The importance of the minimum path criterion in the design of water distribution networks

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ABSTRACT

This paper explores the relationship between the minimum cost design of water distribution networks (WDNs) and the minimum water path criterion (MWPC), according to which the water entering the network through the source nodes should cover the shortest possible paths before being delivered to users. To this end, a three-step linear algorithm for WDN design based on MWPC and set up in the 80 s was applied to many benchmark case studies. The results of the linear three-step algorithm were almost coincident with, and in some cases superior to, those produced by more complex and burdensome algorithms. This represents a solid proof of the strong implications of MWPC for WDN design.

Key words | linear programming, minimum path criterion, network design

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INTRODUCTION

The design of water distribution networks (WDNs) has been one of the most explored topics in the WDN scientific literature, starting from the end of the sixties (Shaake & Lai 1969) up to nowadays (e.g., see Creaco *et al.* 2014, 2016; Wang *et al.* 2015; Liu *et al.* 2016). In particular, the minimum cost design (MCD) is generally carried out considering a single loading condition and referring to lowest source node heads and highest nodal demands. Under these conditions, the objective of MCD is to assign suitable diameters to WDN pipes, which minimize the network cost while respecting constraints on nodal heads and flow velocities. Since the full enumeration of all the possible solutions is almost always impossible due to the big size of the search space, numerous authors have been proposing methods of various kind along the years to tackle this problem. In this context, algorithms based on linear programming (LP) (e.g., Alperovits & Shamir 1977; Ciaponi & Papiri 1983, 1985; Stephenson 1984; Suribabu & Neelakantan 2005) were proposed. Other algorithms of heuristic or metaheuristic kind were also proposed, including local search algorithms, such as the Tabu Search (e.g., Sung *et al.* 2007) and the gradient method

(Fujiwara & Khang 1990; Gomes *et al.* 2009), and global search algorithms, such as the genetic algorithms (GAs) (e.g., Savic & Walters 1997; Shau *et al.* 2005), the Ant Colony (e.g., Maier *et al.* 2003), the Cross Entropy (e.g., Perelman & Ostfeld 2007), the Harmony Search (Geem 2006), the particle swarm (Montalvo *et al.* 2008), the Differential Evolution (Arunachalam & Simonovic 2010), the non linear programming NLP (Kim *et al.* 1994), the mixed integer non linear programming MINLP (Bragalli *et al.* 2012) and the simulated annealing (Cunha & Sousa 1999). Finally, hybrid algorithms were also proposed as the combination of algorithms of various kind, such as those proposed by Krapivka & Ostfeld (2009) and Kadu *et al.* (2008), made up of GA + LP, that proposed by Haghighi *et al.* (2011), made up of GA + integer LP, and that proposed by Eusuff & Lansey (2003), made up of the combination of a global optimization algorithm with a local search method.

Despite being interesting contributions to the field, the nonlinear heuristic and meta-heuristic algorithms mentioned above are often applicable only to oversimplified design problems. Therefore, they have seldom been

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implemented in freeware or licensed pieces of software, which, instead, make use of various network resolution algorithms coming from the research world (e.g., Todini & Pilati 1988). The idea of this work is to use, for analysis and comparison purposes, an algorithm proposed in the 80 s at the same time by Ciaponi & Papiri (1983, 1985) and by Stephenson (1984). This algorithm simplifies the non-linear network design problem by transforming it into the sequence of two LP applications, followed by a third simple corrective step. Since LP can be easily applied to networks of whatever size, the aim of this work is to show that the algorithm mentioned above can bridge the gap between the complex methodologies of limited applicability that exist in the scientific literature and the fully empirical approaches used by practitioners to design WDNs. The analysis and comparison of the results of the algorithm proposed by Ciaponi & Papiri (1983, 1985) and Stephenson (1984) can now be carried out trustworthily, thanks to the large availability of detailed case studies tackled with various methodologies in the scientific literature.

In detail, the algorithm proposed by Ciaponi & Papiri (1983, 1985) is based on the criterion of defining the pipe water discharges in such a way that the water supplied by the source node(s) reaches the demanding nodes covering the shortest path length (minimum water path criterion – MWPC). A positive effect of the application of MWPC lies in the fact that the minimization of the total water path length entails the reduction in the water detention time in the network as well as the better preservation of the water organoleptic and thermic characteristics. Furthermore, MWPC has evident implications of economical optimality, in that the reduction in the total water path length facilitates selection of small pipe diameters, while respecting the constraints of maximum head loss between source and demanding nodes.

