The Origins of Causality Violations In Force-Free Simulations of Black Hole Magnetospheres

Brian Punsly

4014 Emerald Street No.116, Torrance CA, USA 90503

Donato Bini

Instituto per Applicazioni della Matematica C.N.R., Naples, I-80131, Italy and International Center for Relativistic Astrophysics, I.C.R.A., University of Rome La Sapienza, I-00185 Roma, Italy

ABSTRACT

Recent simulations of force-free, degenerate (ffde) black hole magnetospheres indicate that the fast mode radiated from (or near) the event horizon can modify the global potential difference in the poloidal direction orthogonal to the magnetic field, V, in a black hole magnetosphere. By contrast in MHD (meaning perfect magnetohydrodynamics, hereafter), a combination of Alfven and fast waves are required to alter V in a magnetosphere. This distinction is significant because it changes the causal boundary surface for evolving V from the event horizon (the inner fast critical surface in ffde) to the inner Alfven critical surface. Secondly, there is a fundamental contradiction in a wave that alters V coming from near the horizon. The background fields in ffde satisfy the "ingoing wave condition" near the horizon (that arises from the requirement that all matter is ingoing at the event horizon), yet outgoing waves are radiated from this region in the simulation. The resolution of these two issues is important to our understanding of causality in black hole magnetospheres and ffde as a faithful representation of tenuous MHD near a black hole. Studying the properties of the waves in the simulations are useful tools to this end. It is shown that regularity of the stress-energy tensor in a freely falling frame requires that the outgoing (as viewed globally) waves near the event horizon are redshifted away and are ineffectual at changing V. It is also concluded that waves in massless MHD (ffde) are extremely inaccurate depictions of waves in a tenuous MHD plasma, near the event horizon, as a consequence black hole gravity. Any analysis based on ffde near the event horizon is seriously flawed.

1. Introduction

The Blandford-Znajek mechanism of extracting the rotational energy of a black hole (see Blandford and Znajek (1977)) is one of the more celebrated theories in astrophysics. However, certain causality issues have been raised over the years (see Punsly (2001) and references therein). In particular, will a general set of initial conditions in tenuous MHD evolve to this solution? An axisymmetric ffde numerical simulation was presented in Komissarov (2001) to study this question. It assumed ffde and involved a very complicated initial state. The system evolved by adjusting V with waves coming from near the event horizon. It is shown in this Letter that ffde grossly misrepresents the physics near the black hole and the simulations cannot be used to justify causality of the Blandford-Znajek mechanism.

The time evolution of the full MHD system of equations in a black hole magnetosphere is extremely complicated. For simplicity, cold MHD in the limit of zero plasma mass, ffde, is often used. In cold MHD, the causal structure of the magnetosphere is clear, both the fast and Alfven modes are required to modify the poloidal voltage drop across **B**, or equivalently the field line rotation rate (Punsly 2001, 2003; Komissarov 2002). Thus, it must be established outside of the Alfven point of the plasma flowing inward towards the black hole. In the simulation, "the rotation of the magnetic field lines starts near the black hole and propagates outwards in the form of a torsional Alfven wave" (Komissarov 2001). Yet in Komissarov (2002) it is noted that in the initial state, the inner Alfven surface (an effective one-way membrane for Alfven waves that permits traversal by ingoing waves only) is located at the stationary limit surface which is quite far form the horizon. Clearly, the "torsional Alfven wave" must be a pure fast wave. As such the ffde simulation is not in accord with the time evolution of V in MHD that requires two wave polarizations. Likewise in ffde pulsar simulations, the star adjusts V by radiating Alfven waves (Spitkovsky 2003). Komissarov acknowledges in a later paper that the "driving source" for the Blandford-Znajek mechanism "must be located between the inner and outer Alfven surfaces of a black hole magnetosphere and, thus, lay well outside of the horizon"(Komissarov 2002). Consequently, the evolution of V by the fast mode in the simulation is quite paradoxical from a causality perspective.

