



Hypergroups associated with HX -groups

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Abstract

We have already studied a correspondence between HX -groups and hypergroups. Now we extend this correspondence to semigroups of subsets of a group G (and to begin, we considered $\mathbf{Z}_2 \times \mathbf{Z}_3$ as group G). Moreover these subsets can have non empty intersection, they can be contained, one in another one. One calculates the fuzzy grade and we find always $\partial H = 1$.

1 Introduction

In the classic theory of HX -groups, one supposes that the structure from which one starts, is a group G , which is a subgroup of the semigroup $\mathcal{P}^*(K)$ of nonempty subsets of a group K , see for instance [3], [20], [21], [22], [23], [33], [34] and often one supposes also that

$$(1) \quad \forall (A, B) \in \mathcal{P}^*(K) \times \mathcal{P}^*(K), A \neq B \Rightarrow A \cap B = \emptyset.$$

In this paper one has not done always these hypotheses.

In a precedent paper [8] one considered the HX -groups associated with $\mathbf{Z}/n\mathbf{Z}$ for $n \in \{8, 9, 12, 15, 16\}$.

In this paper, we calculate those associated with $\mathbf{Z}_2 \times \mathbf{Z}_3$, without supposing always satisfied the hypothesis (1). In this new situation, we show that the fuzzy grade is always equal to one.

Let us see what means a "fuzzy grade".

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Let (H_0, \circ) be a hypergroupoid.
 Set $\forall(x, y) \in H^2, \forall u \in x \circ y,$

$$m_{x,y}(u) = 1/|x \circ y|, \quad A_1(u) = \sum_{(x,y \in H^2)} m_{x,y}(u)$$

$$Q_1(u) = \{(a, b) \mid u \in a \circ b\}, \quad q_1(U) = |Q_1(U)| \quad \text{and} \quad \mu_1(u) = A_1(u)/q_1(u).$$

Set now $\forall(x, y) \in H^2,$

$$x \circ_1 y = \{z \mid \min\{\mu_1(x), \mu_1(y)\} \leq \mu_1(z) \leq \max\{\mu_1(x), \mu_1(y)\}\}$$

and set denote by H_1 the hypergroupoid (H_0, \circ_1) (which is a join space, see [4]). One goes on in the same way and from H_1 one obtains A_2, q_2, μ_2 defined similarly as A_1, q_1, μ_1 , but starting from H_1 . One obtains another join space (H_2, \circ_2) and so on. The *fuzzy grade* of H_0 , denoted by ∂H_0 is the least k , such that H_k is isomorphic to H_{k+1} .

2 HX -groups on finite groups

If (G, \cdot) is a group and $\mathcal{P}^*(G)$ is the set of all non empty subsets of G then an HX -group is a nonempty subset H of $\mathcal{P}^*(G)$, which is a group, with respect to the hyperoperation (see [3], [20], [22], [21], [23], [33], [34]):

$$\forall(A, B) \in \mathcal{P}^*(G) \times \mathcal{P}^*(G), \quad A \cdot B = \{xy \mid x \in A, y \in B\}.$$

Let us consider $A_0 = \mathbf{Z}_2 \times \mathbf{Z}_3 = \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2)\}$.
 Set

$$B_0 = \{(0, 0), (1, 0)\}, \quad B_1 = \{(0, 0), (0, 1), (0, 2)\}, \quad B_2 = \{(0, 0), (0, 2)\},$$

$$B_3 = \{(0, 1), (1, 1)\}, \quad B_4 = \{(0, 2), (1, 2)\}, \quad B_5 = \{(1, 0), (1, 1), (1, 2)\}.$$

We find

$B_{1,5}$	B_1	B_5	$B_{0,3,4}$	B_0	B_3	B_4
B_1	B_1	B_5	B_0	B_0	B_3	B_4
B_5	B_5	B_1	B_3		B_4	B_0
			B_4			B_3

From $B_{1,5}$ we obtain:

$B_{1,5}$	(0, 0)	(0, 1)	(0, 2)	(1, 0)	(1, 1)	(1, 2)
(0, 0)	B_1	B_1	B_1	B_5	B_5	B_5
(0, 1)		B_1	B_1	B_5	B_5	B_5
(0, 2)			B_1	B_5	B_5	B_5
(1, 0)				B_1	B_1	B_1
(1, 1)					B_1	B_1
(1, 2)						B_1

$B_6 = \{(0, 0), (1, 0), (0, 2), (1, 2)\}$, $B_7 = \{(0, 1), (0, 2), (1, 1), (1, 2)\}$,
 $B_8 = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$.
 $B_{1,5}$ can be written also

