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Non-universality of the absorbing-state phase-transition in a linear chain with power-law diluted long-range connections

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HIGHLIGHTS

- The contact process on chains with power-law distributed connections is investigated.
- The absorbing-state phase transition deviates from the directed percolation universality class.
- We find a crossover from the 1D directed percolation to mean-field-like exponents when the couplings become longer ranged.

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ABSTRACT

In this work we study the critical behavior of the absorbing state phase transition exhibited by the contact process in a linear chain with power-law diluted long-range connections. Each pair of sites is connected with a probability $P(r)$ that decays with the distance between the sites r as $1/r^{\alpha}$. The model allows for a continuous tuning between a standard one-dimensional chain with only nearest neighbor couplings ($\alpha \to \infty$) to a fully connected network ($\alpha = 0$). We develop a finite-size scaling analysis to obtain the critical point and a set of dynamical and stationary critical exponents for distinct values of the decay exponent $\alpha > 2$ corresponding to finite average bond lengths and low average site connectivity. Data for the order parameter collapse over a universal curve when plotted after a proper rescaling of parameters. We show further that the critical exponents depend on α in the regime of diverging bond-length fluctuations ($\alpha < 3$).

1. Introduction

The Contact Process (CP), introduced by Harris a long time ago [1], was one of the first models to exhibit a non trivial critical behavior even in one dimension. Originally proposed to describe epidemic spreading, it has been later used to model a variety of competitive dynamical phenomena [2].

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The CP state is defined on a d-dimensional network, where each site is occupied by an active (infected) or an inactive (healthy) individual. Furthermore, the CP dynamics is a stochastic process obeying a Markovian rule: in a time step, healthy individuals may be infected with a probability p which depends both on the number of active as well as inactive neighboring individuals, while infected individuals may recover at constant probability λ, which is usually considered as a control parameter.

This rule generates a dynamical competition between infection and healing processes, whose output is determined by the value of the control parameter λ. It turns out that the fraction of infected individuals ρ (order parameter) vanishes for large values of λ while, for small values of λ, the ratio ρ remains strictly positive. Therefore, for large values of λ the epidemic is completely eliminated and the population reaches an absorbing healthy state. For small values of λ the system evolves towards a statistically stationary dynamical epidemic configuration with a fluctuating fraction of active individuals. The absorbing state will eventually be reached in systems with a finite size irrespective to the value of λ because random fluctuations will occasionally drive the system towards the healthy vacuum state.

According to the above characteristics, the system then displays a continuous transition from an active state to an absorbing state at a critical transition point λ_c. Many previous works have shown that the CP model belongs to the universality class of directed percolation (DP) [2]. These studies are in agreement with the conjecture of Grassberger and Janssen [3,4] that CP stays in the DP universality class since it only includes short-range interactions and does not have special attributes such as additional symmetries or quenched disorder [5]. All systems that belong to the DP universality class have a continuous phase transition to an absorbing state which is characterized by a strictly positive order parameter.

In recent years, many generalizations of the CP model have been proposed [6–12]. In particular, there has been an increasing interest in model systems with long-range interactions whose critical behavior can depart from the usual DP universality class [13–15]. The most straightforward proposal considered fat-tailed probabilities for the connections, which typically can be Levy or power-law type [16]. Another class of contact processes whose universality class deviates from DP includes models with an explicit particle diffusion process [17,18]. In these systems, the particles can have two distinct states, healthy or infected, and can independently diffuse. The total density of particles acts as the control parameter with the active state being statistically stable only above a critical concentration of particles. Even local diffusion is a relevant ingredient in these models and the critical properties of the absorbing state phase transition depends on the relative diffusivity of particles in distinct states. Recently, a superdiffusive epidemic process model was introduced whose critical exponents were shown to depend on the decay exponent of a Levy-like jump distribution function, exhibiting a crossover from the diffusive epidemic universality class to the mean-field universality class as the jump distribution becomes more long-ranged [19].

