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Real-time flood stage forecasting by Variable Parameter Muskingum Stage hydrograph routing method

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ABSTRACT

The application of a Variable Parameter Muskingum Stage (VPMS) hydrograph routing method for real-time flood forecasting at a river gauging site is demonstrated in this study. The forecast error is estimated using a two-parameter linear autoregressive model with its parameters updated at every routing time interval of 30 minutes at which the stage observations are made. This hydrometric data-based forecast model is applied for forecasting floods at the downstream end of a 15 km reach of the Tiber River in Central Italy. The study reveals that the proposed approach is able to provide reliable forecast of flood estimate for different lead times subject to a maximum lead time nearly equal to the travel time of the flood wave within the selected routing reach. Moreover, a comparative study of the VPMS method for real-time forecasting and the simple stage forecasting model (STAFOM), currently in operation as the Flood Forecasting and Warning System in the Upper-Middle Tiber River basin of Italy, demonstrates the capability of the VPMS model for its field use.

Key words | compound channel, flood, hydrograph, Muskingum stage, real-time forecasting variable parameter

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INTRODUCTION

Many communities owe much of their prosperity to advantages offered by adjacent and nearby streams, the more important being adequate commercial and municipal water supplies, navigation, power development and recreation. Adverse effects, however, are experienced when high flows occur in the form of floods causing loss of life and damage to property which have to be mitigated by employing economically feasible structural measures such as levees, flood walls and channel improvement. However, these types of measures cannot eliminate completely the hydraulic risk, given the impossibility of building larger and larger structures to cope with extremely low probability events. Therefore, an important role remains for non-structural measures to be compared, evaluated and actuated in real time. Flood forecasting is an important non-structural measure for flood damage reduction and for minimising flood-related deaths and, hence, its implementation as an effective tool requires accurate flood forecasting with sufficient lead time. Hence, it is essential that flood forecasting methods should be physically based, less data intensive and, over and above, should be easily understood by the field engineers.

Every flood forecasting model operates in two modes: the simulation mode, and the operation mode (on-line forecasting). A flood forecasting model in the simulation mode attempts to reproduce the response of the system for past recorded precipitation or upstream input flow. The response of the model is compared with the recorded response at the section of forecasting interest and, if they do not match, either

doi: 10.2166/nh.2011.063

the model structure is changed or the parameters are modified until the match is satisfactory. Once the structure of the model and its parameters have been identified during the calibration phase, the model can be used for forecasting purposes and it is said to be used in operational mode. While the basic structure of the model is not changed in the operational mode, the parameters need to be changed to reflect the current catchment conditions due to the variation of the input.

Typically, the flood forecasting models have two components: the deterministic flow component and the stochastic flow component. While the former is determined by the hydrologic/hydraulic model, the latter is determined based on the residual (error) series of the difference between the forecasted flow for a specified lead time and the corresponding observed one. The residual series reflects both the model error, due to the inability of the model used for forecasting to correctly reproduce the flow process, and the observational error while measuring the flow. It is imperative, therefore, to use an appropriate approach to reduce the model error. The adaptive parameter estimation methods employing the Kalman filtering technique may not be worth the effort for real-time flood forecasting (Ahsan & O'Connor 1994; Huang 1999), when the hydrological model employed for forecasting is grossly inadequate to simulate past recorded floods. In such a scenario, the application of the simplified physically based model like the variable parameter Muskingum stage (VPMS) hydrograph routing method along with a simple error updating technique may be found useful for real-time flood forecasting at a river gauging site.

The analysis presented here focuses on this specific aspect by studying the use of a VPMS routing method as a component model of a hydrometric data-based deterministic forecasting model. It will be shown that the use of a physically based component model in a forecasting model enables the use of a simple stochastic error updating model to estimate the forecast additive error. The proposed forecasting model is tested by considering several flood events that occurred along a 15 km river reach of the Tiber River, in Central Italy, bounded by Pierantonio and Ponte Felcino gauging stations and comparing its accuracy with that of a simple Stage Forecasting Model (STAFOM) currently in operation as the Flood Forecasting and Warning System in the Upper-Middle Tiber River basin.

VARIABLE PARAMETER MUSKINGUM STAGE–HYDROGRAPH ROUTING METHOD

The physically based VPMS hydrograph routing method was developed by Perumal & Ranga Raju (1998*a*, *b*) directly from the Saint Venant equations. The form of the routing equation developed is the same as that of the Muskingum method, replacing the discharge variable by the stage variable, which is the reason for adherence to the term "Muskingum". Further, the parameters vary at every routing time interval and they are related to the channel and flow characteristics by the same relationships as established for the physically based Muskingum method (Apollov *et al.* 1964; Cunge 1969; Dooge *et al.* 1982; Perumal 1994*a*, *b*). The detailed development of the method can be found in Perumal & Ranga Raju (1998*a*, *b*) and Perumal *et al.* (2007). Only the equations relevant to this study are presented here.

Using the Approximate Convection–Diffusion equation of the following flow depth formulation (Perumal & Ranga Raju 1999):

$$\frac{\partial y}{\partial t} + c\frac{\partial y}{\partial x} = 0 \tag{1}$$

the Muskingum-type routing equation can be arrived at as (Perumal 1998*a*)

$$y_u - y_d = K\frac{\mathrm{d}}{\mathrm{d}t}[y_d + \theta(y_u - y_d)] \tag{2}$$

where y_u and y_d denote the flow depths at the upstream and downstream sections of the Muskingum reach, respectively. The travel time K can be expressed as

$$K = \frac{\Delta x}{c_3} \tag{3}$$

where Δx is the length of the Muskingum reach and c_3 is the wave celerity.

The weighting parameter θ, after neglecting the inertial terms, can be expressed as

$$\theta = \frac{1}{2} - \frac{Q_3}{2S_0(\partial A/\partial y)_3 c_3 \Delta x} \tag{4}$$

The subscript 3 attached to different variables in Equations (3) and (4) denotes the evaluation of these variables at section 3, at which the normal discharge corresponding to the flow depth at the middle of the Muskingum reach passes during unsteady flow (see Figure 1); Q denotes the discharge; S_0 is the bed slope and $\partial A/\partial y$ is the top width of the water surface.

