Generation of Airy solitary-like wave beams by acceleration control in inhomogeneous media

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Abstract: We investigate the propagation of Airy beams in linear gradient index inhomogeneous media. We demonstrate that by controlling the gradient strength of the medium it is possible to reduce to zero their acceleration. We show that the resulting Airy wave beam propagates in straight line due to the balance between two opposite effects, one due to the inhomogeneous medium and the other to the diffraction of the beam, in a similar way as a solitary wave in a nonlinear inhomogeneous medium. Going even further we were able to invert the sign of the acceleration of the beam.

OCIS codes: (070.7345) Wave propagation; (080.1510) Propagation methods; (000.3860) Mathematical methods in physics; (260.2710) Inhomogeneous optical media; (080.5692) Ray trajectories in inhomogeneous media.

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18. There exist several forms for producing a solution of this equation, for instance by using the Zassenhaus formula [19], or by simplifying the differential equation via a transformation (see for instance [20] for a quadratic term).

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The operators to disentangle the exponential of the sum of two operators can also be given in different orderings of the exponentials involved. Of course, they are all equivalent.
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1. Introduction

Since their proposal [1,2], finite energy optical Airy beams in free space have been a cause of disconcert due to their uncommon accelerating and non-diffracting propagation characteristics, becoming in this way a subject of many investigations. They share several odd properties with the also controversial non-diffracting Bessel beams [3–6] and the Airy quantum wave packets of infinite extent introduced by Berry and Balazs as free particle solutions of the Schrödinger equation [7].

On propagation finite energy two-dimensional Airy beams continuously bend following parabolic trajectories on the plane of propagation extending towards the non-oscillatory region of the beam [1,2]. This was a striking behavior for a two dimensional beam in free space that was different from the non-diffracting transverse parabolic beams previously reported in three dimensions [8,9]. If observed along their parabolic trajectories, infinite extent Airy beams do not change their amplitude shape. It is for this reason they have also been considered as non-diffracting beams.

Although in the optical waveguides literature they are not referred to as Airy beams, it is well known that the propagation modes in inhomogeneous media with a linear gradient refractive index have a profile determined by the Airy function [10,11]. In these media the stationary Airy solution propagates invariant parallel to the propagation axis.

Propagation of Airy beams has recently been studied in plasmons, nonlinear photorefractive materials and photonic crystals [12–16]. In the latter case, it was shown that the nonlinear Airy beam in the photonic crystal can invert the direction of the acceleration by carefully adjusting the phase mismatch of the nonlinear processes involved.

In the present work we demonstrate, theoretically and experimentally, that it is possible to control the transverse acceleration of Airy beams when propagating in inhomogeneous media. By modifying the strength of the gradient index, the acceleration can be reduced to the point of creating a beam that propagates similar to a solitary wave with Airy profile. We show that such propagating behavior is the result of two opposite effects one due to the beam and the other to the medium. Even further, we also show the possibility of inverting the acceleration of the Airy beam.

2. Propagation in inhomogeneous media

I. Fermat's ray theory

The equation of light rays propagating in a medium with refractive index $n = n(x,y,z)$ obtained from Fermat's variational principle is [17],

$$\frac{d}{ds}\left(n\frac{d\mathrm{r}}{ds}\right) = \nabla n, \tag{1}$$

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where **r** is the position vector of a point on a ray function of the length of arc s of the ray. For a material with a linear gradient index (GRIN) $n^2(x) = n_0^2 \pm n_1 x$, with $n_1 << n_0$, and paraxial rays this equation yields