$H_{1,5}^0$	(0, 0)	(0, 1)	(0, 2)	(1, 0)	(1, 1)	(1, 2)
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1, 0)	(1, 0)	(1, 0)
	(0, 1)	(0, 1)	(0, 1)	(1, 1)	(1, 1)	((1, 1)
	(0, 2)	(0, 2)	(0, 2)	(1, 2)	(1, 2)	(1, 2)
(0, 1)		(0, 0)	(0, 0)	(1, 0)	(1, 0)	(1, 0)
		(0, 1)	(0, 1)	(1, 1)	(1, 1)	((1, 1)
		(0, 2)	(0, 2)	(1, 2)	(1, 2)	(1, 2)
(0, 2)			(0, 0)	(1, 0)	(1, 0)	(1, 0)
			(0, 1)	(1, 1)	(1, 1)	((1, 1)
			(0, 2)	(1, 2)	(1, 2)	(1, 2)
(1, 0)				(0, 0)	(0, 0)	(0, 0)
				(0, 1)	(0, 1)	((0, 1)
				(0, 2)	(1, 2)	(1, 2)
(1, 1)					(0, 0)	(0, 0)
					(0, 1)	((0, 1)
					(0, 2)	(1, 2)
(1, 2)						(0, 0) ((0, 1) (0, 2)

We deduce $A_1(0, 0) = A_1(0, 1) = A_1(0, 2) = 29/3$,
 $q_1(0, 0) = q_1(0, 1) = q_1(0, 2) = 18$,
 $\mu_1(0, 0) = \mu_1(0, 1) = \mu_1(0, 2) = 1/3$.
 $A_1(1, 0) = A_1(1, 1) = A_1(1, 2) = 18/3$,
 $q_1(1, 0) = q_1(1, 1) = q_1(1, 2) = 18$,
 $\mu_1(1, 0) = \mu_1(1, 1) = \mu_1(1, 2) = 1/3$.
 It follows that $\partial H_{1,5}^0 = 1$.

$B_{2,1,5}$	B_2	B_1	B_5
B_2	B_1	B_1	B_5
B_1	B_1	B_1	B_5
B_5	B_5	B_5	B_1

One obtains $A_1(0, 0) = A_1(0, 1) = A_1(0, 2) = 25/3 + 9/3 = 34/3$,
 $A_1(1, 0) = A_1(1, 1) = A_1(1, 2) = 30/3$.
 $q_1(0, 0) = q_1(0, 1) = q_1(0, 2) = 34$, $q_1(1, 0) = q_1(1, 1) = q_1(1, 2) = 30$.

$H_{2,1,5}$	(0, 0)	(0, 2)	(0, 0)	(0, 1)	(0, 2)	(1, 0)	(1, 1)	(1, 2)
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1, 0)	(1, 0)	(1, 0)
	(0, 2)	(0, 2)	(0, 2)	(0, 1)	(0, 1)	(1, 1)	(1, 1)	(1, 1)
	(0, 1)	(0, 1)	(0, 1)	(0, 2)	(0, 2)	(1, 2)	(1, 2)	(1, 2)
(0, 2)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1, 0)	(1, 0)	(1, 0)
	(0, 2)	(0, 2)	(0, 2)	(0, 1)	(0, 1)	(1, 1)	(1, 1)	(1, 1)
	(0, 1)	(0, 1)	(0, 1)	(0, 2)	(0, 2)	(1, 2)	(1, 2)	(1, 2)
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1, 0)	(1, 0)	(1, 0)
	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(1, 1)	(1, 1)	(1, 1)
	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(1, 2)	(1, 2)	(1, 2)
(0, 1)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1, 0)	(1, 0)	(1, 0)
	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(1, 1)	(1, 1)	(1, 1)
	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(1, 2)	(1, 2)	(1, 2)
(0, 2)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(1, 0)	(1, 0)	(1, 0)
	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(0, 1)	(1, 1)	(1, 1)	(1, 1)
	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(0, 2)	(1, 2)	(1, 2)	(1, 2)
(1, 0)	(1, 0)	(1, 0)	(1, 0)	(1, 0)	(1, 0)	(0, 0)	(0, 0)	(0, 0)
	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(0, 1)	(0, 1)	(0, 1)
	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(0, 2)	(0, 2)	(0, 2)
(1, 1)	(1, 0)	(1, 0)	(1, 0)	(1, 0)	(1, 0)	(0, 0)	(0, 0)	(0, 0)
	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(0, 1)	(0, 1)	(0, 1)
	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(0, 2)	(0, 2)	(0, 2)
(1, 2)	(1, 0)	(1, 0)	(1, 0)	(1, 0)	(1, 0)	(0, 0)	(0, 0)	(0, 0)
	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(0, 1)	(0, 1)	(0, 1)
	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(0, 2)	(0, 2)	(0, 2)

By consequence, $\mu_1(i, j) = 1/3, \forall(i, j)$ so $H_1 = T$ (the total hypergroup),
whence $\partial(H_{2,1,5}) = 1$.