Following Ciaponi & Papiri (1983, 1985), MWPC is enforced by searching for the pipe water discharges that minimize a linear objective function, while respecting nodal continuity and path head loss constraints, which are linear as well. Similar formulations were proposed by Stephenson (1984) and Suribabu & Neelakantan (2005). In detail, the formulation of Stephenson (1984) leads to identical mathematical expressions to those of Ciaponi & Papiri (1983, 1985), though the objective function of Stephenson

(1984) has a different meaning with network unitary costs being proportional to pipe water discharges. Suribabu & Neelakantan (2005), instead, proposed that further constraints should be used, related to the maximum and minimum water discharge values at each pipe.

After the design configuration of pipe water discharges has been obtained, the pipe optimal diameters are assessed using LP, in a similar way to Alperovits & Shamir (1977).

Being based on two LP applications in a row, the algorithm is conceptually flawed by the fact the results of the former LP application, for the estimation of the design pipe water discharges, affect the results of the latter, aimed at pipe sizing. Therefore, it can be reasonably expected that the solution obtained by this algorithm is different and features higher costs compared to those obtainable by applying other algorithms which minimize the cost function in a single step, as a function of all the decisional variables at stake. This is one of the reasons why this work was aimed at exploring the extent to which the solutions obtainable through the MWPC based algorithm differ in terms of cost from the solutions obtainable through optimization algorithms minimizing directly the WDN cost. Furthermore, the work was aimed at verifying if MWPC is implicitly taken into account by the other optimization algorithms for the WDN design, through the hydraulic analysis of the minimum cost solutions obtained in various benchmark case studies.

In the following subsections, first the MWPC based algorithm (Ciaponi & Papiri 1983, 1985) is described, followed by the applications and the comparisons with the other optimization algorithms.

THE MWPC BASED ALGORITHM

The MWPC based algorithm (Ciaponi & Papiri 1983, 1985) is based on the three following steps:

1. calculation of the design water discharges minimizing the sum of water path lengths, transforming *de facto* a generic looped network into a system of branched networks, each of which fed by a single source;
2. calculation of the pipe diameters minimizing the total cost of the system of branched networks;

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3. re-closure of network loops through minimum diameter pipes.

In the following subsections, each step is thoroughly described.

Step 1 – Calculation of the design water discharges (loop opening)

Let us consider a generic network featuring n_p pipes, n_1 nodes with unknown head and n_0 nodes with fixed head. The number of elementary network loops n_l will be equal to:

$$n_l = n_p - n_1 - n_0 + 1. \tag{1}$$

For the network it is possible to define a topological incidence matrix $\mathbf{A}$ (Todini & Pilati 1988), in which the element $\mathbf{A}\,(i, j)$ can take on the values 0, −1 and 1. In particular, $\mathbf{A}\,(i, j) = 0$ if the i-th pipe does not have the j-th node at one end; if the i-th pipe has the j-th node at one end, $\mathbf{A}\,(i, j) = 1$ or $\mathbf{A}\,(i, j) = -1$ depending on whether the (arbitrarily) assumed flow in the i-th pipe enters or exits the j-th node. Matrix $\mathbf{A}$ can be partitioned into two sub-matrixes, $\mathbf{A_{10}}$ and $\mathbf{A_{12}}$, associated with the nodes of fixed head and the nodes of unknown head, respectively (Todini & Pilati 1988). Continuity equations at demanding nodes of unknown head take on the following vector form:

$$\mathbf{A_{21}} \cdot \mathbf{Q} = \mathbf{q}, \tag{2}$$

where $\mathbf{A_{21}}$ is the transpose matrix of $\mathbf{A_{12}}$, and $\mathbf{Q}$ and $\mathbf{q}$ are the vectors of pipe water discharges Q and nodal demands q respectively. In this context, it should be underlined that q in Equation (2) can also be negative, therefore indicating a water discharge input into the WDN.

