General vectorial decomposition of electromagnetic fields with application to propagation-invariant and rotating fields

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Abstract: A novel decomposition of the transversal part of the electric field vector of a general non-paraxial electromagnetic field is presented, which is an extension of the radial/aximuthal decomposition and is known as $\gamma\zeta$ decomposition. Purely γ and ζ polarized fields are examined and the decomposition is applied to propagation-invariant, rotating, and self-imaging electromagnetic fields. An experimental example on the effect of state of polarization in the propagation characteristics of the field: its is shown that a simple modification of the polarization conditions of the angular spectrum converts a self-imaging field into a propagation-invariant field.

OCIS codes: (350.5500) Propagation; (260.5430) Polarization

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1 Introduction

Self-imaging [1, 2], propagation-invariance [3, 4], and rotation [5, 6, 7] of the intensity distribution of a scalar field are closely related phenomena that have attracted considerable recent interest. In the scalar case the angular spectrum of plane waves associated with the field must be confined on one ring, while in the case of rotating and self-imaging fields two or more rings (known as Montgomery's rings) are involved.

In the case of propagation-invariant fields, the scalar analysis has been extended to the electromagnetic case [8, 9, 10, 11], which is now also rather well understood. Some important new features, such as the broken rotational symmetry of pure Bessel field modes in the non-paraxial domain and the fact that in certain cases two Montgomery's rings may be involved [9], have been uncovered. However, in the case of rotating fields the electromagnetic analysis differs more fundamentally from the scalar analysis [12] because the longitudinal field component does not usually rotate when the transverse cartesian components do so. In paraxial geometries, though, the longitudinal component has an insignificant contribution to the total energy density and the electromagnetic extension of the scalar analysis is accurate [13].

The purpose of this paper is extend a novel decomposition of the electric field into two orthogonal components [14], which we call the $\gamma\zeta$ decomposition, to arbitrary non-paraxial electromagnetic fields, and to investigate the implications of this decomposition in the theory of propagation-invariant, rotating, and self-imaging fields.

The general theory of the $\gamma\zeta$ decomposition is presented in Sect. 2 and applied in Sect. 3 to purely γ and ζ polarized electromagnetic fields. In all cases exact propagation formulas are derived, which are based on the angular spectrum representation of the electromagnetic field. In Sect. 4 the $\gamma\zeta$ decomposition is applied to propagation-invariant fields, including the paraxial case. Some of the results are illustrated experimentally in Sect. 5 and an analysis of rotating fields is provided in Sect. 6.

2 The $\gamma\zeta$ decomposition of the transverse electric field

The propagation of a time-harmonic electromagnetic field in free space is conveniently described by means of the angular spectrum representation [15]. Let us express both the position vector $\boldsymbol{r} = (x, y, z)$ and the wave vector $\boldsymbol{k} = (k_x, k_y, k_z)$ in circular cylindrical coordinates by using the relations

$$\begin{cases} x &= \rho\cos\phi \\ y &= \rho\sin\phi \end{cases} \tag{1}$$

and

$$\begin{cases} k_x &= \alpha\cos\psi \\ k_y &= \alpha\sin\psi \\ k_z &= \beta \end{cases} . \tag{2}$$

The expression for the electric field now takes the form

$$\boldsymbol{E}(\rho, \phi, z) = \int_0^{2\pi}\int_0^{\infty} \boldsymbol{A}(\alpha, \psi)\exp[\mathrm{i}\alpha\rho\cos(\phi - \psi) + \mathrm{i}\beta z]\,\alpha \mathrm{d}\alpha\,\mathrm{d}\psi, \tag{3}$$

where

$$\beta = \begin{cases} \sqrt{k^2 - \alpha^2} & \text{if } \alpha \le k \\ \mathrm{i}\sqrt{\alpha^2 - k^2} & \text{otherwise} \end{cases} \tag{4}$$

and $k = |\boldsymbol{k}|$ is the wave number. The angular spectrum $\boldsymbol{A}(\alpha, \psi)$ is obtained by Fourier-inversion at $z = 0$:

$$\boldsymbol{A}(\alpha, \psi) = \frac{1}{(2\pi)^2}\int_0^{2\pi}\int_0^{\infty} \boldsymbol{E}(\rho, \phi, 0)\exp[-\mathrm{i}\alpha\rho\cos(\phi - \psi)]\rho\,\mathrm{d}\rho\,\mathrm{d}\phi. \tag{5}$$