$$\frac{d^2x}{dz^2} \mp \frac{n_1}{2n_0^2} = 0, \tag{2}$$

whose solutions are rays that, launched parallel to the propagation axis z, follow parabolas whose branches extend in the direction of the gradient. This is, if the gradient is negative the rays bend towards the negative x-axis and vice versa, see Figs. 1a) and 1b). Seen in another way, the rays bend in the direction of the increasing of the refractive index (red dotted line). For each case, the corresponding Airy differential equation has also the same sign of the gradient. This is relevant since the Airy differential equation with the positive sign, $u'' + xu = 0$, is rarely quoted in the literature and its solution is a mirror reflection of the solution of the Airy equation with the negative sign. By looking at the corresponding Airy mode, we notice that if it was to propagate in free space it would move in the opposite direction as that of the rays, see Fig. 1c) and 1d) where the modulus squared of the amplitude is plotted. Then, from this it can be deduced that the gradient index of the medium presents an opposite effect to the propagating Airy beam that can be associated to the opposite force described by Berry and Balazs [7]. This can create a balance such that can make the beam to travel parallel to the longitudinal axis in a similar way as a solitary wave as will be shown below.

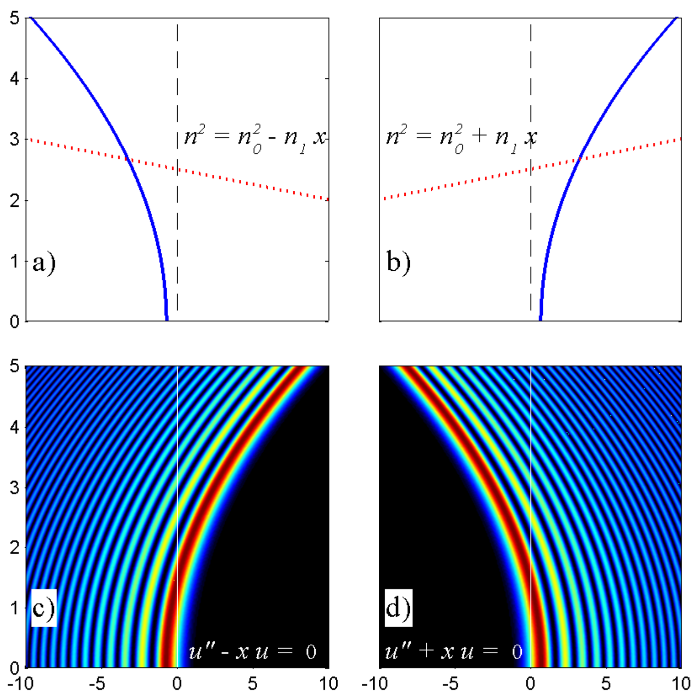

Fig. 1. Top row: Typical behavior of a rays propagating in linear GRIN media. The red dotted line shows qualitatively the gradient of the refractive index and the blue line the corresponding ray trajectory for: a) negative gradient and b) positive gradient. Bottom row: Free space propagation of the Airy mode at the exit of the corresponding inhomogeneous media (see Airy equation at the bottom of the figure) with negative gradient, c), and positive gradient d). The horizontal coordinate corresponds to the transverse coordinate and the vertical one to the evolution coordinate.

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II. Airy beam propagation in linear gradient index media

A wave field propagating in a linear GRIN medium can be described by the next (1+1)D normalized paraxial wave equation

$$i\frac{\partial u}{\partial z}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+k_1 xu. \tag{3}$$

Without loss of generality we consider a positive GRIN medium since the general physical behavior remains unaltered according to the previous section, Fig. 1. The stationary equation associated to this case is $u'' - \xi u = 0$ that is one of the two forms of the Airy differential equation [11]. This is a good reason to use the Airy function as the initial condition below.

In order to solve this equation we use an operational approach by defining the operator p = -i∂/∂x. Performing the following transformation [18-20]

$$u(x,z)=\exp\left(-\tfrac{i}{3k_1}p^3\right)v(x,z), \tag{4}$$

and considering a particular case of the Hadamard lemma [21,22], $e^{\chi A}Be^{-\chi A}=B+\chi[A,B]+(\chi^2/2!)[A,[A,B]]+\ldots$, we simplify the problem of the linear GRIN paraxial wave equation to find the solution of

$$i\frac{\partial v}{\partial z}=\frac{k_1 x}{2}v \tag{5}$$

that can immediately be integrated to give

$$v(x,z)=\exp\left(-i\tfrac{k_1 xz}{2}\right)v(x,0). \tag{6}$$

By considering an Airy initial condition $u(x,0) = Ai(k_x x)$ for the paraxial wave Eq. (3), it is transformed by substitution in Eq. (4) as