In the case of a looped network, the system of Equation (2) is indeterminate when considered standing alone, given that $\mathbf{A_{21}}$ is not of full rank. In fact, the number of rows (equal to the number of demanding nodes n_1) is lower than the number of columns (equal to the number of pipes). This means that an infinite number of water discharge configurations satisfy the system of Equation (2).

The flow distribution corresponding to the MWPC can be obtained by solving an optimization problem featuring the constraints coded in the continuity equations of Equation (2) and the following objective function f to be minimized:

$$f = \mathbf{L}^T \cdot \overline{\mathbf{Q}}, \tag{3}$$

with $\mathbf{L^T}$ is the transpose vector of $\mathbf{L}$ (containing the pipe lengths) and $\overline{\mathbf{Q}}$ is a vector containing the absolute values of Q. Minimizing function f in Equation (3) is equivalent to applying MWPC in the network.

The previous optimization problem can be transformed into an LP problem by duplicating the network pipes, in such a way that opposite flow directions are allowed in the two arcs associated with the generic network pipe. In the network with duplicated pipes, vector $\mathbf{L^d}$ of arc lengths, vector $\mathbf{Q^d}$ of arc flows and matrix $\mathbf{A_{21}^d}$ can be obtained as:

$$\mathbf{L^d} = \left[\ \mathbf{L}^T \quad \vdots \quad \mathbf{L}^T\ \right]^T, \tag{4}$$

$$\mathbf{Q^d} = \left[\ \mathbf{Q}^T \quad \vdots \quad \mathbf{Q}^T\ \right]^T, \tag{5}$$

$$\mathbf{A_{21}^d} = \left[\ \mathbf{A_{12}} \quad \vdots \quad -\mathbf{A_{12}}\ \right]^T, \tag{6}$$

where symbols 'T' and '$\vdots$' indicate the transpose vector/matrix and the vector/matrix vertical concatenation, respectively.

Through the previous matrix operations, the optimization problem in Equations (2) and (3) takes on the new form of function f^d in Equation (7) to be minimized, subject to the linear constraints in Equations (8) and (9), with $\mathbf{Q^d}$ as vector of decisional variables:

$$f^d = \mathbf{L^{dT}} \cdot \mathbf{Q^d}, \tag{7}$$

$$\mathbf{A_{21}^d} \cdot \mathbf{Q^d} = \mathbf{q}, \tag{8}$$

$$Q^d \geq 0. \tag{9}$$

Whether required, further constraints can be added concerning the maximum water discharges that can leave the

source nodes. In detail, these eventual constraints take on the following form:

$$\mathbf{A_{01}^d} \cdot \mathbf{Q^d} \leq \mathbf{q_{0,max}}, \tag{10}$$

where $\mathbf{q_{0,max}}$ is the vector of maximum water discharges leaving the source nodes and $\mathbf{A_{01}^d}$ can be obtained as:

$$\mathbf{A_{01}^d} = \left[-\mathbf{A_{10}} \ \vdots \ \mathbf{A_{10}} \right]^T, \tag{11}$$

if the pipes leaving the sources have the sources themselves as upstream nodes, from the topological point of view.

Constraint in Equation (10) can also be strict equality.

In the solution of the optimization problem, only one of the two arcs associated with the generic pipe can have Q^d greater than 0. In fact, the condition of Q^d greater than 0 in both the arcs would prevent minimization of function f^d. Furthermore, in light of the adopted schematization with outflows allocated only to network nodes, the optimal solution is a configuration of pipe water discharges, in which there are n_l pipes (In some (sporadical) cases, the number of pipes with $Q^d = 0$ in both arcs can be larger or smaller than n_l, though this does not affect the application of step 2, for which the main requirement is to have a pre-assigned distribution of pipe water discharges.) in which $Q^d = 0$ in both arcs. The removal of these pipes transforms the looped network in a group of branched networks (one for each source node), the pipe diameters of which can be sized as explained in the following subsection. This procedure is powerful since it enables fast identification of branched structures with a well-defined characteristic (minimum water path length) even in very complex and huge looped networks with several source nodes.

