Velocity profiles assessment in natural channels during high floods

Tommaso Moramarco, Carla Saltalippi and Vijay P. Singh

ABSTRACT

The accuracy of three different approaches for velocity profiles assessment during high floods, when the velocity points sampling is carried out only in the upper portion of the flow area, has been investigated. The first two methods assume the classical logarithmic law with additional terms, to take account of the dip-phenomenon in the velocity profile. The third one is based on the entropy theory and uses the maximum flow velocity occurring in the flow area. A sample of velocity measurements carried out at Pontelagoscuro gauged section (Po River, Italy), has been considered for the analysis. Six flood events have been selected and the accuracy of the investigated methods has been evaluated in terms of mean error in estimating both the mean velocity along each sampled vertical and the mean flow velocity. For high floods, the logarithmic law and the entropic approach were found quite accurate; however, the ability of the latter in reproducing the velocity profiles only by sampling the maximum flow velocity has been shown. Therefore, a procedure for velocity measurements based on the entropic approach has been proposed. The procedure allows one to both to shorten remarkably the time of the velocity sampling and to quickly estimate the discharge.

Key words | entropy, mean flow velocity, streamflow measurements, velocity profile

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INTRODUCTION

Hydrologic/hydraulic physical processes have been often examined with a deterministic approach. However, many gaps still remain in the analysis and the probabilistic approach can be considered suitable to address them and to find a better response in the analysis. A fundamental probabilistic approach is the entropy theory which was introduced almost sixty years ago by Claude Shannon (1948) in his renowned paper which represents the basis of Information Theory. The Shannon concept was later extended by Janes (1957) who, by introducing the Maximum Entropy Principle, completely modified the approach followed for solving the statistic inference issues. It is well known that the information entropy represents a measure of the uncertainty linked to a probability distribution (Chapman 1986) and it is fundamental for solving several problems based on statistical models, where the absence of data requires general assumptions for parameter estimation (Singh *et al.* 1986). This is the case of the flow velocity distribution at river cross-sections. The velocity distribution has been investigated using deterministic as well as probabilistic approaches. An important probabilistic formulation was developed by Chiu (1987) introducing the formulation of the velocity distribution in the probability domain by considering the random sampling of flow velocity in a channel section. However, as such data are usually not available, Chiu proposed a link between the probability domain and the physical one. He derived possible expressions of the cumulative probability distribution function in terms of the coordinates in the physical space. However, estimation of two-dimensional velocity distribution is not always simple and may require as many as six parameters (Chiu & Chiou 1986). The probability density function of the velocity was then obtained by applying the maximum entropy principle

doi: 10.2166/nh.2011.064

(Barbé *et al.* 1991; Chiu 1987, 1988, 1989). Using this probabilistic formulation, the mean velocity, u_m, can be expressed as a linear function of the maximum velocity, u_{max}, through a dimensionless entropy parameter M (Chiu 1991). Xia (1997) investigated this correlation for some equipped sections along the Mississippi river and he found a perfect linear relationship between mean and maximum velocity. These results were confirmed by Moramarco *et al.* (2004), who analyzed the velocity measurements carried out during a period of 20 years in different gauged river sites of the Upper Tiber basin in Central Italy. They also modified the two-dimensional velocity distribution approach introduced by Chiu & Chiou (1986), so drastically reducing the number of parameters involved. Therefore, the possibility to assess the velocity distribution only considering the maximum velocity and the entropic parameter M can be of fundamental interest in the context of discharge monitoring by traditional technique and, in particular, during high floods. Likewise, there exists a multitude of methods to estimate the velocity distribution in a cross-sectional flow area. Traditional logarithmic approaches describe velocity profiles by using equations with a limited number of parameters which can be determined on the basis of velocity points sampled along each vertical. In particular, these approaches need a number of velocity measurements equal or greater than of the parameters involved, along with the position of the velocity points sampled. Fenton (2002) introduced a modified procedure of the traditional three-point or four-point method to estimate mean velocity along a vertical. In fact, for the proposed procedure, velocity sampling does not need to be performed at fixed heights in the vertical from the bottom. Other interesting approaches were developed, such as that proposed by Ardiclioglu *et al.* (2005), who introduced a dip-correction factor to account the velocity dip phenomenon that always exists close to sidewalls. Although there are a large number of studies on velocity profiles in natural channel, few studies have been performed to estimate the spatial velocity distribution during high flood conditions when it is not possible to sample the whole velocity field and in particular in the lower portion of the flow area. The sampling procedure of velocity measurements in a river cross-section, in this case, could be difficult and dangerous for cableway operators. On the other hand, the value of maximum flow velocity could be more easily sampled since its position is located in the upper portion of the flow area where velocity measurements can be carried out also during high flow conditions. Considering that the rating curve accuracy is strictly connected to experimental data availability which have to be referred both to low and high flow depths, we well know how a quick and accurate determination of flow passing through a river section is fundamental for the rating curve assessment. Therefore, a model able to assess the velocity profiles, also when velocity data are not available in any portion of the flow area should be welcome.