In this paper, we will advance along this research line, considering a Generalized Contact Process (GCP) model in one-dimension with strictly power-law distributed connections and no particle diffusion [20–22]. More precisely, the probability of a link between the sites i and j, namely $P(r_{ij} = |i-j|)$, decays as a power law (see Eq. (1) below). In this sense, the present model differs from those considering the activation process to be governed by interactions with other active individuals, even when they are located at long distances. It has been suggested that the contact process with Levy exchanges belongs to the same universality class of the contact process in models with long-distance interactions [21]. In this class of models, the infection probability decays as a power-law of the distance to the active individuals. Both variants of these models (interaction with all active individuals or just with the nearest one) deviate from the usual DP universality class, exhibiting continuously varying critical exponents [16,22,23].

The present model considers the contact process on a network on which all first-neighbors connections are included together with additional diluted long-distance connections, whose fraction decays as a power-law of the distance. The contact process on complex networks has also been a subject of current interest regarding its critical behavior. In scale-free networks, it has been suggested that the transition should fall in a generalized DP mean-field universality class, although numerical simulations have shown some deviations [24–28]. The contact process on a multi-scale network consisting of linear chains connected by a scale-free one also was shown to belong to a new universality class [29]. More recently, the contact process on chains with quasi-periodic connections has been investigated, and a new set of critical exponents were unveiled with a dependence on the underlying inflation rule [30]. Here, the exponent α governing the decay of the connection probability allows us a continuous variation from the one-dimensional contact process model with just first neighbors couplings ($\alpha \to \infty$), to the mean-field regime consisting in the fully connected network ($\alpha = 0$). Indeed, as we will see later, the parameter α shapes the nature of the interaction, and only for sufficiently large values of α (short-range contacts favored) the model falls in the standard *DP* universality class.

This paper is organized as follows: in Section 2 we describe our short/long range GCP model including the numerical techniques used. Section 3 deals with our results concerning the critical points, emphasizing their main features. The summary and conclusions of this paper are presented in Section 4.

2. Model and numerical methods

We define our GCP model taking into account a linear chain of length L with periodic boundary conditions. Two sites i and j of the chain are connected with a probability $P(r_{ij})$, independent of other pairwise connections, where r_{ij} denotes the smallest distance between sites along the closed chain. This probability is assumed to obey a power-law decay, i.e.

$$P(r_{ij}) = 1/r_{ij}^{\alpha} \tag{1}$$

where r_{ij} is the distance between the sites i and j, and α is the exponent that governs the effective range of the couplings. We remark that, according to this probability, the first neighbor sites are always linked. Furthermore the number of links to a given site, for large L, is in average $2\sum_{r=1}^{\infty} r^{-\alpha} = 2\,\zeta(\alpha)$ (ζ is the Riemann function). Observe also that the average number of connected sites increases for decreasing values of α. In the following, we will develop simulations using several values of $\alpha \geq 2$. The average distance of connected sites is $\zeta(\alpha-1)/\zeta(\alpha)$. The Riemann function $\zeta(\alpha)$ diverges when $\alpha \to 1$, and therefore, when $\alpha = 2$ the average distance of the connected sites $\lim_{\alpha\to 2} \zeta(\alpha-1)/\zeta(\alpha)$ is infinite. In this limiting value, the average number of links *per* site is $2\,\zeta(2) = \pi^2/3$. This means that the most connected case has, in average, an extra link for any site.

The resulting connected lattice is quenched and it is never updated during the dynamics. It can be repeated from the beginning many times with a different realization of the connecting lattice, whose different outputs can be averaged in order to eliminate fluctuations.