Generally, two of the three cartesian components of $\boldsymbol{E}$ may be assumed to be independent. The third is obtained from the Maxwell's divergence equation

$$\nabla\cdot\boldsymbol{E}(\rho, \phi, z) = 0. \tag{6}$$

Let us next decompose the transversal part of the field $E_x\hat{\boldsymbol{x}} + E_y\hat{\boldsymbol{y}}$ into two components by the operation

$$\begin{cases} E_\gamma^{(q)}(\rho, \phi, z) &= \cos(q\phi)E_x(\rho, \phi, z) + \sin(q\phi)E_y(\rho, \phi, z) \\ E_\zeta^{(q)}(\rho, \phi, z) &= -\sin(q\phi)E_x(\rho, \phi, z) + \cos(q\phi)E_y(\rho, \phi, z) \end{cases}, \tag{7}$$

where q is an integer. The longitudinal component of the field vector is retained as original. Equation (7) clearly defines also a new pair of unit vectors, denoted by $\hat{\boldsymbol{\gamma}}$ and $\hat{\boldsymbol{\zeta}}$, which are orthogonal in every point of space. These unit vectors remain invariant in the radial direction, but not in the azimuthal direction, in which they rotate with a constant rate along the ϕ coordinate.

In the following considerations we will call this basis the $\gamma\zeta$ basis. The (vectorial) components $E_\gamma^{(q)}$ and $E_\zeta^{(q)}$ are equivalent to the basis vectors discussed recently in Ref. [14], although the notations used here are different. The rotation of the basis vectors is clearly dependent on the value of q, since they experience a rotation of $q2\pi$ within $0 \le \phi \le 2\pi$. In the case $q = 1$ the basis vectors are rotationally symmetric and are customarily called the radial and azimuthal components denoted by E_ρ and E_ϕ, respectively. For values $q > 1$ the basis vectors experience several full rotations in the counterclockwise direction. On the other hand, for negative values of q the direction of rotation of the basis vectors is reversed. For example, if $q = -2$ the basis vectors experience a rotation of -4π radians in the clockwise direction when $0 \le \phi \le 2\pi$. In the case $q = 0$ the basis vectors represent unmodified cartesian vectors. An example illustrating the directions of the basis vectors with values $q = 1$ and $q = -2$ is given in Fig. 1.

