

BINDING OF MAGNETIC MONOPOLES AND ATOMIC NUCLEI

L. BRACCI and G. FIORENTINI

*Istituto di Fisica dell'Università di Pisa, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, 56100 Pisa, Italy*

Received 28 January 1983

We discuss the possibility that magnetic monopoles bind to atomic nuclei. We estimate the binding energy of these systems to be in the range 10–100 keV and the formation cross section to be $\sigma_{\text{for}} \sim 10^{-28} \text{ cm}^2$. We find that most likely monopoles reaching Earth are bound to a proton and present bounds on the monopole flux.

The formulation of Grand Unified Theories, which allow magnetic monopole-like solutions [1], has stimulated a wide interest in the search of magnetic monopoles and recently evidence has been claimed of their detection [2].

In view of planning further efforts to confirm the existence of magnetic monopoles, it is useful to know the behaviour of these particles when they reach Earth. With this purpose, in this letter we discuss the existence of bound states of magnetic monopoles and atomic nuclei. We estimate the energy levels and the formation rates of these systems. We also comment on some phenomenological implications of the existence of bound states, mainly in connection with the monopole catalysis of proton decay [3]. We consider only electrically neutral monopoles, the case of dyons being treated elsewhere [4]. Only the main results of our investigation will be presented. For an extended discussion of the items touched upon in this letter we refer to a forthcoming paper [5].

We consider a monopole with a magnetic charge q_M $^{\dagger 1}$ $q_M = \epsilon/2e$, $\epsilon = \pm 1$, and a nucleus ${}^A_Z\text{N}$ with mass m , spin $S = 1/2$ and magnetic moment $\mu = S ek/m$. The interaction of the magnetic moment with the field $B(R)$ generated by the monopole,

$$H_{\text{int}} = -\mu \cdot B(R) = -S \cdot R R^{-3} k/2m, \quad (1)$$

$^{\dagger 1}$ We use units such that $\hbar = c = 1$. The electron charge is denoted as $-e = -(137)^{-1/2}$.

can provide an attractive potential for a suitable spin orientation.

Clearly eq. (1) holds only for distances R larger than the nucleus size, a . When $R \lesssim a$ one has to take into account the internal structure of the nucleus and to replace eq. (1) with an equation which takes into account the interaction between the monopole and the constituents of the nucleus. This results in an interaction which is weaker than eq. (1) and which is regular at the origin. The precise form of this interaction is, however, unknown. For $R \leq a$ we approximate the interaction potential with a hard core. Since the true potential, whatever it is, is more attractive than our approximation, the energy values we will obtain are to be regarded as upper bounds.

Within these approximations we find that several stable spin 1/2 nuclei can bind to monopoles. They are listed in table 1, together with the ground state energy. Binding energies are in the range 10–100 KeV and the linear sizes of the bound states are a few tens fermi. It is worth observing that these states are the lowest of infinite families. Within each family the ratio of consecutive energy levels is approximately constant:

$$E_{n+1} = CE_n. \quad (2)$$

The constants C are reported in table 1 $^{\dagger 2}$.

$^{\dagger 2}$ Actually for a few nuclei more than one family of bound states exist. In table 1 we report only the most tightly bound states. See ref. [5] for further details.

Table 1

Binding energies of stable spin 1/2 nuclei ${}^A_Z\text{N}$ to magnetic monopoles with magnetic charge $q_M = -1/2e$ and infinite mass. $\tilde{\mu}$ is the nucleus magnetic moment in units of the nuclear magneton, E_b is the binding energy of the most tightly bound state and C is the ratio between two consecutive energy levels [see eq. (2)].

Nucleus	$\tilde{\mu}$	E_b (keV)	C
${}^1_1\text{H}$	2.79	15.1	4×10^{-4}
${}^3_1\text{H}$	2.98	112	2.2×10^{-2}
${}^3_2\text{He}$	-2.13	13.4	3.85×10^{-3}
${}^{13}_6\text{C}$	0.7	1.8	4.2×10^{-3}
${}^{19}_9\text{F}$	2.63	383	0.25
${}^{31}_{15}\text{P}$	1.13	49.2	0.134
${}^{113}_{48}\text{Cd}$	-0.62	6.3	0.14
${}^{115}_{50}\text{Sn}$	-0.92	29.6	0.3
${}^{117}_{50}\text{Sn}$	-1.0	38.6	0.33
${}^{119}_{50}\text{Sn}$	-1.05	43.9	0.35
${}^{123}_{52}\text{Te}$	-0.74	14.6	0.23
${}^{129}_{54}\text{Xe}$	-0.78	28.3	0.26
${}^{171}_{70}\text{Yb}$	0.49	1.5	9.1×10^{-2}
${}^{195}_{78}\text{Pt}$	0.61	7.6	0.24
${}^{199}_{80}\text{Hg}$	0.50	2.2	0.13
${}^{203}_{81}\text{Tl}$	1.61	94.5	0.57
${}^{205}_{81}\text{Tl}$	1.63	99.4	0.57
${}^{207}_{82}\text{Pb}$	0.59	6.9	0.24