$$v(x,0)=\exp\left(\tfrac{i}{3k_1}p^3\right)Ai(k_x x). \tag{7}$$

By writing the Airy function in its integral form [23]

$$Ai(s)=\tfrac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\exp(it^3/3+st)dt \tag{8}$$

we obtain from (7)

$$v(x,z)=\tfrac{1}{(2\pi)^2}\exp\left(-i\tfrac{k_1 xz}{2}\right)\int_{-\infty}^{\infty}\exp[i(t^3/3+k_x^3t^3/3k_1+k_x xt)]dt \tag{9}$$

and using again (5) we obtain the propagated function

$$u(x,z)=e^{i\Phi(x,z)}Ai\left[k_x x+\frac{k_x z^2}{2}\left(k_1-\frac{k_x^3}{2}\right)\right], \tag{10}$$

where $\Phi(x,z)=\left(k_1-\frac{k_x^3}{2}\right)\left[xz+\frac{z^3}{6}\left(k_1-k_x^3\right)\right]$.

Equation (10) clearly shows that in a medium with linear gradient index it is possible to control the acceleration of the Airy beam, making it zero, and even change its direction depending on the value of the acceleration factor $k_x(k_1 - k_x^3/2)/2$. This is shown in Fig. 2 in which this factor is negative, zero and positive, respectively. A relevant case is that shown in Fig. 2 b) that corresponds to the solitary-like wave propagation of the Airy beam. This behavior is determined by the balance between the free space parabolic wave propagation of Airy beams, characterized by k_x, and the inhomogeneous medium that induces the optical rays

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in the beam to travel along parabolic trajectories in the opposite direction determined by the GRIN strength through k_1. This is demonstrated experimentally in the next section.

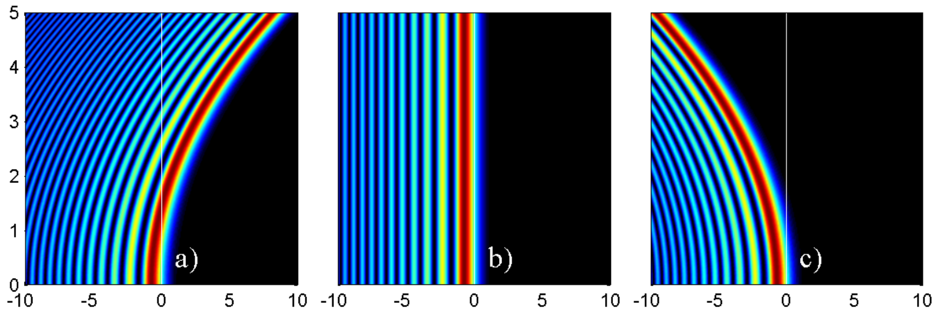

Fig. 2. Propagation of an Airy beam in a medium with positive gradient as function of the value and sign of the acceleration factor, a) negative, b) zero or c) positive. For zero acceleration the beam propagates as an Airy non-diffracting wave field.

3. Experiment

We have implemented experimentally the control of acceleration of an Airy beam propagating in a linear GRIN medium. The medium is a solution of sugar in water, where the strength of the gradient refractive index is controlled by the concentration of sugar. It was previously shown that this solution approximately represents a linear GRIN medium [24,25]. The Airy beam was generated displaying a cubic phase in a Spatial Light Modulator (SLM) [23], which is illuminated with a Gaussian beam. This cubic phase is given by

$$\Psi(x) = \exp[i(2\pi / \alpha)x^3] \tag{11}$$

where α is a constant that characterizes the experimental spatial scale. The cubic phase and the Airy beam are connected by the relation