The objective of the paper is to test, among the aforementioned approaches, the reliability of three methods to estimate the velocity profiles in a natural river section during high floods, when the sampling of velocity points is made only in the upper portion of the flow area. Two methods are based on the logarithmic law and the third one uses the entropy theory. The velocity data collected during six flood events at Pontelagoscuro site, along the Po River, northern Italy, are used for the analysis.

VELOCITY PROFILE DISTRIBUTION MODELS

The classical logarithmic law describing the velocity distribution, u, along a vertical of a cross-sectional flow area, for turbulent flow over a rough bed, can be expressed as (Fenton 2002)

$$u(y) = \frac{u_*}{k}\ln\frac{y}{y_0} + a_1\frac{y}{D} \tag{1}$$

where

- y is the distance from the bottom;
- D is the vertical depth;
- u_* is the shear velocity, $u_* = (gRS)^{0.5}$ (g is the gravitation acceleration, R is the hydraulic radius and S is the energy slope);
- k is the von Karman constant;
- y_0 is the location where the velocity hypothetically equals zero;
- a_1 is a further unknown coefficient, having the same units of velocity, introduced here to take account of the possibility that the velocity profile is deviating from a logarithmic form and that it may present a maximum value at a point below the water surface.

It is worth noting that if one differentiates u, given by Equation (1), with respect to y and equates the derivative to 0 ($u = u_{max}$), the physical meaning of a_1 can be inferred:

$$a_1 = -\frac{u_*}{k}\frac{D}{y_{\max}} \quad (2)$$

$y_{max} = (D\text{-}h)$ is the location where u_{max} occurs and h is its distance from the water surface.

Therefore, if three velocity points, u_1, u_2 and u_3, are sampled at different positions y_1, y_2 and y_3, all three unknown quantities included in Equation (1), u_*/k, y_0 and a_1, can be estimated by calibration procedure.

Equation (1) can be integrated thus obtaining the mean flow velocity value along the vertical:

$$u_v = \int_0^D u(y)dy = \frac{u_*}{k}\left(\ln\frac{D}{y_0} - 1\right) + \frac{a_1}{2} \quad (3)$$

Fenton (2002) introduced an additional quadratic term in Equation (1) to better reproduce the curvature of velocity profile, yielding

$$u\,(y) = \frac{u_*}{k}\ln\frac{y}{y_0} + a_1\frac{y}{D} + a_2\left(\frac{y}{D}\right)^2 \quad (4)$$

where a_2 is the additional unknown coefficient to be found by measurements. In this latter case four velocity points sampled at different positions along each vertical are needed to describe by Equation (4) the entire velocity profile. In this case, the mean flow velocity can be derived as

$$u_v = \int_0^D u(y)dy = \frac{u_*}{k}\left(\ln\frac{D}{y_0} - 1\right) + \frac{a_1}{2} + \frac{a_2}{3} \quad (5)$$

Moramarco *et al.* (2004) allowed the estimation of the velocity profile along a vertical by simplifying the two-dimensional velocity distribution introduced by Chiu (1987, 1988, 1989) and based on the entropy theory:

$$u(y) = \frac{u_{\max_v}}{M}\ln\left(1 + (e^M - 1)\frac{y}{D-h}\exp\left(1 - \frac{y}{D-h}\right)\right) \quad (6)$$

where $u_{\max_v}$ is the maximum velocity sampled along the investigated vertical. M is the entropic parameter, which is a characteristic of the river cross-section.

