlation" does not imply "causation", and that XC is not the best means to measure it [5]. However, according to our knowledge, this is the first attempt to such a scale-wise lag measurement and to ease the analysis, we apply the technique on examples in which what is the cause, and what is the effect is known in advance. Bearing this in mind, scale-wise lags are thus provided for all the IMCs pairs of both "cause" and "effect" signals. Various concepts for the estimation of lags between the same respective frequency ranges signal decompositions were tested. Based on our tests, we observed that most of XC implementations provide good results. In the following we opted to use the 'normalized' XC, due to its numerical efficiency in Matlab, especially when the 'maxlag' option is used.

III. TESTING OF THE IMXC APPROACH

The proposed IMXC approach was tested using the two channel signal shown in Figure 1a, whose components, resolved using MvFIF, are depicted in Figure 2. In particular, this signal was constructed combining 9 non-stationary components, with stationary frequency and non-stationary amplitudes (providing different patterns for each frequency), and applying 9 randomly chosen lags for each component.

To test robustness against noise, the two channels were perturbed using five different levels of noise, measured as SNR in dB using the formula $20\log_{10}(A_{\text{signal}}/A_{\text{noise}})$.

For each level of noise and for each channel, we considered 5 different realizations of Gaussian noise. In Figure 1b we show, as an example, the signal with a Gaussian noise of 13.98 dB SNR. In Figure 3, we report statistics summarizing the performance of the IMXC method in reconstructing the scale by scale lag under different levels of SNR, including the case of clean signal (SNR=Inf). To take into account small perturbations induced by noise, we set a threshold of 5 % accuracy in the reconstructed lag measurements.



(a) (b)

Fig. 1: Clean damped artificial signal (panel a); after adding noise with SNR level 13.98 in dB (panel b).

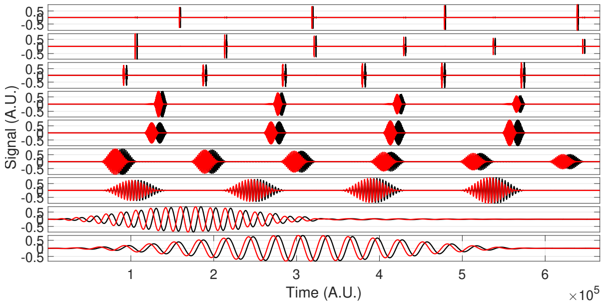


Fig. 2: Components of the two channel noiseless artificial signal in red and black, respectively.

IV. REAL LIFE EXAMPLE - IONOSPHERIC PLASMA

Multi-satellite measurements in the Earth's ionosphere provide ample opportunities to identify the lags, among others induced by the spacecraft distance, that are known to exist but hidden in the data. The only assumption we make here is that the background conditions are stable so that the temporal variation is negligible between the respective satellite measurements above the nearly-same

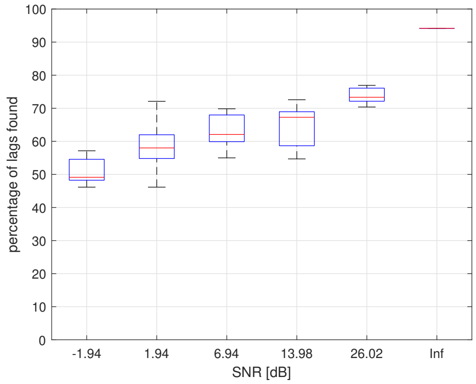


Fig. 3: Box plots summarizing the statistics on the reconstructed lags resolved with 5% accuracy.

TABLE I: Lags identified using IMXC approach on the ionospheric plasma densities measured with Swarm A and C satellites on 1 May 2014 around 15 UT.

	Orbital delay	Crest-like	MSTID
Lag (s)	8.8	22	20
Scale (s)	10–20	50–100	80–100

spatial location.