Along similar lines, bound states of monopoles and nuclei were predicted in ref. [6]. The binding energies reported therein are however incorrect. A comparison between our results and ref. [6] can be found in ref. [5].

Concerning the formation of (MN) bound states, two channels can be envisaged: radiative formation and Auger effect. For the former case,



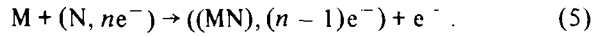
the cross section can be estimated on semiclassical grounds. By calculating the probability of emission of a photon with energy larger than the binding energy along a nucleus trajectory in a potential $V \approx -\mu B(R)$ we get:

$$\sigma_\gamma^{(MN)} \approx Z^2 (\mu/\mu_{\text{nuc}})^2 \cdot 0.3 \times 10^{-28} \text{ cm}^2, \tag{4}$$

where μ_{nuc} is the nuclear magneton.

For monopoles passing through ionized matter reaction (3) is the only process effective for (MN) for-

mation. In non-ionized matter one has to take into account the Auger process:



On the grounds of the experience with exotic atoms [7] we expect that the Auger cross section is much larger than the radiative cross section.

For example, in antiprotonic atoms at velocities $v \lesssim 10^{-3}c$ the ratio of the Auger to the radiative cross section is about 10^8 .

Thus in condensed matter eq. (4) can be considered as a lower limit to the total formation cross section, the real value of the latter being presumably several orders of magnitude larger.

Bound states of monopoles and nuclei could be formed already in the early stages of the Universe. In the hot era, when protons are stable and matter is ionized, (Mp) systems could be formed at a rate

$$\Lambda_{\text{for}} = x m_p^{-5/2} T^{7/2}, \tag{6}$$

where x is the ratio of proton to photon density, $x = 10^{-8} - 10^{-10}$, m_p is the proton mass and T is the Universe temperature, in energy units $^{\dagger 3}$. One sees that Λ_{for} is larger than the expansion rate of the universe, $t_U^{-1} \approx T^2/M_p$, where M_p is the Planck mass, as long as the temperature exceeds 1 keV. Consequently in this era (Mp) systems were formed.

However, a competing process is photodissociation:



A rough estimate, resting on the similarity of photodissociation of hydrogenic systems, yields a dissociation rate

$$\Lambda_{\text{dis}} \approx 10^{12} n_\gamma \exp(-E_b/T) (m_p/E_b)^{7/2} m_p^{-2}, \tag{8}$$

where $n_\gamma \approx 0.25 T^3$ is the photon density and $E_b \approx 10 \text{ keV}$ is the binding energy of (Mp). One sees that Λ_{dis} is larger than Λ_{for} for temperature higher than 100 eV. Thus in the hot era the (Mp) systems were soon dissociated through collisions.

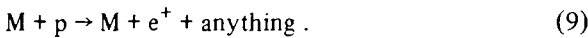
In the cold era, when atoms are stable, the dominant role for bound state formation is played by the Auger effect. A cross section $\sigma_{\text{Au}} \gtrsim 10^{-20} \text{ cm}^2$ is enough to ensure that (Mp) systems are formed in in-

^{†3} For a description of the thermal history of the Universe, see ref. [8].

terstellar or intergalactic space. Moreover, approximately half of the monopoles reach Earth surface after passing through large amounts of water in the Ocean. If monopoles were free when impinging onto the Ocean, there they would bind to a proton ^{†4}.

In conclusion, a monopole reaching Earth is most likely accompanied by a proton and behaves, in any respect, as a dyon. This has to be taken into account when planning a search of magnetic monopoles.

The existence of (MN) bound states affects the rate of the monopole catalyzed proton decay [3]:

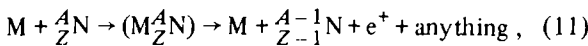


The rate for reaction (9) can be expressed as:

$$\lambda_F = \rho_p w m_p^2 , \tag{10}$$

where ρ_p is the proton density and w is a dimensionless parameter.