Modeling the two-dimensional velocity distribution by the probabilistic formulation and the entropy maximization, Chiu (1988) also showed that the relation between the mean flow velocity, u_m, and the maximum flow velocity, u_{max}, can be expressed by

$$u_m = \Phi(M)u_{\max} \quad (7)$$

where $\Phi(M)$ is (Chiu 1989)

$$\Phi(M) = \frac{u_m}{u_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} \quad (8)$$

Additional details on Equations (7) and (8) can be found in Chiu (1988, 1989).

The entropic parameter M can be estimated, for the investigated gauged river site, on the basis of pairs (u_m, u_{max}) of available data from measurements sampling (Moramarco *et al.* 2004). It's necessary to point out that u_{max} is unknown, but it can be considered as the maximum value in the data set of velocity points sampled during the velocity measurements (Moramarco *et al.* 2004).

Therefore, once M is estimated at gauged section and $u_{\max_v}$ is sampled, for instance by current meter, in the upper portion of flow area, Equation (6) can be applied obtaining the velocity profile along each vertical sampled during the velocity measurement. Obviously, the applicability of Equation (6) depends on the availability of topographical surveys at gauged site which provide the knowledge of the variability of D across the river section. This insight is of great importance if the velocity measurements have to be only addressed in the upper portion of flow area, i.e., during high floods when it is difficult to sample velocity points in the lower portion of flow area.

DATA COLLECTION

To address the velocity distribution analysis during high flood, the velocity measurements data sampled at Pontelagoscuro hydrometric site on Po river, in northern Italy, have been considered. Figure 1 shows the sketch of the gauged section. The sample consists of 48 velocity measurements carried out in the period 1984–97, of which six of them referring to higher floods have been selected for the analysis.

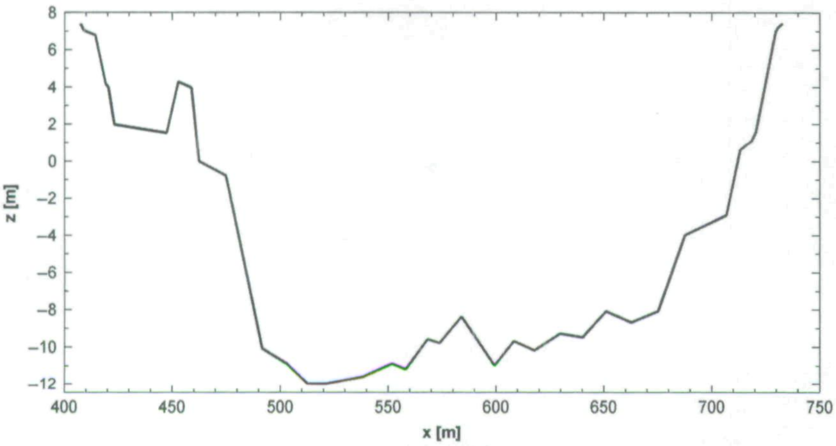

Figure 1 | Topographical survey of the Pontelagoscuro gauged river section.

For each one of the selected measurements, data refer to (1) velocity points sampled along verticals in terms of elevation above the bed and observed value, (2) vertical location respect to left sidewall, (3) hydrometric level, (4) mean flow velocity and (5) discharge. Measurements cover discharge values varying from 500 $m^3\ s^{-1}$ up to 5000 $m^3\ s^{-1}$. The mean flow velocity and the maximum velocity vary in the range (0.5–2) m s^{-1} and (0.8–2.71) m s^{-1}, respectively.

The three velocity distribution equations, Equations (1), (4) and (6), were tested by using the velocity data collected during six flood events, for a total number of verticals and velocity points sampled equal to 80 and 570, respectively. Table 1 summarizes the main characteristics of the selected flood events.

The sampling configuration considers only the availability of the velocity measurements carried out in the upper portion of the flow area. In this way, the sampling during high flood can be represented.

