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Hydrology Research

Hydrology Research Vol 00 No 0, 1 doi: 10.2166/nh.2021.008

Flood routing via a copula-based approach

Mohammad Nazeri Tahroudi[a], Yousef Ramezani[a,*], Carlo De Michele[b] and Rasoul Mirabbasi[c]

[a] Department of Water Engineering, University of Birjand, Birjand, Iran
[b] Department of Civil and Environmental Engineering, Politecnico di Milano, Milano, Italy
[c] Department of Water Engineering, Shahrekord University, Shahrekord, Iran
*Corresponding author. E-mail: y.ramezani@birjand.ac.ir

YR, 0000-0002-8085-9290

ABSTRACT

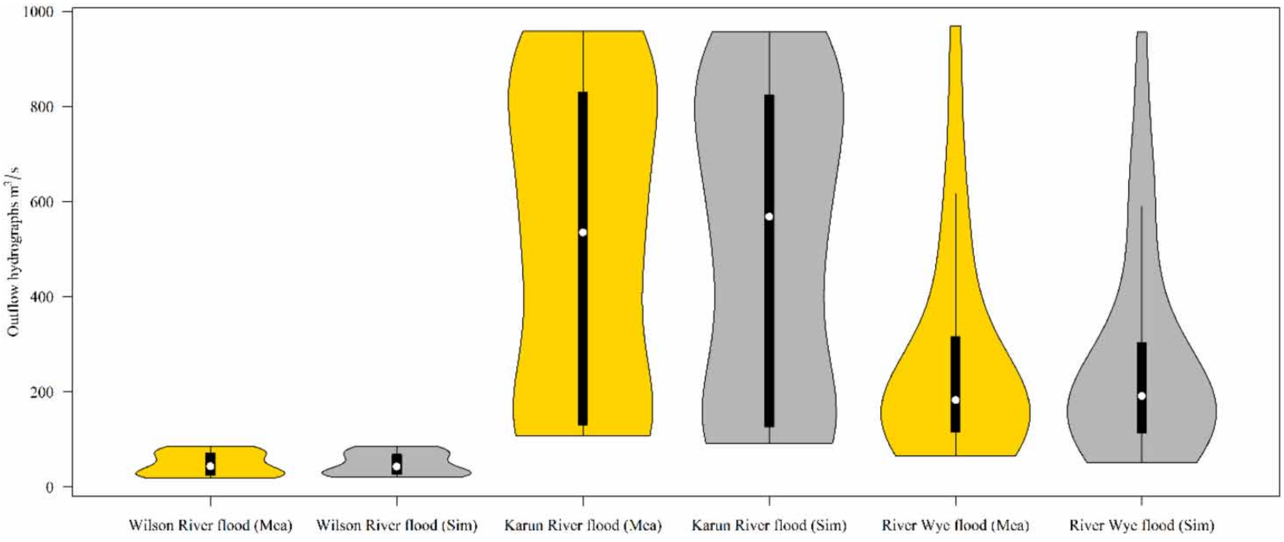
Floods are among the most common natural disasters that if not controlled may cause severe damage and high costs. Flood control and management can be done using structural measures that should be designed based on the flood design studies. The simulation of outflow hydrograph using inflow hydrograph can provide useful information. In this study, a copula-based approach was applied to simulate the outflow hydrograph of various floods, including the Wilson River flood, the River Wye flood and the Karun River flood. In this regard, two-dimensional copula functions and their conditional density were used. The results of evaluating the dependence structure of the studied variables (inflow and outflow hydrographs) using Kendall's tau confirmed the applicability of copula functions for bivariate modeling of inflow and outflow hydrographs. The simulation results were evaluated using the root-mean-square error, the sum of squared errors and the Nash–Sutcliffe efficiency coefficient (NSE). The results showed that the copula-based approach has high performance. In general, the copula-based approach has been able to simulate the peak flow and the rising and falling limbs of the outflow hydrographs well. Also, all simulated data are at the 95% confidence interval. The NSE values for the copula-based approach are 0.99 for all three case studies. According to NSE values and violin plots, it can be seen that the performance of the copula-based approach in simulating the outflow hydrograph in all three case studies is acceptable and shows a good performance.

Key words: conditional density, copula functions, flood hydrograph, Karun River, River Wye, Wilson River

HIGHLIGHTS

- In this study, the application of the conditional density of bivariate copulas was investigated in flood routing via a copula-based approach.
- The results showed that the copula-based approach has a high performance.
- The copula-based approach has been able to simulate the peak flows and the rising and falling limbs of the outflow hydrographs well.

GRAPHICAL ABSTRACT



INTRODUCTION

Flood frequency analysis is a primary method used to specify the design flood, which is important for the construction of hydraulic structures and water resources management (Stedinger *et al.* 1993; Khozeymehnezhad & Tahroudi 2019). In the traditional methods of flood frequency analysis, one of the assumptions for analyzing flood frequency is that hydrologic series are independent and follow the same distribution (Maidment 1993). However, human activities and climate change have influenced all rivers around the world (Jain & Lall 2000; Zhang *et al.* 2008; Vogel *et al.* 2011; Li *et al.* 2013; Hu *et al.* 2015; Machado *et al.* 2015; Tahroudi *et al.* 2019, 2020b; Wei *et al.* 2020; Sharifi *et al.* 2021) as well as non-stationary properties in hydrological phenomena (Jain & Lall 2000). These features affect the base assumptions of flood frequency analysis and the associated reliability (Khozeymehnezhad & Nazeri-Tahroudi 2020).

In general, floods are one of the natural hazards that have caused damage to people worldwide. Therefore, in order to prevent flood damage, structures are usually built near the river. The most important issue for water control structures is to accurately estimate the peak flow in each section of the river and how the flood hydrograph rises and falls in that section. Flood-routing methods are divided into two main groups including hydrological and hydraulic methods. Because hydraulic methods have computational complexities, they are usually difficult to use and require a lot of information. But in contrast to hydraulic methods, simpler relationships can be used, and flood routing can be done at the same time by hydrological methods with proper accuracy. Muskingum is one of the hydrological flood-routing methods. This method has a linear relationship and cannot model nonlinear relationships between inflow, outflow and temporary storage. To overcome this shortcoming, Gill (1978) proposed a nonlinear version of this method and found the parameters of the nonlinear Muskingum method using the least-squares method (LSM).