The state of the system is defined by the string $\sigma_1, \sigma_2, \ldots, \sigma_L$ where the dichotomous variable σ_i describes the state of the individual located at the site i, i.e., $\sigma_i = 1$ indicates an infected individual (active), while $\sigma_i = 0$ a healthy one (inactive). Then, the dynamics updates the state of the system according to the following rules:

(i) We always start with every individual being infected (the fraction of active sites is $\rho = 1$).
(ii) A site i is randomly selected and the status of the individual is checked. Then, if $\sigma_i = 0$, the individual will be infected with a probability $p_i = s_i/n_i$, where n_i and s_i are, respectively, the number of sites and the number of active sites linked to i. Otherwise, if $\sigma_i = 1$, the individual recovers with probability λ.
(iii) We re-determine the fraction of active individuals ρ as $\rho = L^{-1}\sum_{i=1}^{L} \sigma_i$. In order to avoid capture by the absorbing state due to fluctuations, whenever $\rho = 0$ we escape from the trap by infecting a randomly chosen individual. Previous works have shown that this procedure provides an accurate inference of the critical exponents, specially those related to the critical relaxation dynamics when starting from the fully active state because the power-law relaxation process takes place at times much shorter than the typical trapping time [31–33].
(iv) Steps (ii) and (iii) are repeated a number of times enough to reach relaxation.
(v) The whole process is started again with a different realization of the lattice connections.

In these simulations we considered systems of sizes $L = 100, 200, 400, 800$, and 10000. The smaller sizes (from $L = 100$ to $L = 800$) have been used for finite scaling analysis (systems evolve for 10^4 Monte Carlo steps and 5×10^3 independent realizations independently on the size) and data collapse. The larger size $L = 10000$ has been considered for the determination of the exponents which characterize the time relaxation near the critical point. The number of independent realizations for $L = 10000$ (distinct quenched connected networks) was 10^5 for each value of α.

3. Numerical results

Fig. 1(a) shows the behavior of the order parameter ρ as a function of the control parameter λ, for different values of α. We remind that the phase transition to the absorbing state is described, close to criticality, by the relation

$$\langle\rho\rangle \sim (\lambda_c - \lambda)^{\beta}, \tag{2}$$

where β is the exponent that characterizes the transition, and λ_c is the critical point which separates the active and inactive phases. Fig. 1(b) shows the dependence of the critical value λ_c with α. From there we observe that, for increasing values of α, the critical point λ_c converges to the well known critical value 0.30326(1) of CP in chains with only first neighbor couplings [2].

We performed a preliminary estimate of the values λ_c by computing the moment ratio m defined as (3)

$$m = \frac{\langle\rho^2\rangle}{\langle\rho\rangle^2}, \tag{3}$$

which is known to be scale invariant at the absorbing state phase transition [34,35]. In Fig. 2, we plotted the moment ratio m as a function of the control parameter λ, for several values of the network sizes, namely $L = 100, 200, 400$ and 800. In Fig. 2(a) we consider the decay exponent $\alpha = 2.0$, while in Fig. 2(b) we consider $\alpha = 5.0$. The different plots of m, corresponding to different values of the lattice size L, intersect roughly at a common point (m_c, λ_c) where the phase transition occurs. Thus, the intersections in Fig. 2(a) and (b) give us rough estimates of λ_c. This procedure was used for the full set of α values considered in this work.

In order to obtain refined estimates of λ_c and also to calculate the exponent ratio $\beta/z\nu_{\perp}$, we considered the time evolution of the average density ρ of active individuals at the critical point $\lambda = \lambda_c$, which must satisfy the scaling relation

$$\langle\rho(t)\rangle \sim t^{-\beta/z\nu_{\perp}}. \tag{4}$$

Here we have considered a system of size $L = 10000$ and we refined the values of λ_c previously obtained from the scale invariance of the moment ratio by looking for the value of λ that resulted in a power-law decay of the order parameter density over more than three decades. The linear behavior in the log–log scale of Fig. 3 confirms the accuracy of the λ_c values. The average was performed over 10^5 independent realizations of the GCP for $\alpha = 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0$.