Step 2 – Design of the pipes with $Q^d \neq 0$ (branched network(s))

The optimal design of the branched network(s), obtained by removing the pipes with $Q^d = 0$ in both associated arcs, can be carried out for prefixed values of minimum pressure head ($h_{des,k}$) constraints. In this problem, given a set of n_D diameters D_i for each pipe of the branched network(s), the unknown variables are, with reference to the j-th network

pipe, the lengths $l_{j,i}$ of the sub-pipes fitted with diameter D_i. The objective of this design is to minimize the overall cost of the branched network while respecting the minimum acceptable pressure $h_{des,k}$ constraints for the pre-selected configuration of nodal demands. The design entails solving an LP problem, with the following cost function to be minimized:

$$cost_b = \sum_{j=1}^{n_{pb}} \sum_{i=1}^{n_D} c_i l_{ji}, \tag{12}$$

where c_i is the unit cost associated with diameter D_i and n_{pb} is the number of pipes in the branched network(s).

For each network pipe j, the constraint that the sum of sub-pipe lengths $l_{j,i}$ is equal to the length L_j of the whole pipe needs to be considered:

$$L_j = \sum_{i=1}^{n_D} l_{ji}, \tag{13}$$

For the path leading from the reservoir (with head H_{res}) supplying the k-th node (with elevation z_k) to the k-th node itself, the sum of the head losses in the n_k pipes belonging to the path has to be so low as to allow the minimum acceptable pressure $h_{des\ b,k}$ constraint to be respected at node k. This can be expressed as:

$$\sum_{j=1}^{n_k} \sum_{i=1}^{n_D} J_{j,i} l_{j,i} \leq H_{res} - H_{des,k}, \tag{14}$$

where $J_{j,i}$ is the energy friction slope relative to sub-pipe $l_{j,i}$ of pipe L_j; $H_{des,k}$ is the desired total head at node k for full demand satisfaction, being the sum of node elevation z_k and prefixed minimum pressure $h_{des,k}$ of the branched network.

Though all the diameters D_i in the set are simultaneously considered for each pipe in the LP problem, the solution generally yields sub lengths greater than 0 only for one or two diameters.

Other constraints such as those related to minimum and maximum pipe flow velocities can be easily taken into account, since pipe water discharges are pre-assigned in the branched network(s). Actually, pipe sub-lengths $l_{j,i}$ in

which these constraints are not respected due to too small or large diameters (and then too large or small flow velocities) can be set to 0 as a further constraint of LP. The formulation described above can also be extended to include other decisional variables, such as the pressure head at the source nodes.

Step 3 – Loop re-closure

After the design of the pipes in the branched network(s), a diameter needs to be assigned to the pipes that were removed in step 1 during loop opening. In this step, Ciaponi & Papiri (1983, 1985) recommended that the minimum diameter in the list of WDN design diameters be adopted. As for the distribution of pipe flows obtained in steps 1 and 2, the use of the minimum diameter generally prevents large variations from taking place, after network loops have been re-closed. This ensures good preservation of the MWPC prescribed in step 1. Nevertheless, the pipe flow variations due to the loop re-closure may cause minimum pressure head ($h_{des,k}$) or maximum velocity constraints to be violated at some nodes. In this case, the WDN designer can make some adjustments to the pipe design to re-obtain constraint preservation. In order to detect the pipes that need adjustments, step 2 can be tentatively re-carried out with different constraints from before.

Of course, the cost C_3 of the total network is obtained as the sum of the cost of the branched network(s) plus the cost of the pipes used for loop re-closure, following the designer's adjustments.

In the context of step 3, it must be noted that the use of the minimum diameter in the list for loop re-closure is here considered only to pursue the minimum cost. However, in real WDN design applications, other choices could be made, such as the minimum diameter used in Step 2 over all the branched network or for the pipes belonging to same loop as the generic removed pipe. This would yield benefits of increased reliability, at the expense of a larger investment.