From $B_{0,3,4}$ we obtain

$H_{0,3,4}^1$	(0, 0)	(1, 0)	(0, 1)	(0, 2)	(1, 1)	(1, 2)
(0, 0)	B_0	B_0	B_3	B_4	B_3	B_4
(1, 0)		B_0	B_3	B_4	B_3	B_4
(0, 1)			B_4	B_0	B_4	B_0
(0, 2)				B_3	B_0	B_3
(1, 1)					B_4	B_0
(1, 2)						B_3

$H_{0,3,4}^1$ can be written also in the following way:

$H_{0,3,4}^0$	(0, 0)	(1, 0)	(0, 1)	(0, 2)	(1, 1)	(1, 2)
(0, 0)	(0, 0)	(0, 0)	(0, 1)	(1, 2)	(0, 1)	(1, 2)
	(1, 0)	(1, 0)	(1, 1)	(0, 2)	(1, 1)	((0, 2)
(1, 0)		(0, 0)	(0, 1)	(1, 2)	(0, 1)	(1, 2)
		(1, 0)	(1, 1)	(0, 2)	(1, 1)	((0, 2)
(0, 1)			(1, 2)	(0, 0)	(1, 2)	(0, 0)
			(0, 2)	(1, 0)	(0, 2)	(1, 0)
(0, 2)				(0, 1)	(0, 0)	(0, 1)
				(1, 1)	(1, 0)	(1, 1)
(1, 1)					(0, 2)	(0, 0)
					(1, 2)	(1, 0)
(1, 2)						(0, 1)
						(1, 1)

$$A_1(0, 0) = 2/2 + 8/2 = A_1(1, 0) = 5,$$

$$A_1(0, 1) = 2/2 + 10/2 = A_1(1, 1) = 6,$$

$$A_1(0, 2) = 2/2 + 10/2 = A_1(1, 2) = 6.$$

We obtain $q_1(0, 0) = q_1(1, 0) = 10$, $q_1(0, 1) = q_1(1, 1) = q_1(0, 2) = q_1(1, 2) = 12$, whence

$$\mu_1(0, 0) = 1/2, \mu_1(1, 0) = 1/2$$

$\mu_1(0, 1) = \mu_1(1, 1) = \mu_1(0, 2) = \mu_1(1, 2) = 1/2$. Therefore, H^1 is total, whence $\partial(H_{0,3,4}^0) = 1$.

One considers now the semigroup constructed on the set

$$\{B_i \mid 0 \leq i \leq 8\} \cup \{A_0\}.$$

$$B_1 = \{(0, 0), (0, 1), (0, 2)\},$$

$$B_2 = \{(0, 0), (0, 2)\},$$

$$B_5 = \{(1, 0), (1, 1), (1, 2)\},$$

$$B_0 = \{(0, 0), (1, 0)\},$$

$$B_3 = \{(0, 1), (1, 1)\},$$

$$\begin{aligned}
B_4 &= \{(0, 2), (1, 2)\}, \\
B_6 &= \{(0, 0), (1, 0), (0, 2), (1, 2)\}, \\
B_7 &= \{(0, 1), (0, 2), (1, 1), (1, 2)\}, \\
B_8 &= \{(0, 0), (0, 1), (1, 0), (1, 1)\}.
\end{aligned}$$

We have

$$\begin{aligned}
B_1B_6 &= \{(0, 0), (0, 1), (0, 2)\} \cdot \{(0, 0), (1, 0), (0, 2), (1, 2)\} = \\
&\{(0, 0), (1, 0), (0, 2), (1, 1) \mid (0, 1), (1, 1), (0, 0), (1, 0)\} = \\
&\{(0, 2), (1, 2), (0, 1), (1, 1)\} = A_0 \\
B_1B_7 &= \{(0, 0), (0, 1), (0, 2)\} \cdot \{(0, 1), (0, 2), (1, 1), (1, 2)\} = \\
&\{(0, 1), (0, 2), (1, 1), (1, 2) \mid (0, 2), (0, 0), (1, 2), (1, 0)\} = A_0 \\
B_1B_8 &= \{(0, 0), (0, 1), (0, 2)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
&\{(0, 0), (0, 1), (1, 0), (1, 1) \mid (0, 1), (0, 2), (1, 1), (1, 2)\} = A_0 \\
B_2B_5 &= \{(0, 0), (0, 2)\} \cdot \{(1, 0), (1, 1)\} = \\
&\{(1, 0), (1, 1) \mid (1, 2), (1, 0)\} = B_5 \\
B_2B_0 &= \{(0, 0), (0, 2)\} \cdot \{(0, 0), (1, 0)\} = \\
&\{(0, 0), (1, 0) \mid (0, 2), (1, 2)\} = B_6 \\
B_2B_3 &= \{(0, 0), (0, 2)\} \cdot \{(0, 1), (1, 1)\} = \\
&\{(0, 1), (1, 1) \mid (0, 0), (1, 0)\} = B_8 \\
B_2B_4 &= \{(0, 0), (0, 2)\} \cdot \{(0, 2), (1, 2)\} = \\
&\{(0, 2), (1, 2) \mid (0, 1), (1, 1)\} = B_7 \\
B_2B_6 &= \{(0, 0), (0, 2)\} \cdot \{(0, 0), (1, 0), (0, 2), (1, 2)\} = \\
&\{(0, 0), (1, 0), (0, 2), (1, 2) \mid (0, 2), (1, 2), (0, 1), (1, 1)\} = \\
&\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2)\} = A_0 \\
B_2B_7 &= \{(0, 0), (0, 2)\} \cdot \{(0, 1), (0, 2), (1, 1), (1, 2)\} = \\
&\{(0, 1), (0, 2), (1, 1), (1, 2) \mid (0, 0), (0, 1), (1, 0), (1, 1)\} = A_0 \\
B_2B_8 &= \{(0, 0), (0, 2)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
&\{(0, 0), (0, 1), (1, 0), (1, 1) \mid (0, 2), (0, 0), (1, 2), (1, 0)\} = A_0 \\
B_5B_0 &= \{(1, 0), (1, 1), (1, 2)\} \cdot \{(0, 0), (1, 0)\} = \\
&\{(1, 0), (1, 1), (1, 2) \mid (0, 0), (0, 1), (0, 2)\} = A_0 \\
B_5B_3 &= \{(1, 0), (1, 1), (1, 2)\} \cdot \{(0, 1), (1, 1)\} = \\
&\{(1, 1), (0, 1) \mid (1, 2), (0, 2) \mid (1, 0), (0, 0)\} = A_0 \\
B_5B_4 &= \{(1, 0), (1, 1), (1, 2)\} \cdot \{(0, 2), (1, 2)\} \\
&\{(1, 2), (0, 2) \mid (1, 0), (0, 0), (1, 1), (0, 1)\} = A_0 \\
\end{aligned}$$