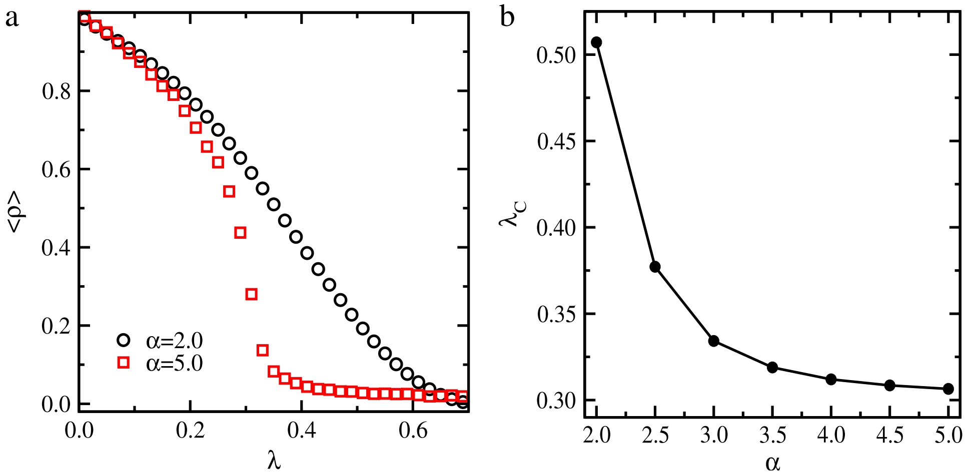

Fig. 1. (Color online) (a) Order parameter $\langle\rho\rangle$ as a function of the control parameter λ for $\alpha = 3.0$ (circles) and 5.0 (squares). Here we have considered $L = 800$ for both values of α. Observe that smaller values of α favor infection; (b) Plot of the critical values of the control parameter λ_c versus the decay exponent α. The points correspond to the intersections of the moment ratio m versus λ as shown in the next figure. When α increases, λ_c approaches to the standard value of the *CP* transition in a linear chain with only first-neighbors couplings.

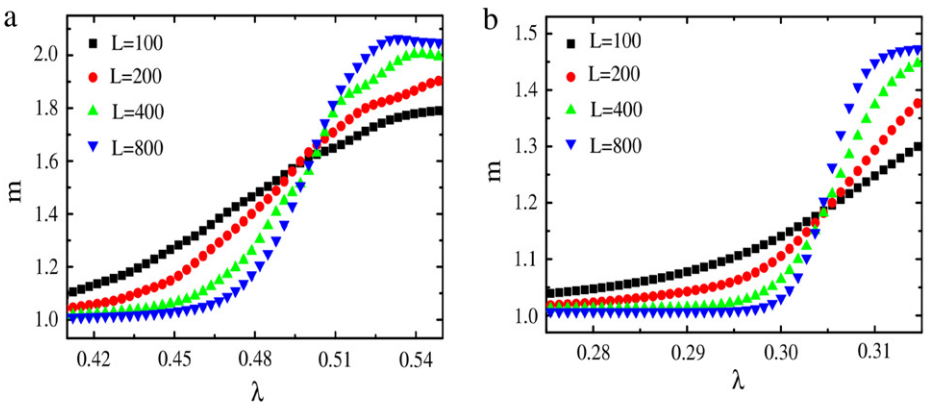

Fig. 2. (Color online) The moment ratio m as a function of the control parameter λ for different sizes L. All curves intersect in the critical point, giving an estimation for λ_c; (a) $\alpha = 2.0 \rightarrow \lambda_c = 0.5071$; (b) $\alpha = 5.0 \rightarrow \lambda_c = 0.3065$.

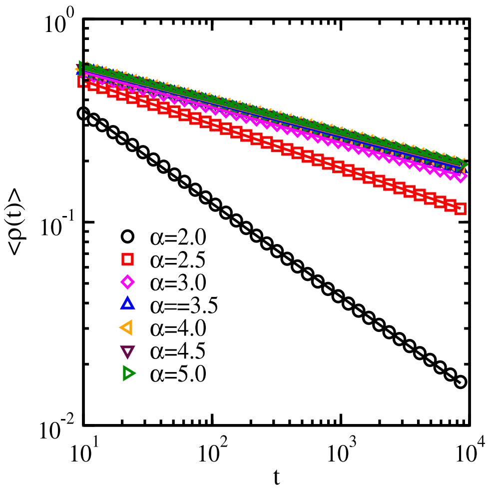

Fig. 3. (Color online) Order parameter $\langle\rho\rangle$ as a function of time t at the critical point. The best fit provides the critical exponents $\beta/z\nu_\perp$ for the different values of α. Data are from simulations on a chain with $L = 10000$ sites.