The rate for the decay-after-formation process,



is

$$\lambda_B^{-1} = (\rho_N \sigma_{for} v)^{-1} + (A w L_N^{-3} m_p^{-2} / 2)^{-1} , \tag{12}$$

where ρ_N is the density of nuclei, σ_{for} is the formation cross section of (MN) system, v is the nucleus-monopole relative velocity and $L_N \approx 20f$ is the dimension of the (MN) system.

By comparing the total induced proton decay rate,

$$\lambda_{tot} = \lambda_F + \lambda_B , \tag{13}$$

with data on proton stability one can set a bound on the monopole flux Φ and on the parameter of the Rubakov effect, w :

$$v_M / \Phi > t_p w m_p^{-2} \{ 1 + [\rho_N L_N^3 + 0.5 A w / (m_p^2 \sigma_{for} v)]^{-1} \} ; \tag{14}$$

where t_p is the proton lifetime, $t_p \geq 3 \times 10^{30}$ yr [9] and v_M is the monopole velocity. For a comparison with experiment on proton decay we take $A = 50$ and $Z = A/2$.

For v we take the value corresponding to the thermal velocity of nuclei, $v = v_T = 10^5$ cm/s.

Fig. 1 shows the allowed regions in the $(w, \phi/v_M)$

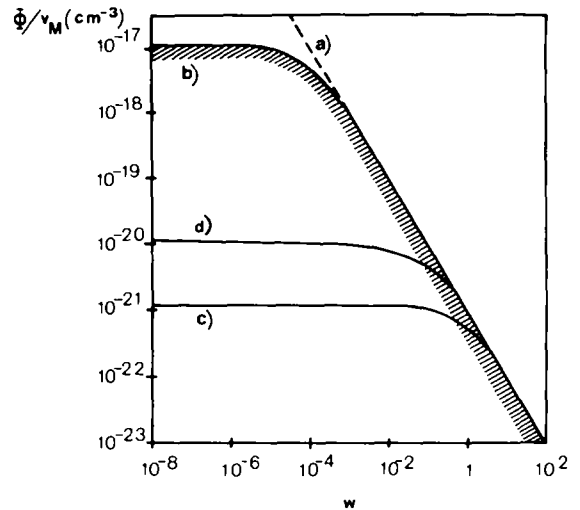


Fig. 1. Bounds on the flux ϕ of monopoles with velocity v_M and on the parameter w of the Rubakov effect [see eq. (10)], from data on proton stability. The allowed regions lie to the left of the curves, which are calculated according to the following hypothesis: (a) for Rubakov effect in flight; (b) taking into account (MN) formation, with formation cross section as in eq. (15); (c) same as in (b) but for a formation cross section 10^4 times larger; (d) same as in (b), with $v_M = 10^8$ cm/s.

plane for some values of σ_{for} . The straight line (a) corresponds to $\sigma_{for} = 0$, i.e. Rubakov effect in flight [eq. (9)] ^{†5}. Curve (b) corresponds to (MN) system formation through the radiative process:

$$\sigma_{for} = \sigma_{\gamma}^{(MN)} \approx Z^2 m_p^{-2} . \tag{15}$$

One sees that the formation of the monopole-nucleus bound states yields more stringent bounds. Curve (c) corresponds to $\sigma_{for} = 10^4 \sigma_{\gamma}^{(MN)}$ and yields even more stringent bounds. One concludes that it is interesting to have more accurate determination of σ_{for} .

It is worth observing that we have been very conservative when assuming $v = v_T$. Generally one has $v = (v_M^2 + v_T^2)^{1/2}$, and correspondingly (MN) formation occurs at a higher rate. Curve (d) corresponds to $v_M = 10^8$ cm/s and a formation cross section as in eq. (15). One concludes that for relatively fast monopoles the radiative formation of (MN) systems enables us to set rather strict bounds on monopole fluxes.

One can envisage several improvements of the pres-

^{†4} The Earth atmosphere is not important for the formation of monopole-nucleus bound states since the most abundant atomic species (¹⁴N and ¹⁶O) do not bind to monopoles.

^{†5} This special case was discussed in ref. [10].

ent discussion. By taking into account the nucleon magnetic form factor a more quantitative description of (Mp) bound states could be provided. In such a frame the existence of nuclear molecules (states of a monopole bound to two or more nuclei) could also be discussed. It is also worth investigating further the atomic and molecular physics processes involved in the formation of the (MN) bound states, as this could provide narrower bounds on the parameters of the Rubakov effect.

One of the authors (G.F.) wishes to express his gratitude to Professors A. Martin and T.E.O. Ericson for interesting conversation. He is also grateful for the CERN Theory Division for the hospitality extended to him while most of this work was done.

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