This Letter points out two physical inaccuracies of an ffde simulation that alters V with waves coming from near the horizon. First of all, any MHD wave near the event horizon requires very strong cross-field currents in order to propagate away from the horizon. This is in direct contradiction to the assumed ffde waves that drive the simulation from near the horizon. These nonforce-free inertial currents are required to move the plasma under the influence of powerful black hole gravity. Even if a plasma is extremely tenuous, black hole gravity is the dominant force near the black hole and it is inappropriate to approximate it away as in ffde. Secondly, the ability of an outgoing wave to alter V gets redshifted away as the horizon is approached.

Many of the points of the Letter will be explored through an axisymmetric numerical simulation presented in Komissarov (2001) that is a time evolution of the magnetosphere of the event horizon of a rotating black hole of mass, M, the space-time of which is described by the Kerr metric. The force-free conditions are given by the following relationships, written covariantly in terms of the Maxwell field strength tensor, $F^{\mu\nu}$, and four-current density, J^{μ}, as well as in component form,

$$F^{\mu\nu}J_{\nu} = 0\ , \quad \rho_e\mathbf{E} + \frac{\mathbf{J}\times\mathbf{B}}{c} = 0\ . \tag{1-1}$$

The simulation is predicated on the following set of equations,

$$F^{\mu\nu}F_{\mu\nu} > 0\ , \quad T^{\mu\nu}{}_{;\nu} = 0\ , \quad *F^{\mu\nu}{}_{;\nu} = 0\ , \quad *F^{\mu\nu}F_{\mu\nu} = 0\ , \tag{1-2}$$

where $T^{\mu\nu}$ is the stress-energy tensor of the electromagnetic field.

The Letter begins with a very brief review of the polarization properties of the two modes that exist in ffde, the Alfven and fast mode. These modes determine the causal structure of an ffde magnetosphere. The third section is a detailed description of the initial state of the Komissarov (2001) simulation. In section 4, the 2-D ffde fast wave-fronts in the Kerr spacetime are calculated for the first time in the literature.

2. Force-Free Discontinuities

The simulation evolves by the propagation of step waves, force-free discontinuities. Since the field is degenerate and magnetic by (1.2), there exists a time-like frame at each point of space-time in which **E** vanishes (Komissarov 2001). This is known as a proper frame (not necessarily a coordinate frame). The structure of the wave-front simplifies in a proper frame, since there is no **E** upstream. In the proper frame, the normal vector of the step wave is **n** (not necessarily a planar wave-front) and the magnetic field upstream (downstream) of the wave-front is designated as $\mathbf{b}_u$ ($\mathbf{b}_d$). The force-free constraint in (1.1), implies that all particles flow parallel to **b**, otherwise the resulting **E** in the frame of the particles would drive large cross-field (nonforce-free) currents.

A wave that propagates an electromagnetic field that is completely transverse to **n** is called the fast mode. By solving, the tangential component of Ampere's law and Faraday's law at the wave-front with the degeneracy condition, one finds that the wave propagates E=B at a velocity c with **E** orthogonal to the **n** - $\mathbf{b}_u$ plane and **B** in the **n** - $\mathbf{b}_u$ plane.

Consider the oblique Alfven mode, with $\mathbf{n}$ at an angle θ to $\mathbf{b}_u$. The wave propagates at a velocity $c\cos\theta$ with variations of $\mathbf{B}$ and $\mathbf{n}\times\mathbf{E}$ orthogonal to the $\mathbf{n}$ - $\mathbf{b}_u$ plane. There is also a normal component of $\mathbf{E}$.

3. A Sample Simulation

In Komissarov (2001), an ffde magnetosphere evolves from an initial state that has only radial and azimuthal components of the magnetic field in Boyer-Lindquist (B-L hereafter) coordinates. Symmetry is used to impose boundaries at the pole and the equatorial plane (the equatorial plane is a current sheet that switches off the azimuthal and poloidal magnetic field). Thus, the problem reduces to an analysis in the upper right quadrant of a plane.

