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Momentum distributions in halo nuclei

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Abstract. From the analogy between the break-up of weakly bound, neutron-rich nuclei and the phenomenon of optical diffraction, it is possible to formulate a model for the momentum distribution of both the core and the valence neutrons of halo nuclei which displays a simple dependence on nuclear structure parameters. The model is applied to the analysis of reactions where ^{11}Be, ^{11}Li and ^{14}Be impinge on ^{12}C, providing an overall account of the experimental findings and predictions for further measurements.

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The probing of the structure of weakly bound, neutron rich halo nuclei has mostly been done by studying their break-up products following collision processes [1]. This is because these nuclei are only available as secondary beams of complex reaction processes and because, as a rule, their excited states are not bound.

The momentum distribution of the break-up products has been shown to display a narrow peak, much narrower than that observed in the fragmentation of well bound nuclei [2–10]. The presence of such narrow components has been interpreted in terms of the very large extension of the wavefunction of the valence neutrons, as compared to that of core nucleons, leading to halo nuclei [2, 3, 11, 12]. In the direction perpendicular to the beam, rather broad components have been observed for the momentum distributions of neutrons detected in coincidence with the core. The presence of broad components in the perpendicular distributions of the cores is controversial: the distributions have in some cases been found consistent with the superposition of a narrow and a broad component [5, 6], while in other experiments a better fit was obtained in term of a single, narrow component [10].

In what follows we present results of a study of these momentum distributions aimed at assessing, in a model independent way, the basis for the experimental findings. For this purpose, the fragmentation of ^{11}Be, ^{11}Li and ^{14}Be impinging on ^{12}C at a variety of bombarding energies ranging from 30 MeV/A to 800 MeV/A has been studied . The choice of a light target was made to emphasize nuclear effects in both one- and two-nucleon halo systems. It will be concluded that while the long tail of the valence, weakly bound neutrons, plays a central role in determining the narrow distributions observed, equally important, in the case of two-nucleon halos, is the presence of single-particle resonances in the potential generated by the cores. Fingerprints of these resonances could, in principle, be observed in detailed studies of the line shape of the narrow component.

It is well-known that if, in the course of encountering an obstacle, either transparent or opaque, a region of a wavefront is altered in amplitude or phase, diffraction will occur. The various segments of the wavefront which propagates beyond the obstacle interfere to cause well known energy-density distributions referred to as diffraction patterns. In the modern view of light, the wavelike nature of photons emerges from the Planck-Einstein relation $E = h\nu$, coupled with the relations $E = cp$ and $\nu = c/\lambda$ for classical electromagnetic waves, leading to $\lambda = h/p$. The assumption made by de Broglie that any moving body may be accompanied by a wave, and that it is impossible to disjoin motion of a body and propagation of a wave, implies that diffraction takes place also in the scattering of particles. In the case of point particles, such as neutrons with wavelengths small as compared to the dimensions of the nucleus, the behaviour is similar to that observed in the diffraction of light by an absolutely black sphere. In the case of composite particles, in addition to elastic scattering similar to diffractional scattering of point particles, diffractional disintegration can also take place, in particular in the case of weakly bound systems.

It is then expected that the narrow collimated beams of neutrons and of heavy particles (cores) observed in coincidence in the scattering of halo systems by atomic nuclei, arise from break-up processes where neither of the two particles suffers any collision, in the classical sense. The reason for this diffractional break-up is to be found in the localization suffered by the wavefunction describing the relative motion of the neutron with respect to the core, as it passes the edge of the target nucleus. This localization has a simple classical picture by exploiting the ansatz implicit in the above discussion that the effects of nuclear transparency can be neglected, and that the target nucleus can be pictured as a black sphere or, more simply, as a black disk lying in a plane perpendicular to the direction of the incident beam.

