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# An optical readout scheme for advanced acoustic GW detectors

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**Abstract**

We have recently proposed a large reading area, optical readout scheme for advanced acoustic gravitational wave (GW) detectors. In this work we focus the analysis on a dual-cylinder detector. A specific configuration is designed and the expected performance is calculated.

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The next generation of acoustic gravitational wave (GW) detectors can play a crucial role in GW observatories, due to their expected high sensitivity in the 1–5 kHz range. In particular, recently proposed configurations are based on two nested masses, either spheres [1] or cylinders [2], and the signal is read in the gap as the differential deformation of the two masses. This design makes no use of resonant mechanical amplifiers, allowing large useful bandwidth (few kHz) and enhanced sensitivity. However, with such a solution the wideband noise of the readout becomes more important, since it must be compared directly with the Brownian noise and the response of the large mass detector. The readout of the weak signal is thus even more challenging.

An important requirement for the transducer is that it must read a large portion of the detector surface, in order to average over 'local' thermal and back-action effects. With an optimal readout, the performance is limited by just the back-action from the vibration modes which are sensitive to GW, and the thermal noise level is negligible.

Optical techniques have recently proved to be a useful tool for the signal readout in resonant GW detectors [3]. The weak vibration changes the length of a Fabry–Perot (FP) cavity, which is compared with a stable reference by means of laser radiation. Here the readout noise contributions come from the back-acting force due to the fluctuations of radiation pressure in the cavity, and from the wideband displacement noise due to the amplitude fluctuations of the laser impinging on the detector. For both contributions a lower limit is given by the radiation shot-noise (quantum limit). Usual, short cavities imply a small beam laser waist, and thus a small interrogation region. The corresponding back-action and Brownian noise are too high for an application to non-resonant acoustic detectors.

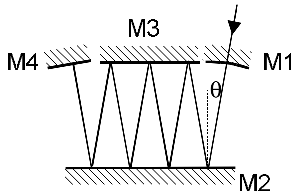


**Figure 1.** Scheme of a folded Fabry–Perot cavity.

We have recently proposed an optical configuration which allows us to overcome this problem [4]. The basic idea is to take a long FP cavity and *fold* the optical path, so that the beam experiences several reflections before getting back to the partially reflecting input mirror. This folded Fabry–Perot (FFP) maintains the sensitivity of the high finesse FP (limited by the losses on one single mirror), but the surface fluctuations are probed by several reflections. It is possible to obtain a huge 'equivalent' beam radius while keeping a compact cavity. In [4] we studied configurations with fixed, small incidence angle and variable total length. We showed that the noise effect is expected to reduce with the number $N$ of spots. More precisely, for large separation between spots, the importance of the Brownian noise spectral power scales as $N^{-1}$, while the radiation pressure effect decreases as $N^{-2}$.

In this paper we focus on the study of the FFP for the readout of the signal from a cylindrical detector. The interesting displacement is radial, and the FFP is developed on a plane containing the cylinder axis. The length of the FFP is fixed by the cylinder height, but the incidence angle and thus the number of bounces can be varied. In this case the optical length is huge even for small $N$. As soon as $N$ increases, the spots come closer, correlation effects become important and the noise reduction is less effective.

In our study the mechanical response of the interrogated area is calculated in the half-space approximation, i.e., considering the mirror as the surface of a half-infinite space. Such an approximation is valid as soon as the overall FFP dimension is small with respect to the detector dimensions: in this case, 'local' effects can be separated from the effect of the main modes in the mechanical response. In the case of a FFP developed along the whole cylinder axis, here considered, the calculation allows us to find the limit for which the longitudinal dependence of the sensing area becomes negligible. At this point, we can consider that a full stripe parallel to the cylinder axis is interrogated. This configuration (a stripe as sensing area, with no longitudinal dependence) is used for the exact calculations reported in [2], which take into account the full set of mechanical modes of the detector, which can thus complement our calculation for optimizing the detector shape and dimensions and estimating the final sensitivity.