The initial state is actually very complicated since it is adhoc. The details will be described in two separate frames for illustrative purposes. One is a global coordinate frame known as the stationary frames at asymptotic infinity. These frames have a four-velocity based on B-L time.[1] The local frame is the orthonormal one carried by the ZAMOs (zero angular momentum observers) which are located at fixed B-L poloidal coordinates, r and θ (Punsly 2001). The toroidal magnetic field density, B^T, is the angular momentum flux per unit magnetic flux in each magnetic flux tube in the steady state and the global potential difference across a narrow tube of magnetic flux, $\delta\Phi$, is ΔV(Punsly 2001). In the initial state,

$$B^T \equiv \alpha\sqrt{g_{\phi\phi}}F_{r\theta} = -\frac{a\sin^2\theta B_0}{\rho^2}\ , \quad \Delta V = -\frac{\Omega_{_F}}{2\pi c}\delta\Phi\ . \tag{3-1}$$

The initial state is characterized by a field line angular velocity in the stationary frames that is zero everywhere, $\Omega_{_F} = 0$. The nonzero components of the electromagnetic field in the ZAMO frames are:

$$B^\phi = -\frac{a}{\rho^2\sqrt{\Delta}}B_0\sin\theta\ , \quad B^P = B^r = \frac{B_0\sin\theta}{\rho\sqrt{g_{\phi\phi}}}\ , \tag{3-2}$$

$$E^\perp = -\beta^\phi_{_F}B^P = E^\theta = \frac{2Mra\sin^2\theta}{\rho^2\sqrt{\Delta g_{\phi\phi}}}B_0\ , \quad \beta^\phi_{_F} = \frac{\Omega_{_F}-\Omega}{c\alpha}\sqrt{g_{\phi\phi}}\ . \tag{3-3}$$

---

[1]B-L coordinates are denoted by (t,r,θ,ϕ). The metric tensor, $g_{\mu\nu}$, is expressed in B-L coordinates throughout the text and is parameterized by the angular momentum per unit mass of the black hole, a. The following standard definitions are used: $\rho^2 \equiv r^2 + a^2\cos^2\theta$, $\Delta \equiv r^2 - 2Mr + a^2$, the lapse function, $\alpha = \sqrt{\Delta\sin^2\theta/g_{\phi\phi}}$, vanishes at the horizon and $\Omega \equiv -g_{\phi t}/g_{\phi\phi}$, where $\beta^\phi_{_F}$ is the azimuthal three-velocity of the corotating frame of the magnetic field as viewed in the ZAMO frames and Ω is the angular velocity of the ZAMOs in the stationary frames. B_0 is a constant introduced in (7) of Komissarov (2001).

The components of the four current in the initial state are $J^\theta = 0$, and

$$\rho_e = -\frac{Mrac(r^2+a^2)\sin\theta\cos\theta}{\pi\sqrt{\Delta g_{\phi\phi}}\rho^6}B_0 \ , \quad J^r = J^r_D - \frac{acB_0\,(r^2+a^2)\cos\theta}{2\pi\sqrt{\Delta}\rho^5} \ . \qquad (3\text{-}4)$$

In pulsar physics, ρ_e is known as the Goldreich-Julian charge density. The displacement current, $\mathbf{J}_D$ and J^ϕ are complicated functions.

The simulation proceeds due to the time evolution of $T^{\mu\nu}$ in equation (1.2). The angular momentum flux, B^T in equation (3.1), is not a constant in the flux tubes. The system evolves to a final state in which B^T equals a constant in each flux tube (the Blandford-Znajek solution). As this occurs, the field line angular velocity changes form zero to approximately one-half of the horizon angular velocity (Komissarov 2001). Consequently by (3.3) and Gauss' law, as the so-called "torsional Alfven wave" propagates outward from the spacetime near the event horizon, it adjusts ρ_e (the global potential) and the poloidal current (that is required to support B^T by Ampere's law).

4. Outgoing Force-Free Fast Waves in the Ergosphere

In this section, we show that outgoing (as viewed globally) ffde fast waves are redshifted away near the horizon. It is also demonstrated that ffde is an inaccurate depiction of plasma waves near the horizon.