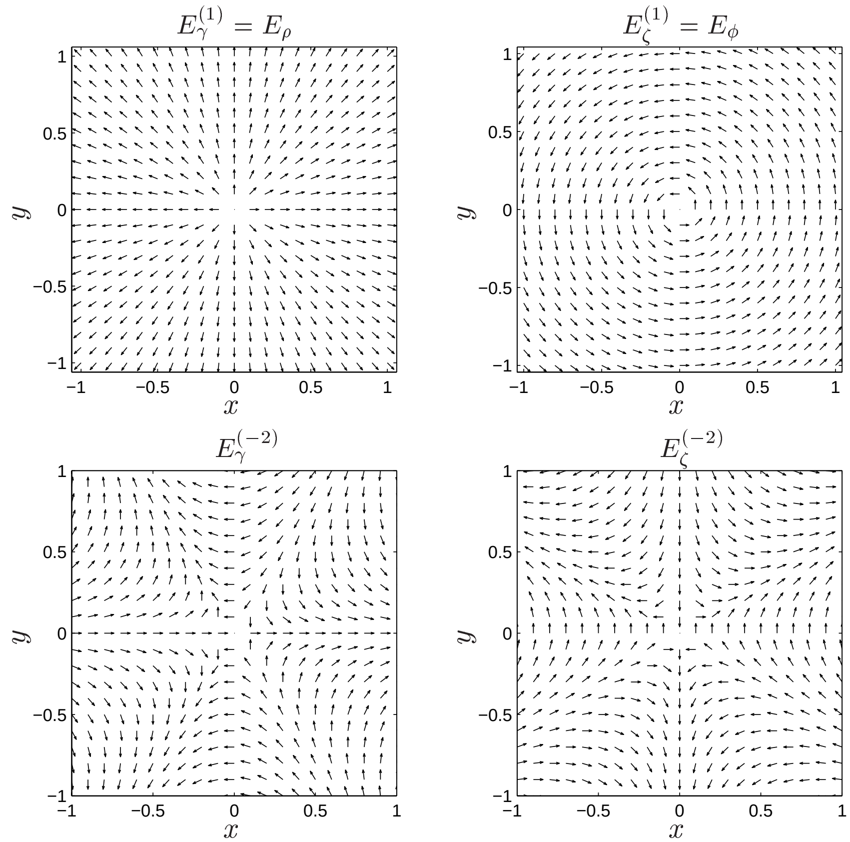

Fig. 1. Directions of the transversal basis vectors in the cases $q = 1$ and $q = -2$.

Combining Eqs. (3) and (7) yields general propagation formulas for the transverse part of the electric field in terms of the γ and ζ components:

$$\begin{aligned} E_\gamma^{(q)}(\rho,\phi,z) &= \sum_{m=-\infty}^{\infty} \exp(\mathrm{i}m\phi) \int_0^\infty J_m(\alpha\rho)\exp(\mathrm{i}\beta z) \\ &\quad \times \left[a_m^x(\alpha)\cos(q\phi) + a_m^y(\alpha)\sin(q\phi)\right] \mathrm{d}\alpha, \end{aligned} \tag{8}$$

and

$$\begin{aligned} E_\zeta^{(q)}(\rho,\phi,z) &= \sum_{m=-\infty}^{\infty} \exp(\mathrm{i}m\phi) \int_0^\infty J_m(\alpha\rho)\exp(\mathrm{i}\beta z) \\ &\quad \times \left[-a_m^x(\alpha)\sin(q\phi) + a_m^y(\alpha)\cos(q\phi)\right] \mathrm{d}\alpha. \end{aligned} \tag{9}$$

In the derivation of Eqs. (8) and (9) we have made use of the Jacobi–Anger expansion [16]

$$\exp(\mathrm{i}\vartheta\cos\tau) = \sum_{m=-\infty}^{\infty} \mathrm{i}^m J_m(\vartheta)\exp(\mathrm{i}m\tau), \tag{10}$$

where J_m denotes the Bessel function of the first kind and of order m, and defined the functions $a_m^j(\alpha)$ as

$$a_m^j(\alpha) = \mathrm{i}^m \alpha \int_0^{2\pi} A_j(\alpha,\psi)\exp(-\mathrm{i}m\psi)\,\mathrm{d}\psi, \tag{11}$$

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where $j = x$ or y.

3 Purely γ- or ζ-polarized fields

In this Section we derive general expressions for fields that have only either a γ component or a ζ component. These results represent a generalization of those given in Refs. [17] and [18], where the field was assumed to be either radially or azimuthally polarized. In addition, we assume no radial symmetry or propagation-invariance of the field. In the following, we assume that $q \neq 0$, since in the case $q = 0$ the field representation is clearly given by Eq. (3).

Let us first consider the case in which the ζ component of the field vanishes. By inserting this requirement into Eq. (9) and rearranging the indices in the summation, we obtain

$$\sum_{m=-\infty}^{\infty} \exp(\mathrm{i}m\phi) \int_0^\infty \left\{ J_{m-q}(\alpha\rho) \left[\mathrm{i}a^x_{m-q}(\alpha) + a^y_{m-q}(\alpha) \right] + J_{m+q}(\alpha\rho) \left[-\mathrm{i}a^x_{m+q}(\alpha) + a^y_{m+q}(\alpha) \right] \right\} \exp(\mathrm{i}\beta z)\, \mathrm{d}\alpha = 0. \quad (12)$$