Using Equations (3) and (4) in Equation (2) and expressing it as a difference equation leads to a form similar to that of the Muskingum routing equation, but using flow depth as the operating variable, and it is expressed as

$$y_{d,j\Delta t} = C_1 y_{u,j\Delta t} + C_2 y_{u,(j-1)\Delta t} + C_3 y_{d,(j-1)\Delta t} \tag{5}$$

where $y_{u,j\Delta t}$ and $y_{d,j\Delta t}$ denote the observed upstream and the estimated downstream flow depths at time $j\Delta t$, respectively; and $y_{u,(j-1)\Delta t}$ and $y_{d,(j-1)\Delta t}$ denote the observed upstream and downstream flow depths at time $(j-1)\Delta t$, respectively. The notation Δt is the routing time interval, and the coefficients C_1, C_2 and C_3 are expressed as

$$C_1 = \frac{-K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \tag{6a}$$

$$C_2 = \frac{K\theta + 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \tag{6b}$$

$$C_3 = \frac{K(1-\theta) - 0.5\Delta t}{K(1-\theta) + 0.5\Delta t} \tag{6c}$$

It has been shown by Perumal et al. (2007) that the VPMS method can be applied for routing in a uniform compound trapezoidal cross-section channel reach consisting of a main channel and a floodplain channel as shown in Figure 2. It has been shown therein that the wave celerity corresponding to flow in the main channel, c_{main}, is expressed as

$$c_{main} = \left[\frac{5}{3} - \frac{2}{3}\frac{R_{main}(\partial P_{main}/\partial y)}{(\partial A_{main}/\partial y)}\right]\left(\frac{Q_{main}}{A_{main}}\right) \quad (y < y_m) \tag{7}$$

where y_m is the main channel depth; A_{main}, P_{main}, and R_{main} represent the flow area, the wetted perimeter and the hydraulic radius for the main channel, respectively; and Q_{main} is the discharge of the main channel section.

The wave celerity for flow in the compound channel is computed as (Perumal et al. 2007)

$$\begin{aligned} c_{compound} = & \left[\left(\frac{5}{3}\frac{\partial A_{main}}{\partial y}\right)v_{main}\right] \Big/ \left[\frac{\partial A_{compound}}{\partial y}\right] \\ & + \left[\left(\frac{5}{3}\frac{\partial A_1}{\partial y} - \frac{2}{3}\frac{A_1}{P_1}\frac{\partial P_1}{\partial y}\right)v_1\right] \Big/ \left[\frac{\partial A_{compound}}{\partial y}\right] \\ & + \left[\left(\frac{5}{3}\frac{\partial A_2}{\partial y} - \frac{2}{3}\frac{A_2}{P_2}\frac{\partial P_2}{\partial y}\right)v_2\right] \Big/ \left[\frac{\partial A_{compound}}{\partial y}\right] \quad (y > y_m) \end{aligned} \tag{8}$$

where v_{main} denotes the velocity of flow in the main channel; v_1 and v_2 are the flow velocities in the floodplains 1 and 2 (shown in Figure 2), respectively; A_1, P_1, A_2 and P_2 denote the flow area and wetted perimeter of the two floodplains, respectively; and $A_{compound}$ is the total flow area of the compound channel.

The flow velocities in the main channel and in floodplains 1 and 2 of the compound channel are evaluated as

$$v_{main} = \frac{\sqrt{S_0}}{n}(R_{main})^{(2/3)}\sqrt{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}} \tag{9a}$$

$$v_1 = \frac{\sqrt{S_0}}{n}(R_1)^{(2/3)}\sqrt{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}} \tag{9b}$$

$$v_2 = \frac{\sqrt{S_0}}{n}(R_2)^{(2/3)}\sqrt{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}} \tag{9c}$$

$$\frac{\partial y}{\partial x} = \frac{y_d - y_u}{\Delta x} \tag{9d}$$

where R_{main}, R_1 and R_2 denote the hydraulic radius of the main channel section and of the floodplains 1 and 2 of the compound channel section, respectively; and n is Manning's roughness coefficient.

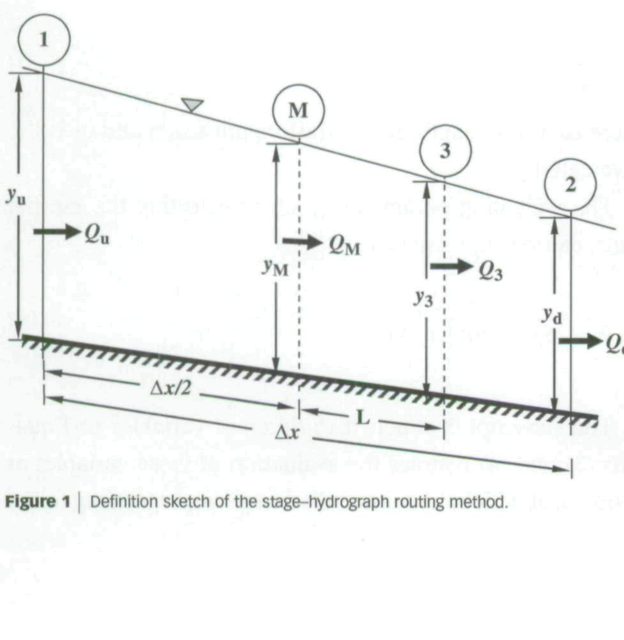

Figure 1 | Definition sketch of the stage–hydrograph routing method.