Since $B_4 \subseteq B_6$, it follows that $B_5B_6 = A_0$.

Since $B_3 \subseteq B_7$, it follows that $B_5B_7 = A_0$.

Since $B_0 \subseteq B_8$, it follows that $B_5B_8 = A_0$.

$$\begin{aligned}
B_0B_0 &= \{(0, 0), (1, 0)\} \cdot \{(0, 0), (1, 0)\} \\
&\{(0, 0), (1, 0) \mid (1, 0), (0, 0)\} = B_0 \\
B_0B_3 &= \{(0, 0), (1, 0)\} \cdot \{(0, 1), (1, 1)\} \\
&\{(0, 1), (1, 1) \mid (1, 1), (0, 1)\} = \{(0, 1), (1, 1)\} = B_3 \\
B_0B_4 &= \{(0, 0), (1, 0)\} \cdot \{(0, 2), (1, 2)\}
\end{aligned}$$

$$\begin{aligned}
& \{(0, 2), (1, 2) \mid (1, 2), (0, 2)\} = \{(0, 2), (1, 2)\} = B_4 \\
& B_0B_5 = \{(0, 0), (1, 0)\} \cdot \{(1, 0), (1, 1), (1, 2)\} \\
& \{(1, 0), (1, 1), (1, 2) \mid (0, 0), (0, 1), (0, 2)\} = A_0 \\
& B_0B_6 = \{(0, 0), (1, 0)\} \cdot \{(0, 0), (1, 0), (0, 2), (1, 2)\} \\
& \{(0, 0), (1, 0), (0, 2), (1, 2) \mid (1, 0), (0, 0), (1, 2), (0, 2)\} = B_6 \\
& B_0B_7 = \{(0, 0), (1, 0)\} \cdot \{(0, 1), (0, 2), (1, 1), (1, 2)\} = \\
& \{(0, 1), (0, 2), (1, 1), (1, 2) \mid (1, 1), (1, 2), (0, 1), (0, 2)\} = B_7 \\
& B_0B_8 = \{(0, 0), (1, 0)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
& \{(0, 0), (0, 1), (1, 0), (1, 1) \mid (1, 0), (1, 1), (0, 0), (0, 1)\} = B_8 \\
& B_3B_6 = \{(0, 1), (1, 1)\} \cdot \{(0, 0), (1, 0), (0, 2), (1, 2)\} = \\
& \{(0, 1), (1, 1), (1, 1), (0, 1) \mid (0, 0), (1, 0), (1, 0), (0, 0)\} = B_8 \\
& B_3B_7 = \{(0, 1), (1, 1)\} \cdot \{(0, 1), (0, 2), (1, 1), (1, 2)\} = \\
& \{(0, 2), (0, 0), (1, 2), (1, 0) \mid (1, 2), (1, 0), (0, 2), (0, 0)\} = B_6 \\
& B_6B_6 = \{(0, 0), (1, 0), (0, 2), (1, 2)\} \cdot \{(0, 0), (1, 0), (0, 2), (1, 2)\} = \\
& \{(0, 0), (1, 0), (0, 2), (1, 2) \mid (1, 0), (0, 0), (1, 2), (0, 2)\} = \\
& \{(0, 2), (1, 2), (0, 1), (1, 1) \mid (1, 2), (0, 2), (1, 1), (0, 1)\} = A_0 \\
& B_4B_8 = \{(0, 2), (1, 2)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
& \{(0, 2), (0, 0), (1, 2), (1, 0) \mid (1, 2), (1, 0), (0, 2), (0, 0)\} = B_6 \\
& B_6B_7 = \{(0, 0), (1, 0), (0, 2), (1, 2)\} \cdot \{(0, 1), (0, 2), (1, 1), (1, 2)\} = \\
& \{(0, 0), (0, 1), (1, 0), (1, 1) \mid (1, 0), (1, 1), (0, 0), (0, 1)\} = A_0 \\
& B_3B_8 = \{(0, 1), (1, 1)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
& \{(0, 1), (0, 2), (1, 1), (1, 2) \mid (1, 1), (1, 2), (0, 1), (0, 2)\} = B_7 \\
& B_4B_6 = \{(0, 2), (1, 2)\} \cdot \{(0, 0), (1, 0), (0, 2), (1, 2)\} = \\
& \{(0, 2), (1, 2), (0, 1), (1, 1) \mid (1, 2), (0, 2), (1, 1), (0, 1)\} = B_7 \\