Using the uniqueness of the Fourier-series representation and the fact that Eq. (12) must hold for all values of ρ and z, we arrive at the requirement that the integrand must vanish identically:

$$J_{m-q}(\alpha\rho) \left[\mathrm{i}a^x_{m-q}(\alpha) + a^y_{m-q}(\alpha) \right] + J_{m+q}(\alpha\rho) \left[-\mathrm{i}a^x_{m+q}(\alpha) + a^y_{m+q}(\alpha) \right] \equiv 0, \quad (13)$$

must hold for all m. However, because the functions J_{m-q} and J_{m+q} are linearly independent for all $m \neq 0$, Eq. (13) implies that only the functions $a^j_m(\alpha)$ with $m = \pm q$ may have non-zero values. In addition, these functions are connected by the relations

$$\begin{cases} a^y_{-q}(\alpha) & = & \mathrm{i}(-1)^q a^x_q(\alpha) \\ a^y_q(\alpha) & = & -\mathrm{i}a^x_q(\alpha) \\ a^x_{-q}(\alpha) & = & (-1)^q a^x_q(\alpha) \end{cases} . \quad (14)$$

Inserting these into Eq. (8) we obtain the propagation formula

$$E^{(q)}_\gamma(\rho, \phi, z) = E^{(q)}_\gamma(\rho, z) = \int_0^\infty f_q(\alpha) J_q(\alpha\rho) \exp(\mathrm{i}\beta z)\, \mathrm{d}\alpha, \quad (15)$$

where

$$f_q(\alpha) = 2a^x_q(\alpha). \quad (16)$$

Thus, the transverse vectorial component of the field may be handled as a rotationally symmetric field, although the propagation integral differs radically from Eq. (5). However, the entire field symmetric only in the case $q = 1$. This is seen also by inserting Eqs. (3) and (14) into Eq. (6), which yields an propagation formula for the longitudinal component of the field:

$$E^{(q)}_z(\rho, \phi, z) = \mathrm{i}\cos[(q-1)\phi] \int_0^\infty \frac{\alpha}{\beta} f_q(\alpha) J_{q-1}(\alpha\rho) \exp(\mathrm{i}\beta z)\, \mathrm{d}\alpha. \quad (17)$$

The absolute value of $E^{(q)}_z(\rho, \phi, z)$ clearly depends on the ϕ-coordinate for all $q \neq 1$.

Derivation of a purely ζ polarized field is performed essentially identically, by assuming that $E_\gamma(\rho, \phi, z) \equiv 0$ and then repeating similar steps as above. The results are

$$E^{(q)}_\zeta(\rho, \phi, z) = E^{(q)}_\zeta(\rho, z) = \int_0^\infty g_q(\alpha) J_q(\alpha\rho) \exp(\mathrm{i}\beta z)\, \mathrm{d}\alpha \quad (18)$$

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and

$$E_z^{(q)}(\rho,\phi,z) = \mathrm{i}\sin[(q-1)\phi]\int_0^\infty \frac{\alpha}{\beta} g_q(\alpha) J_{q-1}(\alpha\rho)\exp(\mathrm{i}\beta z)\,\mathrm{d}\alpha, \tag{19}$$

where

$$g_q(\alpha) = -\mathrm{i} f_q(\alpha). \tag{20}$$

Equations (15) and (18) state, e.g., that a field which is either radially or azimuthally polarized in one transversal plane can retain its polarization state only if it is rotationally symmetric. In addition, this field must be a superposition of J_1 subfields with different radii. This is a generalization of the result given by Jordan and Hall [17], who derived Eq. (18) in the case $q = 1$ assuming that the azimuthally polarized field is rotationally symmetric. The derivation introduced above shows that the rotationally symmetric case examined by Jordan and Hall is the only possible one, which explains the results reported recently by Lapucci and Ciofini [19].

The results derived above are valid only if either the γ or the ζ component of the field vanishes. In all other cases both components are superpositions of Bessel functions with different orders. However, the linearity of Maxwell's equations implies that the results hold also for all linear combinations of $E_\gamma^{(q)}(\rho,\phi,z)$ and $E_\zeta^{(q)}(\rho,\phi,z)$.