Table 1
Estimates of the critical point and critical exponents for different values of α. The values in the last line, corresponding to the $d = 1$ usual DP universality class, are those in Ref. [36].

α	λ_c	$\beta/z\nu_\perp$	$1/z$	$1/z\nu_\perp$	$\beta/\nu_\perp$	$1/\nu_\perp$
2.0	0.5071(1)	0.46(2)	1.20(5)	0.57(5)	0.38(2)	0.48(4)
2.5	0.3772(1)	0.21(1)	0.79(3)	0.40(4)	0.27(2)	0.50(5)
3.0	0.3342(1)	0.17(1)	0.70(3)	0.48(3)	0.26(2)	0.68(1)
3.5	0.3189(1)	0.17(1)	0.68(3)	0.51(3)	0.25(1)	0.75(3)
4.0	0.3120(1)	0.16(1)	0.67(3)	0.54(3)	0.25(1)	0.81(1)
4.5	0.3085(1)	0.16(1)	0.66(3)	0.57(3)	0.25(1)	0.86(4)
5.0	0.3065(1)	0.16(1)	0.65(3)	0.57(2)	0.25(1)	0.86(2)
DP	–	0.159464(6)	0.632613(4)	0.576752(2)	0.25207(1)	0.911698(3)

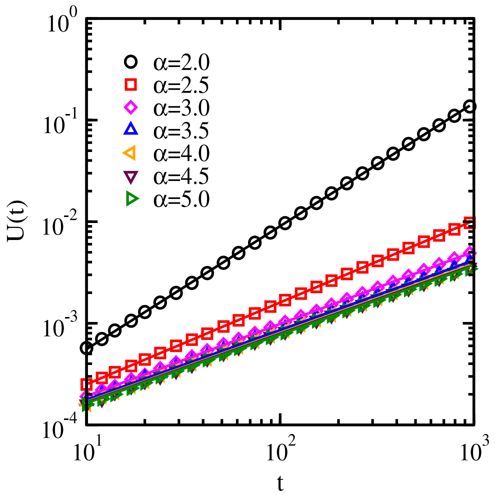

Fig. 4. (Color online) Log–log plot of the moment ratio U versus time t at the critical point for a system of size $L = 10000$. The best fit gives the critical exponent $1/z$ for the different values of α.

The exponent ratio $\beta/z\nu_\perp$ is estimated from the best linear fit of data when plotted in a log–log scale (see estimated values in Table 1). We remark that the exponent ratio for $\alpha \geq 3$ is consistent to the one expected for the directed percolation universality class in $d = 1$ ($\beta/z\nu_\perp = 0.159464(6)$ [36]).

A second dynamical exponent was computed by considering the time evolution of the moment ratio $U(t) = m(t) - 1$ at the critical point $\lambda = \lambda_c$, where it is expected to satisfy the following scaling relation

$$U(t) \sim t^{d/z}, \tag{5}$$

with d being the dimension of the system ($d = 1$ for a linear network). According to the above relation, the slope of the best linear fit of a log–log plot of $U(t)$ versus t gives the critical exponent $1/z$ (see Fig. 4). The estimated values of this dynamical critical exponent are also summarized in Table 1.

A third dynamical critical exponent $1/z\nu_\perp$ can be estimated taking into account that, in the critical point, the derivative of the order parameter logarithm scales as

$$D(t) = \frac{\mathrm{d}|\ln\langle\rho(t)\rangle|}{\mathrm{d}\lambda} \sim t^{1/z\nu_\perp}. \tag{6}$$

The corresponding result is shown in Fig. 5 as a log–log plot. The time interval used in estimating this critical exponent ranges from 10^3 to 10^4. The regime of longer times (up to 10^4) was not included since it turns out that finite difference is a poor approximation of the derivative in this region. The computed values of the exponent $1/z\nu_\perp$ are shown in Table 1.