APPLICATIONS

The applications of this work concerned the 13 WDNs reported in Table 1, in ascending order of number n_p of pipes to be sized. For each network, Table 1 also reports the literature source of the case study, the numbers n_0 and n_1 of source and demanding nodes, the number n_D of available pipe sizes in the design, the source and kind of the optimization algorithm of the reference minimum cost solution used for the analyses and comparisons of this work. In particular, the reference case studies were chosen

Table 1 │ The various case studies and literature source

Network	Source	n_p	n_0	n_1	n_D	Reference solution	Optimization algorithm
Two loop	Alperovits & Shamir (1977)	8	1	6	14	Krapivka & Ostfeld (2015)	GA + LP
Ruey Fang	Shau *et al.* (2005)	16	2	20	16	Shau *et al.* (2005)	GA
Blacksburg	Sherali *et al.* (2001) - Bragalli *et al.* (2012)	30	1	23	14	Bragalli *et al.* (2012)	MINLP
GoYang	Kim *et al.* (1994)	30	1	22	8	Geem (2006)	Harmony Search
Taichung	Sung *et al.* (2007)	31	1	19	13	Sung *et al.* (2007)	Tabu search
Hanoi	Fujiwara & Khang (1990)	34	1	32	6	Perelman & Ostfeld (2007)	Cross entropy
Kadu	Kadu *et al.* (2008)	34	2	24	14	Haghighi *et al.* (2011)	GA + ILP
Fossolo P0	Bragalli *et al.* (2012)	58	1	37	7	Bragalli *et al.* (2012)	MINLP
Fossolo Ir	Bragalli *et al.* (2012)	58	1	37	13	Bragalli *et al.* (2012)	MINLP
Fossolo P1	Bragalli *et al.* (2012)	58	1	37	22	Bragalli *et al.* (2012)	MINLP
R9	Gomes *et al.* (2009)	71	1	61	10	Gomes *et al.* (2009)	Gradient method
Pescara	Bragalli *et al.* (2012)	99	3	71	13	Bragalli *et al.* (2012)	MINLP
Modena	Bragalli *et al.* (2012)	336	4	268	13	Bragalli *et al.* (2012)	MINLP

Number of pipes (n_p), source nodes (n_0) and demanding nodes (n_1), available pipe sizes for the design (n_D) and reference solution considered for comparison.

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among those available in scientific literature based on the fully availability of design data and reference solutions obtained by paper authors.

The simplest case study considered in this work is the two-loop network of Alperovits & Shamir (1977), the layout of which is reported in the following Figure 1.

The network is fed by a single source with head of 210 m and the minimum pressure constraint ($h_{des,k}$) is set to 30 m

to all the network nodes. More information about the network can be found in the referenced work. The application of step 1 of the MWPC based algorithm (Ciaponi & Papiri 1983, 1985) led to removal of pipes 4 and 8, one for either network loop. The minimized value of path objective function f is 872,218 (m L)/s. The application of step 2 of the algorithm led to the pipe design reported in Table 2, which costs 399,473 $. Following loop re-closure with the pipe diameter of 25.4 mm, some adjustments were necessary to eliminate some pressure constraint violations, resulting in the other pipe design reported in Table 2 (step 3). In fact, these small adjustments are required because, by modifying the flows defined in step 1 and taken as granted in step 2, loop re-closure (step 3) can cause slight violations of nodal head constraints.

The cost of the network with re-closed loops is 403,561 $. The value of f for the network with re-closed loops is 872,602 (m L)/s, which is very close to the f value obtained for the network with open loops. In particular, the percentage difference PD_1 can be calculated between the value of f_3 obtained at the end of step 3 and the value of f_2 associated with the network configuration with open loops in step 2. The value of $PD_1 = 100 \ (f_3 - f_2)/f_2$ is equal to 0.04%, attesting to the small flow distribution variations that take place due to loop re-closure.
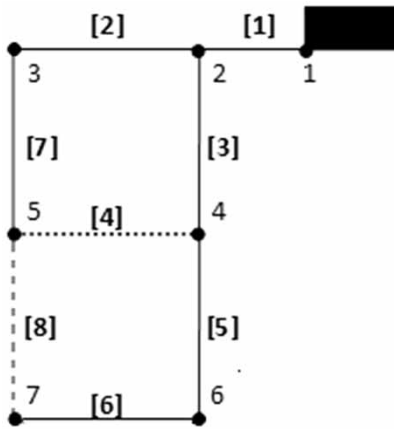

Figure 1 │ Network layout of Alperovits & Shamir (1977). Node numbers close to nodes and pipe numbers close to pipes inside brackets '[ ]'. Dashed pipes are the pipes removed following step 1 of Ciaponi & Papiri (1983, 1985).