4 Paraxial propagation-invariant electromagnetic fields

Probably the most natural definition of propagation-invariance is the condition that the time-averaged energy density of the field $\langle w(\boldsymbol{r},t)\rangle$ remains exactly the same in every transversal plane $z =$ constant [9, 18], i.e.,

$$\langle w(\rho,\phi,z+\Delta z,t)\rangle = \langle w(\rho,\phi,z,t)\rangle \tag{21}$$

for all values of Δz. The condition (21) may also be expressed in the form

$$\frac{\partial}{\partial z}\langle w(\rho,\phi,z,t)\rangle \equiv 0, \tag{22}$$

which is sometimes more suitable.

By comparison of Eqs. (3), (6), and (21) we see that any electromagnetic field with propagation-invariant scalar components is propagation-invariant. However, as pointed out by Turunen and Friberg [9], that condition is sufficient but not necessary when the electromagnetic nature of the field is taken into account. For example, propagation-invariant electromagnetic fields with non-propagation-invariant x- and y-components may be constructed rather easily [9, 18]. These fields may be obtained by assuming that both radial and azimuthal components of the field are propagation-invariant, but not necessarily localized on the same Montgomery's ring. Because the azimuthally polarized field does not have a z-component, the total energy density of this kind of field must remain constant upon propagation [18].

One may expect that this phenomenon may be extended to other cases in which one of the two component-fields does not have a z-component. However, as can be easily verified by inserting Eq. (6) into (3), the condition that the z-component disappears leads immediately to an azimuthally polarized angular spectrum, i.e., $A_x(\rho,\psi)\cos\psi + A_y(\rho,\psi)\sin\psi = 0$ [20]. It is well known that, for such fields, the direction of the electric field vector changes in the radial direction, except for the purely azimuthally polarized field, and hence the orthogonality of the transverse components is possible only if both component-fields are confined to the same Montgomery's ring [18, 21].

Let us next assume that both γ and ζ components of the field are propagation-invariant, i.e.,

$$E_\gamma^{(q)}(\rho,\phi,z+\Delta z) = \exp\left[\mathrm{i}\xi(\rho,\phi,\Delta z)\right] E_\gamma^{(q)}(\rho,\phi,z) \tag{23}$$

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and

$$E_{\zeta}^{(q)}(\rho,\phi,z+\Delta z)=\exp\left[\mathrm{i}\nu(\rho,\phi,\Delta z)\right]E_{\zeta}^{(q)}(\rho,\phi,z)\,, \tag{24}$$

where ξ and ν are as-yet arbitrary real functions. In view of Eqs. (15) and (18), the field expressions are of the form

$$E_{\gamma}^{(q)}(\rho,z)=c_{\gamma}J_{q}\left(\alpha_{\gamma}\rho\right)\exp(\mathrm{i}\beta_{\gamma}z) \tag{25}$$

and

$$E_{\zeta}^{(q)}(\rho,z)=c_{\zeta}J_{q}\left(\alpha_{\zeta}\rho\right)\exp(\mathrm{i}\beta_{\zeta}z), \tag{26}$$

where c_γ and c_ζ are arbitrary complex constants and α_γ and α_ζ denote the constant α_0 for the γ and ζ components, respectively.

The z-component of the field is obtained straightforwardly by using Eqs. (17) and (19) and it takes the form

$$\begin{aligned}E_z(\rho,\phi,z)=&\,\mathrm{i}c_{\gamma}\frac{\alpha_{\gamma}}{\beta_{\gamma}}\cos\left[(q-1)\,\phi\right]J_{q-1}\left(\alpha_{\gamma}\rho\right)\exp\left(\mathrm{i}\beta_{\gamma}z\right)\\&+\mathrm{i}c_{\zeta}\frac{\alpha_{\zeta}}{\beta_{\zeta}}\sin\left[(q-1)\,\phi\right]J_{q-1}\left(\alpha_{\zeta}\rho\right)\exp\left(\mathrm{i}\beta_{\zeta}z\right).\end{aligned} \tag{27}$$

By squaring the absolute value of the field in Eq. (27), it is immediately seen that when $\alpha_\gamma \neq \alpha_\zeta$ the z-component is modified upon propagation in almost all cases, although both γ and ζ components are propagation-invariant. Thus, the electric energy density $w_{\mathrm{e}} \propto \|\mathbf{E}(\rho,\phi,z)\|^2$ (and also the magnetic energy density) is not generally propagation-invariant, but instead self-imaging. Although there exist several special cases, in which the modifications of the electric and magnetic energy densities cancel each other [9, 20], in the case considered here that does not happen, as can be easily verified by a straightforward calculation.