RESULTS

For the application of the Equation (6) the entropic parameter, M, was estimated, on the basis of pairs (u_m, u_{max}) of 48 flow measurements performed during the period 1984–97. In this case, the maximum velocity, u_{max}, has been assumed as the maximum value of sampled velocity points. By Equation (7), $\Phi(M)$ was found to be equal to 0.668 (see Figure 2) and, then, by Equation (8), $M = 2.162$. It is shown that the linear relationship underestimates the actual values of the mean flow velocity, mainly when the maximum velocity is greater than 2.0 m s^{-1}. This is consistent with results obtained by

Table 1 | Stage, flow area (Area), mean velocity, u_m, maximum velocity, u_{max}, and discharge, Q, for the selected events. Percentage errors in estimating the mean flow velocity, u_m, starting from velocity points sampled in the flow area (low flow) and in the upper portion of flow area (high flow) are also shown. For Equations (6)–(9), errors for high flow refer to the sampling of u_{max} only

	Stage	Area	u_m	u_{max}	Q	Errors (%) Low flow			Errors (%) High flow		
Event	(m asl)	(m^2)	(m s^{-1})	(m s^{-1})	($m^3\ s^{-1}$)	Equation (1)	Equation (4)	Equation (6)	Equation (1)	Equation (4)	Equations (6)–(9)
13 February 1985	5.53	2052	1.13	1.8	2358	1.5	−0.3	4	6.3	−2.7	13.8
24 February 1987	4.65	1853	0.94	1.43	1779	5.4	3.1	2.8	12.8	−28.8	7.8
16 October 1987	8.68	2448	2.04	2.71	5026	5	−0.8	−0.3	9.5	−10.9	−4.9
5 July 1988	5.54	2105	1.07	1.59	2283	2.8	1.6	0.1	5.9	−12.4	3.1
27 March 1991	5.38	1882	1.21	1.8	2276	0.4	3.3	0.6	−4	12	4.3
8 May 1991	6.51	1960	1.64	2.15	3218	−6.8	2	−10	7.6	24	−14

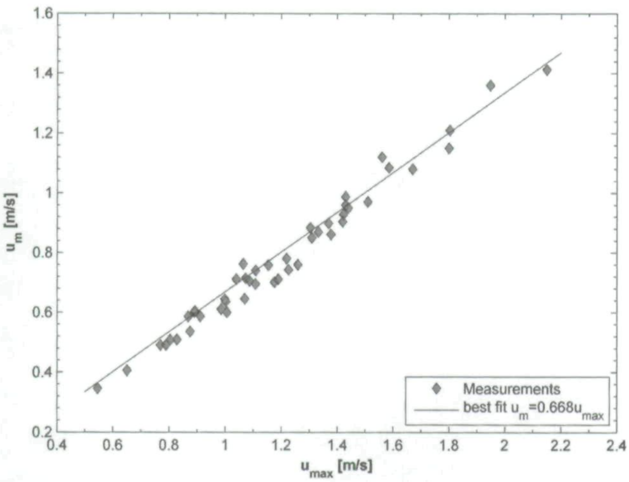

Figure 2 | Relation between mean and maximum velocities at the gauged river section of Pontelagoscuro.

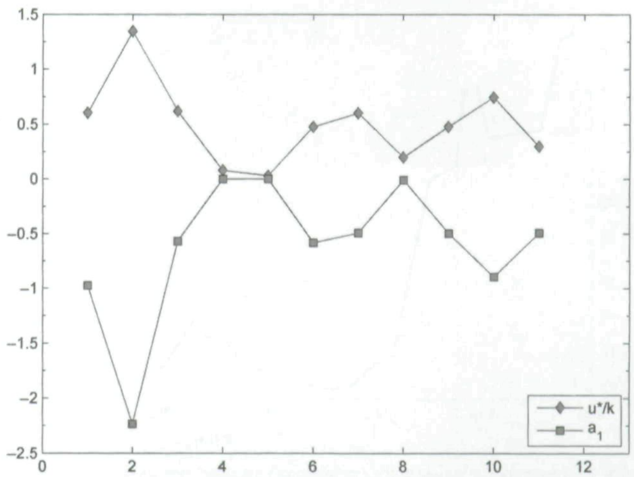

Figure 3 | Comparison between u_*/k and a_1 values referred to each vertical sampled, across the river, for the velocity measurement carried out by current meter on 27 March 1991.

Moramarco *et al.* (2004) on different hydrometric sites located on the Upper Tiber basin in Central Italy.