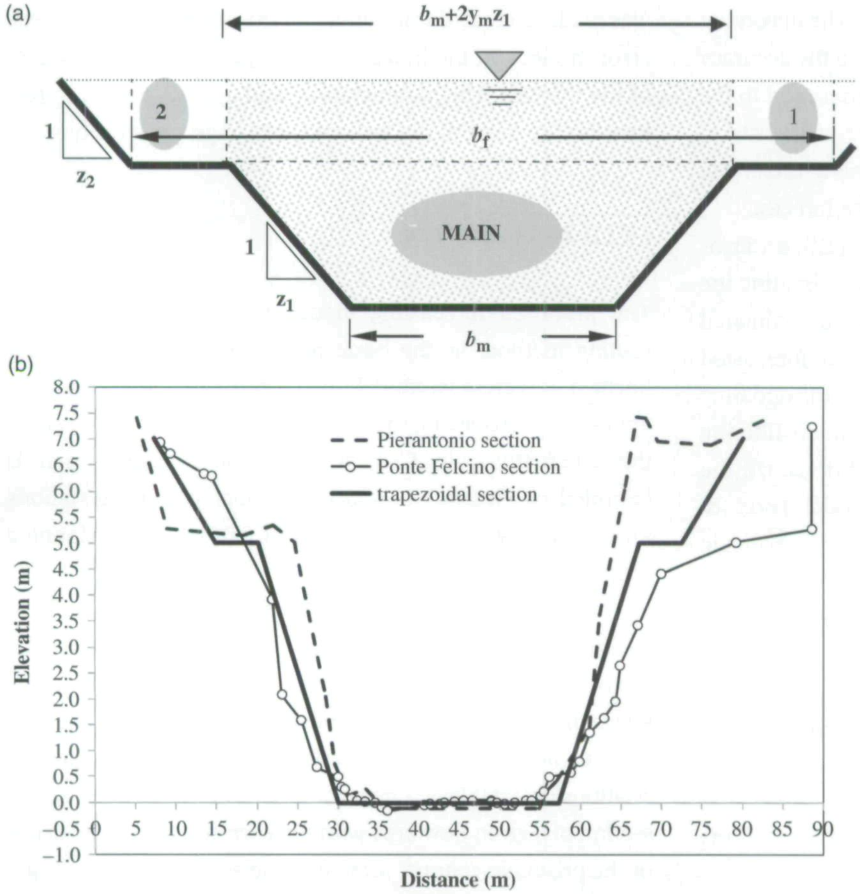

Figure 2 | a) Prismatic compound channel section used for the actual river conceptualization; it is made up of a main channel section (shaded) and two floodplain channels (sections 1 and 2). b) Cross-sections of the Upper Tiber River at Pierantonio (upstream) and Ponte Felcino (downstream) gauging stations with the optimized trapezoidal channel section.

VPMS MODEL FOR REAL-TIME APPLICATION

In order to apply the VPMS method for real-time forecasting purpose, the routing equation given by Equation (5) has to be suitably modified considering a forecast lead time, T_L, as

$$\hat{y}_{d,(j\Delta t+T_L)} = C_1\hat{y}_{u,(j\Delta t+T_L)} + C_2\hat{y}_{u,(j-1)\Delta t+T_L} + C_3\hat{y}_{d,(j-1)\Delta t+T_L} + e_{f,(j\Delta t+T_L)} \quad (10)$$

where $\hat{y}$ denotes the forecast stages, and $e_{f,(j\Delta t+T_L)}$ is the error of forecast, that is, the difference between the observed stage and the corresponding forecasted stage at the site of forecast interest. It can be inferred from Equation (10) that at the time of forecast $j\Delta t$, in order to get the forecast estimate of the downstream stage with a lead time T_L, three different forecast quantities should be available, i.e., $\hat{y}_{u,(j\Delta t+T_L)}$, $\hat{y}_{u,(j-1)\Delta t+T_L}$ and $\hat{y}_{d,(j-1)\Delta t+T_L}$. However, only the last one is known, being the forecast estimate of the downstream stage assessed at the previous time of forecast, $(j-1)\Delta t$. Therefore, in order to apply Equation (10) for estimation of $\hat{y}_{d,(j\Delta t+T_L)}$, the following assumption has to be made based on no-model hypothesis:

$$\hat{y}_{u,j\Delta t+T_L} = \hat{y}_{u,(j-1)\Delta t+T_L} = y_{u,j\Delta t} \quad (11)$$

where $y_{u,j\Delta t}$ is the last upstream observed stage.

Using Equation (11)) in Equation (10), the final forecasting model is expressed as

$$\hat{y}_{d,(j\Delta t+T_L)} = C_1 y_{u,j\Delta t} + C_2 y_{u,j\Delta t} + C_3\hat{y}_{d,(j-1)\Delta t+T_L} + e_{f,(j\Delta t+T_L)} \quad (12)$$

In Equation (12), the minimum T_L is Δt, the routing time interval at which the stage measurements are made, and this corresponds to one time interval ahead forecast. The maximum lead time interval that can be adopted depends on the accuracy of the obtained forecast and that may nearly correspond to the travel time of the upstream discharge to arrive at the site of forecast interest. The use of a larger T_L beyond this approximate travel time would lead to poorer accuracy of the forecast.

In order to estimate $e_{f,(j\Delta t+T_L)}$ in Equation (12), an error updating model also needs to be developed for estimating the forecast error, which when added to the model estimated forecast for a given lead time would yield the final forecasted stage at the site of interest. Note that different error updating techniques of varied complexities such as Kalman filtering (Gelb 1974; Ahsan & O'Connor 1994; Neal *et al.* 2007), the auto-regressive moving average (ARMA) model (Box & Jenkins 1970), and Artificial Neural Networks (e.g., Babovic *et al.* 2001) are available in the literature. Refsgaard (1997) has provided the classification and review of different error updating procedures currently used in real-time flood forecasting. However, for simplicity, it is proposed to use a second-order linear autoregressive error updating model of the following form for forecasting the error at time $(j\Delta t + T_L)$:

$$e_{f,(j\Delta t+T_L)} = a_1\, e_{obs,j\Delta t} + a_2\, e_{obs,(j-1)\Delta t} + \epsilon_{(j\Delta t+T_L)} \tag{13}$$

where $e_{obs,j\Delta t}$ and $e_{obs,(j-1)\Delta t}$ are the forecasting errors estimated at time $j\Delta t$ and $(j-1)\Delta t$, respectively, and $\epsilon_{(j\Delta t+T_L)}$ is the random error (white noise).