Because of the looseness of halo nuclei binding, the internal motion in this system is negligible during the time it requires for the halo nucleus to travel a distance equal to the nucleus diameter. Hence it is meaningful to speak about the instantaneous positions of the neutron and of the core. Consequently, the neutron is not to be found in the lower region of the sphere, which lies in the shadow disk's edge. This implies that the core cannot be located in the opposite defined upper cap, whose volume is properly scaled by the ratio of masses of the neutron and the core. This reduction in volume of the two-particle wavefunction leads to an increase of the kinetic energy, and a certain probability of dissociation. The image of an illuminated hole pierced on a screen is a simple, yet valid picture of the wavefunction after both particles have cleared the edge of the disk.

The intensity or irradiance associated with the diffraction (Fraunhofer diffraction) of classical waves at a circular aperture of radius a is given by $I \approx (J_1(ak\sin\theta)/ak\sin\theta)^2$, a relation first derived by Airy. The quantity $k = 2\pi/\lambda$ is the wavenumber, λ being the wavelength of the wavefront. The quantity $k_\perp = k\sin\theta$ is the value of the wavenumber perpendicular to wave propagation. The expression $J_1(x)/x$ acquires a value of 1/2 for $x = 0$ and a value of $1/2\sqrt{2}$ at $x_{1/2} = 1.6$. Consequently, the full width at half maximum of the transverse momentum distribution associated with the break-up residues (neutron plus core) is

$$(\Delta p_\perp)_{\text{diff}} = \frac{3.2\hbar}{a}, \tag{1}$$

where $a = R_t + R_n$, the quantity $R_t = 1.25A_t^{1/3}$ fm being the radius of the target and $R_n = 0.8$ fm the radius of the neutron (for a target of ^{12}C, $R_t + R_n = 3.6$ fm). Because the wavefunction does not suffer localization along the beam direction, the longitudinal momentum distribution of the break-up products will reflect that of the original wavefunction describing the relative motion of the neutron with respect to the core. For simplicity we shall use the bound state solution of a zero-range potential, that is, $\phi_o(r) = \sqrt{\eta/2\pi}e^{-\eta r}/r$ where the constant η is related to the neutron binding energy E_B through the relation $E_B = \hbar^2\eta^2/2m$. Consequently,

$$(\Delta p_\parallel)_{\text{diff}} = 2\hbar\eta. \tag{2}$$

In the case of stripping, that is in the case where the neutron collides with the target in the classical sense and is absorbed by it, the momentum of the core will reflect the original momentum distribution of the relative wavefunction. Thus,

$$(\Delta p_\parallel)_{\text{str}} = (\Delta p_\perp)_{\text{str}} = 2\hbar\eta \tag{3}$$

In what follows we shall apply the above expressions to reactions involving three halo nuclei (^{11}Be, ^{11}Li and ^{14}Be), all of them impinging on ^{12}C. The nucleus ^{11}Be can be viewed as the core ^{10}Be to which a neutron is bound with a binding energy of 0.5 MeV. Making use of this quantity to calculate η, with $a = 3.6$ fm and the relations given in (1), (2) and (3), one obtains the results displayed in Table 1.

Both the atomic nuclei ^{11}Li and ^{14}Be can be viewed as two neutron systems weakly bound to a core (^{9}Li and ^{12}Be respectively). In keeping with the fact that neither ^{10}Li nor ^{13}Be are bound, we consider the break-up as a two-step process. First, one neutron is either stripped or diffracted leaving the residual nucleus A-1 with an associated momentum width $(\Delta p)_{n_1}$, given by (1), (2) and (3). The other neutron is trapped in a resonant state of the system $A-1$ and is subsequently emitted in flight [13–15]. In order to describe the internal momentum width $2\hbar\eta$ of the first neutron, we have calculated the corresponding value of η using for E_B the two-neutron separation energy (0.3 MeV and 1.3 MeV for ^{11}Li and ^{14}Be respectively). This is in keeping with the fact that the reason for ^{11}Li and ^{14}Be to be stable is to be found in the correlation existing among the two loosely bound neutrons. In the case of ^{11}Li, it has been shown that this ansatz provides a correct description of the tail of the neutron wavefunction, as deduced from a two-neutron correlated wavefunction [12]. To calculate the width of the momentum distribution associated with the second neutron and with the core one should take care of the