& B_4B_7 = \{(0, 2), (1, 2)\} \cdot \{(0, 1), (0, 2), (1, 1), (1, 2)\} = \\
& \{(0, 0), (0, 1), (1, 0), (1, 1) \mid (1, 0), (1, 1), (0, 0), (0, 1)\} = B_8 \\
& B_6B_8 = \{(0, 0), (1, 0), (0, 2), (1, 2)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
& \{(0, 0), (0, 1), (1, 0), (1, 1) \mid (1, 0), (1, 1), (0, 0), (0, 1) \mid \\
& (0, 2), (0, 0), (1, 2), (1, 0) \mid (1, 2), (1, 0), (0, 2), (0, 0)\} = A_0 \\
& B_7B_7 = \{(0, 1), (0, 2), (1, 1), (1, 2)\} \cdot \{(0, 1), (0, 2), (1, 1), (1, 2)\} = \\
& \{(0, 2), (0, 0), (1, 2), (1, 0) \mid (0, 0), (0, 1), (1, 0), (1, 1) \mid \\
& (1, 2), (1, 0), 0, 2), (0, 0) \mid (1, 0), (1, 1), (0, 0), (0, 1)\} = A_0 \\
& B_7B_8 = \{(0, 1), (0, 2), (1, 1), (1, 2)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
& \{(0, 1), (0, 2), (1, 1), (1, 2) \mid (0, 2), (0, 0), (1, 2), (1, 0) \mid \\
& (1, 1), (1, 2), 0, 1), (0, 2) \mid (1, 2), (1, 0), (0, 2), (0, 0)\} = A_0 \\
& B_8B_8 = \{(0, 0), (0, 1), (1, 0), (1, 1)\} \cdot \{(0, 0), (0, 1), (1, 0), (1, 1)\} = \\
& \{(0, 0), (0, 1), (1, 0), (1, 1) \mid (0, 1), (0, 2), (1, 1), (1, 2) \mid \\
& (1, 0), (1, 1), 0, 0), (0, 1) \mid (1, 1), (1, 2), (0, 1), (0, 2)\} = A_0
\end{aligned}$$

B^*	B_1	B_2	B_5	B_0	B_3	B_4	B_6	B_7	B_8	A_0
B_1	B_1	B_1	B_5	A_0	A_0	A_0	A_0	A_0	A_0	A_0
B_2		B_1	B_5	B_6	B_8	B_7	A_0	A_0	A_0	A_0
B_5			B_1	A_0	A_0	A_0	A_0	A_0	A_0	A_0
B_0				B_0	B_3	B_4	B_6	B_7	B_8	A_0
B_3					B_4	B_0	B_8	B_6	B_7	A_0
B_4						B_3	B_7	B_8	B_6	A_0
B_6							A_0	A_0	A_0	A_0
B_7								A_0	A_0	A_0
B_8									A_0	A_0
A_0										A_0

We shall consider now the hypergroupoid B^* on $\{B_i \mid 0 \leq i \leq 8\} \cup \{A_0\}$ and we shall study the corresponding hypergroupoid on

$$H^* = \{(0, 0), (0, 2), (1, 0), (1, 2), (1, 1), (0, 1)\}.$$

From B^* we obtain the following H^* , which we divide into two tables, H^{*1} and H^{*2} .

We first do the following notations:

denote the elements $(0, 0), (0, 1), (0, 2)$ of B_1 by $00_{B_1}, 01_{B_1}, 02_{B_1}$, respectively;

denote the elements $(0, 0), (0, 2)$ of B_2 by $00_{B_2}, 02_{B_2}$, respectively;

denote the elements $(1, 0), (1, 1), (1, 2)$ of B_5 by $10_{B_5}, 11_{B_5}, 12_{B_5}$, respectively;

denote the elements $(0, 0), (1, 0)$ of B_0 by $00_{B_0}, 10_{B_0}$, respectively;

denote the elements $(0, 1), (1, 1)$ of B_3 by $01_{B_3}, 11_{B_3}$, respectively;

denote the elements $(0, 2), (1, 2)$ of B_4 by $02_{B_4}, 12_{B_4}$, respectively;

