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Decoherence in neutron interferometry at low transmission probability

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We present a simplified and improved analysis of some recent experiments of neutron interferometry at low transmission probability. It is shown that both the density fluctuations of the elementary constituents of the absorber and the uncertainties in the sample thickness can be analyzed with the same formalism, and that they lead to a reduction of the visibility of the interference pattern. The effect is quantitatively estimated in the Gaussian case. In the context of quantum measurements, the process can be viewed as a partial dephasing characterized by the *decoherence* parameter. Possible experimental tests are proposed.

Recent advances in neutron interferometry have made possible the investigation of fundamental quantum-mechanical issues. Several experiments that were at a gedanken level until few years ago are now feasible, and the very fundamental postulates of quantum mechanics are now liable to experimental check.

Some experiments performed a few years ago in Vienna [1], by inserting a strong neutron absorber in one of the two routes of the interferometer, yielded results at variance with the theoretical prediction for the visibility. The experimental points lay remarkably below the theoretical curve in the region of very low transmission probability.

A tentative explanation for this discrepancy was proposed [2], and the effect was ascribed to the presence of density fluctuations of the elementary constituents of the neutron absorber. On the other hand, a more thorough analysis is required, in particular because the previous calculation neglected the presence of size uncertainties of the absorber itself. The problem has also been discussed from the measurement-theoretical point of view [3], and an interpretation in terms of the decoherence parameter [4] was put forward.

The aim of this note is threefold: First, we simplify the previous theoretical analysis [2], by showing that the presence of statistical fluctuations and/or size uncertainties leads to a considerable reduction of the visibility of the neutron interference pattern at low transmission probability. Second, we explain the important role played by the decoherence parameter [4] in the interpretation of this experiment. Third, we put forward practical experimental proposals to check the soundness of our analysis.

We shall start by analyzing a typical double slit experiment, and by showing how the decoherence parameter emerges in a natural way when statistical fluctuations are considered. Let the incident neutron wave packet be split into two branch waves ψ_1 and ψ_2, corresponding to the two different routes in the apparatus, and assume that ψ_2 interacts first with a phase shifter (PS) and then with an absorber (A). The first contributes a phase factor $e^{i\delta}$ while the second is assumed to simply multiply the wave function by a transmission coefficient T, so that

$$\psi_2 \to e^{i\delta} T \psi_2 . \quad (1)$$

If ψ_1 and ψ_2 are in phase and $|\psi_1|^2 = |\psi_2|^2 = 1$, the intensity after recombination of the two branch waves is

$$I \propto |\psi_1 + e^{i\delta} T \psi_2|^2 = 1 + |T|^2 + 2\,\mathrm{Re}(T e^{i\delta}) = 1 + t + 2\sqrt{t}\cos(\alpha + \delta) , \quad (2)$$

where we have written $T = |T| e^{i\alpha}$, and have defined the transmission probability $t = |T|^2$. In this way, the visibility of the interference pattern is

$$V = \frac{I_{\mathrm{MAX}} - I_{\mathrm{min}}}{I_{\mathrm{MAX}} + I_{\mathrm{min}}} = \frac{2\sqrt{t}}{1+t} . \quad (3)$$

Notice that in the above formulae the dynamics of the macroscopic apparatuses has been ignored, and the effect of their interaction with the neutron wave function has been "summarized" by introducing two "constants" (δ and T in eq. (1)). Obviously, this is only an approximation, because both the phase shifter and the absorber are macrosystems made up of a huge number of elementary constituents, and characterized by a few macroscopic parameters whose value cannot determine precisely the details of the microscopic motion, so that their fluctuations and/or uncertainties should be taken into account. Our first purpose is to analyze the soundness of approximation (1).

First of all, observe that an interference pattern is made up of a certain number of experimental points, and in turn each of these points is obtained by accumulating the results relative to a very large number of neutrons, that are sent into the interferometer through a weak and steady beam. Each point represents the intensity detected in one of the two channels (say the ordinary one), and is relative to a "precise" value of the phase δ acquired by each neutron after the interaction with the PS. This is obviously a *very reasonable* approximation: Indeed, a "good" phase shifter must yield a constant phase factor for every neutron in the same experimental run. Were this factor not "constant" (up to a very good approximation), the interference experiment itself would be impossible to perform.