Table 1. Comparison of calculated and measured full widths at half maximum (in MeV/c) for the tranverse and longitudinal momentum distributions. For the cores, the first (second) row in the theoretical results refers to the width associated to stripping (diffraction) of the first neutron. For the neutrons, the first (second) row refers to the width associated to the neutron emitted in flight (diffracted). The values obtained from (1–4) are given in parenthesis. In the experimental data, when a two-component fit has been attempted, we report the larger width in the second row. A reference to the experimental work is given in square brackets

	^{10}Be	n	^{9}Li	n	^{12}Be	n
$\Delta p_{\perp,\text{th}}$	50 (61)		53 (55)	37 (35)	85 (104)	52 (53)
	216 (173)	216 (173)	189 (160)	214 (173)	208 (168)	220 (173)
$\Delta p_{\perp,\text{exp}}$	59 ± 9 [5]		49 ± 7[6]	45 ± 2[9]	93 ± 4[26]	50 ± 5[4]
	257 ± 16	181 ± 14[25]	188 ± 10	280	–	–
$\Delta p_{\parallel,\text{th}}$	49 (61)		53 (55)	37 (35)	88 (104)	52 (53)
	79 (61)	79 (61)	66 (55)	56 (47)	110 (104)	104 (98)
$\Delta p_{\parallel,\text{exp}}$	40 [24]		45 ± 2[7]	33 ± 5[9]	88 ± 5[26]	
	–	–	–	–	–	–

recoil associated with the decay in flight of the second neutron, that is, taking into account that the decay is sequential [13],

$$(\Delta p)_f = \sqrt{\left((\Delta p)_{n_1}\frac{A_f}{A-1}\right)^2 + (\Delta p)^2_{\mathrm{intr}}} \quad (4)$$

In (4), f indicates the observed fragment (either the core c, or the second neutron n_2), while A_f is the associated mass number ($A_c = A-2$, $A_{n_2} = 1$). The quantity $(\Delta p)_{\mathrm{intr}}$ is the momentum distribution width associated with the single-particle resonance in system A-1. We shall use $(\Delta p)_{\mathrm{intr}} \approx 35$ MeV/c for ^{11}Li, as indicated from a previous analysis [15] of the angular neutron distribution reported in [3]. In the case of ^{14}Be we have deduced $(\Delta p)_{\mathrm{intr}} \approx 52$ MeV/c from the angular distribution of [4]. The value of $(\Delta p)_f$ is different, depending on whether the first neutron has been diffracted or stripped. This difference is very small for the second neutron, but is large for the core. The corresponding values of $(\Delta p)_{n_2}$ and $(\Delta p)_c$ calculated from (4) are collected in Table 1. The present analysis differs in an important way from previous published work, in which the broad component observed in the transverse direction is interpreted as the reflection of high-momentum components of the intrinsic wavefunction. Diffractive break-up has often been neglected or considered of minor importance, especially at high bombarding energies (cf. e.g. [16–19]). Within such a scenario, it is difficult to understand the differences observed between longitudinal and transverse momentum distributions, which are particularly clear in the neutron momentum distributions (cf. Fig. 1b and 2b), as well as in the angular distributions. Within the present analysis these findings are a natural consequence of the variety of reaction mechanisms at play. Our assumption