denote the elements $(0, 0), (1, 0), (0, 2), (1, 2)$ of B_6 by $00_{B_6}, 10_{B_6}, 02_{B_6}, 12_{B_6}$, respectively;

denote the elements $(0, 1), (0, 2), (1, 1), (1, 2)$ of B_7 by $01_{B_7}, 02_{B_7}, 11_{B_7}, 12_{B_7}$,

respectively and

H^{*1}	00_{B_1}	01_{B_1}	02_{B_1}	00_{B_2}	02_{B_2}	10_{B_5}	11_{B_5}	12_{B_5}	00_{B_0}	10_{B_0}	01_{B_3}	11_{B_3}	02_{B_4}	12_{B_4}
00_{B_1}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_1}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_1}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_2}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	B_6	B_6	B_8	B_8	B_7	B_7
02_{B_2}	B_1	B_1	B_1	B_1	B_1	B_5	B_5	B_5	B_6	B_6	B_8	B_8	B_7	B_7
10_{B_5}	B_5	B_5	B_5	B_5	B_5	B_1	B_1	B_1	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_5}	B_5	B_5	B_5	B_5	B_5	B_1	B_1	B_1	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_5}	B_5	B_5	B_5	B_5	B_5	B_1	B_1	B_1	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_0}	A_0	A_0	A_0	B_6	B_6	A_0	A_0	A_0	B_0	B_0	B_4	B_4	B_6	B_6
10_{B_0}	A_0	A_0	A_0	B_6	B_6	A_0	A_0	A_0	B_0	B_0	B_4	B_4	B_6	B_6
01_{B_3}	A_0	A_0	A_0	B_8	B_8	A_0	A_0	A_0	B_4	B_4	B_4	B_4	B_0	B_0
11_{B_3}	A_0	A_0	A_0	B_8	B_8	A_0	A_0	A_0	B_4	B_4	B_4	B_4	B_0	B_0
02_{B_4}	A_0	A_0	A_0	B_7	B_7	A_0	A_0	A_0	B_6	B_6	B_0	B_0	B_3	B_3
12_{B_4}	A_0	A_0	A_0	B_7	B_7	A_0	A_0	A_0	B_6	B_6	B_0	B_0	B_3	B_3
00_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
10_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
02_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
12_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_6	B_6	B_8	B_8	B_7	B_7
01_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
02_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
11_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
12_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_7	B_7	B_6	B_6	B_8	B_8
00_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
01_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
10_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
11_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	B_8	B_8	B_7	B_7	B_6	B_6
A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0

H^{*2}	00_{B_6}	10_{B_6}	02_{B_6}	12_{B_6}	01_{B_7}	02_{B_7}	11_{B_7}	12_{B_7}	00_{B_8}	01_{B_8}	10_{B_8}	11_{B_8}	A_0
00_{B_1}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_1}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_1}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_2}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_2}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
10_{B_5}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_5}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_5}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_0}	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	A_0
10_{B_0}	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	A_0
01_{B_3}	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	A_0
11_{B_3}	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	B_7	B_7	B_7	B_7	A_0
02_{B_4}	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	A_0
12_{B_4}	B_7	B_7	B_7	B_7	B_8	B_8	B_8	B_8	B_6	B_6	B_6	B_6	A_0
00_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
10_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_6}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
02_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
12_{B_7}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
00_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
01_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
10_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
11_{B_8}	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0
A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0	A_0

Let us calculate now the $\mu_1(i, j)$ for $i \in \{0, 1\}$, $j \in \{0, 1, 2\}$.

We have

$$A_1(0, 1) = 25/3 + 18 \cdot 3 \cdot 2/6 + 2 \cdot 4/4 + 2 \cdot 4/4 + 12 \cdot 2 \cdot 2/6 + 9/3 + 18 \cdot 3 \cdot 2/6 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 4/2 + 2 \cdot 8/4 + 2 \cdot 8/4 + 144/6 = 101 + 25/3 = 328/3,$$

which corresponds to the sets:

- B_1B_1, B_2B_2, B_1B_2 divided by $|B_1|$;
- $B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
- B_2B_3 divided by $|B_8|$;
- B_2B_4 divided by $|B_7|$;
- $B_2(B_6, B_7, B_8)$ divided by $|A_0|$;
- B_5B_5 divided by $|B_1|$;
- $B_1(B_0, B_3, B_4, B_5, B_6, B_7, B_8)$ divided by $|A_0|$;
- B_0B_7 divided by $|B_7|$;
- B_0B_8 divided by $|B_8|$;
- B_4B_6 divided by $|B_7|$;
- B_4B_7 divided by $|B_8|$;

B_4B_4 divided by $|B_3|$;

B_3B_6 divided by $|B_8|$;

B_3B_8 divided by $|B_7|$;

and 144 divided by $|A_0|$, respectively. So, $A_1(0, 1) = 328/3$.