But what about the transmission coefficient T? Is assumption (1) reasonable in this case? Not necessarily: There are two main reasons why the fluctuations of T, unlike those of δ, can be important. First, the absorber thickness D cannot be considered constant, from event to event, because of the sample inhomogeneities and the angular divergence of the beam. Second, even though, during an experimental run, the *macroscopic* state of the absorber A is always the same, each neutron will interact with a slightly different *microscopic* state of it, because the elementary constituents of A are subject to their own internal motion and their positions change all the time; moreover, different neutrons will go through (and interact with) different parts of the absorber, due to the finite lateral size of the beam.

Let us try to take into account the possibility of fluctuation effects in the transmission coefficient and probability, by labelling different incoming neutrons with j ($j = 1, \ldots, N_p$, where N_p is the total number of neutrons in an experimental run): Write T_j for the transmission coefficient of the jth neutron, so that the average transmission coefficient and probability will be

$$\bar{T} = \frac{1}{N_p} \sum_{j=1}^{N_p} T_j , \quad (4)$$

$$t \equiv \bar{t} = \overline{|T|^2} = \frac{1}{N_p} \sum_{j=1}^{N_p} |T_j|^2 , \quad (5)$$

and we have identified $\overline{|T|^2}$ with t, the *experimentally measured* value of the transmission probability. It follows that

$$|\bar{T}|^2 \leqslant \overline{|T|^2} = t , \quad (6)$$

so that it is possible to write

$$|\bar{T}|^2 = t(1-\epsilon) , \quad (7)$$

where ϵ ($0 \leqslant \epsilon \leqslant 1$) has been named *decoherence parameter* [2,4]. Its definition is therefore

$$\epsilon = 1 - \frac{|\bar{T}|^2}{t} . \quad (8)$$

Intensity and visibility (eqs. (2) and (3)) become

$$I' \propto 1+|\bar{T}|^2+2\,\mathrm{Re}(\bar{T}e^{i\delta}) = 1+t+2\sqrt{t}\sqrt{1-\epsilon}\cos(\beta+\delta)\,,$$

$$V'=\frac{2\sqrt{t(1-\epsilon)}}{1+t}=V\sqrt{1-\epsilon}\,, \qquad (9)$$

where V is defined in eq. (3), and we have written $\bar{T}=|\bar{T}|e^{i\beta}$. One sees clearly that for $\epsilon=1$ interference disappears: This represents the case of *total loss of coherence* between the two branch waves.

Our purpose is to give a simple (but rather accurate) estimate for ϵ, and to show that it plays an important role in the analysis of the aforementioned Vienna experiments [1].

Let us start by observing that the standard formula for the transmission probability of a neutron going through an array of (absorbing) scatterers is

$$t_0=\exp(-\sigma_a\langle\rho\rangle\langle D\rangle)=\exp(-\langle n\rangle)\,, \qquad (10)$$

where σ_a is the absorption cross section for the neutron–scatterer interaction, $\langle\rho\rangle$ the (average) density of scatterers, and $\langle D\rangle$ the (average) thickness of the absorber. The quantity $\langle n\rangle$ is interpreted as the average number of scatterers met by the neutron during its interaction with A. In the Vienna experiments, A consisted of a Gd–H_2O solution, so that the neutrons were mostly absorbed by the Gd atoms.

There is a simple-minded (but very effective) interpretation of eq. (10): One assumes that, roughly speaking, *as far as absorption and transmission probabilities are concerned*, each neutron interacts with a small cylinder of Gd–H_2O solution. This cylinder has height roughly equal to the length of the absorber, and base roughly equal to the neutron–Gd absorption cross section. Notice that $l=(\sigma_a\langle\rho\rangle)^{-1}$ is the mean free path of a neutron for absorptive scattering by Gd atoms. We are neglecting the role of water in the process, because water does not strongly absorb neutrons. Moreover, the density fluctuations of the water molecules are completely negligible when compared to Gd [2].