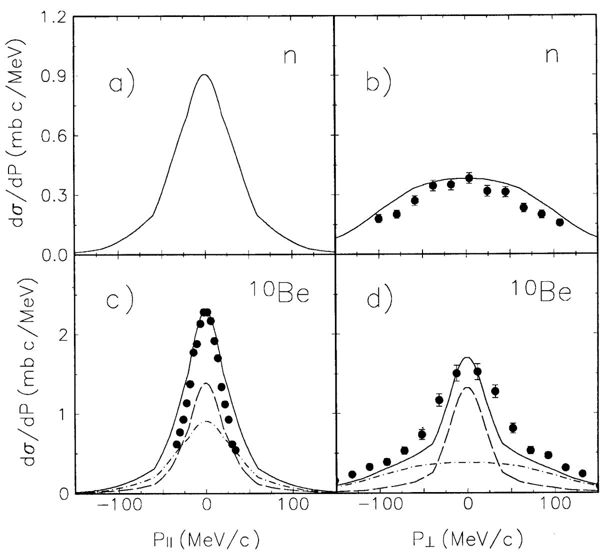

Fig. 1a–d. The momentum distributions for the break-up of ^{11}Be are shown. **a** and **b** refer to the neutron distribution in the longitudinal and transverse direction respectively. **c** and **d** refer to the core distribution in the longitudinal and transverse directions respectively. The *solid curve* is the total distribution, the *dashed* (*dash-dotted*) *curve* is the contribution from the events in which the neutron has been stripped (diffracted) by the target. The experimental data are taken from [25] (b), [24] (c) and [5] (d)

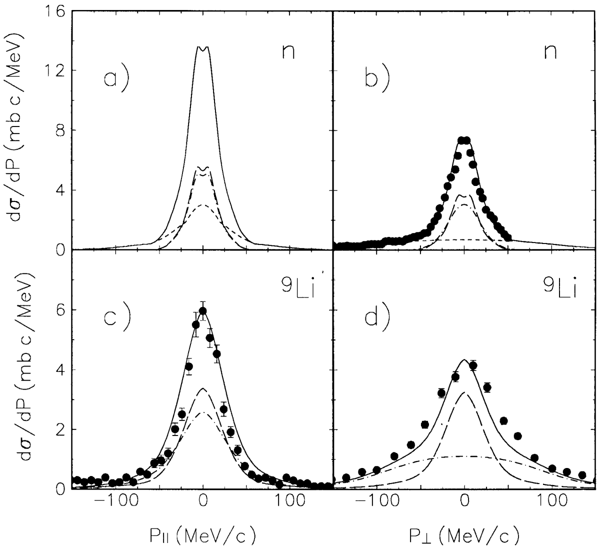

Fig. 2a–d. The same as Fig. 1, for the break-up of ^{11}Li. The curves in **a** and **b** refer to the total neutron distribution (*solid curve*), to the contribution from the first diffracted neutron (*short-dashed curve*) and from the neutron emitted in flight after diffraction of the first neutron (*dash-dotted curve*) or after stripping of the first neutron (*long-dashed curve*). The data are taken from [9] (b), [7] (c) and from [6] (d)

of complete absorption, however, enhances the contribution of diffraction to the total break-up cross section. A more accurate calculation, taking into account the transparency of the nucleus, may lead to a reduction of the diffractive cross section, introducing also a dependence on the beam energy.

Moreover, our consideration of the role played by the single-particle resonances, leading to sequential break-up, is in contrast with the frequently made assumption, that the two valence neutrons are emitted together as a dineutron (cf. e.g. [19–21]).

The momentum distributions of the reactions discussed above have also been calculated making use of a generalization of the expressions originally proposed by Serber, by Glauber and by Akhiezer and Sitenko to describe the break-up of energetic deuterons scattered off atomic nuclei [16] (cf. also e.g. [19–21]). The relations for the stripping cross section $d\sigma_{\mathrm{str}}/d^3p$, and the diffraction cross section $d\sigma_{\mathrm{diff}}/d^3p$ in the projectile frame are,

$$\frac{d\sigma_{\mathrm{str}}}{d^3p} = \frac{N_{\mathrm{val}}}{(2\pi\hbar)^3}\int_{\rho_n < R_t + R_n} d^2\rho_n \left|\int d^3r\, e^{-i/\hbar \vec{p}\vec{r}} S_c(|\vec{\rho}_n - \vec{r}_\perp|)\phi_o(r)\right|^2 \quad (5)$$

and

$$\frac{d\sigma_{\mathrm{diff}}}{d^3p} = \frac{N_{\mathrm{val}}}{(2\pi\hbar)^3}\int_{\rho_c > R_t + R_c} d^2\rho_c \left|\int d^3r\, \phi^*_{\vec{p}}(\vec{r})\, S_n(|\vec{\rho}_c + \vec{r}_\perp|)\phi_o(r)\right|^2 \quad (6)$$