In order to estimate some stationary critical exponents, we performed a collapse of stationary order parameter data [37]. The resulting scaled data is illustrated in Fig. 6(a) for $\alpha = 2.0$, as well as in Fig. 6(b) where we considered $\alpha = 5.0$. Other values of these stationary critical exponents can be found in Table 1. Note that all critical exponents deviate from the usual DP values for $\alpha < 3$, approaching the corresponding mean-field values as α decreases.

We close this section by considering the extreme case $\alpha = 0$, where all possible links are active and the model assumes a mean-field character. Each of the $(1 - \rho)L$ healthy individuals has a probability ρ of being infected, while the recover probability of one of the ρL active individuals is λ. Therefore, the number of infected individuals increases of one with

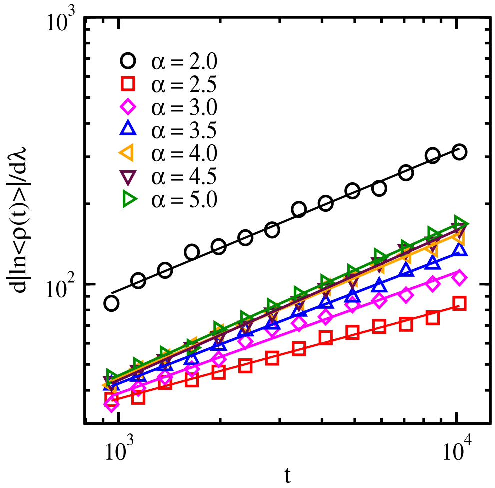

Fig. 5. Log–log plot of the derivative of $d|\ln\langle\rho\rangle|/d\lambda$ versus time t at the critical point for a system of size $L = 10000$. The best fit gives the critical exponent ratio $1/z\nu_\perp$.

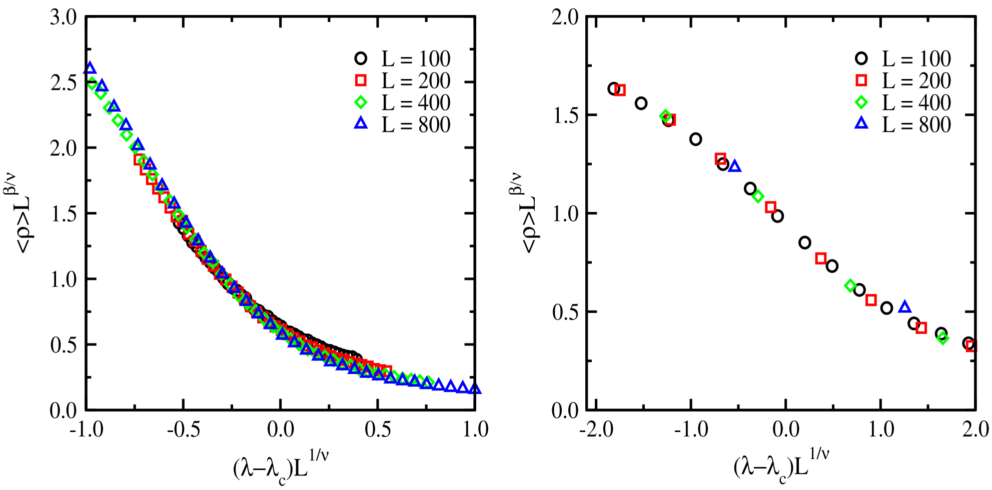

Fig. 6. (Color online) Data collapse of the order parameter, calculated for $L = 100$, 200, 400, and 800. The stationary regime was reached after 10^4 Monte Carlo steps and 5×10^3 independent realizations were considered. (a) $\alpha = 2.0$: best collapse with $\lambda_c = 0.5071$, $\beta/\nu_\perp = 0.38$ and $1/\nu_\perp = 0.48$; (b) $\alpha = 5.0$: best collapse with $\lambda_c = 0.3065$, $\beta/\nu_\perp = 0.25$ and $1/\nu_\perp = 0.86$.

probability $\rho(1-\rho)$, and decreases of one with probability $\lambda\rho$. In the continuous time limit

$$\frac{d\rho}{dt} = \rho(1-\rho-\lambda), \tag{7}$$

the critical point is $\lambda_c = 1$, while the steady state has

$$\rho = \lambda_c - \lambda, \tag{8}$$

yielding $\beta = 1$. Furthermore, at the critical point

$$\rho(t) \sim t^{-1}, \tag{9}$$

and therefore $\beta/z\nu_\perp = 1$, which also implies $1/z\nu_\perp = 1$.