As far as the application of the logarithmic distribution is concerned, i.e., Equations (1) and (4), unknown parameters have been estimated by sampling along each vertical, at equal distance, three and four velocity points, respectively. Table 1 shows the percentage errors for estimating the mean flow velocity of selected events. As can be seen, the approach performance is quite satisfactory, even though Equations (4) and (6) outperform Equation (1).

Table 2 shows the statistical properties of three quantities in Equation (1), u_*/k, y_o and a_1 in terms of mean, RMSE and variance. In particular, by comparing the mean values of u_*/k and a_1, it is noticeable as the location of u_{max} strongly influence the a_1 value. By way of example, Figure 3 shows the comparison between u_*/k and a_1 values across the river site, for the velocity measurements carried out on 27 March 1991. It can be seen that for highest values of u_*/k, for which the location of u_{max} can be expected to move towards the river bottom, a_1 values, in accordance with Equation (2), reach their maximum negative.

The three velocity distributions have been also applied considering the velocity points only sampled in the upper portion of the flow area. Considering the application of Equations (1) and (4), the third and fourth velocity point, respectively, is represented from the bottom velocity that is surmised equal to zero. The statistical properties of the involved parameters are shown in Table 2, except a_2 which does not have a direct physical meaning.

In order to drastically reduce the sampling period during the measurement, we assume that Equation (6) is applied only considering the maximum velocity point in the flow area, u_{max}, and assuming the behaviour of the maximum velocity quantity in the cross-sectional flow area represented through an elliptical curve:

$$u_{\max_v}(x) = u_{\max}\sqrt{1-\left(\frac{x}{x_S}\right)^2} \quad (9)$$

Table 2 | Statistical quantities of three parameters of Equation (1), u_*/k, y_0 and a_1, for the case of sampling during low flow, LF, and high flow, HF

	u_*/k		y_0		a_1	
	LF	HF	LF	HF	LF	HF
Mean	0.250	0.442	0.025	0.135	−0.290	−0.492
RMSE	0.1026	0.3617	0.0556	0.1535	0.3421	0.6503
Variance	0.01056	0.13086	0.00309	0.02358	0.11701	0.42294

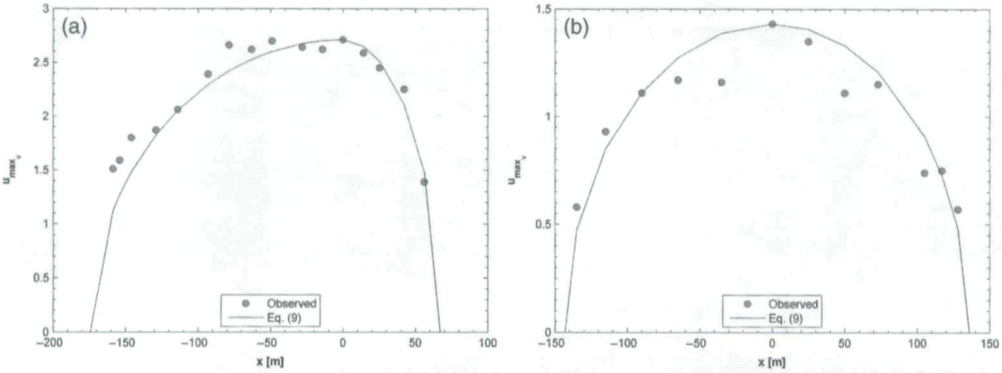

Figure 4 | Elliptical distribution, Equation (9), of the maximum velocities, $u_{\max_v}$, along verticals plotted against the observed ones for the flood event of (a) 16 October 1987 and (b) 24 February 1987.

where $x_S = x_{SX}$ or $x_S = x_{DX}$ represents the distance from the right or left sidewall of the vertical, with reference to $x = 0$, along which the maximum velocity, u_{max}, is sampled, respectively.

Equation (9) can be derived by Chezy's formula and assuming a depth distribution $\frac{D}{D_{\max}} = 1 - \left(\frac{x}{x_s}\right)^2$, with D_{max} the flow depth along the vertical where u_{max} is sampled. It is important to note that for narrow river sections, as shown in Moramarco *et al.* (2004), Equation (9) should be modified considering a depth distribution raised to power of 1 instead of 0.5, thus obtaining for $u_{\max_v}$ a representation in terms of parabolic curve.