Many researchers have used various algorithms to calculate the parameters of the nonlinear Muskingum method, including Pattern Search and Hook–Jeeves in combination with Linear Regression, Conjugate Gradient and Davidon–Fletcher–Powell (HJ+DEP) (Tung 1985), combining the nonlinear method with the LSM (NL-LSM) (Yoon & Padmanabhan 1993), Genetic Algorithm (GA) (Mohan 1987), Harmony Search (HS) (Kim *et al.* 2001), Lagrange Multiplier Method (LMM) (DAS 2004), Broyden–Fletcher–Goldfarb–Shanno (BFGS) method (Geem 2006), Immune Clonal Selection Algorithm (ICSA) (Luo & Xie 2010), Parameter-Setting-Free and HS (PSF-HS) (Geem 2011), Differential Evolution (DE) (Xu *et al.* 2011), combining HS and BFGS method (HS-BFGS) (Karahan *et al.* 2012), Artificial Bee Colony method (ABC) (Vafakhah *et al.* 2015) and Ant Colony Optimization (ACO) (Zeinali & Pourreza-bilondi 2018). The mentioned methods are data-driven and the statistical distribution of data is not considered. It seems that by presenting a method according to the joint distribution of inflow and outflow hydrographs, the accuracy of results can be increased. The complex hydrological events such as flood, drought and storm are often characterized by a number of correlated random variables. Copula functions can model the dependence structure independently from the marginal distribution functions and create multivariate distributions with different margins and dependence structures.

In recent years, the copula functions have been used in simulation and modeling of meteorological and hydrological variables and their efficiency and accuracy have been confirmed. For example, Tahroudi *et al.* (2020a) introduced a new approach to simulate the occurrence of related variables based on the conditional density of copulas. The proposed approach was adopted to investigate the dynamics of hydrological and meteorological droughts in the Zarinehroud basin, Lake Urmia, Iran. Tahroudi *et al.* (2020b) used the conditional behavior of two signatures to analyzing the signatures of rainfall deficiency and groundwater-level deficiency in the Naqadeh sub-basin, Lake Urmia Basin, Iran, based on copulas. They showed that the presented conditional density function was an alternative method to the conditional return period. Nazeri *et al.* (2020) investigated the frequency analysis of the suspended sediment load of the Zarinehrood basin, Lake Urmia, Iran, given by the peak flow at the Chalekhmaz hydrometric station using copula functions. The results showed that the simulated suspended load is closer to the measured suspended load of the Chalekhmaz station in bivariate analysis. Copula-based models have recently become very popular due to the use of joint distribution and the involvement of effective parameters in equations. Copula-based simulations and modeling are also important due to the high accuracy of the simulations. This model has a high ability in simulating and modeling of the meteorological and hydrological parameters (Tahroudi *et al.* 2020a, 2020b). In this study, the accuracy of the copula-based approach in flood routing and flood hydrograph simulation was investigated and compared with previous researches. The proposed copula-based approach, by combining the conditional density of copula functions and the diagonal section of copula functions, attempts to simulate flood hydrographs such as the Wilson River flood (WRF) in 1974 (USA), the River Wye flood (RWF) in 1960 (England) and the Karun River flood (KRF) in 2008 (Iran). Flood routing in these rivers has already been studied by various researchers using various methods including the Muskingum method, meta-heuristic methods and optimization algorithms. The main objective of this study is to investigate the accuracy of the copula-based approach in flood routing based on the conditional density of the copula functions.

MATERIALS AND METHODS

Studied floods

In this study, three floods studied in previous researches including the WRF in 1974 (USA), the River Wye flood in 1960 (England) and the KRF in 2008 (Iran) were considered to investigate the accuracy of the proposed copula-based approach and its conditional density. This data set has also been extensively studied by others (Gill 1978; Tung 1985; Yoon & Padmanabhan 1993; Mohan 1997; Kim *et al.* 2001; Das 2004; Geem 2006, 2011; Luo & Xie 2010; Xu *et al.* 2011; Karahan 2012; Vafakhah *et al.* 2015; Zeinali & Pourreza-Bilondi 2018). Also these case studies present a pronounced nonlinear relationship between weighted flow and storage volume. The 69.75-km stretch of the River Wye from Erwood to Belmont has no tributaries and a very small lateral inflow. It is, thus, an excellent case study to demonstrate the use of flood-routing techniques (Natural Environment Research Council (NERC) 1975; Bajracharya & Barry 1997; Karahan *et al.* 2013). The Karun River basin is located in the southern part of Iran between longitudes of 48°15′ and 52°30′ east, latitudes of 30°17′ and 33°49′ north with a basin area of 67,000 km^2. Flood data from 30 November 2008 to 3 December 2008 are considered for the purpose (Vafakhah *et al.* 2015). Using the proposed approach, the outflow hydrographs of the studied floods were simulated. The hydrographs of the studied floods are presented in Figure 1.

Copula functions

The introduction of copulas is attributed to Sklar (1959), which describes in a theorem how 1-D distribution functions can be combined in the form of multivariate distributions. For 2-D continuous random variables X_1 and X_2 with marginal distribution functions $F_{X_1(x_1)}$ and $F_{X_2(x_2)}$, the joint distribution of the variables can be expressed as follows:

$$H_{X_1,X_2}(x_1, x_2) = P[X_1 \le x_1, X_2 \le x_2] \quad (1)$$

Copula is a function that joins the univariate marginal distribution functions to create a bivariate or multivariate distribution function. Thus, Sklar (1959) described that the probability multivariate distribution of H using the marginal distribution functions and dependence structure can be represented by the copula function C:

$$C(F_{X_1}(x_1), F_{X_2}(x_2)) = H_{X_1,X_2}(x_1, x_2) \quad (2)$$

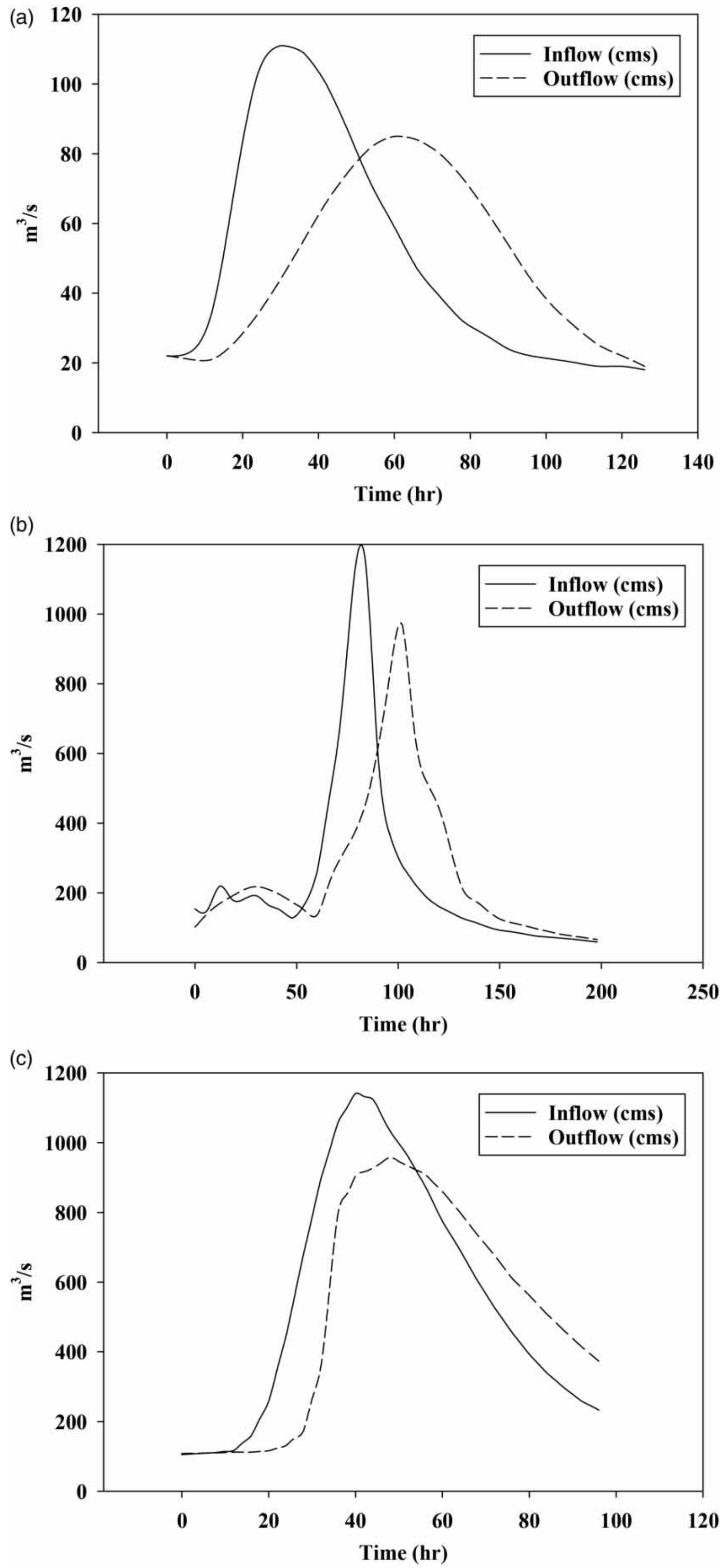


Figure 1 | Outflow and inflow hydrographs: (a) WRF, (b) RWF and (c) KRF.

Since the cumulative marginal distribution functions for continuous random variables are non-decreasing from zero to one, the copula of C can be considered as a transform H_{X_1,X_2} from $(-\infty, +\infty)^2$ to $[0, 1]^2$. This transformation divides marginal distribution functions. Therefore, the copula function of C only relates to the relationship between the variables and a complete description of the dependence structure achieved (Nelsen 2006; Salvadori *et al.* 2007). For 2-D copulas, Sklar's theorem is as follows.

Suppose H is the joint distribution of variables X_1 and X_2 with cumulative distributions $u = F_{X_1}(x_1)$ and $v = F_{X_2}(x_2)$. There exists a 2-D copula in the set of real numbers and is shown in the following equation:

$$H(x_1, x_2) = C(u, v) = C(F_{X_1}(x_1), F_{X_2}(x_2)) \tag{3}$$

The 2-D copula function has the following properties:

A. For each u and v in [0, 1]:

$$C(u, 0) = C(0, v) = 0 \tag{4}$$

$$C(u, 1) = u, \quad C(1, v) = v \tag{5}$$

This feature is called the boundary condition of the 2-D copula. Considering these boundary conditions, it can be concluded from Equation (3) that if one of the marginal distribution functions has a value of zero, then the value of the copula function is zero (same conclusion for Equation (5)).

B. For u_1, u_2 $(u_1 \leq u_2)$ in U_1 and v_1, v_2 $(v_1 \leq v_2)$ in U_2, this non-equation is obtained:

$$C(u_2, v_2) - C(u_2, u_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0 \tag{6}$$

Before applying the copulas, the dependence between the variables must be investigated. There are some coefficients for evaluating the dependence structure such as the Pearson coefficient, Kendall's tau (τ) and Spearman's rank correlation (ρ). To overcome the problems presented by the Pearson coefficient, some non-parametric measures such as Kendall's τ and Spearman's rank correlation (ρ) have been applied. The main advantage of using the Kendall's τ over the Spearman and Pearson coefficients is that the Kendall's τ method can interpret its value as a direct measurement of concordance and discordance pairs. The disadvantages related to the Pearson coefficient are: (1) it measures the linearity between variables and (2) it exists only if the standard deviations of the two variables exist finite. In addition, they cannot be used to diagnose dependency when involved with more than two variables (see also Gauthier 2001; De Michele *et al.* 2005; Nazeri Tahroudi *et al.* 2021). In this study, the Kendall's τ is applied to assess the dependence between the two variables. For more information about copula functions, see the following references (De Michele *et al.* 2005; Nelsen 2006; Salvadori & De Michele 2007; Salvadori *et al.* 2007; Mirabbasi *et al.* 2012; Ramezani *et al.* 2019; Khozeymehnezhad & Nazeri-Tahroudi 2020).

Copula-based simulation

Copula-based simulations were first discussed in Bedford & Cooke (2001) as well as Bedford & Cooke (2002). To obtain the sample $u_1, \ldots, u_d$ from a d dimensional copula, the following steps are performed:

$$w_j \overset{i,i,d}{\sim} U[0; 1], \quad j = 1, \ldots, d \tag{7}$$

Then,

$$\begin{aligned} u_1 &= w_1 \\ u_2 &:= C_{2|1}^{-1}(w_2|u_1) \\ &\vdots \\ u_d &:= C_{d|d-1,\ldots,1}^{-1}(w_d|u_{d-1}, \ldots, u_1) \end{aligned} \tag{8}$$