Forecasting using Equation (13) can be made only after the lapse of certain initial period of the forecasting event, known as the warm-up period. The difference between the observed stage and the VPMS routed stage in the warm-up period is considered as the actual error and its series is assumed to be stochastic in nature. The initial parameters a_1 and a_2 of the error update model are assessed using this error series estimated in the warm-up period. The duration of initial warm-up period considered for developing the error update model should not be too long to avoid that the forecasting exercise becomes of no practical use for forecasting the given event, and, at the same time, it should not be too short resulting in numerical problem while estimating the parameters a_1 and a_2 using the least squares approach. However, in this study, the error updating model given by Equation (13) has been applied without generating the random error component. It may be noted that no attempt was made to study the sensitivity of the order of the stochastic error model and the initial warm-up period on the estimates of the forecast. The parameters a_1 and a_2 are updated in real time on the basis of the last available stage observations.

FIELD APPLICATION

The proposed forecasting model consisting of the VPMS routing method, as the basic model, and the second-order linear autoregressive model, as the error updating model, is applied for forecasting the flow stage in a 15 km reach along the Tiber River, in Central Italy. The selected reach is bounded by Pierantonio and Ponte Felcino gauging stations and has an average bed slope S_0 of 0.0016 and a Manning roughness coefficient $n = 0.039$.

Note that the approximation of the VPMS method for routing a given stage hydrograph in a river reach requires the use of an equivalent prismatic channel reach; this involves the approximation of the actual river reach sections at the two ends to an equivalent prismatic section with a one-to-one relationship established between the flow depth of the actual section of a given flow area with the corresponding flow depth of the prismatic channel section of the same flow area. Based on the surveyed cross-sections at the ends of the actual river reach, it was considered appropriate to approximate the actual reach by a compound trapezoidal section reach. Accordingly, the surveyed cross-sections of the actual reach were overlapped and a two-stage trapezoidal compound section geometry was assessed paying particular attention to the flow area reproduction. In particular, once the floodplain level, y_m, (see Figure 2) is assumed on the basis of the properties of the two channel ends, the section parameters b_m, b_f, z_1 and z_2 (see Figure 2 for symbols) are assessed by minimizing the mean square error in the real mean flow area estimate (see Perumal *et al.* 2010). Based on this criterion, a compound trapezoidal section with $b_m = 27.31$ m, $y_m = 5.0$ m, $b_f = 57.6$ m, $z_1 = 1.98$ and $z_2 = 3.8$ was identified. Therefore, the relationships between the actual flow depths and the equivalent trapezoidal section ones at the channel ends were developed in order to have the same value of the actual mean flow area, yielding

$$y_{u-trap} = 0.916\, y_{u-actual} + 0.065 \tag{14}$$

$$y_{d-trap} = 1.079\, y_{d-actual} - 0.067 \tag{15}$$

where $y_{u\text{-}trap}$ and $y_{d\text{-}trap}$ are the equivalent upstream and downstream flow depths in the trapezoidal channel section corresponding to the flow depths $y_{u\text{-}actual}$ and $y_{d\text{-}actual}$ in the actual river section. Using the upstream section relationship, the observed stage hydrograph of any event was converted to equivalent trapezoidal section stage hydrograph to enable the routing using the VPMS method and, using the relationship ($y_{d\text{-}actual} = 0.927\ \ y_{d\text{-}trap} + 0.062$) developed on the basis of the downstream site properties, the routed hydrograph of the equivalent trapezoidal section was converted to the actual end section estimated hydrograph.

To study the applicability of the proposed forecasting model, 12 flood events recorded concurrently at Pierantonio and Ponte Felcino stations were used. The details of these events, each recorded at half -hour intervals, are shown in Table 1, where also the details of wave travel time, percentage of lateral flow and actual and equivalent trapezoidal peak flow depths at both the stations are reported. As can be seen, on the basis of the selected events, the mean flood wave travel time for the investigated river reach is nearly equal to 1.5 hours.

The accuracy of the proposed forecasting model was studied using a warm-up period of 5 hours and considering five different forecast lead times (1.0, 1.5, 2.0, 2.5 and 3.0 hours). The efficiency of the forecast was evaluated using two criteria: (1) the Nash–Sutcliffe (NS) efficiency coefficient (Nash & Sutcliffe 1970) and (2) the Persistence Criterion (*PC*). As the NS coefficient is well known in hydrological literature (ASCE 1993), only the Persistence Criterion is explained here. It compares the prediction of the proposed model against that obtained by the no-model, which assumes a steady state over the forecasting lead time, and is evaluated as

$$PC = \left(1 - \frac{\sum_i (y_{i\Delta t} - \hat{y}_{i\Delta t})^2}{\sum_i \left(y_{i\Delta t} - y_{(i\Delta t - T_L)}\right)^2}\right) \times 100 \qquad (16)$$

where y and $\hat{y}$ denote the observed and the forecasted flow depth values, respectively.

Further, to investigate the reliability of the proposed VPMS model for flood forecasting a comparative study between the VPMS solution and the corresponding stage hydrographs forecasted by STAFOM (Moramarco *et al.* 2006; Barbetta *et al.* 2008), the model currently in operation as the Flood Forecasting and Warning System in the Upper-Middle Tiber River basin, was carried out. STAFOM involves a physically based approach incorporating the lateral flow contribution with an additive error component that is updated using the stage observations available in real-time (Barbetta *et al.* 2008). The model requires the estimation of four parameters if the downstream rating curve is unknown, otherwise only two parameters have to be determined.