Table 2 │ Diameters [mm] assigned to the network pipes of the network of Alperovits & Shamir (1977)

| Pipe | Ciaponi & Papiri (1983, 1985) | | Krapivka & Ostfeld (2009) |
	Step 2	Step 3	
1	457.2	457.2	457.2
2	304.8 ($L = 219.22$ m)	**304.8 ($L = 207.08$ m)**	304.8 ($L = 207.00$ m)
	254.0 ($L = 780.78$ m)	**254.0 ($L = 792.92$ m)**	254.0 ($L = 793.00$ m)
3	406.4	406.4	406.4
4	–	25.4	25.4
5	406.4 ($L = 682.29$ m)	**406.4 ($L = 693.61$ m)**	406.4 ($L = 693.24$ m)
	355.6 ($L = 317.71$ m)	**355.6 ($L = 306.39$ m)**	355.6 ($L = 306.76$ m)
6	254.0 ($L = 986.13$ m)	**254.0 ($L = 989.52$ m)**	254.0 ($L = 989.46$ m)
	203.2 ($L = 13.87$ m)	**203.2 ($L = 10.48$ m)**	203.2 ($L = 10.54$ m)
7	254.0 ($L = 909.18$ m)	**254.0 ($L = 902.31$ m)**	254.0 ($L = 902.27$ m)
	203.2 ($L = 90.82$ m)	**203.2 ($L = 97.69$ m)**	203.2 ($L = 97.73$ m)
8	–	25.4	25.4

Solution at the end of steps 2 and 3 of the algorithm of Ciaponi & Papiri (1983, 1985) and reference solution of Krapivka & Ostfeld (2009). In bold, adjustments following loop re-closure.

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For the network of Alperovits & Shamir (1977), the reference solution considered in this work is the design configuration obtained by Krapivka & Ostfeld (2009) (see Table 2) by applying a hybrid algorithm made up of GA + LP. The value of the path function f_r for the reference solution is 872,611 (m L)/s, which is very close to f_2. In particular, the percentage difference $PD_2 = 100 (f_r\text{-}f_2)/f_2$ is equal to 0.04%, highlighting the fact that the reference solution also respects the MWPC, though being obtained without considering explicitly this criterion inside the optimization process. The total cost C_r of the reference solution is of 403,572 $, which is slightly superior to that obtained through the algorithm of Ciaponi & Papiri (1983, 1985). In particular, the percentage difference $PD_3 = 100 (C_r\text{-}C_3)/C_3$ is equal to 0.003%.

Table 3 reports, for the network of the various case studies, the values of function f and cost obtained by the algorithm of Ciaponi & Papiri (1983, 1985) at the end of steps 2 and 3 and those associated with the reference solutions.

First, it must be remarked that, except for Blacksburg and Fossolo Pl, all the branched networks sized on the basis of MWPC have a lower cost than the looped networks sized by means of other optimization algorithms. However,

this result has no application since networks must deliver water to users connected along the pipes. Therefore, no pipes can be removed from the designed layout. The loop re-closure through pipes of minimum diameters (step 3 in the algorithm of Ciaponi and Papiri), indispensable for restoring the pipes removed in step 1, does not alter significantly the pipe flow distribution and the minimized flow path length f obtained for the network with open loops (Figure 2). In fact, PD_1 is always quite small, below 4% in all but 3 case studies (Go Yang, Fossolo P0 and Fossolo Ir) in which PD_1 is around 7%. The higher PD_1 values in these case studies take place because, in correspondence to some of the pipes removed in step 1, there are not negligible ground elevation differences between the end nodes. These differences cause some not negligible flow distribution variations when the loops are reclosed. This unpleasant effect in the Fossolo network is attenuated in the Fossolo P1 case study, where loops are re-closed with 16 mm diameter pipes, rather than 60 mm diameter pipes (as in Fossolo P0 and Fossolo Ir).