To determine the conditional distribution functions $C_{j|j-1,\ldots,1},\ j=1,\ \ldots,d$ required for the pair copula structure, a feedback relation for the conditional distribution function with h function is used. For a bivariate copula with parameter θ_{ij}, the h functions are defined as follows:

$$h_{i\backslash j}(u_i|u_j;\ \theta_{ij}) := \frac{\partial}{\partial u_j} C_{ij}(u_i,\ u_j;\ \theta_{ij}) \tag{9}$$

$$h_{j\backslash i}(u_j|u_i;\ \theta_{ij}) := \frac{\partial}{\partial u_i} C_{ij}(u_i,\ u_j;\ \theta_{ij}) \tag{10}$$

Model performance

To evaluate the performance of the approach, the root-mean-square error (RMSE), the sum of squared errors (SSE) and the Nash–Sutcliffe efficiency coefficient (NSE) were used.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(\widehat{Q}_i - Q_i)^2}{n}} \tag{11}$$

$$\mathrm{SSE} = \sum_{i=1}^{n}(\widehat{Q}_i - Q_i)^2 \tag{12}$$

$$\mathrm{NSE} = 1 - \frac{\sum_{i=1}^{N}(\widehat{Q}_i - Q_i)^2}{\sum_{i=1}^{N}(Q_i - \bar{Q}_i)^2} \tag{13}$$

Lower RMSE and SSE, and higher NSE indicate higher accuracy of the model.

In the above relations, Q_i, $\widehat{Q}_i$ and $\bar{Q}_i$ are the measured, simulated and mean discharges of the outflow hydrograph, respectively, and n is the number of data (Tahroudi *et al.* 2019; Akbarpour *et al.* 2020; Shahidi *et al.* 2020).

RESULTS AND DISCUSSION

Here, the dependence between inflow and outflow hydrographs of the case studies were investigated using the Kendall's τ. The dependence results for WRF, RWF and KRF along with data scatter are presented in Figure 2.

In Figure 2, I is the inflow hydrograph and O the outflow hydrograph. Figure 2 provides the data histogram in the upper left and lower right panels: the upper right panel gives the value of the Kendall's τ, and the lower left panel gives the empirical contour lines. The highest dependence is related to KRF with 0.82, and the lowest value is related to WRF with 0.34. These values are all statistically significant according to the independence test with a confidence level of 95%. According to these results, it is possible to use copula functions for the joint analysis and simulation of outflow hydrographs. Each copula function is capable of modeling a particular range of dependencies. Some are suitable for weak dependencies and some can model the whole range of dependencies. For example, the Gumbel–Hougaard copula can only be used for positive dependence. The Ali-Mikhail-Haq copula is suitable only for weak dependence ($-0.1807 < \tau < 0.3333$), and the Farlie–Gumbel–Morgenstern copula is suitable for $-2.9 < \tau < 2.9$, while the Clayton and Frank copulas are suitable for both positive and negative dependencies (Nelsen 2006; Salvadori *et al.* 2007).

Selection of the copula function

In this study, copula functions were applied to the simulation and modeling of the outflow hydrographs. In this regard, different copula functions (Clayton, Ali-Mikhail-Haq, Farlie–Gumbel–Morgenstern, Frank, Gumbel, Gumbel–Hougaard, Placket, Gaussian and Joe) and their rotational states were examined. For this purpose, the combination of conditional density of bivariate copulas with the diagonal section of the copula was used. While examining different copula functions, the copula was selected based on Bayesian information criterion (BIC), Akaike information criterion (AIC) and log-likelihood

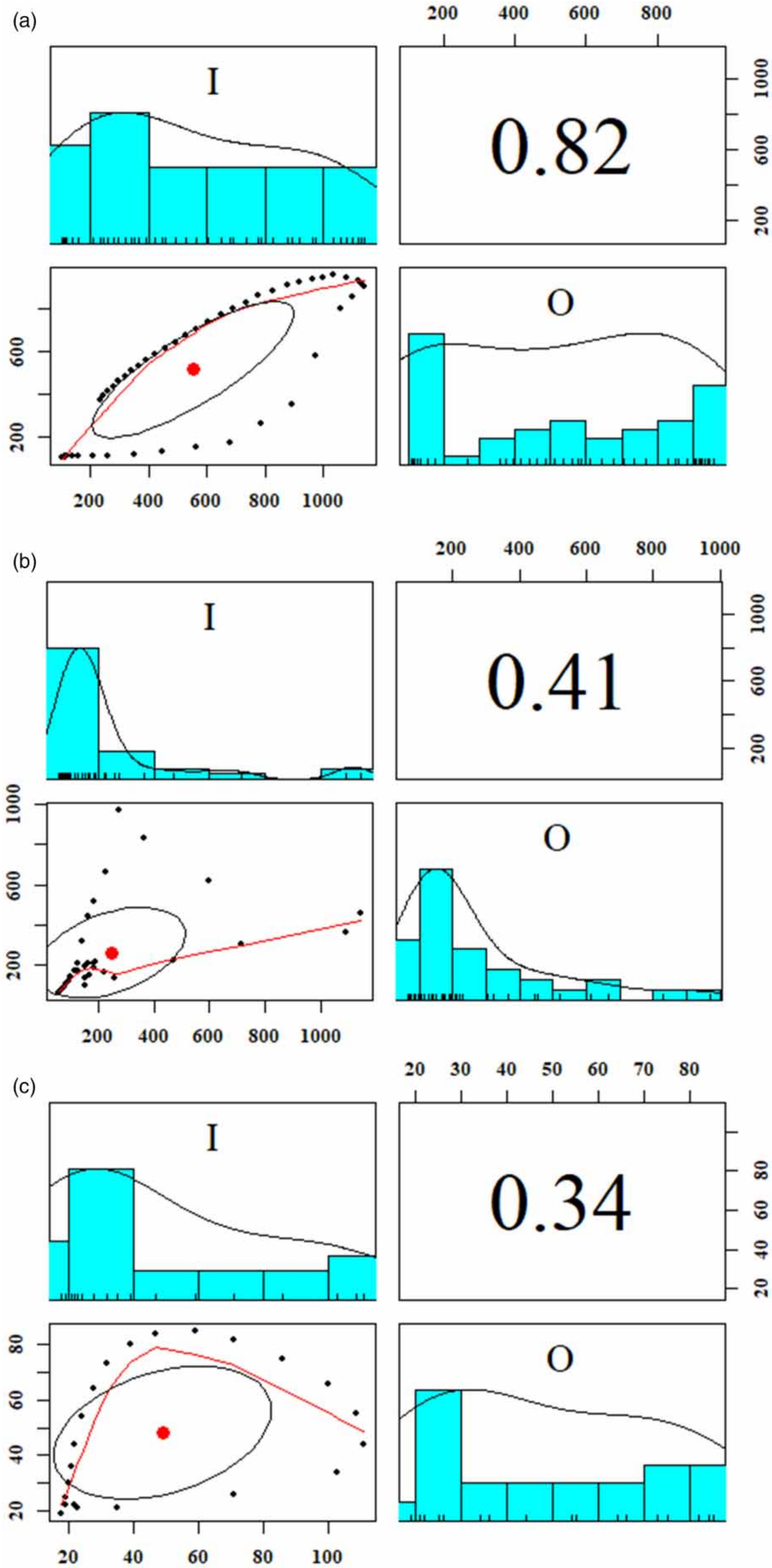


Figure 2 | Kendall's τ, empirical contour lines and histogram of KRF (a), RWF (b) and WRF (c).