In our work, we use the closely-separated measurements of the European Space Agency's Swarm Alpha (A) and Charlie (C) satellites [6] that use identical Langmuir probe instruments to sample the ionospheric plasma density in the topside ionosphere (about at 460 km altitude). At low- and mid-latitudes, Swarm A and Swarm C have a longitudinal and latitudinal separations of about 146 km and 62 km, respectively. As Swarm A and C fly at about 7 km/s in those regions, latter separation translates into a lag of about 8.8 s between the two satellites. The selected case is the passage of the two close-by satellites flying over the Japanese longitudinal range on 1 May 2014 around 15 UT. Over Japan, they encounter a plasma enhancement due to the passage of a Medium-scale Travelling Ionospheric Disturbance (MSTID), as reported by [7]. Swarm satellites pass fast over the MSTID, as it moves at velocity of the order of a few hundreds of meters per second [8], therefore, we can consider the MSTID frozen in that frame. Additionally, we consider for the same Swarm A and C tracks, the passage through a peak in the plasma density, due to northern crest of the equatorial ionospheric anomaly (EIA), that reaches the Swarm altitudes (see, e.g. [10]).

The nighttime MSTIDs, identified by [7], occur during the local midnight, and are caused by electro-dynamical forces, such as the Perkins instability, as supported by [11]. What we aim at in this work is the identification *on top* of the 8.8 s delay between the satellites, the delay associated with the respective ionospheric structure, and the corresponding scales at which this lag is found.

The electron density (Ne) measurements from Swarm A and Swarm C are shown in panels a and b of Figure 4. The cross correlation was computed IMC per IMC by considering 5-minute time windows, which are highlighted in Figure 4, panel b, as the green and yellow shaded areas, respectively. Panels c and d in Figure 4 show the obtained lags cross-correlation as a function of the IMC temporal scale. In these panels we report only positive lags, i.e., when what we consider here as the effect (Swarm A) comes after the cause (Swarm C).

Besides the 8.8 s orbital delay (highlighted in panel c and d of Figure 4 by blue ellipses), the IMXC approach was able to recover also a 22 s lag (a crest-like delay, highlighted in panel c, Figure 4, by a red ellipse) and a 20 s lag (the MSTID delay, highlighted in panel d, Figure 4, by a green ellipse). These last two lags are physically reasonable delays between peaks in Ne latitudinal/time profiles modified by the satellites traversing extensive ionospheric structures, the former by the dip equator and the latter by the MSTID. Identified lags are reported in table I, together with the corresponding scales at which they are found. The orbital delay-related lags are generally associated with shorter scales in range of 10–20 s while the lags associated with the actual ionospheric features are covered at longer scales of 50–100 s. Concerning the orbital delay, if we consider that the satellites are flying at about 7 km/s, the corresponding spatial scale is of the order of 100 km, which is, as expected, at the same order of magnitude of the spatial separation between the satellites.

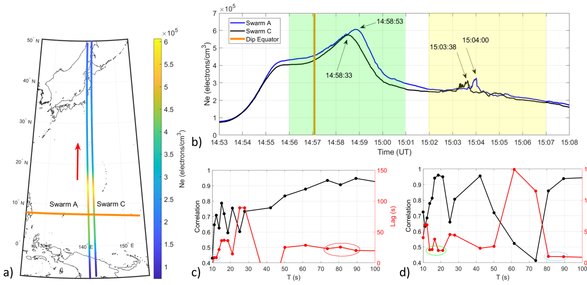


Fig. 4: (panel a) Geographic distribution of the electron density measured by Swarm A and C. Orange line indicates the position of the magnetic equator, while the red arrow indicates the flying direction of the satellites. (panel b) Electron density as a function of time measured by Swarm A (blue) and Swarm C (black). Times of plasma peaks are also indicated. Orange line indicates the time at which the satellites cross the magnetic equator, while green and yellow shaded areas indicate the time windows used to produce the plots in panel c and d, respectively. (panel c) Correlation between IMCs and the corresponding measured lags in the EIA crest region (green area in panel b). (panel d) Same as panel c, but for the MSTID region (yellow area in panel b).