The only known cases in which the field confined on two Montgomery's rings may be propagation-invariant are the case in which either α_γ or α_ζ is equal to zero [9] and the case $q = 1$, i.e., when E_γ and E_ζ represent radial and azimuthal components, respectively [18]. In the paraxial limit $\alpha_j/\beta_j \to 0$ the contribution to the energy density from the z-component of the field may be neglected and the field becomes propagation-invariant.

5 Experimental results on propagation-invariance

A simple way to demonstrate finite-aperture approximations of propagation-invariant fields is to use a ring aperture, whose far-field diffraction pattern is a Bessel field. The radius of the ring corresponds to the radius of the Montgomery's ring in the Fourier-plane.

In the case of an aperture consisting of two concentric rings, a self-imaging field is generally produced [1]. However, as we saw in the previous Section, the field may be almost propagation-invariant if the transversal parts of the sub-fields confined on different rings are orthogonal. This situation may be achieved, for example, by using the setup illustrated in Fig. 2: An annular half-wave plate is used to alter the polarization state of the sub-field confined to the outer ring. A Fourier-transform of the field is then performed by using a simple refractive lens.

The orientation of the optical axis of the half-wave plate with respect to the direction of linear polarization of the incident field, denoted by θ, directly affects the polarization state of the field. If a linearly polarized input field is used, the angle $\theta = \pi/4$ converts an x-polarized wave into a y-polarized wave and *vice versa*. On the other hand, if the optical axis is parallel (or perpendicular) to the direction of polarization, the polarization state is not modified. These cases are illustrated in Fig. 3.

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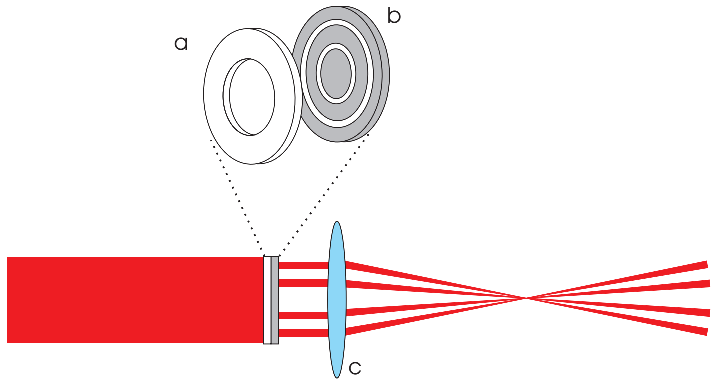

Fig. 2. The principle of the experimental setup. Here a and b denote an annular half-wave plate and a ring aperture, respectively, and c is a thin lens.

We made the experiments by using a setup illustrated in Fig. 2. The used light source was a Helium–Neon laser with $\lambda = 632.8\,\text{nm}$, whose phase-front was made approximately planar by using a lens-system (not shown in the Figure). The diameters of the rings in the aperture were 6 and 10 mm, whereas the (inner) diameter of the annular half-wave plate was 8 mm. The focal length of the lens was $f = 1000\,\text{mm}$.

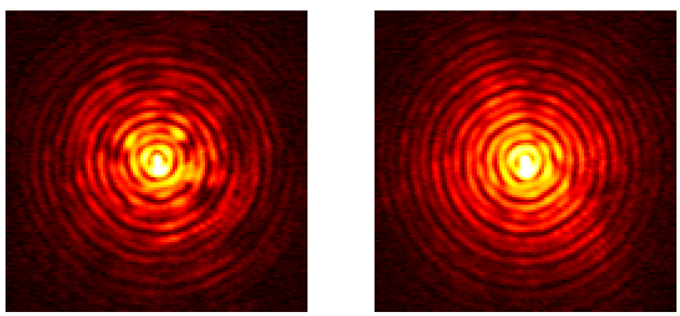

Fig. 3. (1.21 MB) An animation of the experimental fields within one self-imaging distance $z_{\mathrm{T}} \approx 20\,\text{mm}$. The field in the left-hand side is obtained with parallel polarization states and the field in the right-hand side with orthogonal polarization states.