(Log-Like) criteria. The results of the examining different copula functions or joint analysis of KRF (inflow and outflow hydrographs), Clayton copula with a dependence parameter of 4.90 and AIC, BIC and Log-Like values of −86.1, −84.2 and 44, respectively, was selected. For RWF, Gaussian copula with a dependence parameter of 0.85 and AIC, BIC and Log-Like values of −35.8, −34.2 and 18.9, respectively, and for WRF also Gaussian copula with a dependence parameter of 0.61 and corresponding evaluation criteria of −527, −4.18 and 3.64 were selected. According to the results, it can be concluded that the Clayton and Gaussian copulas have the best performance for the studied floods. This is due to differences in the marginal functions. In general, the type of selected copula function depends on the dependence structure of the river flood, and for different rivers, different copula functions may be specified as the best fitted one.

After selecting the best fitted copulas to describe the dependence structure between the variables of the inflow and outflow hydrographs, these functions can be used to estimate the conditional density of copulas to evaluate the conditional state of the considered variables. The graphs of conditional density ($c(u,v)$) were studied using the second derivative of $C(u|v)$ and diagonal section of copulas for the studied variables.

In this case, u represents the inflow hydrograph (I) and v also represents the outflow hydrograph (O) at the copula scale. For a certain amount of u, a graph of different values of v is provided. The maximum value of this graph is the O in copula scale. The reason for using the diagonal section of the copula was to reduce the computational complexity. These steps were implemented for WRF, RWF and KRF. Figure 3 for c=0.5 is given as an example. In this figure, the copula values were calculated for the different values of c and fixed values of outflow hydrograph (O). The maximum value of x-axis equals to the expected value in outflow hydrograph (O) and selected as predicted one. These stages were calculated for all values of c to obtain a new series of conditional outflow hydrograph (O). The calculated new outflow hydrograph (O) values indicate the conditional state of outflow hydrograph (O) corresponding to the inflow hydrograph (I).

Simulation of the outflow hydrograph of the WRF

Inflow and outflow hydrographs of the WRF were analyzed according to the conditional density of the copula functions and using the selected copula, the outflow hydrograph of the WRF was simulated. Corresponding to each input data, output data were simulated using the copula-based approach and its conditional density. The results of the simulation of the outflow hydrograph of the WRF are presented in Figure 4.

The results of the simulation indicated that the simulated outflow hydrograph of the WRF lays between the 95% confidence interval. These results indicate the acceptable accuracy of the copula-based approach in the simulation of the outflow hydrograph. Also, as can be seen in Figure 4, the simulated outflow hydrograph of the WRF fit well with the measured data. The proposed copula-based approach has a higher accuracy than the different univariate and multivariate models due to the use of the marginal distribution of the studied variables. The connection of the statistical distribution of the studied variables increases the certainty of the simulations. The accuracy of the proposed approach in simulating and modeling of the different variables has been confirmed in various researches such as Tahroudi *et al.* (2020a, 2020b). To compare and evaluate the performance of the copula-based approach, the obtained results in this study were examined with the results of other researchers for the studied floods. In the case of the WRF, various researchers have examined different methods for estimating the outflow hydrograph. The most studies in this field have used different optimization methods to estimate the coefficients of the

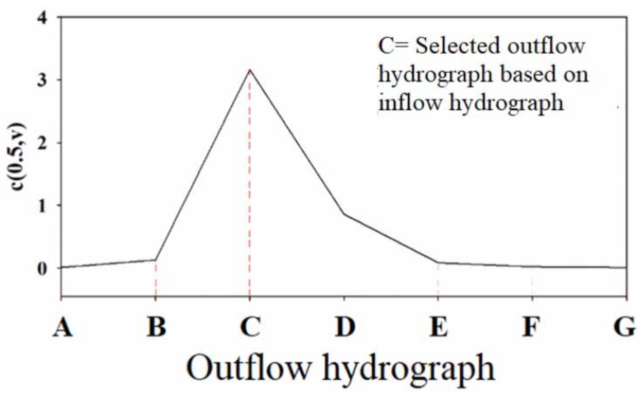


Figure 3 | Sketch map of conditional density of outflow hydrograph (O) for c=0.5.

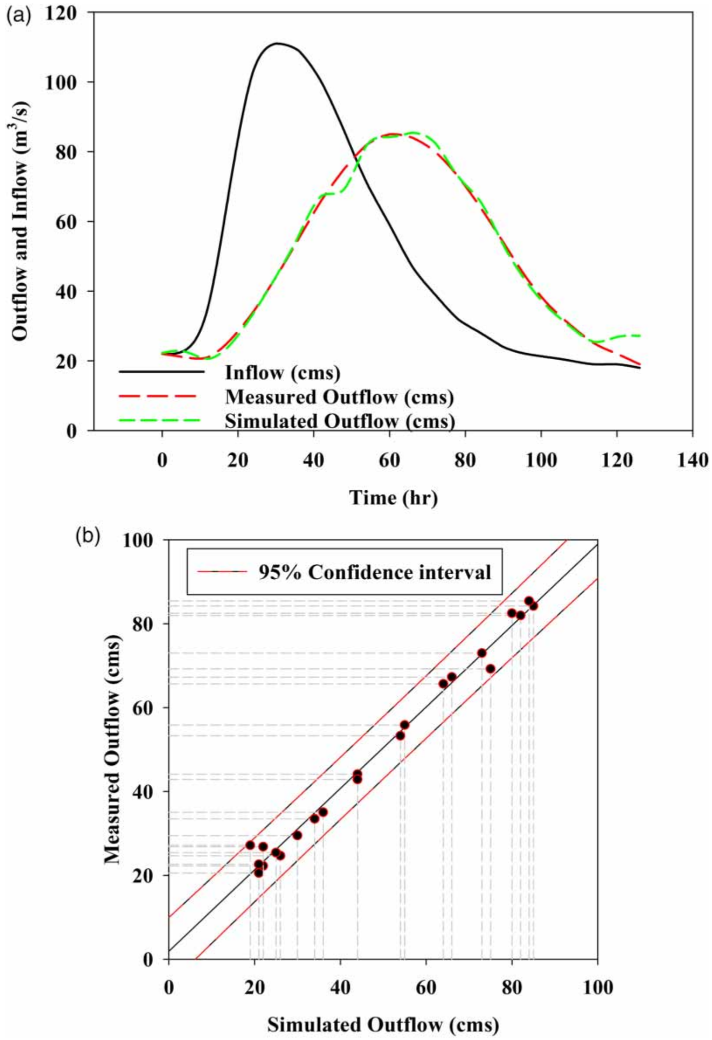


Figure 4 | Simulation results of the outflow hydrograph of the WRF (a) and 95% confidence interval of the simulation results (b) using the proposed copula-based approach.