V. CONCLUSIONS AND OUTLOOK

The proposed scale-wise lag reconstruction technique, called IMXC, was found to perform well when applied to couples of artificial and real life signals, even in presence of high level of noise. It is important to point out that the correlation analysis alone without a preliminary decomposition performed via MvFIF, is unable to provide any of the correct lags for both the artificial and the natural signals, when employed on the original data sets (not shown). The key result presented in this work can be summarized as follows. When we deal with a multicomponent signal, which is associated with a multiscale process that has different lags at different scales, one way to reconstruct physically meaningful lags is by first decomposing the signal into well separated scales and then apply a scale by scale analysis.

As reminded in the introduction, to ensure that two signals under study are in a cause-effect relationship, one has to use standard measures like Granger causality or entropic principles. More advanced concepts like the fluctuation-response protocol, proposed in [5], are applicable even in presence of weak nonlinear terms. After this preliminary analysis has confirmed a cause-effect relationship between two measurements, the presented technique allows to investigate quantitatively the information process. In a future work, we plan to apply the proposed approach to the study of physical delays involved in the solar wind-magnetosphere-ionosphere-thermosphere coupling (see, e.g. [1]).

Finally, we point out that the IMXC method, as it is, does not allow to analyze all the cases in which the cause, in order to initiate any effect, has to build up to reach over some threshold. An example from the space physics is given by the geomagnetic storm, which starts after some duration of specific level of the geoeffective southward interplanetary magnetic field. We plan to work in the future to extend the IMXC technique to cover also this kind of phenomena.

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REFERENCES

[1] M. Materassi, B. Forte, A. J. Coster, and S. Skone (Eds.). The Dynamical Ionosphere: A Systems Approach to Ionospheric Irregularity. Elsevier, 2019

[2] A. Cicone, and E. Pellegrino. Multivariate Fast Iterative Filtering for the decomposition of nonstationary signals, arXiv math.NA, arXiv 1902.04860v2.

[3] A. Cicone, Iterative filtering as a direct method for the decomposition of nonstationary signals, Numer Algor, *85*, pp. 811–827, 2020.

[4] A. Cicone, and H. Zhou. Numerical Analysis for Iterative Filtering with New Efficient Implementations Based on FFT, Numer Math (Heidelb), emph147, 1,1-28, 2021

[5] M. Baldovin, F. Cecconi and A. Vulpiani, Understanding causation via correlations and linear response theory, Phys. Rev. Research, *2*, 4, 043436, 2020

[6] E. Friis-Christensen, H. Lühr, D. Knudsen, and R. Haagmans, Swarm – an Earth observation mission investigating geospace. Adv. Space Res., emph41, 1, 210-216, 2008

[7] H. Kil and L. J. Paxton, Global distribution of nighttime medium-scale traveling ionospheric disturbances seen by Swarm satellites, Geophys. Res. Lett., *44*, 18, 9176-9182, 2017.

[8] R. Leitinger, and M. Rieger, The TID model for modulation of large scale electron density models, Ann. Geophys., *48*, 3, 2005.

[9] M. Piersanti, M. Materassi, A. Cicone, L. Spogli, H. Zhou, and R. G. Ezquer. Adaptive Local Iterative Filtering: a promising technique for the analysis of non-stationary signals, J. Geophys. Res. Space Phys., *123*, 1, 1031-1046, 2018

[10] L. Spogli, D. Sabbagh, M. Regi, C. Cesaroni, L. Perrone, L. Alfonsi, et al., Ionospheric response over Brazil to the August 2018 geomagnetic storm as probed by CSES-01 and Swarm satellites and by local ground-based observations, J. Geophys. Res. Space Phys., *126*, 2, e2020JA028368, 2021.

[11] Y. Otsuka, A. Shinbori, T. Tsugawa et al., Solar activity dependence of medium-scale traveling ionospheric disturbances using GPS receivers in Japan, Earth Planets Space *73*, 22, 2021.