$$q_1(0, 1) = 25 + 108 + 8 + 8 + 48 + 9 + 108 + 16 + 16 + 16 + 16 + 4 + 16 + 16 + 144 = 558;$$

$$\mu_1(0, 1) = 328 / (3 \cdot 558) = 0.1959.$$

$A_1(1, 1) = 2 \cdot 9/3 + 18 \cdot 3 \cdot 2/6 + 2 \cdot 2 \cdot 3/3 + 2 \cdot 4/4 + 2 \cdot 4/4 + 12 \cdot 2 \cdot 2/6 + 9/3 + 18 \cdot 3 \cdot 2/6 + 4/2 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 144/6 = 6 + 4 + 18 + 2 + 2 + 8 + 3 + 18 + 4 + 4 + 4 + 4 + 2 + 4 + 4 + 24 = 111$, which corresponds to the sets:

B_1B_5 divided by $|B_5|$;

$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;

B_2B_5 divided by $|B_5|$;

B_2B_3 divided by $|B_8|$;

B_2B_4 divided by $|B_7|$;

$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;

B_5B_5 divided by $|B_1|$;

$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;

B_4B_4 divided by $|B_3|$;

B_3B_6 divided by $|B_8|$;

B_3B_8 divided by $|B_7|$;

B_4B_6 divided by $|B_7|$;

B_4B_7 divided by $|B_8|$;

B_0B_7 divided by $|B_7|$;

B_0B_8 divided by $|B_8|$;

and 144 divided by $|A_0|$, respectively. So, $A_1(1, 1) = 111$.

$$q(1, 1) = 18 + 12 + 108 + 8 + 8 + 48 + 9 + 108 + 16 + 16 + 16 + 16 + 4 + 16 + 16 + 144 = 563$$

$$\mu_1(1, 1) = 111 / 563 = 0.19716.$$

$A_1(1, 0) = 2 \cdot 9/3 + 18 \cdot 3 \cdot 2/6 + 2 \cdot 3 \cdot 2/3 + 2 \cdot 4/4 + 2 \cdot 4/4 + 12 \cdot 2 \cdot 2/6 + 18 \cdot 3 \cdot 2/6 + 4/2 + 2 \cdot 4/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 4/2 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 144/6 = 114$, which corresponds to the sets:

B_1B_5 divided by $|B_5|$;

$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;

B_2B_5 divided by $|B_5|$;

B_2B_0 divided by $|B_6|$;

B_2B_3 divided by $|B_8|$;

$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;

$B_5(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;

B_0B_0 divided by $|B_0|$;

B_0B_4 divided by $|B_6|$;

B_0B_6 divided by $|B_6|$;

B_0B_8 divided by $|B_8|$;

B_3B_4 divided by $|B_0|$;

B_3B_6 divided by $|B_8|$;

B_3B_7 divided by $|B_6|$;

B_4B_7 divided by $|B_8|$;

B_4B_8 divided by $|B_6|$

and 144 divided by $|A_0|$, respectively. So, $A_1(1, 0) = 114$.

$$q_1(1, 0) = 18 + 108 + 12 + 8 + 8 + 48 + 108 + 4 + 8 + 16 + 16 + 16 + 8 + 16 + 16 + 16 + 144 = 570;$$

$$\mu_1(1, 0) = 114/570 = 0.2.$$

$$A_1(1, 2) = 2 \cdot 9/3 + 18 \cdot 3 \cdot 2/6 + 2 \cdot 3 \cdot 2/3 + 2 \cdot 4/4 + 2 \cdot 4/4 + 12 \cdot 2 \cdot 2/6 + 18 \cdot 3 \cdot 2/6 + 2 \cdot 4/2 + 2 \cdot 4/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 4/2 + 144/6 = 114,$$

which corresponds to the sets:

B_1B_5 divided by $|B_5|$;

$B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;

B_2B_5 divided by $|B_5|$;

B_2B_0 divided by $|B_6|$;

B_2B_4 divided by $|B_7|$;

$B_2(B_6, B_7, B_8)$ divided by $|A_0|$;

$B_5(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;

B_0B_3 divided by $|B_4|$;

B_0B_4 divided by $|B_6|$;

B_0B_6 divided by $|B_6|$;

B_0B_7 divided by $|B_7|$;

B_4B_6 divided by $|B_7|$;

B_4B_8 divided by $|B_6|$;

B_3B_7 divided by $|B_6|$;

B_3B_8 divided by $|B_7|$;

B_3B_3 divided by $|B_4|$

and 144 divided by $|A_0|$, respectively. So, $A_1(1, 2) = 114$.