Muskingum method, all of which have led to the simulation and modeling of the outflow hydrographs. The results of comparing the performance of the proposed copula-based approach with other pervious researches are presented in Table 1. Abbreviations are introduced in Supplementary Appendix A.

As can be seen from Table 1, with the exception of the LSM, NL-LSM and LMM methods, the SSE value in the other methods varies between 36.3 and 45.6. Based on the Table 1, the SSE of the proposed copula-based approach to simulate the outflow hydrograph of the WRF is equal to 145.57. According to the SSE, the copula-based approach has higher accuracy compared to the LSM and NL-LSM models (mentioned in Table 1), but has lower accuracy than other considered models. However, the performance of the copula-based approach, according to NSE=0.99 and RMSE=2.57 m^3/s, is acceptable and shows a good performance. As can be seen from Figure (4a), the copula-based approach has been able to simulate the peak flow and the rising and falling limbs of the outflow hydrograph of the WRF well. Also, all simulated data are at the 95% confidence interval (Figure (4b)), which indicate the high performance of the proposed approach.

Simulation of the outflow hydrograph of the RWF

The outflow hydrograph of the RWF was also simulated using the proposed copula-based approach. The simulation results of the outflow hydrograph of the RWF are shown in Figure 5.

The results of Figure 5 show visually that the proposed copula-based approach is able to simulate the outflow hydrograph of the RWF. The peak flow of the outflow hydrograph is well simulated. Exception of one data, other simulated data are laid at

Table 1 | Results of the previous researches on the WRF, RWF and KRF comparison with the proposed copula-based approach

Case study	Researcher	Used method: Muskingum+	SSE
WRF	Gill (1978)	LSM	145.96
SSE (proposed copula-based approach)=145.57	Tung (1985)	HJ+DFP	45.61
	Yoon & Padmanabhan (1993)	NL-LSM	156.44
	Mohan (1997)	GA	38.24
	Kim *et al.* (2001)	HS	36.30
	Das (2004)	LMM	130.49
	Geem (2006)	BFGS	36.77
	Luo & Xie (2010)	ICSA	36.80
	Geem (2011)	PSF-HS	36.77
	Xu *et al.* (2011)	DE	36.77
	Karahan *et al.* (2012)	HS-BFGS	36.77
	Vafakhah *et al.* (2015)	ABC	35.62
	Zeinali & Pourreza-Bilondi (2018)	ACO	36.77
RWF	Karahan *et al.* (2012)	HS-BFGS	37,944.14
SSE (proposed copula-based approach) =12,968.34	Zeinali & Pourreza-Bilondi (2018)	ACO	37,944.15
KRF	Vafakhah *et al.* (2015)	ABC	177,161.40
SSE (proposed copula-based approach) =10,655.55	Zeinali & Pourreza-Bilondi (2018)	ACO	144,691.73

the 95% confidence interval. The proposed copula-based approach has been able to establish a good relationship between the inflow and outflow hydrographs of the RWF and simulate the outflow hydrograph. The RWF has also been studied in previous researches by different methods. The results of comparing the performance of the proposed copula-based approach with other Muskingum-based models are presented in Table 1.

The two studies presented in Table 1 used the Muskingum-based method and the optimization algorithms. The SSE for the simulation of the outflow hydrograph of the RWF using the copula-based approach is obtained as 12,968.34, which shows better performance than the two studies with SSE of 37,944.

Simulation of the outflow hydrograph of the KRF

Finally, the last flood is KRF. As with other studied floods, the copula-based approach and its conditional density were implemented. The simulation results of the outflow hydrograph of the KRF are presented in Figure 6. The proposed copula-based approach simulates the outflow hydrograph of the KRF and its peak flow well. All simulated points are at the 95% confidence interval (Figure 6(b)). According to Figure 6, the accuracy of the proposed approach in the simulation of the outflow hydrograph of the KRF is confirmed. The results of comparing the proposed approach with the results of other previous researches in the simulation of the outflow hydrograph of the KRF are presented in Table 1.

The SSE for the simulation of the outflow hydrograph of the KRF using the copula-based approach is obtained as 10,655.65, which shows much better performance than the two studies (Vafakhah *et al.* (2015) with an SSE value of 177,161.40 and Zeinli & Pourreza-Bilondi (2018) with an SSE value of 144,691.73). The RMSE values of WRF, RWF and KRF are 2.57, 19.53 and 14.75 m^3/s, respectively. Therefore, the accuracy of the simulation in all three case studies is confirmed. In this study, the violin plot was used to evaluate the certainty of the proposed approach. The violin plots related to the simulation of the outflow hydrograph of the studied floods are presented in Figure 7.

According to the results of the violin plot, it can be seen that the copula-based approach has been able to cover the range of data changes, and quarters one and three. As can be seen from Figure 7, the range of changes in the studied floods, as well as 5 and 95% of the values simulated by the copula-based approach, is closer to the measured values, which confirms the reliability of the approach. Since the simulated value of peak flow is so important in flood warning systems, thus the simulated peak flow of flood routing should not be much different from the measured peak flow. On the other hand, there is always a possibility of over-estimation or under-estimation in the simulation of the outflow hydrograph; therefore, it is necessary to choose a model that has the highest accuracy. According to the results presented in this study and comparing with other