Due to the aforementioned reasons, the number of elementary constituents n met by every neutron fluctuates around its average value $\langle n\rangle$, so that the transmission probability for a single event should be written

$$t_n=\exp(-n)\,, \quad n=\sigma_a\rho D\,, \qquad (11)$$

where

$$n=\langle n\rangle+\delta n\,. \qquad (12)$$

We require the Gaussian properties for the fluctuating components,

$$\langle\delta n\rangle=0\,, \quad \langle(\delta n)^2\rangle=g\langle n\rangle\,, \qquad (13)$$

where g $(0\leqslant g\leqslant 1)$ is a parameter characterizing the fluctuation: The limiting cases $g=0$, 1, correspond to absence of fluctuations and Poissonian fluctuations, respectively. The latter case represents the dilute-solution limit, or alternatively an ideal-gas correlation function [5,2]. The parameter g represents the strength of the fluctuations, or alternatively, the size of the uncertainties in some macroscopic parameter, such as D. In principle, the ρ-dependence of g can be determined theoretically, but, in general, this is not an easy task because it would require, among others, a statistical-mechanical investigation of the molecular theory of a two-component liquid. On the other hand, the uncertainties of D depend on the actual frabrication technique. As will be shown, however, the value of g can be readily determined from experimental data. Note that the present definition of g is slightly different from the one proposed in ref. [2], where the role of water was not neglected. The main conclusions, however, will be essentially unaltered.

In the above equations, $\langle ...\rangle$ is a statistical ensemble average over the absorber microstates. We shall make the following ergodic hypothesis,

$$\overline{...}=\langle ...\rangle\,, \qquad (14)$$

where $\overline{...}$ is the average over many particles, introduced in eqs. (4) and (5). From eqs. (11)–(14), and the Gaussian reduction formula, it is easy to obtain

$$t=\bar{t}=\langle t_n\rangle=t_0\exp(\tfrac{1}{2}g\langle n\rangle)\,. \qquad (15)$$

This is the first, important consequence of the approach we propose: We infer that, unlike what is usually believed,

$$t\neq t_0=\exp(-\sigma_a\langle\rho\rangle\langle D\rangle)=\exp(-\langle n\rangle)\,. \qquad (16)$$

Equation (16) is liable to experimental check: The values of the parameters t, σ_a, $\langle\rho\rangle$ and $\langle D\rangle$ are indeed all *directly* measurable. If t and t_0 are found to be different, eq. (15) yields an experimental estimate for g.

In this context, it is useful to observe that, as already remarked in ref. [2], the typical value $\langle\rho\rangle \simeq 5\times10^{26}$ [m^{-3}], repeatedly used previous papers [1,2], is *not* correct, because it is obtained by making use of formula (10), and not (15).

Notice that, in the above analysis, we have *not* specified the origin of the of the fluctuations of n. Since, by definition, $n=\sigma_a\rho D$, its fluctuations can be ascribed to density (ρ) fluctuations of the Gd atoms in the water solution, as well as to uncertainties in the sample thickness D. This is true, of course, within the limits of validity of eq. (10), which is based on the Goldberger formula (see the following equation). A more exhaustive quantum-mechanical analysis should start from the Dyson series of the interaction Hamiltonian for the neutron–Gd interaction, as outlined in ref. [2].

In order to analyze the effect of statistical fluctuations on the transmission coefficient, we start from the well-known Goldberger formula [6]

$$T_n = \exp[-(\mathrm{i}\lambda b_R + \tfrac{1}{2}\sigma_a)\rho D] = \exp[-\tfrac{1}{2}n(1+2\mathrm{i}\lambda b_R/\sigma_a)]\,, \qquad (17)$$

where b_R is the real part of the scattering length of the elastic neutron–Gd collision, and σ_a the absorption cross section for neutron–Gd. Once again, for the sake of simplicity, we are neglecting the role of water. This assumption is sound, because water contributes an almost constant factor in the transmission coefficient.