4.1. The Propagation Vector of 2-D Fast Waves in the Proper Frame

In this subsection, we compute the axisymmetric 2-D fast wave-fronts near the horizon. This is necessary so that one can determine the Poynting flux of the waves in the proper frame of the plasma. This provides a meaningful physical constraint on the waves that is used in the next subsection to quantify the effects of gravitational redshift. From (3.2) and (3.3), an orthonormal proper frame is realized by a radial inward boost with velocity, v^r, relative to the ZAMO frames. Denote the ZAMO basis vectors as $\hat{e}_\mu$ and the proper frame basis vectors as $\bar{e}_\mu$:

$$\bar{e}_0 = \gamma[\hat{e}_0 + v^r\hat{e}_r] \ , \quad \bar{e}_r = \gamma[v^r\hat{e}_0 + \hat{e}_r] \ , \quad \bar{e}_\theta = \hat{e}_\theta \ , \quad \bar{e}_\phi = \hat{e}_\phi \ , \qquad (4\text{-}1a)$$

$$v^r = -\frac{2Mr\sin\theta}{\rho\sqrt{g_{\phi\phi}}} \ , \quad \gamma = \sqrt{\frac{\Delta(\rho^2+2Mr)+4M^2r^2}{\Delta(\rho^2+2Mr)}} \ . \qquad (4\text{-}1b)$$

In order for n_μ, to be the normal vector field to a 2-D fast wave-front in curved space requires that the hypersurface orthogonality condition be satisfied, $n_{[\mu;\nu}n_{\lambda]} = 0$: it is a geodesic and $n_\mu = hf_{,\mu}$, where f=constant defines the world-surface of the wave-front and h is an arbitrary function (Lightman et al 1975). The geodesic normal vectors can be described by Carter's equations of geodesic motion (Punsly 2001). There are 4 constants of motion for a null geodesic, the mass is zero, m is the angular momentum about the symmetry axis of the black hole, ω is the energy of the geodesic and K^2 is Carter's fourth constant of motion which represents the relativistic total angular momentum of the geodesic. By axisymmetry of the wave front and the light-like velocity of the wave we have $m = 0$. Using these definitions in the hypersurface orthogonality condition implies that the outgoing (in the stationary frames) fast ffde wave-fronts around a Kerr black hole are defined by $f=$ constant surfaces and $h = 1$,

$$f = \pm \int \sqrt{K^2 - \omega^2 a^2 \sin^2\theta}\, d\theta + \int \frac{\sqrt{\omega^2\left(r^2+a^2\right) - \Delta K^2}}{\Delta}\, dr - \omega t\ . \qquad \text{(4-2)}$$

The quantities ω and K^2 are constants along the wave surface in order to satisfy the hypersurface orthogonality condition (Note that there are no outgoing spherical fast waves near the black hole in the Kerr geometry). In the proper frame, $n_\mu \equiv (-\mid \bar{\mathbf{n}} \mid,\ \bar{\mathbf{n}})$. Since $m = 0$, $\bar{n}^\phi = 0$. The other components of the wave normal vector field are found by using (4.1) and (4.2) to evaluate $f_{,\mu}$:

$$\bar{n}^r = \frac{\gamma}{\rho\sqrt{\Delta}}\left[2Mr\omega + \sqrt{\omega^2\left(r^2+a^2\right)^2 - \Delta K^2}\right]\ ,\ \bar{n}^\theta = \pm\frac{1}{\rho}\sqrt{K^2 - \omega^2 a^2 \sin^2\theta}\ . \qquad \text{(4-3)}$$

4.2. Causal Stress-Energy Constraints

The equivalence principle demands that $T^{\mu\nu}$ of the waves is well-behaved in a freely falling frame, this is the fundamental physical reason why a black hole can not radiate an **arbitrary** spectrum and flux of light waves (Candelas 1980). In this subsection, the same result is demonstrated for ffde fast waves. It is straightforward to evaluate the structure of a transverse step wave in the **upstream** proper frame. From the polarization properties in section 2, the fields transported by the fast wave are determined by $\bar{\mathbf{n}}$ and $\mathbf{b}_u$ in terms of a potential function, $\bar{E}_\theta$,

$$\bar{\mathbf{E}} = \left[-\frac{\bar{n}_\theta}{\bar{n}_r}\bar{e}_r + \bar{e}_\theta + \frac{\bar{n}_\theta b^r}{\bar{n}_r b^\phi}\bar{e}_\phi\right]\bar{E}_\theta\ , \bar{\mathbf{B}} = \left[\frac{\bar{n}_\theta^2 b^r}{\bar{n}_r \mid \bar{\mathbf{n}} \mid b^\phi}\bar{e}_r - \frac{\bar{n}_\theta b^r}{\mid \bar{\mathbf{n}} \mid b^\phi}\bar{e}_\theta + \frac{\mid \bar{\mathbf{n}} \mid}{\bar{n}_r}\bar{e}_\phi\right]\bar{E}_\theta\ . \text{(4-4)}$$