$$\Im\{\Psi(x)\} = Ai(qk), \tag{12}$$

where α denotes the Fourier transform, Ai represents the Airy beam, k is the conjugate variable (spatial frequency) corresponding to the spatial coordinate x, and q=[$\alpha/(6\pi)]^{1/3}$. The phase optical field $\Psi(x)$ is generated with a phase SLM [26], and the Fourier transform of the phase field $\Psi(x)$ [in Eq. (12)] is implemented with a lens of positive power. The experimental setup, depicted in Fig. 3, shows a beam expander (BE) that conditions the beam of a solid state laser (verdi V8, 532 nm) to illuminate the total active area of the SLM ($\cong$1cm^2). The illumination on the SLM is oblique, avoiding the necessity of a beam splitter. The lens (L) that produces the Fourier transform of the cubic phase has a focal distance f = 50cm. The dimensions of the solution container (SC) were 60 cm (length), 2.5 cm (height) and 5 cm (width). The back focal plane of the lens, where the Airy beam is obtained, coincides with the front end of the SC. The parameter $\alpha=1.287\text{x}10^{10}$ μm^3, of the generated phase modulation Ψ(x), was chosen to achieve an Airy beam deflection of approximately 2 mm in a propagation distance of 600mm, when no sugar is added to the water. We employed 3 different concentrations of sugar: C_1 (with no added sugar), C_2 (with 6.5 gr of sugar) and C_3 (with 13 gr of sugar). To obtain the concentrations C_2 and C_3, the water with the added sugar is gently stirred for approximately 15 seconds. After that, the solution is allowed to stabilize before capturing images of the propagated beams.

The generated Airy beam arrives to the solution container at 10 mm below the interface air-water. Propagation trajectories of the reference Gaussian beam for increasing solution concentrations appear at the left column of images in Fig. 4 (a,c,e). Meanwhile, the propagation trajectories of the Airy beams for different concentrations are shown at the right column of images in Fig. 4 (b,d,f). In order to enhance the deviations of the beams, which

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propagate from left to right, the displayed images present a zoom 10× along the vertical direction. The corresponding length scales are shown in the small arrows. For each image of Fig. 4, the top bright line corresponds to the interface air-water and the second bright line (which appears curved in some cases) corresponds to the propagated beam. As expected, the Gaussian beam deviation increases with the sugar concentration in the solution. On the other hand, the Airy beam shows an upward deflection when the medium is homogeneous (no added sugar), see Fig. 4 b). When the medium has concentration C2 the Airy beam shows zero acceleration propagating as a solitary wave would do in a nonlinear medium, Fig. 4 d). Increasing the concentration to C3 the beam shows inversion of its acceleration, Fig. 4 f), confirming the theoretical predictions of the previous section. In particular, the apparent width of the last beam in which its acceleration was inverted, is larger than that propagating in the homogeneous medium in concordance with the tehoretical result shown in Fig. 2c). From the mechanical analogous and Eq. (2), the acceleration of each Airy beam in the solution is obtained to be equals to $2\Delta x/\Delta z^2 = (n_1/2n_0^2)$ where Δx is the vertical deviation of the beam and Δz is the length of the container. Notice that from this expression we can obtain the gradient index strength coefficient n_1. The estimated accelerations for the experimentally generated Airy beams, in Fig. 4 (b,d,f), are 5.5×10-6 mm-1, 0, and −13.8×10-6 mm-1, respectively.

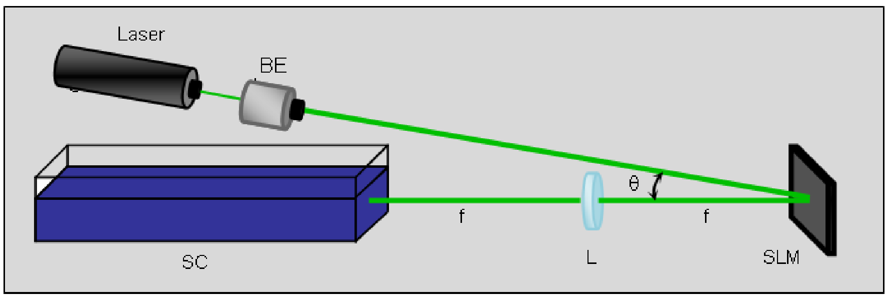

Fig. 3. Components of the experimental setup: Laser, beam expander (BE), SLM, Fourier transforming lens (L), and solution container (SC).