PD_2 has the same order of magnitude as PD_1 (Figure 3), pointing out that the search for the minimum cost solution leads to network configurations with low values of f, even when MWPC is not explicitly taken into account inside

Table 3 | For the various networks, values of the path and cost objective functions obtained by Ciaponi & Papiri (1983, 1985) at the end of steps 2 and 3, and in the reference solution

| Network	f [L m/s]			Network cost		
	Ciaponi and Papiri Step 2	Ciaponi and Papiri Step 3	Reference Solution	Ciaponi and Papiri Step 2	Ciaponi and Papiri Step 3	Reference Solution
Two loop	872,218	872,602	872,611	399,473 $	403,561 $	403,572 $
Ruey Fang	294,887	298,107	312,223	$8,497 \times 10^3$ $	$10,334 \times 10^3$ $	$11,593 \times 10^3$ $
Blacksburg	99,185	99,293	99,528	119,714 $	120,142 $	118,251 $
GoYang	15,702	16,765	16,726	$112,758 \times 10^3$ won	$177,382 \times 10^3$ won	$177,136 \times 10^3$ won
Taichung	205,691	209,481	213,687	$5,629 \times 10^3$ $	$7,255 \times 10^3$ $	$8,775 \times 10^3$ $
Hanoi	$34,108 \times 10^3$	$34,304 \times 10^3$	$35,100 \times 10^3$	$6,027 \times 10^3$ $	$6,215 \times 10^3$ $	$6,080 \times 10^3$ $
Kadu	6,591,526	6,660,273	$6,698 \times 10^3$	$106,810 \times 10^3$ rupie	$124,189 \times 10^3$ rupie	$131,313 \times 10^3$ rupie
Fossolo P0	16,766	18,071	17,055	$50,755 \times 10^3$ £	$78,359 \times 10^3$ £	$70,681 \times 10^3$ £
Fossolo Ir	16,766	17,997	17,904	98,547 €	181,856 €	178,494 €
Fossolo P1	16,766	16,891	17,049	29,460 €	30,398 €	29,117 €
R9	$2,660 \times 10^3$	$2,666 \times 10^3$	$2,741 \times 10^3$	$191,865 \times 10^3$ Cr$	$198,185 \times 10^3$ Cr$	$199,390 \times 10^3$ Cr$
Pescara	900,212	937,902	944,282	1,228,663 €	1,856,557 €	1,820,263 €
Modena	786,210	810,230	821,564	$2,161 \times 10^3$ €	$2,537 \times 10^3$ €	$2,577 \times 10^3$ €
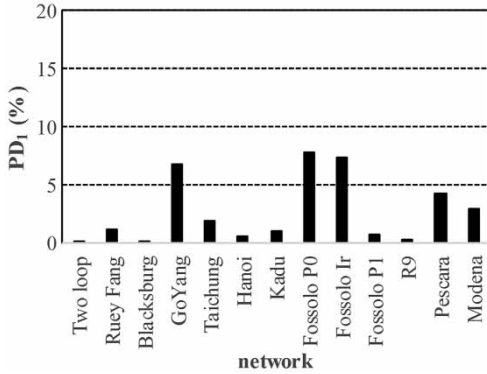
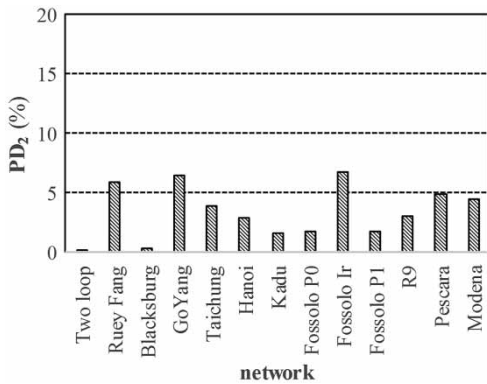**Figure 2** | Values of PD_1 for the various networks.

Figure 3 | Values of PD_2 for the various networks.

the optimization process. In fact, the reference solutions in the various case studies were obtained through algorithms that do not take account of MWPC in network design.