By (4.1), (4.4) and (3.3) if the wave propagates variations, $\delta\Omega_{_F}$ then $\bar{E}_\theta$ transported by the wave in the upstream proper frame is

$$\bar{E}_\theta = -\frac{\bar{n}^r \sqrt{g_{\phi\phi}} B^P}{\gamma \left[\bar{n}^r + v^r \mid \bar{\mathbf{n}} \mid\right] c\alpha} \delta\Omega_{_F} + O\left(\left[\frac{\bar{n}^\theta}{\mid \bar{\mathbf{n}} \mid}\right]^2\right) , \tag{4-5}$$

where the error terms represent the small changes to B^P transported by the wave. According to (4.4) and (4.5), outgoing fast waves ($\bar{n}^r > 0$) are blueshifted near the horizon in a proper frame: $\bar{B}_\phi \approx \bar{E}_\theta \sim \delta\Omega_{_F}\alpha^{-2}$. By contrast for ingoing waves, the fields are well behaved in the proper frame $\sim \alpha^0$: this is the essence of the "ingoing wave condition" that is used as a constraint on the background fields near the horizon in ffde. The Poynting flux of the outgoing wave has two divergent contributions in the upstream proper frame, the linear term, $\mathbf{E}\times\mathbf{b}_u \sim \alpha^{-2}$ and the dominant, quadratic pure wave contribution $T^{\mu\nu} \sim n^\mu n^\nu$, $\mathbf{E}\times\mathbf{B} \sim \alpha^{-4}$. The same result occurs in a freely falling frame by (3.19) of Punsly (2001). Alternatively stated, the regularity of $T^{\mu\nu}$ in a freely falling frame near the horizon requires that a globally outgoing fast wave near the horizon can only affect changes in $\delta\Omega_{_F} \sim \alpha_0^2$, where the subscript "0" means to evaluate at the point wave emission (a similar result is true for the poloidal current). The exact same scaling is found from detailed calculations of outgoing MHD waves near the horizon (see Table 6.1 of Punsly (2001)). Formally in ffde, the event horizon cannot radiate changes in $\Omega_{_F}$, or the poloidal current.

This is a significant result in the steady state. By (1.1), $\mathbf{J}\cdot\mathbf{E} = 0$, so there is no transfer of energy from the plasma to the fields in the magnetosphere. Thus the poloidal current that supports the Poynting flux cannot be created within the magnetosphere in the steady state solution (in contrast to MHD). It must be injected at the boundaries. The only boundaries on the field aligned current are the event horizon and asymptotic infinity. But, the horizon cannot change the poloidal current, creating another causality paradox.

The simulation evolves through large changes to $\Omega_{_F}$ that emerge from near the horizon. The pathology of these waves cannot be dismissed by moving the point of emission just outside the horizon. It arises from physical contradiction of large amplitude force-free waves emerging from a region that is inertially dominated - gravity determines the global particle trajectories irrespective of all external forces (Punsly 2001). One can understand this inconsistency by considering the effect of one of these waves, emerging from near the horizon, on the plasma upstream in the proper frame. By (4.5) the fields transported by the wave diverge like α^{-2}, and such a perturbation drastically alters the direction of $\mathbf{b}$. Thus, there are large accelerations of the particles as they transition from one force-free state to another as the wave-front passes: flowing parallel to $\mathbf{b}_u$ upstream to flowing parallel to the perturbed $\mathbf{b}_d$, downstream. The physics of this interaction cannot be described within ffde. These large accelerations imply strong forces as well, no matter how tenuous the plasma.