$$q_1(1, 2) = 18 + 108 + 12 + 8 + 8 + 48 + 108 + 8 + 8 + 16 + 16 + 16 + 16 + 16 + 16 + 4 + 144 = 570;$$

$$\mu_1(1, 2) = 114/570 = 0.2.$$

$$A_1(0, 0) = 25/3 + 18 \cdot 2 \cdot 3/6 + 4 \cdot 2/4 + 4 \cdot 2/4 + 12 \cdot 2 \cdot 2/6 + 9/3 + 18 \cdot 3 \cdot 2/6 + 4/2 + 2 \cdot 4/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 4/2 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 144/6 = 346/3,$$

$$q_1(0, 0) = 25 + 108 + 8 + 8 + 48 + 9 + 108 + 4 + 8 + 16 + 16 + 8 + 16 + 16 + 16 + 16 + 144 = 574;$$

$$\mu_1(0, 0) = 346/(3 \cdot 574) = 0.2009.$$

$$A_1(0, 2) = 25/3 + 18 \cdot 2 \cdot 3/6 + 4 \cdot 2/4 + 4 \cdot 2/4 + 12 \cdot 2 \cdot 2/6 + 9/3 + 18 \cdot 3 \cdot 2/6 + 2 \cdot 4/4 + 2 \cdot 4/2 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/2 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 2 \cdot 8/4 + 144/6 = 346/3,$$

which corresponds to the sets:

- $(B_1 + B_2) \cdot (B_1 + B_2)$ divided by $|B_1|$;
- $B_1(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
- B_2B_0 divided by $|B_6|$;
- B_2B_4 divided by $|B_7|$;
- $B_2(B_6, B_7, B_8)$ divided by $|A_0|$;
- B_5, B_5 divided by $|B_1|$;
- $B_5(B_0, B_3, B_4, B_6, B_7, B_8)$ divided by $|A_0|$;
- B_0B_4 divided by $|B_6|$;
- B_0B_3 divided by $|B_4|$;
- B_0B_6 divided by $|B_6|$;
- B_0B_7 divided by $|B_7|$;
- B_3B_7 divided by $|B_6|$;
- B_3B_8 divided by $|B_7|$;
- B_4B_6 divided by $|B_7|$;
- B_4B_8 divided by $|B_6|$;
- B_3B_3 divided by $|B_4|$

and 144 divided by $|A_0|$ respectively. So, $A_1(0, 2) = 346/3$.

$$q_1(0, 2) = 25 + 108 + 8 + 8 + 48 + 9 + 108 + 8 + 8 + 16 + 16 + 16 + 16 + 16 + 4 + 144 = 574;$$

$$\mu_1(0, 2) = \mu_1(0, 0) = 346/(3 \cdot 574) = 0.200929.$$

So, we have

$$\mu_1(0, 1) = 0.1959$$

$$\mu_1(1, 1) = 0.19716$$

$$\mu_1(1, 0) = \mu_1(1, 2) = 0.2$$

$$\mu_1(0, 0) = \mu_1(0, 2) = 0.200929.$$

Let us denote

$$(0, 1) \text{ by } 1, \quad (1, 1) \text{ by } 2,$$

$$(1, 0) \text{ by } 3, \quad (1, 2) \text{ by } 4,$$

$$(0, 0) \text{ by } 5, \quad (0, 2) \text{ by } 6.$$

Let H be the set $\{1, 2, 3, 4, 5, 6\}$.

Hence we obtain the following structure:

H_1^*	1	2	3	4	5	6
1	1	1, 2	1, 2, 3, 4	1, 2, 3, 4	H	H
2		2	2, 3, 4	2, 3, 4	2, 3, 4, 5, 6	2, 3, 4, 5, 6
3			3, 4	3, 4	3, 4, 5, 6	3, 4, 5, 6
4				3, 4	3, 4, 5, 6	3, 4, 5, 6
5					5, 6	5, 6
6						5, 6

From H_1^* we obtain

$$A_2(2) = 1 + 2/2 + 4/4 + 4/3 + 4/5 + 4/6 = 29/5.$$

$$q_2(2) = 2 + 1 + 4 + 4 + 4 + 4 = 19$$

$$\mu_2(2) = 29/(5 \cdot 19) = 0.305263.$$

$$A_2(3) = A_2(4) = 4/2 + 4/4 + 4/3 + 4/5 + 8/4 + 4/6 = 39/5.$$

$$q_2(3) = q_2(4) = 4 + 4 + 4 + 4 + 4 + 8 = 28$$

$$\mu_2(3) = \mu_2(4) = 39/(5 \cdot 28) = 0.27857.$$

$$A_2(5) = A_2(6) = 4/2 + 8/3 + 4/5 + 4/6 = 82/15.$$

$$q_2(5) = q_2(6) = 4 + 8 + 4 + 4 = 20$$

$$\mu_2(5) = \mu_2(6) = 82/(15 \cdot 20) = 0.27333.$$

$$A_2(1) = 1 + 2/2 + 4/4 + 4/6 = 11/3$$

$$q_2(1) = 11, \quad \mu_2(1) = 11/(11 \cdot 3) = 0.3333$$

Notice that

$$\mu_2(1) = 0.3333 > \mu_2(2) = 0.305263 > \mu_2(3) = \mu_2(4) = 0.27857 > \mu_2(5) = \mu_2(6) = 0.27333.$$

Thus, $H_2 \simeq H_1$, whence $\partial H_1^* = 1$.

3 Conclusion

Clearly, H_2 and H_1 are isomorphic and by consequence, the fuzzy grade of H^* is one.

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