Table 1 | Pertinent characteristics of the flood events studied

		Pierantonio section			Ponte Felcino section	
Event	**Wave travel time (h)**	**Lateral inflow (%)**	**Actual peak stage (m)**	**Equivalent trapezoidal peak stage (m)**	**Actual peak stage (m)**	**Equivalent trapezoidal peak stage (m)**
December 96	1.50	1.90	4.74	4.32	4.22	4.33
April 97	1.50	6.50	5.07	4.62	4.57	4.70
November 97	1.00	5.40	4.22	3.86	3.81	3.90
February 99	2.00	4.40	5.06	4.61	4.52	4.65
December 99	0.00	24.70	2.71	2.52	2.79	2.82
December 00	2.00	Flooding	5.92	5.37	5.25	5.42
April 01	2.00	0.20	3.68	3.38	3.23	3.29
November 05	2.50	Flooding	7.10	6.42	6.92	7.19
3rd December 05	1.00	3.60	5.10	4.64	4.42	4.55
5th December 05	1.00	5.70	5.49	4.99	4.76	4.91
30th December 05	2.00	1.90	4.99	4.54	4.34	4.46
February 06	1.50	28.40	2.28	2.14	2.64	2.66

Table 2 | Forecasting model results provided by the VPMS and STAFOM models for a lead time of 1 hour (err_y_{peak} = percentage error in peak stage; err_t_{peak} = error in time to peak stage)

	VPMS model				STAFOM model			
Event	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)
December 96	0.11	−1.50	99.80	93.24	0.12	−1.50	99.42	78.69
April 97	−0.11	−0.50	99.95	97.97	0.01	−0.50	99.77	88.54
November 97	1.00	−3.00	99.87	96.17	1.46	−3.00	99.73	90.52
February 99	−0.87	−0.50	99.90	96.62	−0.82	−1.00	99.26	72.42
December 99	2.00	1.00	99.78	77.82	2.43	1.00	99.61	58.70
December 00	−0.75	1.50	99.80	90.27	−0.33	0.00	99.44	69.10
April 01	−0.70	0.50	99.63	95.10	−1.84	1.00	98.48	77.39
November 05	0.06	0.00	99.87	90.46	−0.85	0.00	99.60	68.25
3rd December 05	−1.40	−0.50	99.73	94.93	−3.07	−1.00	99.45	88.57
5th December 05	−0.17	0.50	99.79	93.24	−0.37	0.50	99.74	85.20
30th December 05	0.30	0.50	99.91	92.21	0.45	−0.50	99.74	76.68
February 06	1.49	1.00	99.62	81.51	0.46	1.00	99.23	58.59
Mean absolute value	**0.75**	**0.92**	**99.80**	**91.63**	**1.02**	**0.92**	**99.46**	**76.05**

RESULTS AND DISCUSSION

Tables 2–6 show the forecasting results provided by both the proposed approach and STAFOM for the peak flow stage forecast at Ponte Felcino station for all the selected flood events and for all the investigated lead times. The results also include the accuracy of peak reproduction, error in time to peak, Nash–Sutcliffe (NS) efficiency and Persistence Criterion (*PC*) efficiency. The two most significant events studied herein are characterized by flooding (December 2000 and November 2005) with flow spilled over the main channel, almost in the entire stretch of the

Table 3 | As Table 2, but for a lead time of 1.5 hours

	VPMS model				STAFOM model			
Event	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)
December 96	0.54	−1.00	99.68	95.11	0.86	−1.00	99.47	91.59
April 97	−0.79	0.00	99.87	97.49	−0.99	0.00	99.86	96.91
November 97	1.89	−2.50	99.81	97.32	2.26	−2.50	99.73	95.99
February 99	−0.10	0.50	99.94	99.01	0.02	−0.50	99.52	92.23
December 99	2.60	1.50	99.49	75.60	3.22	1.50	99.37	69.29
December 00	−0.84	−1.00	99.68	92.69	−0.35	−3.50	99.37	84.59
April 01	0.77	0.00	99.60	97.57	−1.21	0.00	99.03	93.77
November 05	−0.38	0.00	99.67	89.05	−0.80	1.00	99.30	76.01
3rd December 05	−0.31	−0.50	98.88	90.54	−1.09	−0.50	99.13	92.13
5th December 05	0.32	0.00	99.57	93.83	−0.09	0.00	99.58	93.61
30th December 05	0.91	0.50	99.86	94.71	1.35	−0.50	99.75	90.08
February 06	3.50	0.00	98.88	74.48	0.93	1.50	98.26	58.52
Mean absolute value	**1.08**	**0.63**	**99.58**	**91.45**	**1.10**	**1.04**	**99.36**	**86.23**

Table 4 | As Table 2, but for a lead time of 2.0 hours

	VPMS model				STAFOM model			
Event	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)
December 96	0.98	−0.50	99.36	94.35	1.23	−0.50	99.51	95.63
April 97	−0.28	−2.00	99.42	93.33	−0.48	0.50	99.79	97.51
November 97	2.50	−3.50	99.44	95.61	3.02	−1.50	99.58	96.67
February 99	0.50	0.50	99.64	96.75	0.15	0.50	99.85	98.64
December 99	3.00	2.00	98.8	66.6	3.48	2.00	99.04	73.18
December 00	−0.20	−8.50	99.28	90.26	−0.02	−2.50	99.26	89.94
April 01	3.50	0.00	97.85	92.51	0.87	0.50	99.26	97.43
November 05	−0.65	1.00	99.38	88.29	1.51	0.50	99.15	84.00
3rd December 05	1.87	−8.00	95.59	78.22	0.65	0.00	97.40	87.01
5th December 05	1.43	−3.00	98.6	88.51	0.35	−3.00	99.15	92.93
30th December 05	1.27	0.50	99.67	92.78	1.34	0.50	99.76	94.59
February 06	5.80	0.50	97.25	63.27	9.29	0.50	96.66	55.57
Mean absolute value	**1.83**	**2.50**	**98.69**	**86.71**	**1.87**	**1.04**	**99.03**	**88.59**

reach and, also received unaccounted lateral flow (see Table 1). It can be inferred from Tables 2–6 that the proposed approach and STAFOM are characterized by similar and high accuracy. However, it can be observed that the VPMS method provides, on average, more accurate forecast stage values for a forecasting lead time, T_L, of 1.0 and 1.5 hours, whereas for higher T_L values, the STAFOM estimates seem to be more reliable.