From eqs. (17), (12)–(14), and the Gaussian reduction formula, we immediately obtain

$$\bar{T} = \langle T_n\rangle = T_0 \exp[\tfrac{1}{8}g\langle n\rangle(1+2\mathrm{i}\lambda b_R/\sigma_a)^2]\,,$$

$$T_0 \equiv \exp[-\tfrac{1}{2}\langle n\rangle(1+2\mathrm{i}\lambda b_R/\sigma_a)]\,, \qquad (18)$$

where, obviously, $|T_0|^2 = t_0$. From eqs. (15) and (18) we readily obtain

$$|\bar{T}|^2 = t^{1+\gamma}, \quad \gamma = \frac{2g}{2-g}\,\frac{(\lambda b_R)^2+\sigma_a^2}{\sigma_a^2}\,, \qquad (19)$$

and the visibility can be rewritten in terms of γ as

$$V' = \frac{2t^{(1+\gamma)/2}}{1+t} = Vt^{\gamma/2}\,, \qquad (20)$$

where V is defined in eq. (3). It is worth stressing that the above equation is liable to *direct* experimental check: By inferring the value of g from eq. (15), we can test the validity of eq. (20).

Let us briefly discuss the main consequences of our analysis. We have shown that, if uncertainties and fluctuations are taken into account, the usually accepted relation between transmission coefficient and probability ($|T|^2=t$) is not valid anymore, and must be replaced by eq. (19). Accordingly, the value of the visibility is reduced by a factor $t^{\gamma/2}$, as shown by eq. (20). This immediately suggests how the effect we are anticipating could be checked experimentally: Indeed, the correction to the visibility expressed by eq. (20) is negligible when $t\simeq1$, but becomes dramatically important when $t\to0$. This makes us understand why it is reasonable to expect a reduction of the visibility at extremely low values of the transmission probability, and is in agreement with some preliminary experimental data [1]. We stress that the present analysis, though simplified with respect to a previous one [2], yields essentially the same results, as far as the role played by water can be neglected.

An experimental verification could also be achieved by measurements with grained absorbing phase shifters inserted in one beam path of a neutron interferometer of by using an absorbing material near a critical transition point. In the first case, different beam paths through the sample have distinctly different absorption probabilities, and the expectation value $\langle\ldots\rangle_\chi$ has to be taken over different beam paths contributing to the interference pattern. Similarities to the Christiansen filter method where the real part of the phase shift is varied are obvious [7]. In the second case, a time average $\langle\ldots\rangle_\tau$ has to be considered, because the critical fluctuations cause varying absorption probabilities due to the marked density fluctuations which appear as an enhanced visibility of the interference pattern near to the phase transition point. In the latter case, the theoretical treatment must be modified to account for the presence of long relaxation time and correlation length.

It is also very interesting to discuss the above results from a measurement-theoretical point of view. The decoherence parameter is readily evaluated by eq. (8) as

$$\epsilon = 1 - \exp\{-\tfrac{1}{4}g\langle n\rangle[1+4(\lambda b_R/\sigma_a)^2]\}$$
$$= 1 - \exp\{-g\langle n\rangle[(\lambda b_R)^2+\sigma_a^2]/\sigma_a^2\}$$
$$= \epsilon(g, \langle n\rangle)\,, \tag{21}$$

and can be expressed in terms of t and γ as

$$\epsilon = 1 - t^{\gamma}\,. \tag{22}$$

Analogously to the case of the visibility, discussed before, this implies that even though, at high transmission probability ($t \simeq 1$) fluctuation effects are not observable, they become dramatically important when $t \to 0$. In such a case, $\epsilon \to 1$, and quantum coherence is totally lost. Observe that this effect is completely independent of the fact that one of the two branch waves is (almost) totally absorbed: Indeed, even if t is extremely small (say, of the order of 10^{-5}), both branch waves are still *present* in the interferometer, and always give rise to interference. The point is that this interference is *drastically reduced* with respect to its expected value (3).

The above formulae show that fluctuations existing *intrinsically* in any apparatus influence the experimental outcome of any experiment. Their influence becomes very pronounced in interference experiments when a very weak signal emerging from one beam path and having intrinsically strong fluctuations determines the interference phenomena. In this sense, one can state that perfect measurements are *impossible* even *in principle*, when the microscopic structure of the absorber is taken into account. This point is important both from an epistemological and measurement-theoretical point of view [4].

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