4. Summary and conclusions

In summary, we introduced in this paper a new model exhibiting an absorbing state phase transition with continuously varying critical exponents. The model is a generalization of the contact process on a one-dimensional chain in which additional connections between sites at a distance r are randomly distributed with a probability decaying as $1/r^\alpha$. The binding exponent α allows a continuous tuning between the standard one-dimensional contact process with just first neighbors interactions ($\alpha \to \infty$), and the mean-field regime is achieved in the limit of a fully connected network ($\alpha = 0$).

We developed numerical simulations on finite networks, and employed a finite-size scaling analysis to locate the critical point and exponents for distinct values of the binding exponent α. The average length of the long-distance connections diverges for $\alpha \leq 2$. Our simulations were restricted to the regime of $\alpha \geq 2$. Our results show that significant deviations from the usual directed percolation universality class appear as α decreases. Although the model incorporates quenched disorder in the lattice topology, the transition remains a second order one, contrasting with the Griffiths behavior reported to occur in some other models with a quenched disorder distribution of recovery rates [38], but in full agreement with previous studies of absorbing-state phase-transitions in complex networks with an heterogeneous distribution of site connectivities [13].

The scale invariance of the second moment of the stationary order parameter density was used to precisely locate the critical point. Further, the short time dynamics of the critical order parameter, its logarithmic derivative and moment ratio, were used to evaluate three independent dynamical critical exponents. Stationary critical exponents were also independently estimated by employing a collapse of stationary order parameter data in the vicinity of the transition. The reported results show that the critical exponents are roughly constant for $\alpha > 3$, the regime with finite variance of the bond-length. Below this particular value the exponents vary continuously with α approaching the mean-field behavior as α decreases. This finding adds to the general belief that long-range interactions can indeed lead to a non-equilibrium critical behavior with continuously varying exponents. It is important to stress that it has been previously demonstrated that long-range interactions can lead to the breakdown of universality [22]. Further, continuously varying exponents have also been reported to take place in scale-free networks, depicting a power-law distribution of connectivities [24–28]. It is important to stress that a similar trend has been recently observed in the superdiffusive contact process on which a crossover to mean-field critical behavior is achieved when the jump distribution governing the diffusion process becomes more long-ranged [19]. However, the model considered in the present work belongs to a distinct universality class because it does not contain an explicit particle diffusion process. Therefore the present results indicate that long-range processes promote a crossover to mean-field behavior irrespective to the universality class of the corresponding short-range model and its universality class.

Our model thus provides a simple framework to continuously investigate the crossover from one-dimensional to mean-field critical behavior of the contact process phase transition. The long-range character of the connections is able to modify the critical behavior with respect to the usual directed percolation universality class. Analytical perturbative and field theoretical approaches would be in order to describe the critical behavior in the distinct coupling regimes. Furthermore, the regime of small values of $\alpha < 2$ deserves a more extensive simulation study. In this regime, the average distance of connected sites strongly diverges as the system size increases. Under these conditions, finite-size scaling arguments have to be properly adapted to deal with such non-extensive scenario. We hope the present results can stimulate future investigations along these lines.