Figures 3–6 show some typical forecasted events for various lead times. The given inflow hydrograph and the corresponding observed outflow hydrograph are also shown in these figures. It is inferred from the results given in Tables 2–6 that, up to a lead

Table 5 | As Table 2, but for a lead time of 2.5 hours

	VPMS model				STAFOM model			
Event	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)	err_y_{peak} (%)	err_t_{peak} (h)	NS (%)	PC (%)
December 96	3.80	−4.50	97.89	87.65	1.77	0.00	98.81	93.34
April 97	0.80	−1.50	98.00	84.94	0.47	−1.50	99.08	93.50
November 97	5.77	−4.00	98.25	91.07	3.30	0.00	98.96	95.00
February 99	3.40	−4.00	98.20	89.37	0.93	1.50	99.50	97.17
December 99	3.74	2.50	97.09	46.13	4.51	2.50	98.41	71.26
December 00	3.95	−8.50	98.20	83.92	1.25	−8.00	98.71	89.13
April 01	8.10	−1.00	90.91	79.01	4.64	−0.50	96.51	92.44
November 05	−0.94	2.00	98.88	86.38	−0.95	1.50	98.47	81.91
3rd December 05	8.17	−7.50	86.39	55.22	3.21	−7.00	92.64	76.77
5th December 05	5.96	−4.00	94.82	72.21	3.27	−4.00	97.64	87.66
30th December 05	1.63	0.50	98.90	84.15	1.81	−2.00	99.47	92.66
February 06	7.98	1.50	94.16	48.03	4.05	1.50	94.48	53.52
Mean absolute value	**4.52**	**3.46**	**95.97**	**75.67**	**2.51**	**2.50**	**97.72**	**85.36**

Table 6 | As Table 2, but for a lead time of 3.0 hours

	VPMS model				STAFOM model			
Event	**err_y_{peak} (%)**	**err_t_{peak} (h)**	**NS (%)**	**PC (%)**	**err_y_{peak} (%)**	**err_t_{peak} (h)**	**NS (%)**	**PC (%)**
December 96	7.95	−5.00	94.72	78.00	4.66	−4.00	96.44	86.51
April 97	2.74	−7.50	95.53	76.02	1.10	−0.50	97.32	87.46
November 97	10.10	−3.50	96.24	86.48	4.82	−3.50	97.35	91.40
February 99	11.69	−3.50	95.26	80.00	3.26	−2.00	97.98	92.05
December 99	4.10	4.50	95.68	42.98	4.50	4.50	97.61	69.56
December 00	9.10	−8.50	96.24	75.95	4.67	−7.50	97.32	84.85
April 01	13.8	−0.50	78.80	64.70	9.35	0.00	90.33	85.78
November 05	−1.22	2.50	98.17	84.20	−0.90	2.50	97.75	82.00
3rd December 05	13.46	−6.50	74.61	39.25	7.42	−6.50	87.32	72.46
5th December 05	10.60	−3.50	90.50	63.89	6.76	−3.50	94.71	81.07
30th December 05	2.53	0.00	97.76	77.28	2.69	0.00	98.77	88.29
February 06	10.00	2.00	90.68	39.90	5.69	2.00	91.32	49.58
Mean absolute value	**8.11**	**3.96**	**92.02**	**67.39**	**4.65**	**3.04**	**95.35**	**80.92**

time of 3.0 h, the flood event on 3 December 2005 characterized by a complex shape of the peak region and the two flood events on December 1999 and February 2006 could not be successfully forecasted as reflected by their *PC* estimates (<50%). However, for the last two events, significant lateral flows (>25% of inflow hydrograph volume) affected the model performance. As the proposed forecasting model has been developed using the assumption of no lateral flow in the considered reach, it is expected that the efficiency of the model would be poorer in forecasting the flow depth when that event is associated with significant lateral flow. Although the error update model can, to some extent, improve the forecasts in the event of experiencing lateral flow, it may not give reliable forecasts when there is significant lateral flow in the reach. The minimum *PC* estimated for the forecasted events is greater than 60%, except for three events (December 1999, 3 December 2005 and February 2006), out of which two events are characterized by significant lateral flow.

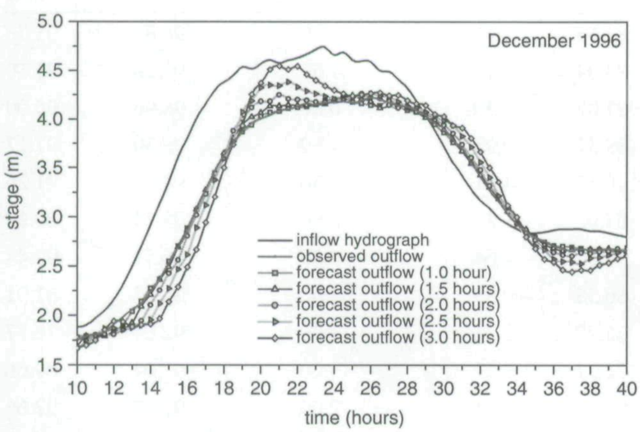

Figure 3 | December 1996 event: comparison between observed and forecast stage hydrographs for different lead times at Ponte Felcino section. The input stage hydrograph at Pierantonio site is also shown.

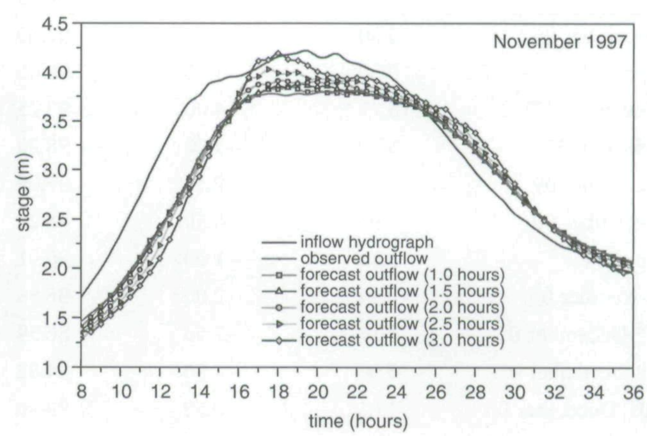

Figure 4 | As Figure 3, but for the event of November 1997.