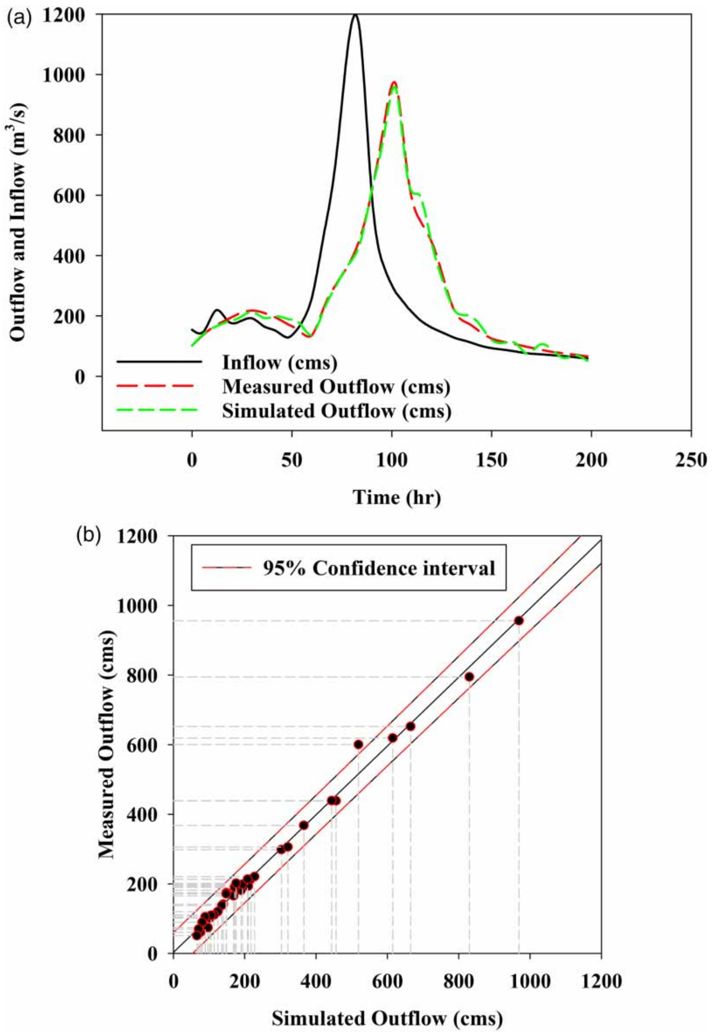


Figure 5 | Simulation results of the outflow hydrograph of the RWF (a) and 95% confidence interval of the simulation results (b) using the proposed copula-based approach.

studies implemented by different methods, it can be concluded that the proposed approach based on conditional density and diagonal section of copulas is a suitable approach to simulate the outflow hydrograph of the flood.

In addition, in order to evaluate the accuracy of the copula-based approach, the histogram of the measured and simulated values was also examined and the results were presented in Figure 8. The results of Figure 8 showed that the histogram of the simulated and measured values are similar and the contour lines and scattering of these values are the same. Figure 8 also shows a high correlation between the simulated and measured data.

CONCLUSION

In this study, the accuracy of the proposed copula-based approach and its conditional density in flood routing was investigated. The performance of the proposed approach was evaluated considering some floods used in the literature: WRF, RWF and KRF. The results of the simulation of outflow hydrograph for the three case studies showed that the accuracy and performance of the proposed copula approach are acceptable according to the 95% confidence interval and the comparison of violin plots. The simulated outflow hydrographs fit well the measured outflow hydrographs and the hydrograph peak flows were well simulated, which are the most important parameters in the design of hydraulic structures. The performances of the copula-based approach, according to the NSE, are satisfying showing a value of 0.99 in all three case studies. This is also confirmed by the comparison of violin plots between measured and simulated outflows. These make us confident in the

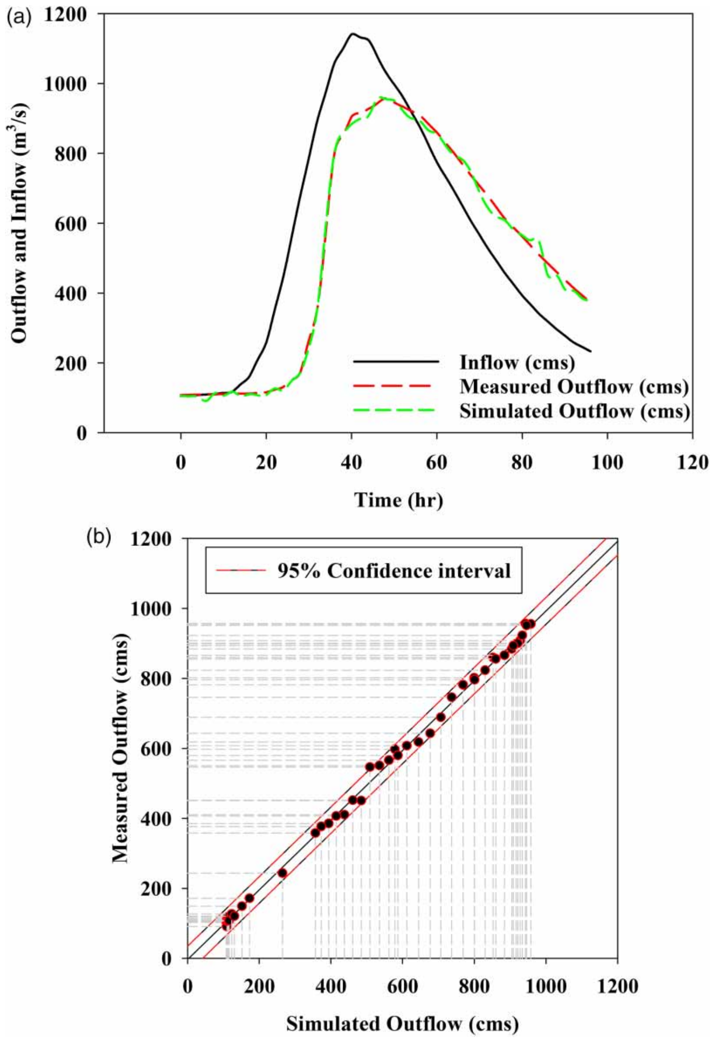


Figure 6 | Simulation results of the outflow hydrograph of the KRF (a) and 95% confidence interval of the simulation results (b) using the proposed copula-based approach.