Figures 4(a) and (b) show the comparison between the maximum velocity, $u_{\max_v}$, sampled along each vertical and the computed one by the elliptical approach, Equation (9), for the measurements carried out during the flood events that occurred on 16 October 1987 and 24 February 1987, respectively. As can be seen, the elliptical trend reproduces with a fair accuracy the behavior of the maximum velocities sampled in the flow area, for both events. Figure 4(b) shows, in the central portion of the section, an irregular distribution of the maximum velocities most probably due to secondary flows that, obviously, cannot be modeled by Equation (9). Secondary flows are due to presence across the river of piers of the bridge where velocity measurements have been carried out by current meter.

Applying Equation (6) coupled with Equation (9) for each vertical, the location, h, below the water surface where $u_{\max_v}$ is sampled, is assumed constant and corresponding to location of the maximum velocity, u_{max}.

This assumption could be inappropriate mainly in portions of flow area close to sidewalls giving, however, errors not significant for the cross-sectional mean flow velocity assessment. As regards the vertical depth, D, it can be estimated on the basis of topographical surveys of the river section, which are generally available during the working period of the gauged site. Figure 5 shows, for the event on 24 February 1987, a comparison between the observed spatial distribution of the velocity in the flow area and the reconstructed one by using Equations (1) and (6). For Equation (6), results obtained by using the maximum velocity sampled along each vertical are also shown. An overall overview shows the field of velocity fairly represented by Equation (6) in terms of both direction and module; whereas Equation (1) provided a poor representation at the same way of Equation (4). These insights can be also inferred from Figure 6 where the spatial distribution of percentage errors, in magnitude, is shown.

Figure 7 shows, for the three approaches, the percentage errors in estimating the mean velocity along each vertical with respect to dimensionless distance from the location where the maximum velocity is sampled ($x = 0$). The mean error was found about 11% for Equations (1) and (6) and 23% for Equation (4).

Figure 8 shows the cumulative frequency of the percentage error in magnitude. As can be seen, both Equation (6) coupled with Equations (9) and (1) have a similar trend with an error lower than 20% for 92% and 86% of sampled verticals, respectively. The slightly lower accuracy of Equation (6) is due to Equation (9) which is unable to take account of secondary flow effects that for some verticals determined a reduction of maximum velocity along verticals, $u_{\max_v}$. As might be expected, Equation (4) by using velocity points