By examining Fig. 3, we notice that the field produced by orthogonally polarized rings is propagation-invariant, whereas the parallel case leads to an ordinary self-imaging field, as expected.

6 Rotating intensity distributions

The orthogonality of the γ and ζ polarized fields may be used to produce not only propagation-invariant fields, but rotating fields as well. The condition for rotation of an electromagnetic field can be defined as [12]

$$\langle w(\rho, \phi + \eta\Delta z, z + \Delta z, t)\rangle = \langle w(\rho, \phi, z, t)\rangle \tag{28}$$

for all Δz. Here η is a constant that defines both the direction of propagation and the self-imaging distance. Contrary to the case of propagation-invariant fields, the energy-density distributions of electromagnetic extensions of rotating scalar fields do not generally rotate [12]. However, in the paraxial domain the scalar theory predicts the behavior of the transverse components of the field accurately, and hence in many practical applications the prediction of the scalar theory coincides with the experimental results [13].

It is clear that scalar fields are not the only solutions whose intensity distributions rotate in the paraxial domain, for it is possible to combine non-rotating scalar x- and y- components of the field to form rotating intensity distributions. However, the task of finding that kind of fields directly from Eq. (28) appears to be a very demanding task and hence some other method must be used. In the following, we use the $\gamma\zeta$ decomposition to find a class of such solutions.

In view of Eq. (28) it is clear that the intensity distribution of the paraxial field which fulfills the conditions

$$\left\{ \begin{array}{lcl} E_\gamma^{(q)}(\rho,\phi+\eta\Delta z, z+\Delta z) & = & \exp[\mathrm{i}\xi(\rho,\phi,\Delta z)]E_\gamma^{(q)}(\rho,\phi,z) \\ E_\zeta^{(q)}(\rho,\phi+\eta\Delta z, z+\Delta z) & = & \exp[\mathrm{i}\xi(\rho,\phi,\Delta z)]E_\zeta^{(q)}(\rho,\phi,z) \end{array} \right. , \tag{29}$$

where $\xi(\rho,\phi,\Delta z)$ is an arbitrary real function, rotates. Inserting this requirement into Eqs. (8) and (9) yields, similarly to the case of exactly rotating fields [12], the connections

$$a_m^y(\alpha) = \mathrm{i}s a_m^x(\alpha) \tag{30}$$

and

$$\beta_{ms} = \beta_0 - (m+sq)\eta, \tag{31}$$

where s is a sign function which can assume values $s=-1$ or $s=+1$ for each m, and β_0 is a constant. Naturally, in the cases $q=0$ or $q=1$, we obtain the rotation conditions for scalar or electromagnetic fields, respectively [12].

By inserting Eqs. (30) and (31) into Eqs. (8) and (9), we obtain the following expressions for the transversal components:

$$E_\gamma^{(q)}(\rho,\phi,z) = \sum_{m\in\mathcal{M}}\sum_{s=-1,1} a_{ms} J_m(\alpha_{ms}\rho)\exp\{\mathrm{i}[(m+sq)\phi+\beta_{ms}z]\} \tag{32}$$

and

$$E_\zeta^{(q)}(\rho,\phi,z) = \sum_{m\in\mathcal{M}}\sum_{s=-1,1} \mathrm{i}s a_{ms} J_m(\alpha_{ms}\rho)\exp\{\mathrm{i}[(m+sq)\phi+\beta_{ms}z]\}, \tag{33}$$

where $\mathcal{M}$ denotes the set of values of m leading to real values of β_{ms}, in view of Eqs. (4) and (31). The coefficients a_{ms} are defined by $a_{ms} = a_m^x(\alpha_{ms})/\alpha_{ms}$. An example of the field satisfying Eqs. (30) and (31) is illustrated in Fig. 4. It can be clearly seen that the x-component of the field is not rotating. However, the γ-component, as well as the ζ-component (which is not shown), do rotate. This means that the contribution to the intensity form the transversal part of the field rotates.