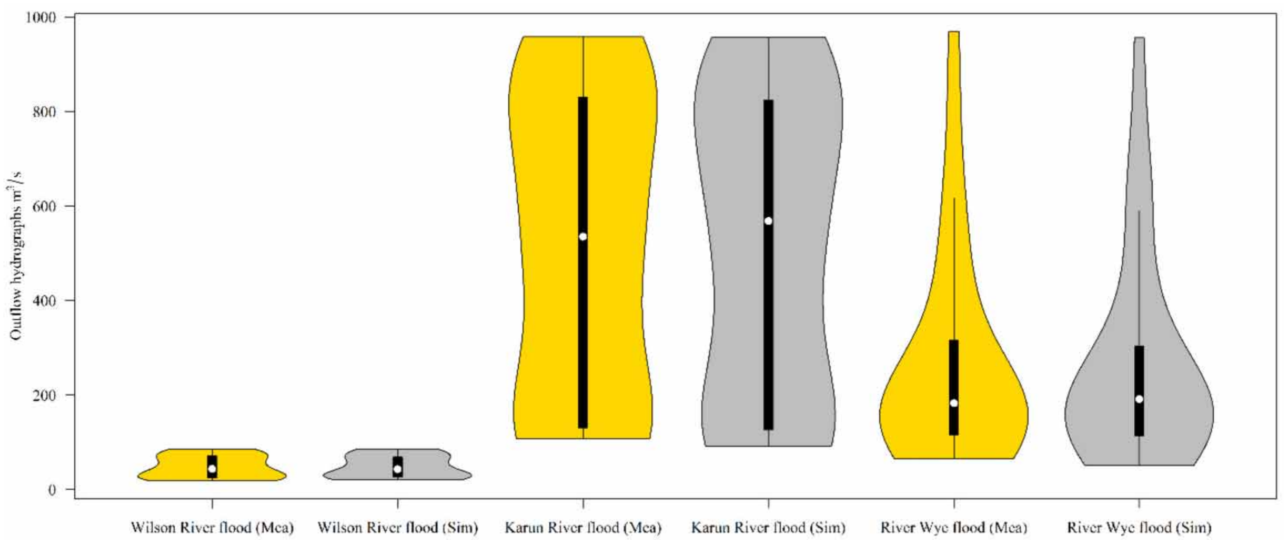


Figure 7 | Violin plots of measured (Mea) and simulated (Sim) outflow hydrographs for the case studies.

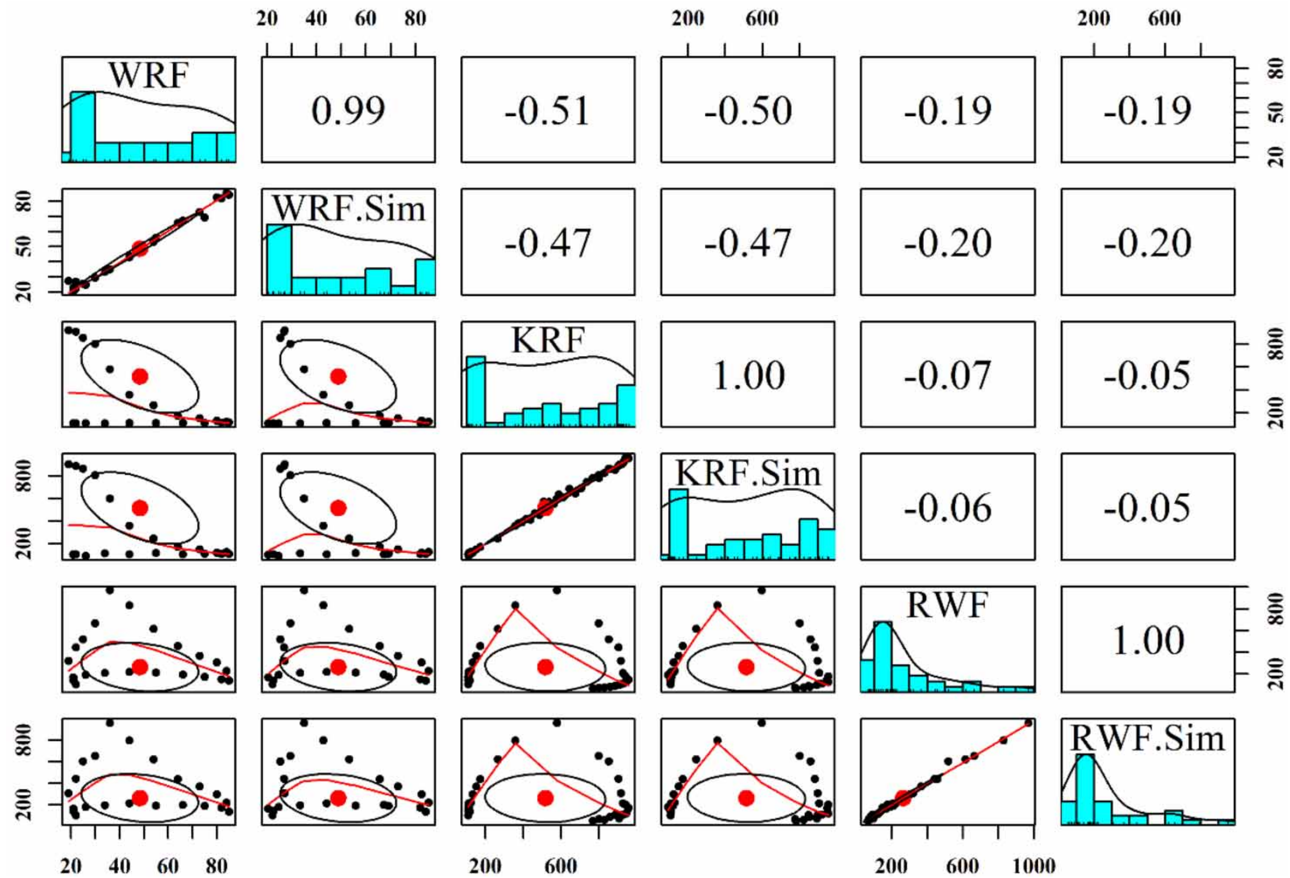


Figure 8 | Histogram, contour lines and correlation of measured and simulated (Sim) outflow hydrographs for the case studies.

application of the proposed approach in other case studies. Due to the fact that the data-driven models, such as artificial intelligence, rely only on data, while the copula-based approach considers the dependence structure of data and uses the joint and marginal distributions of data, and it is more reliable than data-driven models. The results also showed that there is no limitation to the number of data used in this approach. The same sample size is not necessary for comparison. In addition to the presented results, the results of correlation and histogram analysis of the simulated and measured data show that the histogram of the pair variables in all three case studies is similar.

ACKNOWLEDGEMENTS

The authors would like to thank the Politecnico di Milano for providing the facilities to the first author as a visiting researcher. Also, the authors would like to thank the Iran Water Resources Management Company for providing the data.

DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

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First received 18 December 2020; accepted in revised form 2 September 2021. Available online 17 September 2021