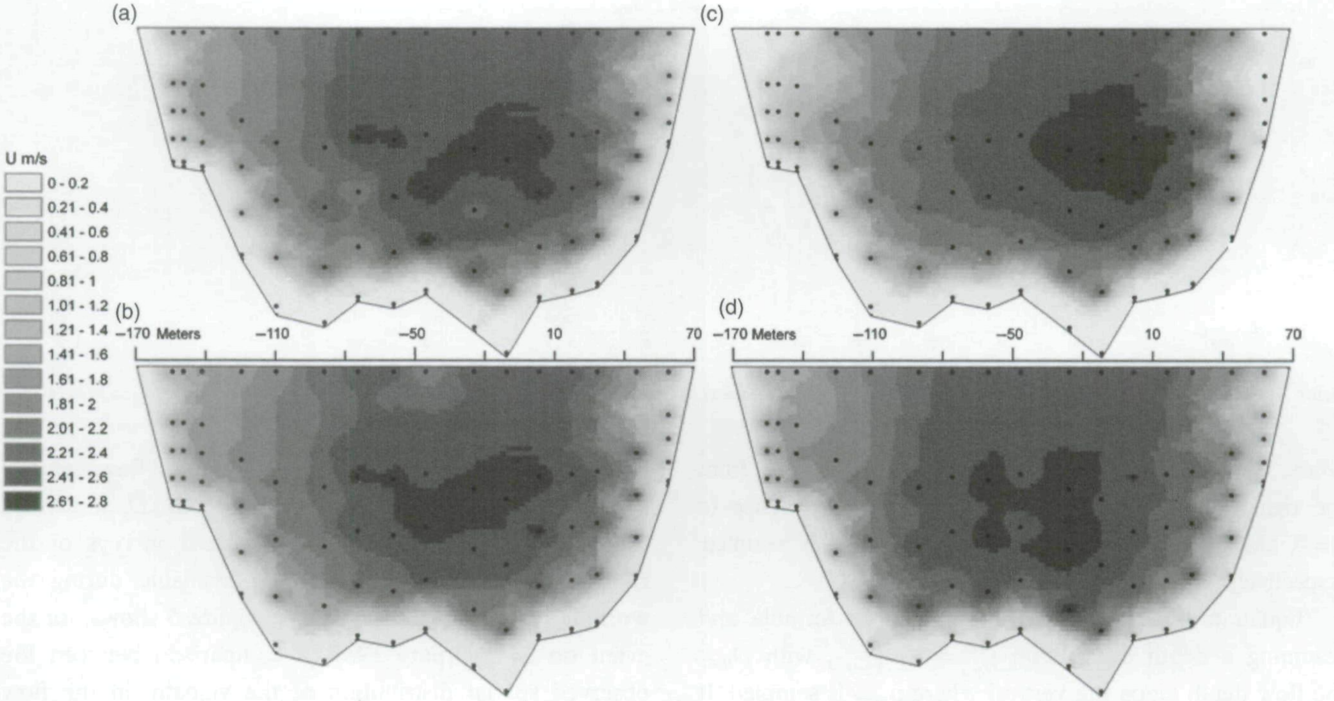

Figure 5 | Flood Event on 27 February 1987. Spatial distribution of flow velocity obtained by (a) sampled velocity points, (b) Equation (6) using the velocity points sampled in the upper portion of the flow area, (c) Equation (6) coupled to Equation (9), and (d) Equation (1). Sampled velocity points are also shown.

sampled in the upper portion of flow area, was found to be poorly accurate with a percentage error exceeding 20% for 46% of verticals.

In terms of error in mean flow velocity estimation, from Table 1 it can be inferred that Equation (6) coupled with Equation (9) provided a mean error lower than 5%; whereas for Equations (1) and (4) it increased, in magnitude, up to 9% and 14%, respectively.

CONCLUSIONS

Based on the results obtained, the following conclusions can be drawn:

(a) the logarithmic methods for high flood conditions produced, along each vertical, percentage errors comparable with the ones corresponding to the application of the entropic approach;

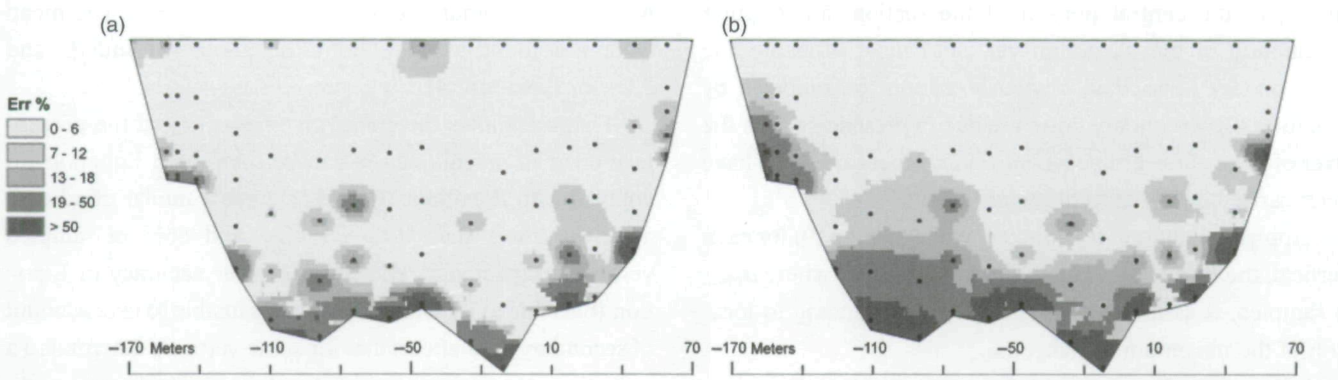

Figure 6 | Flood Event on 27 February 1987. Percentage errors in estimating the flow velocity spatial distribution by using (a) Equation (6) with maximum velocity sampled along each vertical and (b) Equation (1).