This information is lost in ffde by setting the mass equal to zero in MHD. However, full MHD calculations capture this strong force. Even in the most tenuous of plasma states, any MHD fast wave requires very strong inertial, cross-field (nonforce-free) currents in order to propagate outwards from near the event horizon (see (6.93) of Punsly (2001) or Hirotani et al (1993)). These currents translate to a very strong $F^{\mu\nu}J_{\nu}$ force in the proper frame. **This pathology of ffde is a direct consequence of the momentum equations of the plasma near the horizon, the plasma attains an infinite inertia in a global sense (Punsly 2001) which is the opposite of the force-free assumption that assumes plasma inertia is negligible.**

5. Conclusion

In this paper two concerns about ffde simulations were raised. First of all the event horizon cannot effect V due to the gravitational redshift. Secondly, if V is modified by fast waves that emerge from just outside the horizon then they are created by a plasma that cannot support them self-consistently. The plasma state is set by the gravitational field (inertially dominated) and outgoing, large amplitude, plasma waves interact with a real plasma in this region through strong forces in the proper or freely falling frames.

The Blandford-Znajek solution is the unique ffde steady state solution since there is no other ffde solution that conserves energy and angular momentum in the flux tubes: B^T and Ω_F are a constant in each flux tube. Any simulation restricted to ffde either finds no solution or this one. The only ambiguity is the very minor modifications to the initial poloidal field noted by Komissarov (2001). If the ffde restriction is relaxed then there are many MHD solutions and if the MHD restriction is removed then there are others as well (Punsly 2001). The paradox of the outgoing "torsional Alfver wave" described in the Introduction is an indication that simulations required to seek this unique ffde solution are unphysical. The calculations of section 4 demonstrate that the simulation evolves by means of unphysical waves and currents. Furthermore, the results of this paper conflict with the strong waves that are instantaneously created in the simulation as a consequence of $\mathbf{J}_D$ in the initial state.

Because of their diametrically opposite outgoing wave properties (very strong inertial forces versus force-free) near the horizon, the magnetically dominated limit of a full MHD simulation should differ substantially from this ffde treatment. The results of the MHD simulations of Koide et al (2002), Camenzind and Khanna (2000), Semenov et al (2002) and the time stationary solution in Chapter 9 of Punsly (2001) indicate that this is the case. This fact is apparent in all of the above in which relativistic inertia imparted to the plasma by the gravitational field dominates the dynamics in the ergosphere, regardless of the degree of inertial dominance imposed in the initial state. The interaction drives very strong cross-field currents (note that these currents cannot be dismissed as transients in Punsly (2001) in which they are eternal and Semenov et al (2002) in which they are persistent for long simulations and the driving force never goes away). Not coincidentally, this is precisely the region in which unphysical waves emerge in the ffde simulation.

In summary, the force-free assumption does not apply to physics near the event horizon. Even if an initial MHD state is chosen by hand to be force-free, the pathology of force-free physics near the horizon becomes evident when the outgoing MHD perturbations of the initial state (such as the waves discussed above) are analyzed.

REFERENCES

Blandford, R. and Znajek, R. 1977, MNRAS. **179**, 433

Camenzind, M. and Khanna, R. 1999, Nuovo Cimento **B115**, 815

Candelas, P. 1980, Phys. Rev. D **21**, 2185

Hirotani, K., Tomimatsu, A. and Takahashi, M. 1993, PASJ **45**, 431

Koide, S., Shibata, K., Kudoh, T. and Meier, D. 2002, Science **295**, 1688

Komissarov, S. 2001, MNRAS. **326** L41

Komissarov, S. 2002, in *Proceedings of the Third Sakharov Conference on Physics, Moscow, 2002*

Lightman, A., Press, W., Price, R. and Teukolsky, S. 1975, *Problem Book in Relativity and Gravitation* (Princeton University Press, Princeton)

Punsly, B. 2001, *Black Hole Gravitohydromagnetics* (Springer-Verlag, New York)

Punsly, B. 2003, ApJ **583**, 842

Semenov, V. 2002, et al, Physica Scripta **65**, 13

Spitkovsky, A. 2003 to appear in *Young Neutron Stars and Their Environments*, (IAU Symposium Vol.218), astro-ph/0310731

AAS LaTeX macros v5.0.