In the FFP the input mirror is partially reflecting and the others are high reflectors. The input (M1) or the end (M4) mirror is concave, with radius $R$, and the beam experiences several reflections between two intermediate flat mirrors M2 and M3 before reaching M4 and being reflected back (figure 1). If we call $D$ the distance between the mirrors M2 and M3, $\theta$ the incidence angle and $N$ the number of bounces on M2 (there are $N - 1$ bounces on M3) we get a cavity optical length $L = 2ND/\cos\theta$. For small incidence angles, the cavity linewidth and the shot-noise limited sensitivity in the detection of the displacement between mirrors M2 and M3 are the same as the ones of a simple Fabry–Perot cavity of length $D$, made with the same mirrors M1 and M4 [4].

Both thermal noise and radiation pressure effects can be calculated from the susceptibility $\chi(\omega)$ describing the mechanical response to an exerted pressure, as performed in [4]. In the half-space approximation and for small incidence angle, the effective susceptibility of a mirror with $N$ spots is [5]

$$\chi_N = \frac{1-\sigma^2}{\pi^{1/2} w Y} \left\{ N + 2 \sum_{n=2}^{N} \sum_{q=1}^{n-1} \exp\left( -\frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2} \right) I_0 \left( \frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2} \right) \right\}, \qquad (1)$$

where $\mathbf{r}_n$ is the position of the $n$th spot, $w$ is the beam radius, assumed as constant, $I_0$ is the modified Bessel function of the first kind, $\sigma$ is the Poisson coefficient and $Y$ is the Young modulus of the mirror material. The Brownian noise spectrum (one sided) for a FFP which has $N$ spots on M2 and $N'$ on M3 can be written as

$$S_{\mathrm{Br}}^{\mathrm{FFP}}(\omega) = \left( \frac{D}{2L} \right)^2 \frac{4k_{\mathrm{B}}T}{\omega} \phi \left( 4\chi_N + 4\chi_{N'} + 2\frac{1-\sigma^2}{\pi^{1/2} w Y} \right), \qquad (2)$$

where $\phi$ is the mirror loss angle (we suppose $\phi \ll 1$), $k_{\mathrm{B}}$ is the Boltzmann constant and $T$ is the temperature. The factor $(D/2L)^2$ transforms fluctuations of the roundtrip length (considered in the spectra given in [4]) into fluctuations of the mirror distance $D$, which is the quantity to be measured. For the radiation pressure effect, we can write

$$S_{\mathrm{rp}}^{\mathrm{FFP}} = \left( \frac{D}{2L} \right)^2 \left( \frac{2}{c} \right)^2 2h\nu \left( \frac{F}{\pi} \right)^2 P_{\mathrm{in}} \left| 2\chi_N + 2\chi_{N'} + 2\frac{1-\sigma^2}{\pi^{1/2} w Y} \right|^2, \qquad (3)$$

where $c$ is the speed of light, $F = \pi/\Sigma_{\mathrm{TOT}}$, $\Sigma_{\mathrm{TOT}} = 2(N + N' + 1)\Sigma$ are the total round-trip losses, $\Sigma$ are the average losses (for absorption and scattering) on each mirror and a shot-noise limited laser with power $P_{\mathrm{in}}$ and optical frequency $\nu$ is resonant with the cavity. Here we have considered that, for an optimal Pound–Drever–Hall detection [6], the input mirror transmission is equal to the total round-trip losses.

The above expressions must be completed with the geometrical properties of the FFP here considered. The distance between spots is given by $H/N$, where $H$ is the cylinder height; while the beam waist is $w = \sqrt{z_R \lambda/\pi}$, where $\lambda$ is the laser wavelength (1.064 $\mu$m for our Nd:YAG laser) and $z_R = \sqrt{L(R-L)}$ is the confocal parameter. The constant $w$ approximation is valid as soon as $L < z_R$.