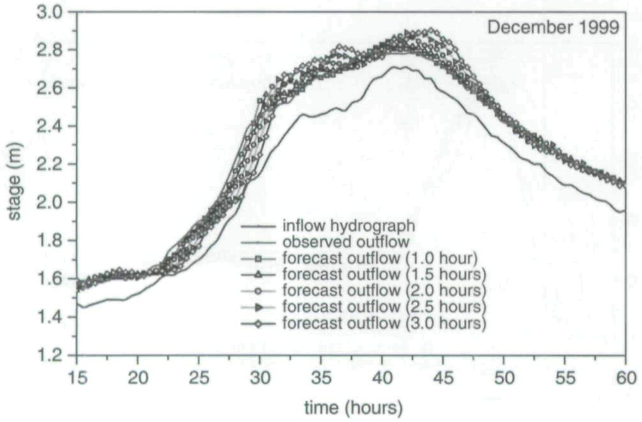

Figure 5 | As Figure 3, but for the event of December 1999.

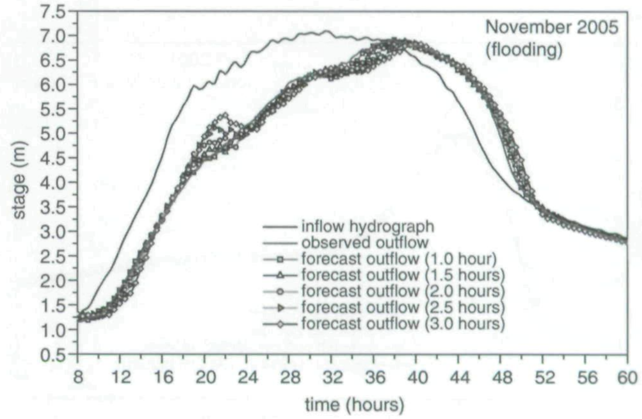

Figure 6 | As Figure 3, but for the event of November 2005.

Further, Figures 7 and 8, illustrating the comparison between the observed downstream stage hydrograph and those forecasted by both the VPMS and STAFOM models for the flood events that occurred on April 1997 and April 2001, with lead times of 1.0 and 3.0 hours, reveal that the VPMS model has a comparable accuracy with STAFOM in flood-stage forecasting.

In order to investigate the role of the error updating model in the assessment of the forecasted stage, a comparative analysis was carried out between the results obtained by the proposed approach and that by Equation (12), neglecting the term quantifying the error of forecast, $e_{f,(j\Delta t+T_L)}$. The analysis showed that the adjustment due to the error updating model is particularly significant during the advancement of rising limb of the hydrograph, when the rate of increase of rising limb suddenly decreases, and at the flood peak region, providing overestimates of the forecasted stage around this time zone. This was observed for almost all the events studied and can be seen from Figures 3–6 and from the forecast results of other events (not shown here).

Figure 9 illustrates a typical comparison between the observed stage hydrograph and those forecasted by the real-time VPMS model with and without considering the error updating model (Equation (13)) for the floods that occurred on December 1996 and November 1997 with a lead-time of 3 hours. It can be seen that the forecasting error, $e_{f,(j\Delta t+T_L)}$, has an important role within the forecasting procedure, significantly improving the forecasting accuracy during the advancement of rising limb and, also, as underlined above, producing an overestimation during the peak phase.

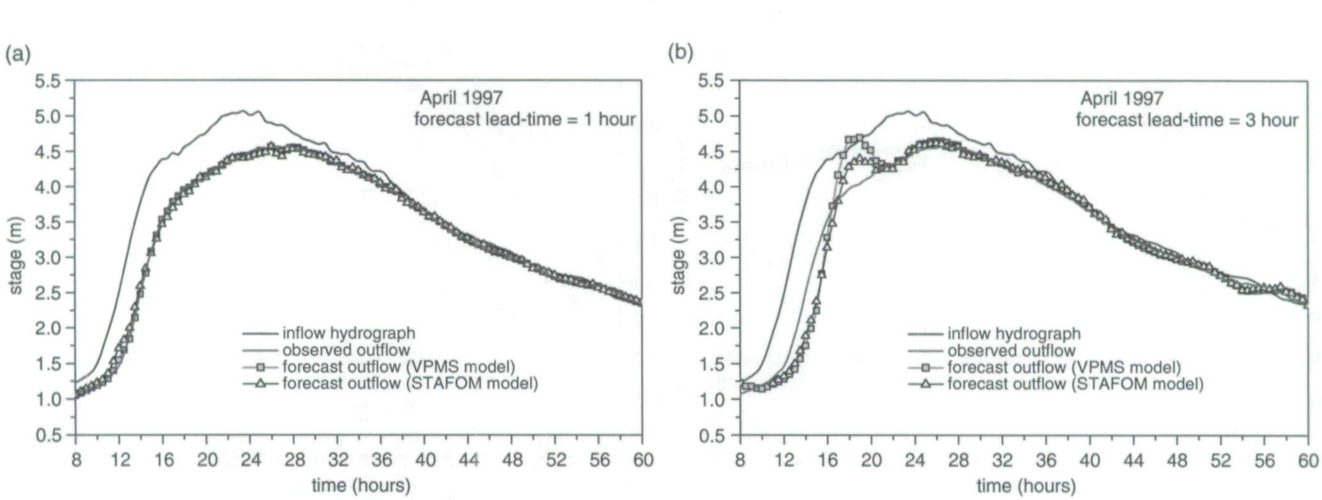

Figure 7 | April 1997 event: stage forecasting by the VPMS and the STAFOM models for two lead times of a) 1 hour and b) 3 hours at Ponte Felcino section. The input stage hydrograph at Pierantonio site is also shown.