In the above equations, we have indicated the impact parameters of the core and of one valence neutron by $\vec{\rho}_c$ and $\vec{\rho}_n$. The factor N_{val} is equal to the number of halo neutrons, $N_{\mathrm{val}} = 2$ for ^{11}Li and ^{14}Be, $N_{\mathrm{val}} = 1$ for ^{11}Be. The wavefunctions $\phi_{\vec{p}}(\vec{r})$ are the continuum solutions of the zero range potential already used to calculate ϕ_o. The S-matrices for core and neutron scattering used in the calculations are given by

$$S_i(\rho) = 0, \quad \rho < R_t + R_i;$$
$$S_i(\rho) = 1, \quad \rho > R_t + R_i; \quad i = c, n. \qquad (7)$$

The momentum distributions associated to the neutron decaying in flight are obtained folding the cross sections (5) or (6) with the intrinsic shape of the resonance, assuming the values given above for the width $(\Delta p)_{\rm intr}$. The transverse momentum distributions are obtained integrating over the direction parallel to the beam and over one direction perpendicular to it. The resulting momentum distributions, and the associated width are compared to the experimental findings in Figs. 1–3 and in Table 1. The experimental distributions are given in arbitrary units, and we have scaled them to the maximum of the theoretical ones. The results of the detailed calculations are rather similar to those calculated through the simple relations given in (1-4), which are also displayed in Table 1. The shapes of the momentum distributions are determined by the interplay of the three mechanisms discussed above, namely, diffraction, stripping and decay in flight. The latter mechanism is of course not present in the case of ^{11}Be. The core (^{10}Be) transverse momentum distribution displays a narrow peak associated with neutron stripping, superimposed to a broad component arising from diffraction, while for the neutron only the diffractive contribution is present. In the longitudinal direction the contributions arising from diffraction and stripping have a similar shape ($(\Delta p_\parallel)_{\rm diff} \approx (\Delta p_\parallel)_{\rm str}$, cf. (2) and (3)). The core distributions for ^{11}Li and ^{14}Be are analogous to those of ^{11}Be, while the neutron distributions display a narrow peak which is directly determined by the shape of the single-particle resonance via the decay in flight. This is because $(\Delta p)_{\rm intr} > (\Delta p)_{n_1}/(A-1)$ (cf. (4)).

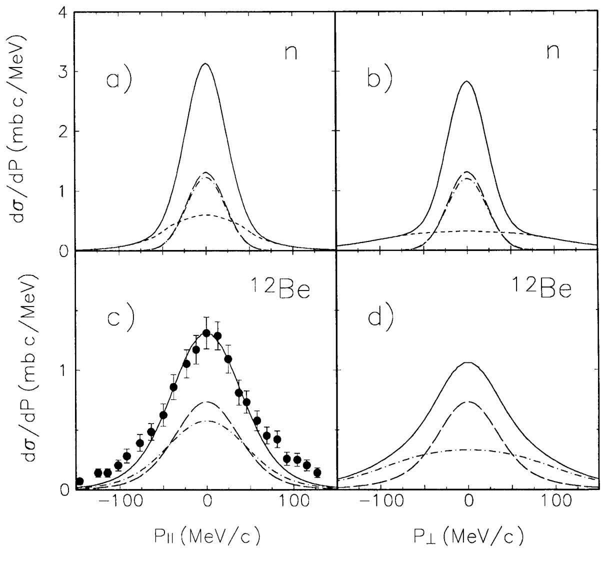

Fig. 3a–d. The same as Fig. 2, for the break-up of ^{14}Be. The data in **c** are taken from [26]

We conclude that the observed momentum distributions of the break-up products of neutron-rich, halo nuclei, can be explained in terms of the interplay of two reaction mechanisms acting with similar importance, namely stripping and diffraction. The differences observed between one- and two-neutron systems can be traced back to the sequential decay of the neutrons.

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