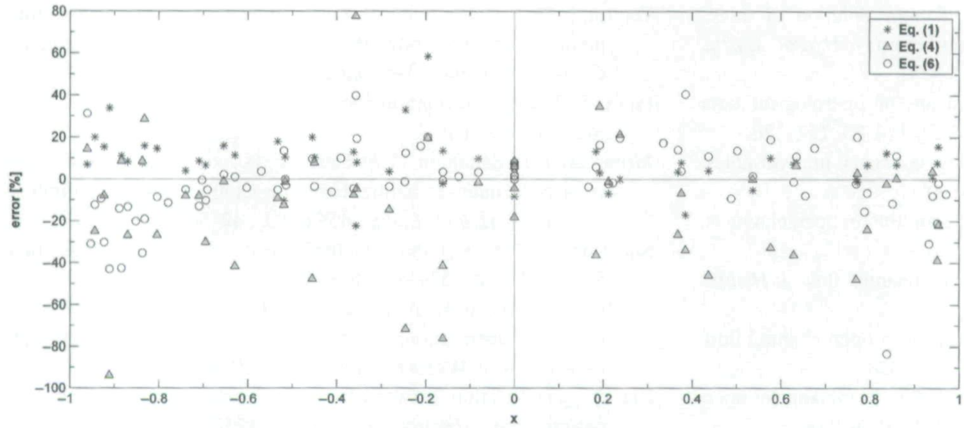

Figure 7 | Percentage error in estimating the mean velocity along the 80 sampled verticals assuming velocity points sampled only in the upper portion of flow area (high flow). Distance x' represents the dimensionless horizontal distance of each vertical from that in which u_{max} was observed (x = 0).

(b) the velocity profiles reconstructed by the modified entropic approach, Equation (6), were found to be very accurate using the velocity points sampled in the upper portion of the flow area, and fairly accurate through the sampling of only the maximum velocity, whose value is used to define the trend of maximum velocities across the river such as expressed by the elliptical distribution, Equation (9). If secondary flows occur, Equation (9) might fail and, hence, its reliability should be always verified by using the velocity points sampled in the upper portion of flow area;

(c) the procedure examined for velocity measurements during high floods and based on the sampling of u_{max} only, without losing the accuracy in estimating the mean flow velocity, allows both operation in safe conditions and reduction of the time of measurement which is fundamental for high floods. This aspect is fundamental for practical hydrology because the monitoring of the maximum velocity, nowadays, can be done by using a portable radar unit, which makes possible a very quick measurement and, hence, for the same flood more gauged river sites can be monitored, a situation that cannot be accomplished by using traditional techniques such as the one based on the use of a current meter.

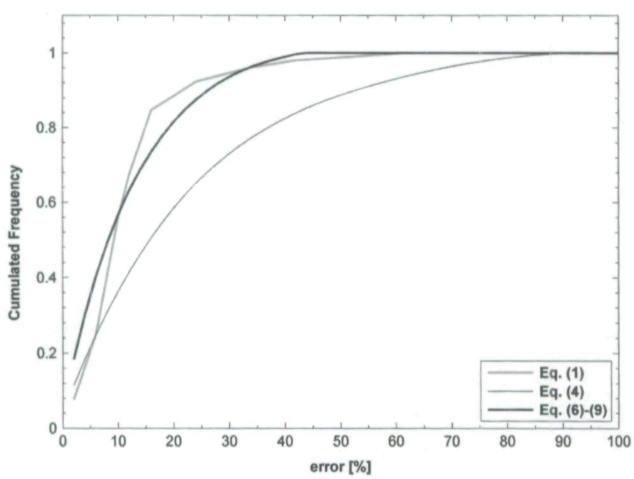

Figure 8 | Cumulated frequency of the percentage error, in magnitude, in estimating the mean velocity along the 80 investigated verticals, considering the sampling of velocity points in the upper portion of flow area (high flow). Errors referring to Equations (6) and (9) are assessed considering the sampling of u_{max} only.

ACKNOWLEDGEMENTS

The authors thank ENEL, Mestre for providing the correntometric data used in this study. This work was partly funded by the National Research Council of Italy, with the financial support of the Agenzia Interregionale per il Fiume Po. The suggestions of reviewers for significantly improving the contents of the paper are gratefully acknowledged. Finally, the authors also thank the hydrology group of CNR-IRPI for their scientific and technical assistance.

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First received 1 June 2009; accepted in revised form 11 January 2010. Available online February 2011