Table 1. The parameters assumed in Fig. 4. The constant β_c must be less than k but is otherwise arbitrary.

m	β_m	n	a_{mn}
0	$\beta_c/2$	1	i
-2	$\beta_c/2$	-1	1
-1	β_c	1	$-$i
-3	β_c	-1	1

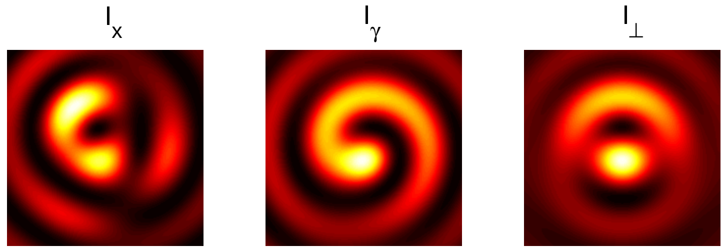

Fig. 4. (1.38 MB) An animation of the intensity distributions of the central parts of the x- and γ-components and the total electric energy density as functions of the z-coordinate, calculated with the parameters given in Table 1. The movie is best appreciated if viewed repeatedly.

Let us next consider the case $q = 1$, which provides also rotating z-components, of course assuming that both conditions (30) and (31) are met. Now the z-component of the electric field takes the form [12]

$$E_z(\rho, \phi, z) = \sum_{m\in\mathcal{M}} \sum_{s=-1,1} s a_{ms} \frac{\alpha_{ms}}{\mathrm{i}\beta_{ms}} J_{m+s}(\alpha_{ms}\rho) \exp\left\{\mathrm{i}\left[(m+s)\phi + \beta_{ms} z\right]\right\}. \tag{34}$$

If we now retain only one value of m in Eqs. (32) and (33), we find that the transversal part of the field takes the form

$$\begin{aligned} \boldsymbol{E}_\perp(\rho, \phi, z) &= \exp\left(\mathrm{i}\beta_{m,1} z\right) a_{m,1} J_m\left(\alpha_{m,1}\rho\right) \exp\left(\mathrm{i}m\phi\right) \begin{bmatrix} 1 \\ \mathrm{i} \end{bmatrix} \\ &\quad + \exp\left(\mathrm{i}\beta_{m,-1} z\right) a_{m,-1} J_m\left(\alpha_{m,-1}\rho\right) \exp\left(\mathrm{i}m\phi\right) \begin{bmatrix} 1 \\ -\mathrm{i} \end{bmatrix}, \end{aligned} \tag{35}$$

where the column vectors are the usual Jones vectors. Thus, the left- and right-handed components are of propagation-invariant form. Hence, if this kind of a field is paraxial, the intensity distribution is approximately propagation-invariant, although the field is exactly rotating. Such a field may be easily demonstrated by using the setup discussed in Section 6, but by using a circularly polarized input field. In that case, the half-wave plate inverts the handedness of the field and hence a field of the form of Eq. (35) is produced.

7 Conclusions

In conclusion, we have introduced some new aspects into the theory of propagation-invariant, rotating, and self-imaging electromagnetic fields. Starting from a general and exact $\gamma\zeta$ decomposition of an arbitrary non-paraxial field we have considered the propagation and the properties of purely γ and ζ polarized fields, or which radially and azimuthally polarized fields are important special cases. It was then shown that if both the γ and ζ components are propagation-invariant, so is the electric energy density even though the two components are confined on Montgomery's rings with different radii. A generalized class of paraxial rotating electromagnetic fields was also introduced, and an experimental demonstration illustrating a dramatic effect of the polarization properties of the angular spectrum into the propagation characteristics of electromagnetic fields: an apparently minor modification of the polarization properties converts a self-imaging field into a propagation-invariant field.

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