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References

[1] T.E. Harris, Ann. Probab. 2 (1974) 969.
[2] J. Marro, R. Dickman, in: V. Privman (Ed.), Non-Equilibrium Statistical Mechanics in One Dimension, Cambridge University Press, Cambridge, 1997.
[3] H.K. Janssen, Z. Phys. B 42 (1981) 151.
[4] P. Grassberger, Z. Phys. B 47 (1982) 365.
[5] H. Hinrichsen, Adv. Phys. 49 (2000) 815.
[6] N. Inui, A. Tretiakov, Phys. Rev. E 85 (2012) 021123.
[7] M.M. de Oliveira, R. Dickman, Phys. Rev. E 84 (2011) 011125.
[8] P.C. da Silva, M.L. Lyra, L.R. da Silva, G. Corso, U.L. Fulco, Internat. J. Modern Phys. C 22 (2011) 563.
[9] F.L. Santos, R. Dickman, U.L. Fulco, J. Stat. Mech. (2011) P03012.
[10] N.V. da Costa, U.L. Fulco, M.L. Lyra, I.M. Gleria, Phys. Rev. E 75 (2007) 031112.
[11] P.C. da Silva, M.L. Lyra, U.L. Fulco, L.R. da Silva, Internat. J. Modern Phys. C 15 (2004) 1279.
[12] G. Ódor, Universality in Nonequilibrium Lattice Systems, World Scientific, Singapore, 2008.
[13] C. Castellano, R. Pastor-Satorras, Phys. Rev. Lett. 96 (2006) 038701.
[14] S.C. Ferreira, R.S. Ferreira, R. Pastor-Satorras, Phys. Rev. E 83 (2011) 066113.
[15] K. Gong, M. Tang, H. Yang, M. Shang, Chaos 21 (2011) 043130.
[16] H. Hinrichsen, J. Stat. Mech. (2007) P07006.
[17] R. Kree, B. Schaub, B. Schmittmann, Phys. Rev. A 39 (1989) 2214.
[18] F. van Wijland, K. Oerding, H.J. Hilhorst, Physica A 251 (1988) 179.
[19] M.B. da Silva, A. Macedo-Filho, E.L. Albuquerque, M. Serva, M.L. Lyra, U.L. Fulco, Phys. Rev. E 87 (2013) 062108.
[20] M.C. Marques, A.L. Ferreira, J. Phys. A: Math. Gen. 27 (1994) 3389.
[21] E.V. Albano, Phys. Rev. E 54 (1996) 3436.
[22] F. Ginelli, H. Hinrichsen, R. Livi, D. Mukamel, A. Torcini, J. Stat. Mech. (2006) P08008.
[23] C.E. Fiore, M.J. de Oliveira, Phys. Rev. E 76 (2007) 041103.
[24] R. Pastor-Satorras, A. Vespignani, Phys. Rev. Lett. 86 (2001) 3200; Phys. Rev. E 63 (2001) 066117.
[25] C. Castellano, R. Pastor-Satorras, Phys. Rev. Lett. 100 (2008) 148701.
[26] S.C. Ferreira, R.S. Ferreira, R. Pastor-Satorras, Phys. Rev. E 83 (2001) 066113.
[27] J.D. Noh, H. Park, Phys. Rev. E 79 (2009) 056115.

[28] S.C. Ferreira, R.S. Ferreira, C. Castellano, R. Pastor-Satorras, Phys. Rev. E 84 (2011) 066102.
[29] S.C. Ferreira, M.L. Martins, Phys. Rev. E 76 (2007) 036112.
[30] R. Juhász, G. Ódor, Phys. Rev. E 80 (2009) 041123.
[31] U.L. Fulco, D.N. Messias, M.L. Lyra, Phys. Rev. E 63 (2001) 066118.
[32] U.L. Fulco, D.N. Messias, M.L. Lyra, Physica A 295 (2001) 49.
[33] A. Macedo-Filho, G. Corso, U.L. Fulco, M.L. Lyra, J. Stat. Mech. 10 (2010) P04027.
[34] R. Dickman, J.K.L. da Silva, Phys. Rev. E 58 (1998) 4266.
[35] S. Lübeck, R.D. Willmann, J. Stat. Phys. 115 (2004) 1231.
[36] I. Jensen, J. Phys. A: Math. Gen. 32 (1999) 5233.
[37] D.P. Landau, Phys. Rev. B 13 (1976) 2997.
[38] T. Vojta, M. Dickison, Phys. Rev. E 72 (2005) 036126.