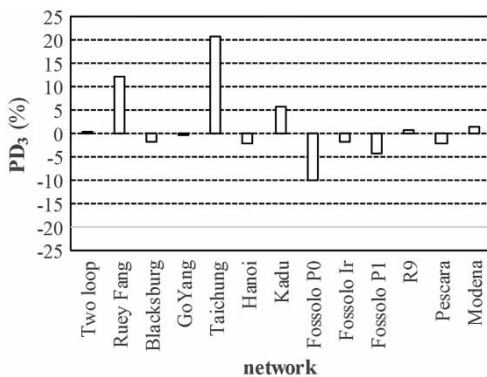Finally, the closeness of PD_3 to 0 (Figure 4) attests to the good performance of the algorithm based on MWPC. This

Figure 4 | Values of PD_3 for the various networks.

algorithm is able to provide solutions with close (and in some cases even inferior) network cost to those obtained through more complex and burdensome optimization algorithms. The only network configuration in which the algorithm of Ciaponi & Papiri (1983, 1985) does not provide a cost effective solution is the Fossolo network with the P0 set of available pipe diameters. The poor performance in this case study may be ascribed to the issue highlighted above, related to the higher flow distribution variations taking place after loop re-closure. Another reason for the poor performance could be the presence of stringent maximum flow velocity constraints in this case study. This could have led to an oversizing of the pipes where water discharges are concentrated in the branched network in step 2 of the algorithm, compared to solutions obtained by minimizing directly the network cost.

A final remark can be made about the cost increase that takes place during loop-closure (step 3), due to the re-introduction of pipes that were removed during step 1. This increase strongly depends on the total lengths of the removed pipes compared to the total length of the original layout with closed loops. For instance, in the case of the Fossolo network, the total length of the removed pipes is equal to 5,408 m, that is almost half of the total length of the original layout (11,145 m). However, despite the influence of loop-closure on network cost, it must be acknowledged that loop closure does not alter flow distribution much, which stays close to that obtained by applying MWPC (step 1).

CONCLUSIONS

This work has confirmed that, in WDN design, imposing the minimum path length on the water flowing from the source nodes to the demanding nodes has strong economic consistency, besides representing a valid design criterion for the conservation of water qualitative properties.

In a looped network model with demands allocated to the nodes, the application of the MWPC causes a number of pipes equal to the number of elementary loops to be removed in the network layout. Therefore, the looped network layout is transformed into a system of branched networks. The application of the MCD to branched

networks yields the absolute minimum cost in almost all the case studies considered, thus proving the existence of a strong relationship between MCD and MWPC. The strong relationship between the MCD and the MWPC is only slightly affected by the need to assign a hydraulic function to the pipes that should theoretically be excluded from the network based on a criterion of pure economic optimization (but that cannot be removed due to obvious reasons). In fact, the total flow path length (function f) varies only slightly due to loop re-closure. Furthermore, the WDN design carried out with various optimization methodologies always leads to network configurations featuring f values close to the lowest value obtained applying the MWPC, even when the methodologies do not take account of MWPC inside the optimization process.

It entails that MCD, which strictly speaking should be tackled by means of algorithms aimed at minimizing in a single step the network cost as a function of the whole set of decisional variables a stake, can be faced through two optimization steps:

1. assessment of the design pipe water discharges that minimize the water path length;
2. calculation of the diameters that minimize the network cost.

Both the steps can be solved by means of LP, following well established formulations (Ciaponi & Papiri 1983, 1985; Stephenson 1984; Suribabu & Neelakantan 2005), and completed by a third step for network loop re-closure.

The calculations reported in this work confirmed that the three-step linear algorithm described above is very effective. In fact, it yields solutions comparable (and in some cases superior) to those obtainable through non-linear and heuristic algorithms, which are more burdensome from the computational viewpoint.

The three-step algorithm described above enabled solving the nonlinear WDN design problem through LP, which can be successfully applied to networks of whatever size, thanks to its lightness, robustness and effectiveness. Therefore, algorithms like that used in this work can bridge the gap between the complex methodologies of limited applicability that exist in the scientific literature and the fully empirical approaches used by practitioners to design WDNs.

The strong economic consistency of MWPC and its major role in WDN design was highlighted here in case studies characterized by regular ground elevations (the maximum difference in the ground elevation ranges from 0 to 27 m). The application of MWPC needs further investigation into the cases of strong ground elevation gradients, where flow distributions between nodes with very different ground elevations are affected by available heads rather than flow lengths. In fact, nodal ground elevations are actually neglected in the first step of the algorithm. Hence, the loop re-closure (step 3) can alter significantly the flow paths obtained in steps 1 and 2, thus worsening the performance of the algorithm in its current version.

A development of this work will also concern the extension of the MWPC to networks operating under multiple loading conditions.

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Uncorrected Proof

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