Solitary-like propagation and switching of acceleration of Airy beams has been observed in nonlinear crystals [12–16]. Such manipulation requires the fabrication of the crystals, whose refractive index is modified by complex nonlinear processes. At some point these crystals can be seen as diffused waveguides with a GRIN index as the one described in our experiments.

4. Conclusions

We have demonstrated for the first time the solitary-like long-distance wave propagation of Airy beams in inhomogeneous media. Using Fermat's principle for rays and wave propagation we provided the physics of this behavior as the result of the balance between the free space acceleration of Airy beams in one direction and the effect of the gradient index that induces a deflection in the opposite direction. The Airy beam solutions in GRIN media was fully described theoretically using an operational method. The solution also shows the possibility of inversion of the acceleration by manipulating the gradient index strength that was demonstrated experimentally.

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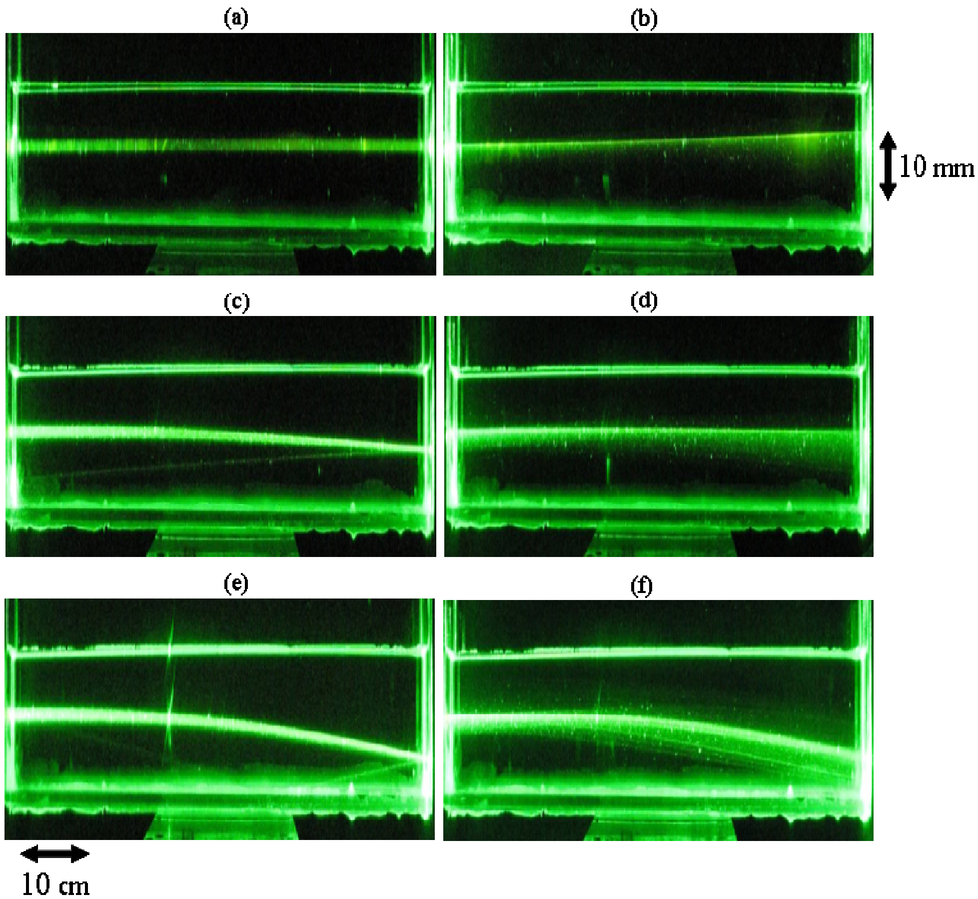

Fig. 4. Left side images: trajectories of the Gaussian beam for solution concentrations (a) C1, (c) C2, and (e) C3. Right side images: trajectories of the Airy beam for solution concentrations (b) C1, (d) C2, and (f) C3.

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