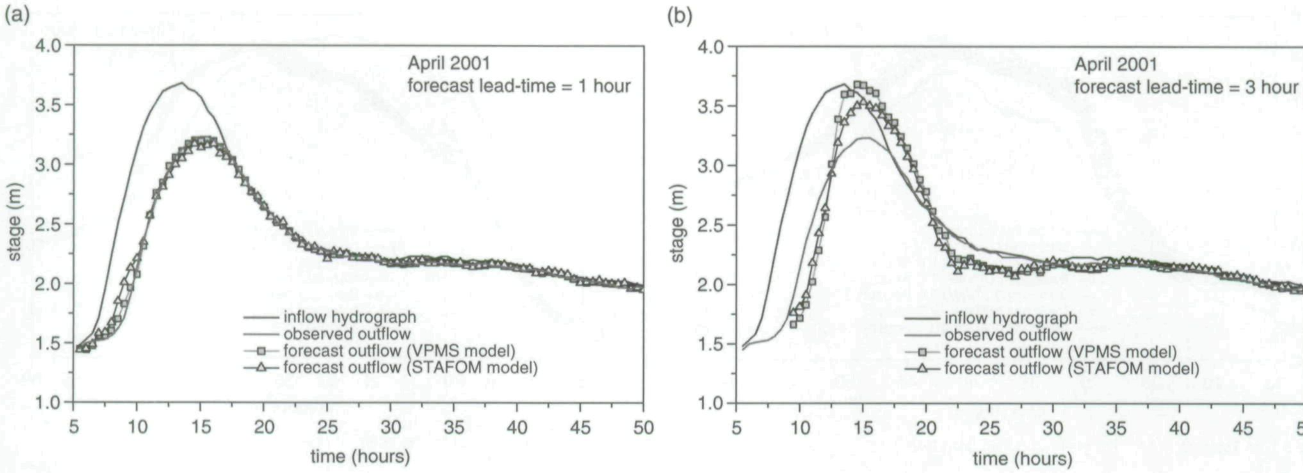

Figure 8 | As Figure 7, but for the event of April 2001.

The significant effect of the updating error technique may be attributed to the consideration of the simplified model structure and the assumption introduced.

CONCLUSIONS

The application of a VPMS hydrograph routing method for real-time flood forecasting at a river gauging site is demonstrated in this study. Based on the forecasting performance for several investigated events, one can infer that the proposed model has the potential for practical forecasting applications in hydrometric data-based modelling provided that the adopted forecasting lead time is not longer than the mean wave travel time of the selected river reach, which for the investigated case study can be assumed equal to 1.5–2.0 hours. Further investigations on different case studies have to be carried out in order to verify the proposed forecasting model accuracy and, furthermore, it would be advisable to extend the model formulation to take into account significant lateral flow contribution entering along the selected river reach.

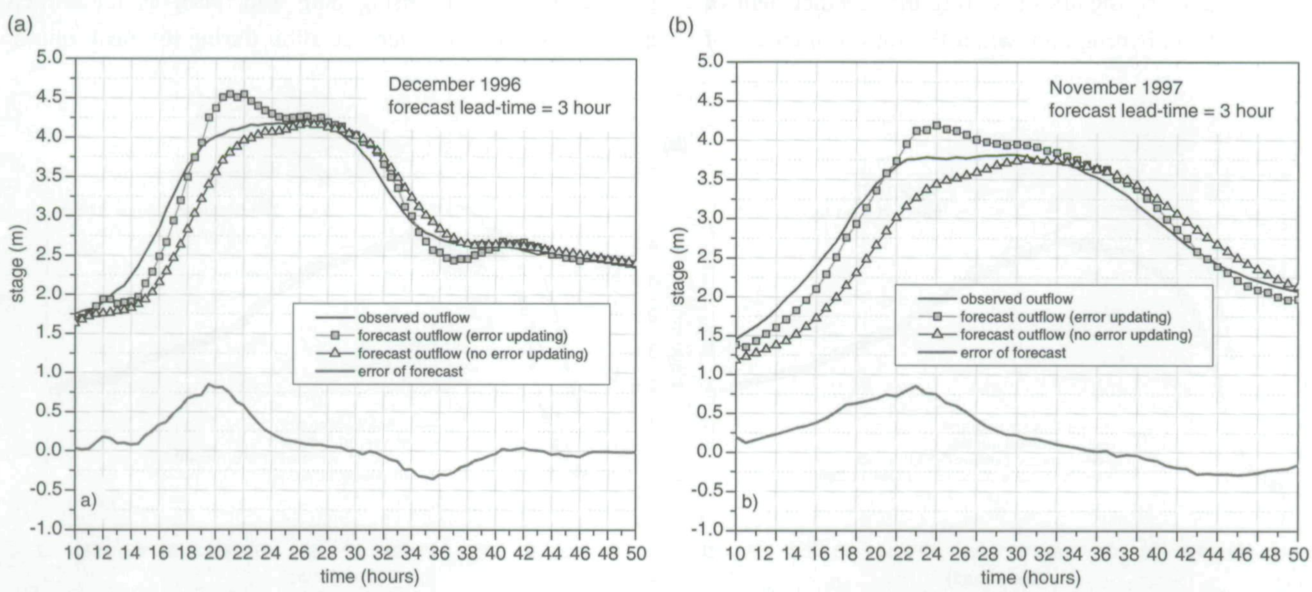

Figure 9 | Comparison between the observed stage hydrograph and those forecast by the VPMS model with and without the error updating technique for a lead-time of 3 hours for the flood events that occurred on a) December 1996 and b) November 1997 at Ponte Felcino section. The error of forecast computed by Equation (13) is also shown.

ACKNOWLEDGEMENTS

The authors are grateful for the financial support from the "International short-term mobility programme for scientists/researchers from Italian and foreign institutions" granted to the first author by the CNR (National Research Council) of the Italian Government to carry out part of this research at the CNR–IRPI Office of Perugia, Italy. The authors are also grateful to Umbria Region for providing most of the analysed data.

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First received 29 May 2009; accepted in revised form 6 December 2009. Available online February 2011