We consider sapphire substrate mirrors at the temperature of 1 K, with the following material parameters: $\phi = 3 \times 10^{-9}, \sigma = 0.25$ and $Y = 4 \times 10^{11}$ Pa. The cylinder height is $H = 2.35$ m (the one considered for a molybdenum dual detector in [2]), and the mirror losses are $\Sigma = 1$ ppm. The Brownian noise is calculated at the frequency of $\omega/2\pi = 1$ kHz.

In figure 2 we show the displacement noise spectral power due to radiation pressure and the Brownian noise, together with the shot-noise limited displacement sensitivity (dashed), for different values of the input laser power and for two cavity configurations. We remark that the Brownian fluctuations are always below the sensitivity level. It means that weaker constraints on the mirror loss angle can be accepted, thus including also the losses on the coating. The curves have a weaker dependence on $N$ with respect to the similar ones reported in [4]. Indeed, due to the fixed total length $H$, the dependence of the optical length $L$ on $N$ is here much weaker, above all for small $N$.

As expected, the value of $N$ which is necessary to bring the radiation pressure noise below the detection sensitivity level increases with the laser power. For an input power of 1 W, the back action falls below the displacement sensitivity even for few spots. However, we remark that the cavity optical length, and thus the spot size, are already large. A simple 10 cm FP cavity would yield a radiation pressure induced noise of $2.7 \times 10^{-41}$ m$^2$ Hz$^{-1}$, well above the sensitivity limit.

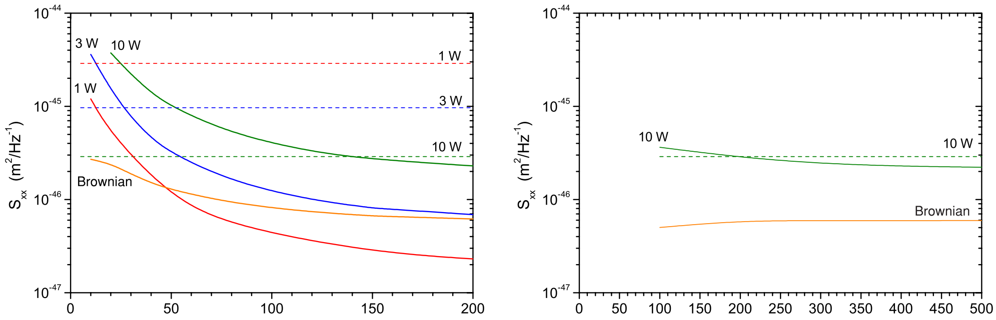


**Figure 2.** Displacement noise spectral power due to radiation pressure noise, for different values of impinging power, and Brownian noise (at 1 kHz), as a function of $N$. Left: $D = 10$ cm, $R = 100$ m; right: $D = 1$ cm, $R = 50$ m.

(This figure is in colour only in the electronic version)

The optimal displacement sensitivity calculated in [2] for a molybdenum dual detector is $10^{-45}$ m$^2$ Hz$^{-1}$ (single side). With such a figure, in a quantum-limited detection, the shot-noise effects in the detection and in the radiation pressure give roughly the same displacement fluctuations, as calculated with the exact cylinder susceptibility and for a stripe interrogation area. This sensitivity is obtained for an input power of about 3 W, which can be considered as our final target. For this value, the radiation pressure effect gets lower than the sensitivity level if $N > 26$ (for $D = 10$ cm), in our half-space approximation. Above this $N$, such approximation is probably no more accurate, but the interrogation area can be considered as a full stripe and the calculations of [2] can be applied. For larger power, a better displacement sensitivity can be achieved, as could be required for further detectors design. Both values of $D$ considered allows us to obtain a sufficient noise reduction, showing a good versatility of the FFP. The suggested configurations are summarized in the following table:

| $D$ (mm) | $R$ (m) | $N$ | $w$ (mm) | $H/N$ (mm) | $L$ (m) | $z_R$ (m) | $\theta$ (rad) |
|---|---|---|---|---|---|---|---|
| 100 | 100 | 200 | 4.1 | 11.8 | 40 | 49 | 0.06 |
| 10 | 50 | 450 | 2.0 | 5.2 | 9.3 | 12 | 0.25 |

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